

CALCULATION OF NETWORK MODELS CONTAINING NULLATORS AND NORATORS

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Networks containing coupled two-poles (controlled generator, gyrator, ideal transformer, negative impedance converter) can be modelled without coupled branches, by using nullator and norator [1, 2, 3].

As known, the nullator (Fig. 1a) is a two-pole with zero current and voltage. The norator in turn (Fig. 1b) represents no restriction on current and voltage. Accordingly, the insertion of a nullator into a network consisting of impedances and generators makes the equations of the network redundant, while that of the norator makes them indefinite. In the case of an identical number of nullators and norators as many linearly independent equations can be written as there are branches in the network, i.e. unknown quantities of the analysis.

The equivalent circuits made by using nullators and norators can be calculated according to [4] by the method of node potentials. In the method of node potentials, equations are first written for the network obtained by omitting nullators and norators. In these equations nullators and norators can be taken into consideration by some modifications.

In the following a method based on the use of the loop and cut-set matrices of the graph of the network is presented.

Lossy generators of the network can be taken into consideration by the equivalent circuits of Thevenin or Norton. These can be regarded as a single branch containing an impedance, or as two branches containing an ideal generator and an impedance, respectively. In the following the latter will be employed. For the calculations let us select a tree of the graph of the network in which a twig corresponds to each nullator and ideal voltage generator of the network, while a link to each norator and ideal current generator. (Such a selection is always possible.)

Class the branches of the network into the following six groups:

1. links containing ideal current generator,
2. links containing norator,
3. links containing impedance,

4. twigs containing impedance,
5. twigs containing nullator,
6. twigs containing ideal voltage generator.

The number of branches in each groups are in sequence: $b_1, b_2, b_3, b_4, b_5, b_6$. It should be noted that the number of branches in groups 2 and 5 is identical, accordingly $b_2 = b_5$.

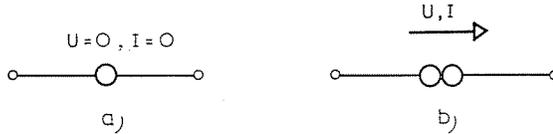


Fig. 1

Let us number the branches in the order of grouping. Loops of the loop system generated by the selected tree are numbered according to the respective links, the cut-set system generated by this tree is numbered in the order of the respective twigs. Using the loop matrix \mathbf{B} of the loop system, the loop equations of the network are

$$\mathbf{B}\mathbf{U} = \mathbf{0} \quad (1)$$

where \mathbf{U} is the column matrix of branch voltages.

Partition \mathbf{B} and \mathbf{U} according to the six groups of branches. Thus (1) can be written in the form:

$$\begin{array}{c}
 b_1 \quad b_2 \quad b_3 \quad b_4 \quad b_5 \quad b_6 \\
 b_1 \quad \left[\begin{array}{cccccc}
 \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{11} & \mathbf{F}_{12} & \mathbf{F}_{13} \\
 \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{F}_{21} & \mathbf{F}_{22} & \mathbf{F}_{23} \\
 \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{F}_{31} & \mathbf{F}_{32} & \mathbf{F}_{33}
 \end{array} \right] \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \\ \mathbf{U}_3 \\ \mathbf{U}_4 \\ \mathbf{0} \\ \mathbf{U}_0 \end{bmatrix} = \mathbf{0} \quad (2)
 \end{array}$$

The numbers of the columns of the individual blocks are indicated above the matrix, those of the rows beside the matrix. It has been taken into consideration that $\mathbf{U}_6 = \mathbf{U}_0$ is the column matrix of the source voltage of voltage generators, and $\mathbf{U}_5 = \mathbf{0}$ is the voltage of nullators. From (2) we have

$$\mathbf{U}_1 + \mathbf{F}_{11}\mathbf{U}_4 + \mathbf{F}_{13}\mathbf{U}_0 = \mathbf{0} \quad (3)$$

$$\mathbf{U}_2 + \mathbf{F}_{21}\mathbf{U}_4 + \mathbf{F}_{23}\mathbf{U}_0 = \mathbf{0} \quad (4)$$

$$\mathbf{U}_3 + \mathbf{F}_{31}\mathbf{U}_4 + \mathbf{F}_{33}\mathbf{U}_0 = \mathbf{0} \quad (5)$$

