CALCULATION OF NETWORKS CONTAINING IDEAL GENERATORS BY USING THE TOPOLOGICAL MATRICES

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The topology of networks can be characterized by various matrices, such as the incidence matrix **A**, the cut-set matrix **Q**, or the loop matrix **B**. Network calculation methods permitting the elaboration of general computer programs, independent of the concretely given network, are well known [1, 2]. These methods can be employed, if the ideal generators contained in the network are either exclusively ideal voltage generators, or exclusively ideal current generators. A process suited for the analysis of networks containing both types of ideal generators is also known [3], the aim of this, however, is the solution of a more general problem, the calculation of networks containing also dependent generators.

In the following two methods of calculating linear invariant networks containing ideal voltage and current generator are described. One of the calculation processes supplies the current or voltage of all impedances directly. With the other method the number of unknown quantities is lower, identical with the number of twigs or links corresponding to impedances in the graph of the network. The methods to be described permit to take into consideration such branches of the network which represent a short-circuit or break, since the short-circuit can be considered as an ideal voltage generator having zero source voltage, and the break as an ideal current generator having zero source current.

In the course of the calculations the circuit of the network, the source currents or voltages of the generators, as well as the impedance values are taken as given quantities.

Determination of currents and voltages

In the following the individual lossy generators are considered, according to the substitution image of Thevenin or Norton, as two branches: one consisting of an ideal generator, and the other of an impedance. Arranging the branches of the network in three groups, in the first group branches containing an ideal current generator, further the fictitious branches which can be taken into consideration by a break, the voltage of which is to be determined, should be included. Into the second group passive branches having impedances different from both 0 and ∞, while into the third group ideal voltage generators and short-circuits are classified. The branches are given order numbers in the sequence of grouping, accordingly branches contained in the first group obtain the serial numbers $1, 2, \ldots, b_\alpha$, those in the second group $b_\alpha + 1, b_\alpha + 2, \ldots, b_\alpha + b_\beta$, while those of the third group the serial numbers $b_\alpha + b_\beta + 1$, $b_\alpha + b_\beta + 2, \ldots, b_\alpha + b_\beta + b_\gamma = b$.

Let us select for the calculations a tree of the graph of the network in which branches of the first group should be links, those of the third group twigs. (This can be done in every real case, namely if the ideal voltage generators form a loop, or the ideal current generators a cut-set, then the problem is redundant.) The loop and cut-set system generated by the tree selected in this way is used. Let us give order numbers to the loops in the sequence of numeration of the links, to the cut-sets according to the twigs. The pertaining loop matrix is designated by $\mathbf{B}$, the cut-set matrix by $\mathbf{Q}$.

Designate the column matrices formed of the voltage and of the current of the branches by $\boldsymbol{U}$ and $\boldsymbol{I}$, respectively, and partition these according to the three groups of branches

$$\boldsymbol{U} = \begin{bmatrix} \boldsymbol{U}_\alpha \\ \boldsymbol{U}_\beta \\ \boldsymbol{U}_\gamma \end{bmatrix} = \begin{bmatrix} \boldsymbol{U}_\alpha \\ \boldsymbol{U}_\beta \\ \boldsymbol{U}_0 \end{bmatrix}; \qquad \boldsymbol{I} = \begin{bmatrix} \boldsymbol{I}_\alpha \\ \boldsymbol{I}_\beta \\ \boldsymbol{I}_\gamma \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_0 \\ \boldsymbol{I}_\beta \\ \boldsymbol{I}_\gamma \end{bmatrix} \tag{1}$$

where $\boldsymbol{U}_0$ is the column matrix formed of source voltages, $\boldsymbol{I}_0$ that of source currents.

