THE STATE EQUATION OF LINEAR NETWORKS CONTAINING TWO-PORTS AND COUPLED TWO-POLES

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In a former publication [1] it has been demonstrated that by characterizing two-ports of the network by the chain matrices, a homogeneous description can be given, valid also for degenerated two-ports. While there the calculation of the stationary state was considered, now the method will be employed for examinations in the time domain.

1. Stating the problem

Consider a linear time invariant network. Network parameters, the time function of source quantities and the initial ($t = +0$) or starting ($t = -0$) values are given. The time function of branch voltages and branch currents is required.

Our basic aim is to construct the state equation:

$$\dot{\mathbf{x}}(t) = \overline{\boldsymbol{A}}\mathbf{x}(t) + \overline{\boldsymbol{B}}\mathbf{z}(t), \tag{1}$$

where $\mathbf{x}$ is the column matrix of the state variables, $\mathbf{z}$ that of the source quantities, while $\overline{\boldsymbol{A}}$ and $\overline{\boldsymbol{B}}$ are matrices characterizing the network. Including the other variables into a matrix $\mathbf{y}$, they can be expressed in terms of the state variables and the excitations as:

$$\mathbf{y}(t) = \overline{\boldsymbol{C}}\mathbf{x}(t) + \overline{\boldsymbol{D}}\mathbf{z}(t). \tag{2}$$

The problem is, on the one hand, the selection of the state variables $\mathbf{x}$, on the other hand the elaboration of a systematic process for the determination of matrices $\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}}, \overline{\boldsymbol{C}}, \overline{\boldsymbol{D}}$ and of Eqs (1) and (2).

2. The network elements

The examined network may contain the following linear invariant elements (the symbols in brackets denote both the respective element and the number of the respective element; the meaning of the capitals will be clarified later).

a) Voltage sources ($v = V$)
b) Current sources ($a = A$)
c) Resistances ($r = R + G$)
d) Condensers ($c = C + D$)
e) Uncoupled or coupled inductors ($l = L + \Gamma$)
f) Resistive two-ports ($p = P + Q$, $s = S + T$, $p = s$).

The voltage sources are assumed not to form a loop, the current sources not to form a cut-set, further the two-ports are regarded two-ports with justification (they are not general four-poles).

The individual elements are characterized by the following equations:

$$u_{vk} = f_k(t), \qquad i_{ak} = g_k(t) \tag{3}$$

where f and g are given functions;

$$u_{rk} = R_k \, i_{rk} \quad \text{or} \quad i_{rk} = G_k \, u_{rk} \tag{4}$$

where $G_k = 1/R_k$;

$$i_{ck} = C_k \, \dot{u}_{ck} \tag{5}$$

$$u_{lk} = L_k \, \dot{i}_{lk} \quad \text{and} \quad u_{lk} = \sum_j L_{kj} \, \dot{i}_{lj} \tag{6}$$

where $L_{kj} = L_{jk}$;

$$\begin{aligned} u_{pk} &= H_{kj} \, u_{sj} + K_{kj} \, i_{sj}, \\ i_{pk} &= M_{kj} \, u_{sj} + N_{kj} \, i_{sj}, \end{aligned} \tag{7}$$

where p indicates the primary port of the two-port, while s the secondary port. (For details see [1].) Parameters denoted by a capital letter are time-independent constants.

3. The state variables

As known, state variables of linear invariant networks are very simple to choose, if there are no capacitive ($c + v$) loops and inductive ($l + a$) cut-sets. In this case the state variables are the voltage of condensers and the current of inductors.

In a more general case there are two possibilities. On the one hand, independent condenser voltages and independent inductor currents can be regarded as state variables. This means that in each independent capacitive loop the voltage of one of the condensers, and in each independent inductive cut-set the current of one of the inductors is not regarded as state variable. Let C and D denote those condensers, the voltage of which is, or is not a state variable, respectively, and similarly L and Γ those coils, the current of which is, or is not a state variable, respectively ($c = C + D$, $l = L + \Gamma$). In this case a proper tree can be chosen in the graph of the network in such a way that all C and Γ branches are twigs, and all D and L branches are links.

On the other hand, the charge of the cut-sets generated by twigs C and the flux at loops generated by links L, or the voltages and currents, respectively, proportional to these, can also be chosen as state variables.

The first mentioned method is simpler, though it has the disadvantage that the continuity of state variables is not ensured if the time-function of the excitations contains jumps.

For the sake of comprehensibility, the case without capacitive loops or inductive cut-sets will be examined first, thereafter the more general case.

4. The "regular" network

The *Kirchhoff* laws for voltages and currents are:

$$\boldsymbol{B}\mathbf{u} = \mathbf{0}\,, \quad \underset{\sim}{\boldsymbol{Q}}\mathbf{i} = \mathbf{0} \tag{8}$$

where $\boldsymbol{B}$ denotes the loop matrix and $\underset{\sim}{\boldsymbol{Q}}$ the cut-set matrix. If the fundamental loop and cut-set system generated by a tree is chosen, then, as known,

$$\boldsymbol{B} = [\boldsymbol{F} \;\; \mathit{1}]\,, \qquad \underset{\sim}{\boldsymbol{Q}} = [\mathit{1} \;\; \boldsymbol{E}]\,,\ \boldsymbol{E} = -\boldsymbol{F}^{+} \tag{9}$$

where $\mathit{1}$ is the unit matrix and $^{+}$ denotes the transpose.