Write the cut-set equations

$$\mathbf{QI} = \mathbf{0} \tag{6}$$

where \mathbf{Q} is the matrix of the cut-set system generated by the selected tree, according to the previous numbering, and \mathbf{I} is the column matrix of the branch currents of the network. Partition also these according to the six groups of branches. Since in the case of the above numbering of branches, loops and cut-sets

$$\mathbf{B} = [\mathbf{1} \ \mathbf{F}] \quad \text{and} \quad \mathbf{Q} = [-\mathbf{F}^+ \ \mathbf{1}] \tag{7}$$

where \mathbf{F}^+ designates the transpose of \mathbf{F} , (6) can be written as follows:

$$\begin{bmatrix} -\mathbf{F}_{11}^+ & -\mathbf{F}_{21}^+ & -\mathbf{F}_{31}^+ & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ -\mathbf{F}_{12}^+ & -\mathbf{F}_{22}^+ & -\mathbf{F}_{22}^+ & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ -\mathbf{F}_{13}^+ & -\mathbf{F}_{23}^+ & -\mathbf{F}_{33}^+ & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \mathbf{I}_0 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \\ \mathbf{I}_4 \\ \mathbf{0} \\ \mathbf{I}_6 \end{bmatrix} = \mathbf{0} \tag{8}$$

where \mathbf{I}_0 is the column matrix of the source current of the current generators and $\mathbf{I}_5 = \mathbf{0}$ the current of nullators. Hence

$$-\mathbf{F}_{11}^+ \mathbf{I}_0 - \mathbf{F}_{21}^+ \mathbf{I}_2 - \mathbf{F}_{31}^+ \mathbf{I}_3 + \mathbf{I}_4 = \mathbf{0} \tag{9}$$

$$-\mathbf{F}_{12}^+ \mathbf{I}_0 - \mathbf{F}_{22}^+ \mathbf{I}_2 - \mathbf{F}_{32}^+ \mathbf{I}_3 = \mathbf{0} \tag{10}$$

$$-\mathbf{F}_{13}^+ \mathbf{I}_0 - \mathbf{F}_{23}^+ \mathbf{I}_2 - \mathbf{F}_{33}^+ \mathbf{I}_3 + \mathbf{I}_6 = \mathbf{0}. \tag{11}$$

Currents and voltages of the branches can be determined from the above equations e.g. in the following way. It is seen from (2) that \mathbf{F}_{22} is a quadratic matrix. If it is not singular then from (10)

$$\mathbf{I}_2 = -\mathbf{F}^{+^{-1}} \mathbf{F}_{12}^+ \mathbf{I}_0 - \mathbf{F}_{22}^{+^{-1}} \mathbf{F}_{32}^+ \mathbf{I}_3. \tag{12}$$

Substituting this into (9) we find that

$$(\mathbf{F}_{21}^+ \mathbf{F}_{22}^{+^{-1}} \mathbf{F}_{32}^+ - \mathbf{F}_{31}^+) \mathbf{I}_3 + \mathbf{I}_4 = (\mathbf{F}_{11}^+ - \mathbf{F}_{21}^+ \mathbf{F}_{22}^{+^{-1}} \mathbf{F}_{12}^+) \mathbf{I}_0 \tag{13}$$

the unknown quantities being \mathbf{I}_3 and \mathbf{I}_4 the currents of links and twigs containing impedance, while in equation (5) the voltages of the same branches. These will be used in the following calculations.

The relationships between the current and voltage of impedances can be written as:

$$\mathbf{U}_3 = \mathbf{Z}_3 \mathbf{I}_3 \quad \mathbf{I}_3 = \mathbf{Y}_3 \mathbf{U}_3 \quad \mathbf{Y}_3 = \mathbf{Z}_3^{-1} \tag{14}$$

$$\mathbf{U}_4 = \mathbf{Z}_4 \mathbf{I}_4 \quad \mathbf{I}_4 = \mathbf{Y}_4 \mathbf{U}_4 \quad \mathbf{Y}_4 = \mathbf{Z}_4^{-1} \tag{15}$$

\mathbf{Z}_3 and \mathbf{Z}_4 denotes the branch impedance matrix of branches in groups 3 and 4, respectively. In the network there are no coupled branches since cou-

plings are eliminated by the nullator—norator model. Thus, \mathbf{Z}_3 and \mathbf{Z}_4 are diagonal matrices.