Write the loop equations of the network

$$\mathbf{B}\boldsymbol{U} = \boldsymbol{0}\,.$$

Partition matrix $\mathbf{B}$ of the equation according to the grouping of branches By considering relationship (1),

$$\begin{bmatrix} \mathbf{1} & \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{0} & \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_\alpha \\ \boldsymbol{U}_\beta \\ \boldsymbol{0} \end{bmatrix} = - \begin{bmatrix} \mathbf{1} & \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{0} & \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{U}_0 \end{bmatrix} \tag{3}$$

where $\mathbf{1}$ is the unit matrix of b_α order, and the number of columns in $\mathbf{B}_{11}$ and $\mathbf{B}_{21}$ is b_β. ($\mathbf{B}$ obtained this form on account of the numbering of branches and loops as indicated above.) From our equation

$$\boldsymbol{U}_\alpha + \mathbf{B}_{11}\,\boldsymbol{U}_\beta = - \,\mathbf{B}_{12}\,\boldsymbol{U}_0 \tag{4}$$

$$\mathbf{B}_{21}\,\boldsymbol{U}_\beta = - \,\mathbf{B}_{22}\,\boldsymbol{U}_0\,. \tag{5}$$

Kirchhoff's cut-set equations are:

$$\mathbf{Q}\boldsymbol{I} = \boldsymbol{0}\,. \tag{6}$$

Partition also $\mathbf{Q}$ according to the grouping of the branches. By considering (1)

$$\begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} & \mathbf{0} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I}_\beta \\ \boldsymbol{I}_\gamma \end{bmatrix} = - \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} & \mathbf{0} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_0 \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \tag{7}$$

where $\mathbf{1}$ is the unit matrix of b_γ order, and the number of columns in $\mathbf{Q}_{11}$ is b_α. From (7) we obtain

$$\mathbf{Q}_{12}\boldsymbol{I}_\beta = -\mathbf{Q}_{11}\boldsymbol{I}_0 \tag{8}$$

$$\mathbf{Q}_{22}\boldsymbol{I}_\beta + \boldsymbol{I}_\gamma = -\mathbf{Q}_{21}\boldsymbol{I}_0\,. \tag{9}$$

$\boldsymbol{U}_\beta$ and $\boldsymbol{I}_\beta$ are the column matrices composed of voltage and current of passive elements, respectively. These are connected by the relationship

$$\boldsymbol{U}_\beta = \mathbf{Z}_\beta \boldsymbol{I}_\beta \tag{10}$$

where $\mathbf{Z}_\beta$ is the impedance matrix of the part of the network consisting of branches of not extreme impedances. (In the main diagonal of $\mathbf{Z}_\beta$, the impedances of the individual branches, while outside the main diagonal, the respective mutual impedances are seen.)

Eqs (5) and (8) can accordingly be summarized as:

$$\begin{bmatrix} \mathbf{B}_{21}\mathbf{Z}_\beta \\ \mathbf{Q}_{12} \end{bmatrix} \boldsymbol{I}_\beta = - \begin{bmatrix} \mathbf{B}_{22}\boldsymbol{U}_0 \\ \mathbf{Q}_{11}\boldsymbol{I}_0 \end{bmatrix} \tag{11}$$

and from this the current of passive elements can be expressed. Namely in (11) the multiplier of $\boldsymbol{I}_\beta$ is a quadratic matrix, and if the inverse of this can be formed, then

$$\boldsymbol{I}_\beta = - \begin{bmatrix} \mathbf{B}_{21}\mathbf{Z}_\beta \\ \mathbf{Q}_{12} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_{22}\boldsymbol{U}_0 \\ \mathbf{Q}_{11}\boldsymbol{I}_0 \end{bmatrix}. \tag{12}$$

$\boldsymbol{U}_\beta$ can be expressed in a similar way

$$\boldsymbol{U}_\beta = - \begin{bmatrix} \mathbf{B}_{21} \\ \mathbf{Q}_{12}\mathbf{Z}_\beta^{-1} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_{22}\boldsymbol{U}_0 \\ \mathbf{Q}_{11}\boldsymbol{I}_0 \end{bmatrix}. \tag{13}$$

