TWO-PORT MODELS WITH NULLATORS AND NORATORS

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Introduction

Recently, several publications [1, 2, 3] have discussed the modelling of two-ports with controlled generators and other extreme parameters, by using nullators and norators. These models permit to calculate networks by topological methods and to solve them by computer programming.

In the following a systematic process is described for modelling a linear two-port with arbitrary parameters.

The concept of nullator and norator

The nullator is a two-pole with zero current and voltage. The norator involves no restriction with respect to current and voltage. Their symbols are shown in Figs 1a and 1b.

A network analysis problem can be solved unequivocally, if an unambiguous relationship can be established between currents and voltages of the two-poles forming the network. The nullator in turn represents two restrictions. Namely the insertion of a nullator into a real circuit makes the analysis problem redundant, the number of the possible independent Kirchhoff equations being increased by one, while the number of relationships of voltages and currents by two. The insertion of a norator into the circuit adds another independent Kirchhoff equation leaving the number of restrictions for voltages and currents unchanged. Accordingly the insertion of a norator makes the problem indefinite. For an equal number of inserted nullators and norators, the network calculation problem can be solved.

By connecting a nullator and norator we obtain a nullor. The nullor is a two-port with the primary side connected to a nullator, and the secondary side to a norator (Fig. 2). The nullor can be regarded as the model of the ideal transistor. Thus the nullor can approximately be realized by a transistor (Fig. 3).

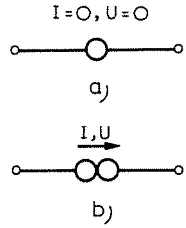


Fig. 1

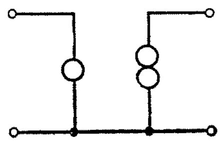


Fig. 2

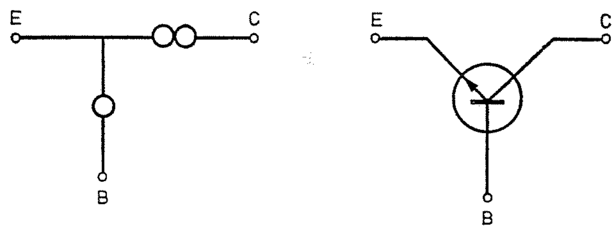


Fig. 3

Equivalent circuits containing nullator and norator for two-ports with extreme parameters

Models relying for ideal controlled generators on nullator and norator are known [1, 2]. For each type an equivalent circuit can be found, where the nullator—norator pair forms a nullor, hence they can be realized by an ideal transistor. In the following these circuits will be made use of. Fig. 4 shows models for each type, for the cases of two opposite reference directions of secondary side voltage or current. (The modelled network can of course be calculated also with impedances having negative real part.) From circuit diagrams it is obvious that the indicated relationships are met.

On the basis of ideal controlled generators two-ports can be modelled so that two elements in the main diagonal of a parameter matrix are zero (Figs 5 and 6). Each of the equivalent circuits can be decomposed to two network sections connected to the ports by nullator or norator. If such a network section is connected to both ports by nullators, then the prescribed relationship