It is advisable to express \mathbf{U}_3 or \mathbf{I}_4 from the above equations, possible by inverting a matrix of order b_3 and b_4 , respectively.

For calculating \mathbf{U}_3 we have from (13) and (15):

$$\mathbf{U}_4 = -\mathbf{Z}_4(\mathbf{F}_{21}^+ \mathbf{F}_{22}^{+^{-1}} \mathbf{F}_{32}^+ - \mathbf{F}_{31}^+) \mathbf{I}_3 + \mathbf{Z}_4(\mathbf{F}_{11}^+ - \mathbf{F}_{21}^+ \mathbf{F}_{22}^{+^{-1}} \mathbf{F}_{12}^+) \mathbf{I}_0. \quad (16)$$

Substitute this into (5) by using (14):

$$\mathbf{U}_3 = [\mathbf{1} - \mathbf{F}_{31} \mathbf{Z}_4 (\mathbf{F}_{21}^+ \mathbf{F}_{22}^{+^{-1}} \mathbf{F}_{32}^+ - \mathbf{F}_{31}^+) \mathbf{Y}_3]^{-1} [\mathbf{F}_{31} \mathbf{Z}_4 (\mathbf{F}_{21}^+ \mathbf{F}_{22}^{+^{-1}} \mathbf{F}_{12}^+ - \mathbf{F}_{11}^+) \mathbf{I}_0 - \mathbf{F}_{33} \mathbf{U}_0] \quad (17)$$

If \mathbf{U}_3 is known (14) yields \mathbf{I}_3 , hence (16) \mathbf{U}_4 and (15) \mathbf{I}_4 .

Similarly, for the calculation of \mathbf{I}_4 we express \mathbf{I}_3 from (5), by using (14) and (15).

$$\mathbf{I}_3 = -\mathbf{Y}_3 \mathbf{F}_{31} \mathbf{Z}_4 \mathbf{I}_4 - \mathbf{Y}_3 \mathbf{F}_{33} \mathbf{U}_0. \quad (18)$$

Substituting this into (13) and arranging:

$$\mathbf{I}_4 = [\mathbf{1} - (\mathbf{F}_{21}^+ \mathbf{F}_{22}^{+^{-1}} \mathbf{F}_{32}^+ - \mathbf{F}_{31}^+) \mathbf{Y}_3 \mathbf{F}_{31} \mathbf{Z}_4]^{-1} [(\mathbf{F}_{21}^+ \mathbf{F}_{22}^{+^{-1}} \mathbf{F}_{32}^+ - \mathbf{F}_{31}^+) \mathbf{Y}_3 \mathbf{F}_{33} \mathbf{U}_0 + (\mathbf{F}_{11}^+ - \mathbf{F}_{21}^+ \mathbf{F}_{22}^{+^{-1}} \mathbf{F}_{12}^+) \mathbf{I}_0]. \quad (19)$$

In the knowledge of \mathbf{I}_4 , \mathbf{I}_3 can be expressed from (18), while \mathbf{U}_3 and \mathbf{U}_4 on the basis of (14) and (15).

Thus, the voltage and current of impedances have been determined in two different ways.

The other currents and voltages can also be calculated from our equations. Thus the current of the norators \mathbf{I}_2 can be written from (12), the voltage \mathbf{U}_2 from (4), the voltage of current generators \mathbf{U}_1 from (3), while the current of voltage generators \mathbf{I}_6 from (11) and (12).

The calculation method is presented on two examples.

a) In the network shown in Fig. 2

$$\begin{aligned} R &= 1 \text{ k}\Omega; & R_1 &= 56 \text{ k}\Omega; & R_2 &= 25 \text{ k}\Omega; \\ R_c &= 1.5 \text{ k}\Omega; & R_e &= 0.5 \text{ k}\Omega; & R_f &= 0.8 \text{ k}\Omega, \end{aligned}$$

and the hybrid parameters characterizing the transistor, at high frequency are

$$h_{11} = 0.95 \cdot 10^{-3} \Omega; \quad h_{12} = 5.4 \cdot 10^{-4}; \quad h_{21} = 50; \quad h_{22} = 100 \cdot 10^{-6} \text{ S}.$$

Let us determine the voltage amplification factor $\mathbf{U}_2/\mathbf{U}_1$.