In the knowledge of $\boldsymbol{U}_\beta$ and $\boldsymbol{I}_\beta$ the voltage of current generators and the current of voltage generators can be determined on the basis of Eqs (4) and (9):

$$\boldsymbol{U}_\alpha = -\mathbf{B}_{11}\boldsymbol{U}_\beta - \mathbf{B}_{12}\boldsymbol{U}_0 = \mathbf{B}_{11} \begin{bmatrix} \mathbf{B}_{21} \\ \mathbf{Q}_{12}\mathbf{Z}_\beta^{-1} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_{22}\boldsymbol{U}_0 \\ \mathbf{Q}_{11}\boldsymbol{I}_0 \end{bmatrix} - \mathbf{B}_{12}\boldsymbol{U}_0 \tag{14}$$

$$\boldsymbol{I}_\gamma = -\mathbf{Q}_{22}\boldsymbol{I}_\beta - \mathbf{Q}_{21}\boldsymbol{I}_0 = \mathbf{Q}_{22} \begin{bmatrix} \mathbf{B}_{21}\mathbf{Z}_\beta \\ \mathbf{Q}_{12} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_{22}\boldsymbol{U}_0 \\ \mathbf{Q}_{11}\boldsymbol{I}_0 \end{bmatrix} - \mathbf{Q}_{21}\boldsymbol{I}_0\,. \tag{15}$$

Reduction of equations

By developing results obtained so far, a grouping of required branch currents and branch voltages is possible, permitting the reduction of the number of unknown quantities and equations. Also for this calculation a tree is selected in which a link corresponds to each ideal current generator, a twig to each voltage generator, while impedances may be either links or twigs. The branches are classified into four groups:

1. links consisting of ideal current generator (of a number b_1),
2. links consisting of impedance (of a number b_2),
3. twigs consisting of impedance (of a number b_3),
4. twigs consisting of ideal voltage generator (of a number b_4).

Branches containing an impedance should be possibly grouped into twigs and links in such a way that no mutual impedance occurs between the twig and the link.

The numbering of branches is done according to the sequence of the grouping. For the calculation the loops in the loop system generated by the selected tree are numbered in the sequence of the order number of the respective links, while the cut-sets of the system of cut-sets in that of the respective twigs.

The loop equations of the network, as given by the matrices partitioned according to the four groups of branches, are given by

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{F}_{11} & \mathbf{F}_{12} \\ \mathbf{0} & \mathbf{1} & \mathbf{F}_{21} & \mathbf{F}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_1 \\ \boldsymbol{U}_2 \\ \boldsymbol{U}_3 \\ \boldsymbol{0} \end{bmatrix} = - \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{F}_{11} & \mathbf{F}_{12} \\ \mathbf{0} & \mathbf{1} & \mathbf{F}_{21} & \mathbf{F}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{U}_0 \end{bmatrix} \tag{16}$$

where the number of rows of $\mathbf{F}_{11}$ is b_1, the number of columns is b_3, the number of columns in $\mathbf{F}_{12}$ is b_4, the number of rows in $\mathbf{F}_{21}$ is b_2. From (16) we obtain that

$$\boldsymbol{U}_1 + \mathbf{F}_{11}\boldsymbol{U}_3 = -\mathbf{F}_{12}\boldsymbol{U}_0 \tag{17}$$

and

$$\boldsymbol{U}_2 + \mathbf{F}_{21}\boldsymbol{U}_3 = -\mathbf{F}_{22}\boldsymbol{U}_0 . \tag{18}$$

Let us write also the cut-set equations in terms of the partitioned matrices and take into consideration the relationship between the loop matrix and cut-set matrix expressed by $\mathbf{QB}^{+} = \mathbf{0}$. (Here $^{+}$ denotes the transpose of the matrix.)

$$\begin{bmatrix} -\mathbf{F}_{11}^{+} & -\mathbf{F}_{21}^{+} & \mathbf{1} & \mathbf{0} \\ -\mathbf{F}_{12}^{+} & -\mathbf{F}_{22}^{+} & \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I}_2 \\ \boldsymbol{I}_3 \\ \boldsymbol{I}_4 \end{bmatrix} = - \begin{bmatrix} -\mathbf{F}_{11}^{+} & -\mathbf{F}_{21}^{+} & \mathbf{1} & \mathbf{0} \\ -\mathbf{F}_{12}^{+} & -\mathbf{F}_{22}^{+} & \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_0 \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \tag{19}$$

hence

$$-\mathbf{F}_{21}^{+}\,\boldsymbol{I}_2 + \boldsymbol{I}_3 = \mathbf{F}_{11}^{+}\,\boldsymbol{I}_0 \tag{20}$$
$$-\mathbf{F}_{22}^{+}\,\boldsymbol{I}_2 + \boldsymbol{I}_4 = \mathbf{F}_{12}^{+}\,\boldsymbol{I}_0\,. \tag{21}$$