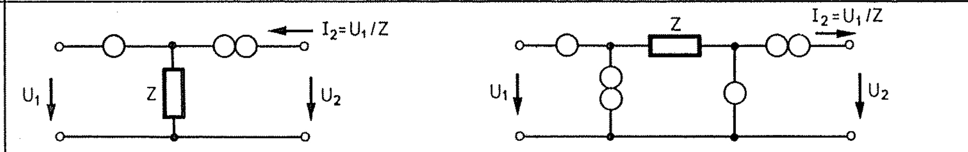
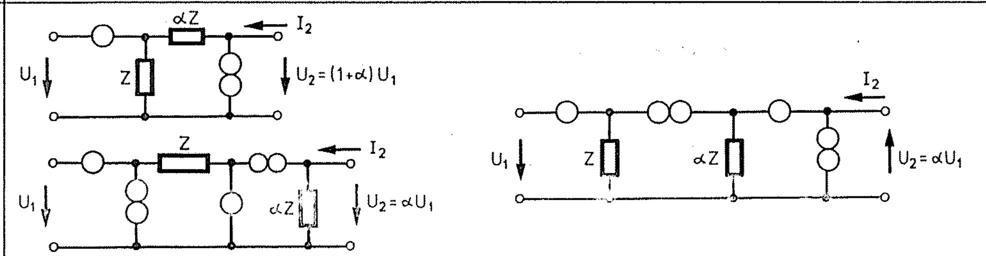
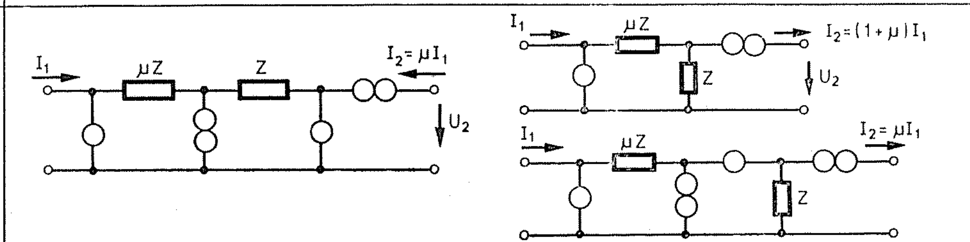
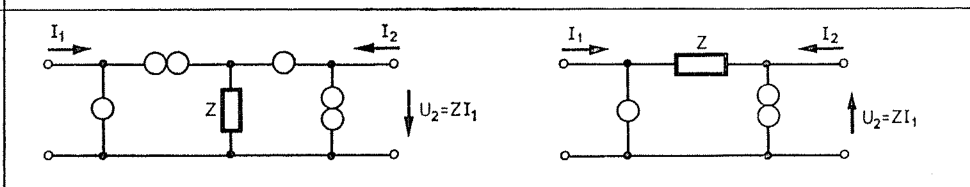
name	equations	equivalent circuits
voltage-controlled current-source	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \frac{1}{Z} & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$	
voltage-controlled voltage-source	$\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1+\alpha & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$ $\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$	
current-controlled current-source	$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \mu & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$ $\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1+\mu & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$	
current-controlled voltage-source	$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ Z & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$	

Fig. 4

between input and output voltage exists. If it is connected to the ports by norators, then the prescribed relationship between the currents of the two ports is provided. If it is connected to the ports by a nullator and by a norator, then it produces the prescribed relationship between the voltage at one port and the current at the other. Circuits have been traced on the basis of the models of controlled generators. Accordingly, Fig. 5 shows the equivalent circuits containing nullator and norator of two-ports, where the impedance, admittance, chain, and inverse chain parameters in the main diagonal are zero. Let us consider now the cases where each of the non-zero parameters have non-negative, one, or both negative real parts.

Impedances in the circuits in Fig. 5 can be expressed by two-port parameters in the following way:

with impedance parameters $Z_1 = z_{12}\ ;\ Z_2 = z_{21}\ ;$

with admittance parameters $Z_1 = \frac{1}{y_{21}}\ ; Z_2 = \frac{1}{y_{12}}\ ;$

with chain parameters $Z_1 = a_{12};\quad Z_2 = \frac{1}{a_{21}}\ ;$

with inverted chain parameters $Z_1 = \frac{1}{b_{21}}\ ; Z_2 = b_{12}\ .$

Similarly, Fig. 6 shows some models of two-ports characterized by hybrid parameters. In the circuits $h_{12} = Z_1/Z_2$ and $h_{21} = Z_3/Z_4$.

In addition to models in Figs 5 and 6, several other equivalent circuits containing nullator and norator, can be modelled.