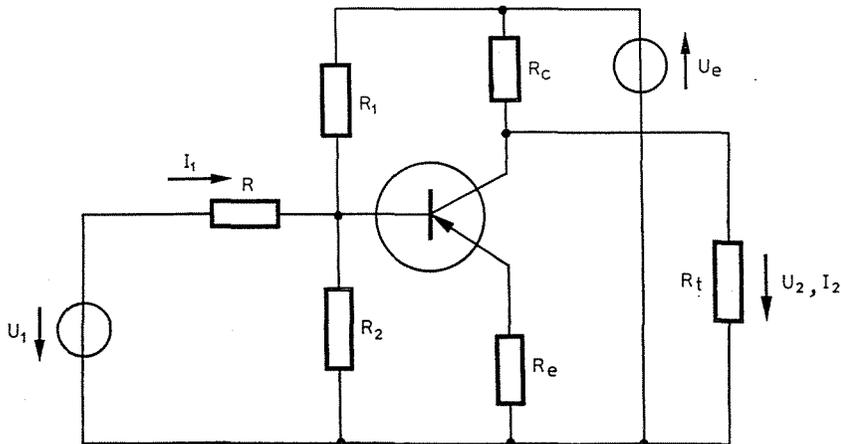


Fig. 2

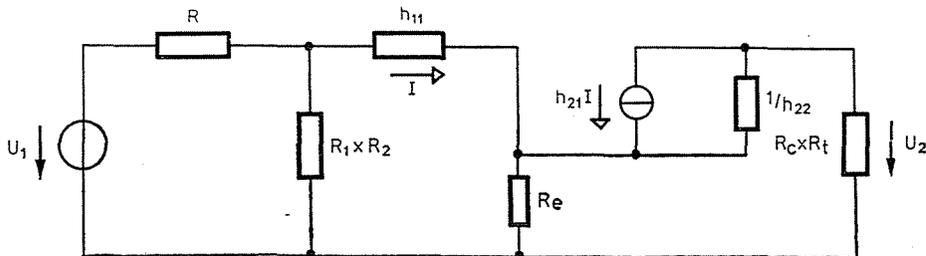


Fig. 3

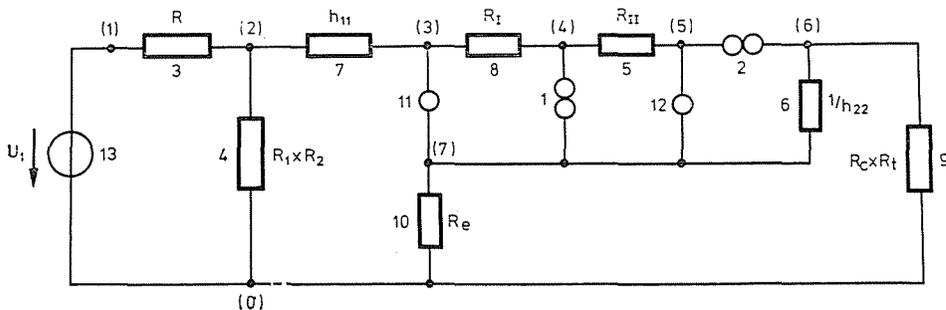


Fig. 4

From the aspect of high-frequency signals the direct voltage generator U_e can be regarded as a short-circuit. Neglecting the reaction of the collector-emitter voltage on the base-emitter voltage ($h_{12} \approx 0$), the transistor can be substituted by a current controlled current generator (Fig. 3). A calculation model of this circuit is shown in Fig. 4. Here $h_{21} = R_I/R_{II}$. In our calculation let $R_I = 10 \text{ k}\Omega$, then $R_{II} = 0.2 \text{ k}\Omega$. The graph of the network with the fore-