First restrict our calculations to the case where there is no mutual impedance between twigs and links. Then we can write that

$$\boldsymbol{U}_2 = \mathbf{Z}_2\,\boldsymbol{I}_2; \quad \boldsymbol{I}_2 = \mathbf{Y}_2\,\boldsymbol{U}_2\,; \quad \mathbf{Y}_2 = \mathbf{Z}_2^{-1} \tag{22}$$

$$\boldsymbol{U}_3 = \mathbf{Z}_3\,\boldsymbol{I}_3; \quad \boldsymbol{I}_3 = \mathbf{Y}_3\,\boldsymbol{U}_3; \quad \mathbf{Y}_3 = \mathbf{Z}_3^{-1} \tag{23}$$

and thus, by using (20):

$$\boldsymbol{U}_3 = \mathbf{Z}_3\,\boldsymbol{I}_3 = \mathbf{Z}_3\,\mathbf{F}_{11}^{+}\,\boldsymbol{I}_0 + \mathbf{Z}_3\,\mathbf{F}_{21}^{+}\,\mathbf{Y}_2\,\boldsymbol{U}_2\,. \tag{24}$$

Substituting this into (18), and expressing $\boldsymbol{U}_2$:

$$\boldsymbol{U}_2 = -\,[\mathbf{1} + \mathbf{F}_{21}\,\mathbf{Z}_3\,\mathbf{F}_{21}^{+}\,\mathbf{Y}_2]^{-1}\,[\mathbf{F}_{22}\,\boldsymbol{U}_0 + \mathbf{F}_{21}\,\mathbf{Z}_3\,\mathbf{F}_{11}^{+}\,\boldsymbol{I}_0]\,. \tag{25}$$

In the knowledge of $\boldsymbol{U}_2$, $\boldsymbol{U}_3$ can be determined from (24) and with this $\boldsymbol{U}_1$ from (17).

Similarly from (18), by using (22) we have

$$\boldsymbol{I}_2 = -\mathbf{Y}_2\,\mathbf{F}_{21}\,\boldsymbol{U}_3 - \mathbf{Y}_2\,\mathbf{F}_{22}\,\boldsymbol{U}_0. \tag{26}$$

Substituting this into (20):

$$\boldsymbol{I}_3 = [\mathbf{1} + \mathbf{F}_{21}^{+}\,\mathbf{Y}_2\,\mathbf{F}_{21}\,\mathbf{Z}_3]^{-1}\,[\mathbf{F}_{11}^{+}\,\boldsymbol{I}_0 - \mathbf{F}_{21}^{+}\,\mathbf{Y}_2\,\mathbf{F}_{22}\,\boldsymbol{U}_0]\,. \tag{27}$$

In the knowledge of $\boldsymbol{I}_3$, $\boldsymbol{I}_2$ can be determined from (26) and with this, $\boldsymbol{I}_4$ from (21). Thus, the required quantities have been determined.

If there is a mutual impedance between links and twigs, then

$$\begin{bmatrix} \boldsymbol{U}_2 \\ \boldsymbol{U}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{22} & \mathbf{Z}_{23} \\ \mathbf{Z}_{32} & \mathbf{Z}_{33} \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_2 \\ \boldsymbol{I}_3 \end{bmatrix} \tag{28}$$

accordingly

$$\begin{aligned} \boldsymbol{U}_2 &= \mathbf{Z}_{22}\,\boldsymbol{I}_2 + \mathbf{Z}_{23}\,\boldsymbol{I}_3 \\ \boldsymbol{U}_3 &= \mathbf{Z}_{32}\,\boldsymbol{I}_2 + \mathbf{Z}_{33}\,\boldsymbol{I}_3\,. \end{aligned} \tag{29}$$