Models of two-ports

By using the models shown in Figs 5 and 6 (or other equivalent models) the equivalent circuit of two-ports characterized by the impedance, admittance, or hybrid parameters, can be easily given. Namely by inserting an immittance connected in series or parallelly at the primary and secondary side, in conformity to elements having non-negative real part in the main diagonal, we obtain the model characterized by the prescribed parameters. As an example, the equivalent circuit of the two-port characterized by the impedance and hybrid parameters of non-negative real part is shown in Fig. 7.

On the basis of the foregoing, in Fig. 8 the models of the ideal transformer, the negative impedance converter, and the gyrator are shown. It should be noticed that the equivalent circuit of the gyrator is also described in [3].

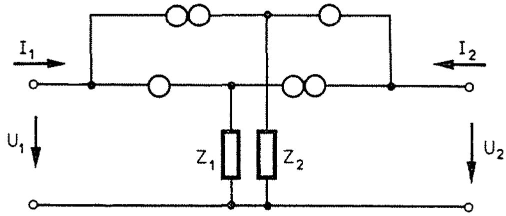
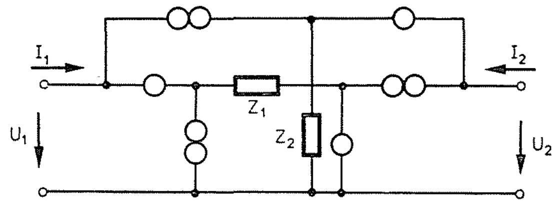
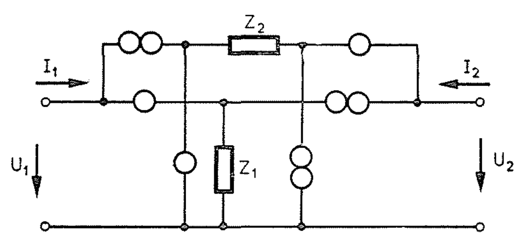
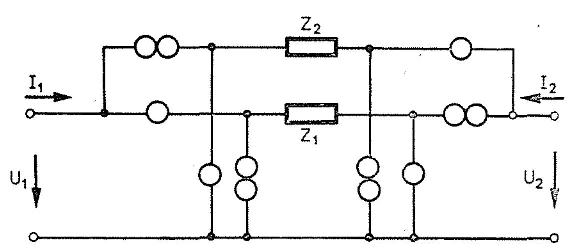
equations	equivalent circuit
$\begin{bmatrix}U_1\\U_2\end{bmatrix}=\begin{bmatrix}\bigcirc & z_{12}\\z_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}I_1\\I_2\end{bmatrix}$ $\begin{bmatrix}I_1\\I_2\end{bmatrix}=\begin{bmatrix}\bigcirc & y_{12}\\y_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}U_1\\U_2\end{bmatrix}$ $\begin{bmatrix}U_1\\I_1\end{bmatrix}=\begin{bmatrix}\bigcirc & a_{12}\\a_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}U_2\\I_2\end{bmatrix}$ $\begin{bmatrix}U_2\\I_2\end{bmatrix}=\begin{bmatrix}\bigcirc & b_{12}\\b_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}U_1\\I_1\end{bmatrix}$	
$\begin{bmatrix}U_1\\U_2\end{bmatrix}=\begin{bmatrix}\bigcirc & -z_{12}\\z_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}I_1\\I_2\end{bmatrix}$ $\begin{bmatrix}I_1\\I_2\end{bmatrix}=\begin{bmatrix}\bigcirc & y_{12}\\-y_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}U_1\\U_2\end{bmatrix}$ $\begin{bmatrix}U_1\\I_1\end{bmatrix}=\begin{bmatrix}\bigcirc & -a_{12}\\a_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}U_2\\I_2\end{bmatrix}$ $\begin{bmatrix}U_2\\I_2\end{bmatrix}=\begin{bmatrix}\bigcirc & b_{12}\\-b_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}U_1\\I_1\end{bmatrix}$	
$\begin{bmatrix}U_1\\U_2\end{bmatrix}=\begin{bmatrix}\bigcirc & z_{12}\\-z_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}I_1\\I_2\end{bmatrix}$ $\begin{bmatrix}I_1\\I_2\end{bmatrix}=\begin{bmatrix}\bigcirc & -y_{12}\\y_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}U_1\\U_2\end{bmatrix}$ $\begin{bmatrix}U_1\\I_1\end{bmatrix}=\begin{bmatrix}\bigcirc & a_{12}\\-a_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}U_2\\I_2\end{bmatrix}$ $\begin{bmatrix}U_2\\I_2\end{bmatrix}=\begin{bmatrix}\bigcirc & -b_{12}\\b_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}U_1\\I_1\end{bmatrix}$	
$\begin{bmatrix}U_1\\U_2\end{bmatrix}=\begin{bmatrix}\bigcirc & -z_{12}\\-z_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}I_1\\I_2\end{bmatrix}$ $\begin{bmatrix}I_1\\I_2\end{bmatrix}=\begin{bmatrix}\bigcirc & -y_{12}\\-y_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}U_1\\U_2\end{bmatrix}$ $\begin{bmatrix}U_1\\I_1\end{bmatrix}=\begin{bmatrix}\bigcirc & -a_{12}\\-a_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}U_2\\I_2\end{bmatrix}$ $\begin{bmatrix}U_2\\I_2\end{bmatrix}=\begin{bmatrix}\bigcirc & -b_{12}\\-b_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}U_1\\I_1\end{bmatrix}$	