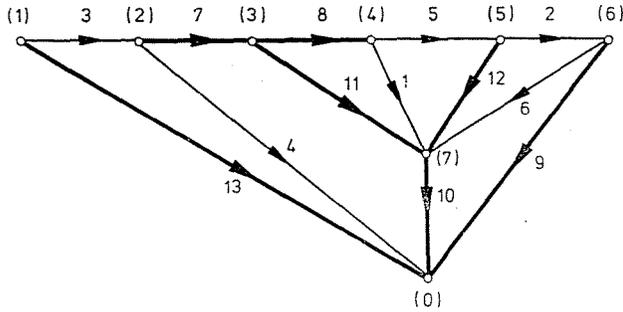


Fig. 5

going numbering is shown in Fig. 5. Twigs are indicated by thick lines. Since there is no current generator in the network, $b_1 = 0$, further, according to the number of nullators and norators $b_2 = b_5 = 2$, and, since there is one voltage generator in the network, $b_6 = 1$. The number of nodes is 8, accordingly there are 7 twigs. Thus $b_3 = b_4 = 4$. The matrix of the loop system generated by the selected tree is:

$$\mathbf{B} = \begin{bmatrix}
 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & 0 \\
 \hline
 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & -1 \\
 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & -1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0
 \end{bmatrix}$$

accordingly

$$\mathbf{F}_{21} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}; \quad \mathbf{F}_{22} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}; \quad \mathbf{F}_{23} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mathbf{F}_{31} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}; \quad \mathbf{F}_{32} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ -1 & 1 \\ 0 & 0 \end{bmatrix}; \quad \mathbf{F}_{33} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix};$$

$$\mathbf{I}_0 = \mathbf{O}; \quad \mathbf{U}_0 = \mathbf{U}_1$$

$$\mathbf{Z}_3 = \langle R \quad R_1 \times R_2 \quad R_{II} \quad 1/h_{22} \rangle = \langle 1 \quad 17.28 \quad 0.2 \quad 10 \rangle 10^3 \Omega$$

$$\mathbf{Z}_4 = \langle h_{11} \quad R_I \quad R_c \times R_t \quad R_e \rangle = \langle 0.95 \cdot 10^{-6} \quad 10 \quad 0.522 \quad 0.5 \rangle 10^3 \Omega$$

$$\mathbf{Y}_3 = \langle 1 \quad 0.0579 \quad 5 \quad 0.1 \rangle 10^{-3} \text{ S}$$

On the basis of Eq. (17), from the above values we obtain

$$U_3 = \begin{bmatrix} 0.0918 \\ 0.908 \\ -0.392 \\ -1.836 \end{bmatrix} U_1 \text{ and } I_3 = Y_3 U_3 = \begin{bmatrix} 0.0918 \\ 0.0526 \\ -1.96 \\ -0.184 \end{bmatrix} U_1 \cdot 10^{-3} \text{ S}$$

$$U_4 = \begin{bmatrix} 0.0372 \cdot 10^{-6} \\ 0.392 \\ -0.927 \\ 0.857 \end{bmatrix} U_1$$

The required voltage U_2 is the third element of U_4 , accordingly $U_2/U_1 = -0.93$.

b) In Fig. 6 the equivalent circuit of the negative impedance converter closed by resistance R , generated by generator of voltage U_0 is shown. Let us calculate the current of the voltage generator.

For the calculations the branches are numbered according to Fig. 6. Branches 1, ..., 5 are links containing norator, branches 6, 7, 8 are links containing impedance, 9, 10 are twigs containing impedance, 11, ..., 15 being twigs with nullator, 16 a twig containing an ideal voltage generator (Fig. 7). The matrix of the fundamental loop system generated by this tree is

$$B = \begin{bmatrix} 1 & 0 & F_{21} & F_{22} & F_{23} \\ 0 & 1 & F_{31} & F_{32} & F_{33} \end{bmatrix} =$$

$$= \left[\begin{array}{ccccc|ccccc|ccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \end{array} \right]$$

accordingly

$$F_{21}^+ = \begin{bmatrix} 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}; \quad F_{31}^+ = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix};$$

$$F_{22}^{+-1} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}; \quad F_{32}^+ = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

$$Y_3 = \left\langle \frac{1}{R} \quad \frac{1}{R_1} \quad \frac{1}{R_1} \right\rangle \text{ and } Z_4 = \langle R_2 R_2 \rangle .$$