By using these and (20), we obtain from (18)

$$\begin{aligned} \boldsymbol{I}_2 = -\,[\mathbf{Z}_{22} + \mathbf{F}_{21}\,\mathbf{Z}_{32} + (\mathbf{Z}_{23} + \mathbf{F}_{21}\,\mathbf{Z}_{33})\,\mathbf{F}_{21}^{+}]^{-1}\,[\mathbf{F}_{22}\,\boldsymbol{U}_0 + \\ + (\mathbf{Z}_{23} + \mathbf{F}_{21}\,\mathbf{Z}_{33})\,\mathbf{F}_{11}^{+}\,\boldsymbol{I}_0]\,. \end{aligned} \tag{30}$$

In the knowledge of $\boldsymbol{I}_2$, $\boldsymbol{I}_3$ can be calculated from (20) and $\boldsymbol{I}_4$ from (21). With these $\boldsymbol{U}_2$, $\boldsymbol{U}_3$ can be expressed on the basis of (29) and thus also $\boldsymbol{U}_1$ from (17).

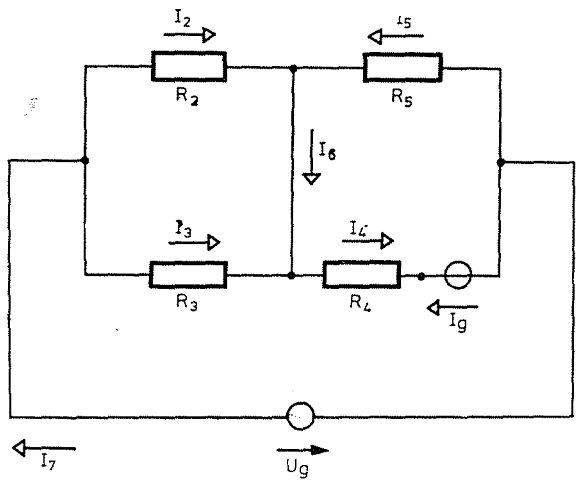

Fig. 1

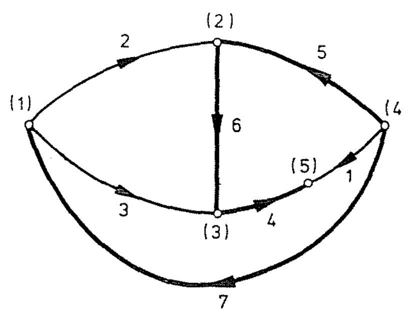

Fig. 2

Example

Calculate the branch currents of the network shown in Fig. 1 by each of the methods described above. The graph of the network is shown in Fig. 2 and here the twigs of the tree selected for the calculation are indicated by thick lines.

According to the method described in the first part, twig 1 is classified into the first group, twigs 2, 3, 4, 5 into the second group, while twigs 6, 7 are classified into the third group. The matrices of the cut-set and loop system generated by the tree are:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} & 0 \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} & \mathbf{1} \end{bmatrix} = \left[\begin{array}{c|cccc|cc} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ \hline 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\mathbf{B} = \begin{bmatrix} 1 & \mathbf{B}_{11} & \mathbf{B}_{12} \\ 0 & \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} = \left[\begin{array}{c|cccc|cc} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ \hline 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & -1 & 1 \end{array}\right].$$

Dotted lines indicate the partitioning in Eqs (7) and (3), respectively. For the calculations still the following matrices are necessary:

$$\mathbf{Z}_\beta = \langle R_2 \quad R_3 \quad R_4 \quad R_5 \rangle$$

$$\boldsymbol{U}_0 = \begin{bmatrix} 0 \\ -U_g \end{bmatrix}; \quad \boldsymbol{I}_0 = I_g$$

$$\mathbf{B}_{21}\,\mathbf{Z}_\beta = \begin{bmatrix} R_2 & 0 & 0 & -R_5 \\ 0 & R_3 & 0 & -R_5 \end{bmatrix}$$

$$\mathbf{B}_{22}\,\boldsymbol{U}_0 = \begin{bmatrix} -U_g \\ -U_g \end{bmatrix}; \qquad \mathbf{Q}_{11}\boldsymbol{I}_0 = \begin{bmatrix} I_g \\ I_g \end{bmatrix}.$$