Fig. 5

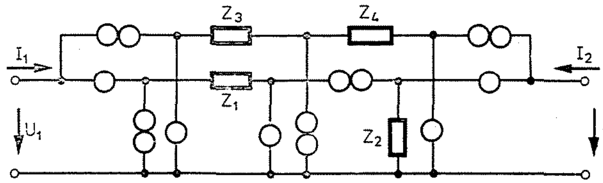
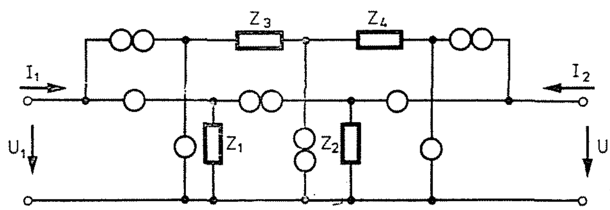
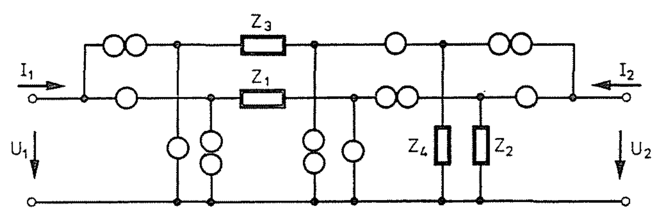
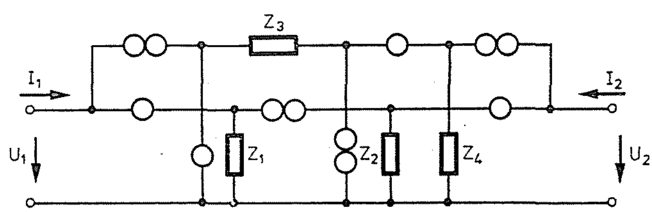
equations	equivalent circuit
$\begin{bmatrix}U_1\\I_2\end{bmatrix}=\begin{bmatrix}\bigcirc & h_{12}\\ h_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}I_1\\U_2\end{bmatrix}$	
$\begin{bmatrix}U_1\\I_2\end{bmatrix}=\begin{bmatrix}\bigcirc & -h_{12}\\ h_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}I_1\\U_2\end{bmatrix}$	
$\begin{bmatrix}U_1\\I_2\end{bmatrix}=\begin{bmatrix}\bigcirc & h_{12}\\ -h_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}I_1\\U_2\end{bmatrix}$	
$\begin{bmatrix}U_1\\I_2\end{bmatrix}=\begin{bmatrix}\bigcirc & -h_{12}\\ -h_{21} & \bigcirc\end{bmatrix}\begin{bmatrix}I_1\\U_2\end{bmatrix}$	