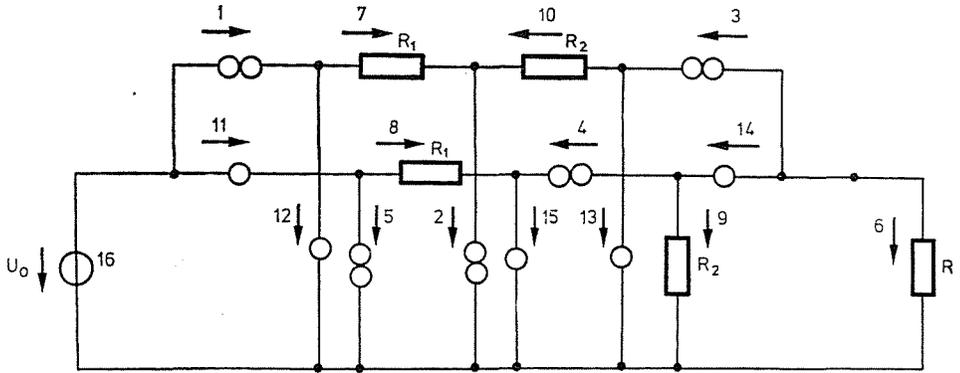


Fig. 6

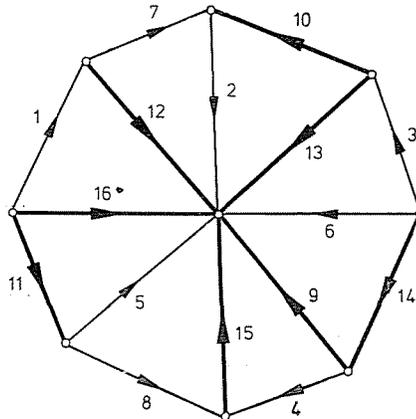


Fig. 7

From these, on the basis of (19) we find:

$$\mathbf{I}_4 = \begin{bmatrix} I_9 \\ I_{10} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{R_2}{R} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{U_0}{R_1} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{U_0}{R} \\ -\frac{R_2}{R_1} \frac{U_0}{R} \end{bmatrix} = \frac{U_0}{R} \begin{bmatrix} 1 \\ -1/k \end{bmatrix}.$$

Substituting \mathbf{I}_4 into (18) we obtain:

$$\mathbf{I}_3 = \begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix} = \frac{U_0}{R} \begin{bmatrix} R_2/R \\ -(R_2/R_1)^2 \\ R/R_1 \end{bmatrix}.$$

From (12)

$$\mathbf{I}_2 = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \frac{U_0}{R} \begin{bmatrix} -(R_2/R_1)^2 \\ -R_2/R - (R_2/R_1)^2 \\ -R_2/R \\ -R/R_1 \\ -R/R_1 \end{bmatrix}.$$

Substituting I_2 and I_3 into (11)

$$I_0 = I_{16} = \left(\frac{R_2}{R_1}\right)^2 \frac{U_0}{R} = \frac{1}{k^2} \frac{U_0}{R},$$

as it has been expected.

Summary

A calculation method for network models containing nullators and norators is described. The calculation is based on use of loop and cut-set matrices of the circuit. For the determination of voltages and currents a system of equations containing as many unknown quantities as there are twigs and links containing impedances, respectively, has to be solved.

References

1. DAVIES, A. C.: Nullator—norator equivalent networks for controlled sources. *Proc. of IEEE* 1967, pp. 722—723.
2. MITRA, S. K.: *Analysis and Synthesis of Linear Active Networks*. Wiley, New York, 1969.
3. VÁGÓ, I., HOLLÓS, É.: Two-port models with nullators and norators. *Periodica Polytechnica Electr. Eng.* 17, 301—309. (1973).
4. DAVIES, A. C.: Matrix analysis of networks containing nullators and norators. *Electronics Letters* 1966, Vol. 2, No. 2, pp. 48—49.

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