With these the required branch currents can be expressed on the basis of (12):

$$\boldsymbol{I}_2 = \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = -\begin{bmatrix} \mathbf{B}_{21}\mathbf{Z}_\beta \\ \mathbf{Q}_{12} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_{22}\boldsymbol{U}_0 \\ \mathbf{Q}_{11}\boldsymbol{I}_0 \end{bmatrix} =$$

$$= \frac{1}{R_2R_3 + R_2R_5 + R_3R_5} \begin{bmatrix} R_3U_g - R_3R_5I_g \\ R_2U_g - R_2R_5I_g \\ -(R_2R_3 + R_2R_5 + R_3R_5)\,I_g \\ -(R_2 + R_3)\,U_g - R_2R_3I_g \end{bmatrix}.$$

From (5) we obtain:

$$\boldsymbol{I}_\gamma = \begin{bmatrix} I_6 \\ I_7 \end{bmatrix} = -\,\mathbf{Q}_{22}\boldsymbol{I}_2 - \mathbf{Q}_{21}\boldsymbol{I}_0 =$$

$$= \frac{1}{R_2R_3 + R_2R_5 + R_3R_5} \begin{bmatrix} -R_2U_g - R_3\,(R_2 + R_5)\,I_g \\ (R_2+R_3)\,U_g - (R_2+R_3)\,R_5I_g \end{bmatrix}.$$

Thus the required currents have been determined. In the course of the calculations a matrix of order four had to be inverted.

In the following the previous problem will be solved by using the other method described. In this case the loop matrix is partitioned according to the four groups of branches.

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & \mathbf{F}_{11} & \mathbf{F}_{12} \\ 0 & 1 & \mathbf{F}_{21} & \mathbf{F}_{22} \end{bmatrix} = \left[\begin{array}{c|cc|cc|cc} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ \hline 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & -1 & 1 \end{array}\right].$$

Here

$$\mathbf{Z}_2 = \langle R_2 \quad R_3 \rangle; \qquad \mathbf{Z}_3 = \langle R_4 \quad R_5 \rangle.$$

Using these, and on the basis of (27),

$$\boldsymbol{I}_3 = \begin{bmatrix} I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{R_2 R_3 + R_2 R_5 + R_3 R_5}{R_2 R_3} \end{bmatrix}^{-1} \begin{bmatrix} -I_g \\ -\dfrac{R_2 + R_3}{R_2 R_3} U_g - I_g \end{bmatrix} =$$

$$= \begin{bmatrix} -I_g \\ \dfrac{-R_2 R_3 I_g - (R_2 + R_3) U_g}{R_2 R_3 + R_2 R_5 + R_3 R_5} \end{bmatrix}.$$

Here a matrix of order two is to be inverted.

In the knowledge of $\boldsymbol{I}_3$ from (26) we obtain:

$$\boldsymbol{I}_2 = \begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = \frac{U_g - R_5 I_g}{R_2 R_3 + R_2 R_5 + R_3 R_5} \begin{bmatrix} R_3 \\ R_2 \end{bmatrix}$$

and from (21)

$$\boldsymbol{I}_4 = \begin{bmatrix} I_6 \\ I_7 \end{bmatrix} = \frac{1}{R_2 R_3 + R_2 R_5 + R_3 R_5} \begin{bmatrix} -R_2 U_g - (R_2 + R_5) R_3 I_g \\ (R_2 + R_3)(U_g - R_5 I_g) \end{bmatrix}.$$

The result is in accordance with the one obtained by the previous method.

Summary

The paper describes a process of graph theory which can be employed, as against several known methods, also for the calculation of networks containing both ideal voltage generator and ideal current generator. The structure of networks is characterized by the loop and cut-set matrix.

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