Fig. 6

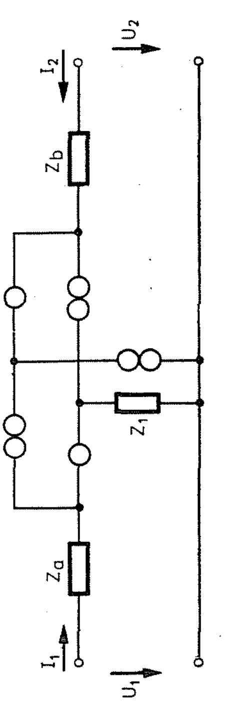
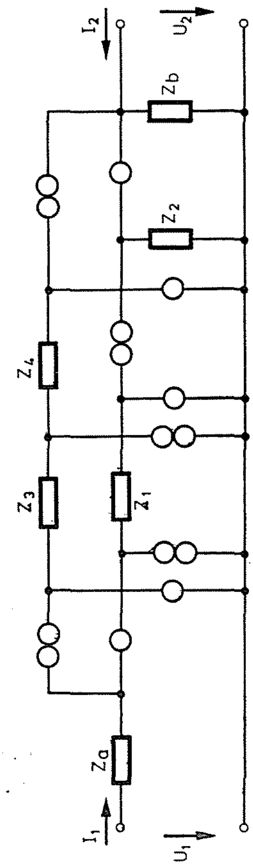
equations	equivalent circuit
$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$	 $z_{11} = Z_a$, $z_{12} = Z_1$, $z_{21} = Z_2$, $z_{22} = Z_b$
$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$	 $h_{11} = Z_a$, $h_{12} = \frac{Z_1}{Z_2}$, $h_{21} = \frac{Z_3}{Z_4}$, $h_{22} = \frac{1}{Z_b}$

Fig. 7

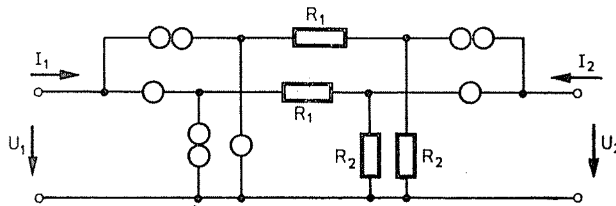
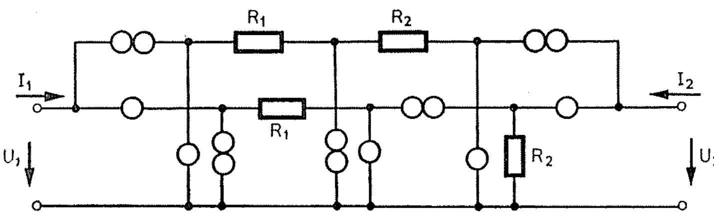
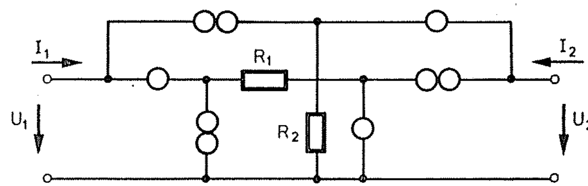
name	equations	equivalent circuit
ideal transformer	$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & n \\ n & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$ $n > 1$	 $n = 1 + \frac{R_1}{R_2}$
negative impedance converter	$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & k \\ k & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$ $k > 0$	 $k = \frac{R_1}{R_2}$
gyrator	$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 0 & -R_1 \\ R_2 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$	

Fig. 8

Summary

The paper describes the models for two-ports containing nullator—norator pairs. In the knowledge of two-port parameters, models can be formed on the basis of the equivalent circuits of controlled generators. Each of the described models can be composed of impedances and ideal transistors.

References

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