Let us first examine the case where there are neither capacitive loops, nor inductive cut-sets in the network. Choose a tree in the following way. All v and c branches are twigs ($v = V$, $c = C$, $D = 0$), all a and l branches are links ($a = A$, $l = L$, $\Gamma = 0$), what is possible, according to the condition. We should select maximal number of r branches as twigs (R-branches) and minimal number as links (G-branches). Among the p-branches (primary ports), in turn, possibly minimal number should be chosen as twigs (P-branches) and maximal number as links (Q-branches). Among the s-branches (secondary ports) those designed as twigs are denoted by S, those designed as links by T.

Accordingly, the form of matrix $\boldsymbol{F}$ interpreted in (9), partitioned to blocks is found to be

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{F}_{AV} & \boldsymbol{F}_{AC} & \boldsymbol{F}_{AR} & \boldsymbol{F}_{AS} & \boldsymbol{F}_{AP} \\ \boldsymbol{F}_{LV} & \boldsymbol{F}_{LC} & \boldsymbol{F}_{LR} & \boldsymbol{F}_{LS} & \boldsymbol{F}_{LP} \\ \boldsymbol{F}_{QV} & \boldsymbol{F}_{QC} & \boldsymbol{F}_{QR} & \boldsymbol{F}_{QS} & \boldsymbol{F}_{QP} \\ \boldsymbol{F}_{TV} & \boldsymbol{F}_{TC} & \boldsymbol{F}_{TR} & \boldsymbol{F}_{TS} & \boldsymbol{F}_{TP} \\ \boldsymbol{F}_{GV} & \boldsymbol{F}_{GC} & \boldsymbol{F}_{GR} & \boldsymbol{F}_{GS} & \boldsymbol{F}_{GP} \end{bmatrix} \begin{matrix} A \\ L \\ Q \\ T \\ G \end{matrix} \Bigg\} \text{loops} \qquad \begin{matrix} V & O & R & S & P \leftarrow \text{cut-sets} \end{matrix} \tag{10}$$

The structure of matrix $\boldsymbol{E} = -\boldsymbol{F}^{+}$ already follows from this ($\boldsymbol{E}_{VA} = -\boldsymbol{F}^{+}_{AV}$, etc.).

It follows from the structure of the tree that there are no S- and P-twigs in the loops generated by the G-branches (resistive links), and no T-links in the cut-sets generated by the P-branches (primary twigs), accordingly

$$\begin{aligned} \boldsymbol{F}_{GS} = 0, \quad \boldsymbol{F}_{GP} = 0, \quad \boldsymbol{F}_{TP} = 0\ , \\ \boldsymbol{E}_{SG} = 0, \quad \boldsymbol{E}_{PG} = 0, \quad \boldsymbol{E}_{PT} = 0\ . \end{aligned} \tag{11}$$

The first group of *Kirchhoff* laws, on the basis of A-loops and V-cut-sets is found to be

$$\begin{aligned} \mathbf{u}_A &= -(\boldsymbol{F}_{AV}\,\mathbf{u}_V + \boldsymbol{F}_{AC}\,\mathbf{u}_C + \boldsymbol{F}_{AR}\,\mathbf{u}_R + \boldsymbol{F}_{AS}\,\mathbf{u}_S + \boldsymbol{F}_{AP}\,\mathbf{u}_P)\,, \\ \mathbf{i}_V &= -(\boldsymbol{E}_{VA}\,\mathbf{i}_A + \boldsymbol{E}_{VL}\,\mathbf{i}_L + \boldsymbol{E}_{VQ}\,\mathbf{i}_Q + \boldsymbol{E}_{VT}\,\mathbf{i}_T + \boldsymbol{E}_{VG}\,\mathbf{i}_G)\,. \end{aligned} \tag{12}$$

These can be calculated in knowledge of the quantities in the right-hand side. The second group of *Kirchhoff* laws is given by the loops Q, T, and G, and cut-sets R, S, and P. Taking also (11) into consideration,

$$\begin{aligned} \mathbf{u}_Q + \boldsymbol{F}_{QR}\,\mathbf{u}_R + \boldsymbol{F}_{QS}\,\mathbf{u}_S + \boldsymbol{F}_{QP}\,\mathbf{u}_P &= -(\boldsymbol{F}_{QV}\,\mathbf{u}_V + \boldsymbol{F}_{QC}\,\mathbf{u}_C)\,, \\ \mathbf{u}_T + \boldsymbol{F}_{TR}\,\mathbf{u}_R + \boldsymbol{F}_{TS}\,\mathbf{u}_S &= -(\boldsymbol{F}_{TV}\,\mathbf{u}_V + \boldsymbol{F}_{TC}\,\mathbf{u}_C)\,, \\ \mathbf{u}_G + \boldsymbol{F}_{GR}\,\mathbf{u}_R &= -(\boldsymbol{F}_{GV}\,\mathbf{u}_V + \boldsymbol{F}_{GC}\,\mathbf{u}_C)\,, \\ \mathbf{i}_R + \boldsymbol{E}_{RQ}\,\mathbf{i}_Q + \boldsymbol{E}_{RT}\,\mathbf{i}_T + \boldsymbol{E}_{RG}\,\mathbf{i}_G &= -(\boldsymbol{E}_{RA}\,\mathbf{i}_A + \boldsymbol{E}_{RL}\,\mathbf{i}_L)\,, \\ \mathbf{i}_S + \boldsymbol{E}_{SQ}\,\mathbf{i}_Q + \boldsymbol{E}_{ST}\,\mathbf{i}_T &= -(\boldsymbol{E}_{SA}\,\mathbf{i}_A + \boldsymbol{E}_{SL}\,\mathbf{i}_L)\,, \\ \mathbf{i}_P + \boldsymbol{E}_{PQ}\,\mathbf{i}_Q &= -(\boldsymbol{E}_{PA}\,\mathbf{i}_A + \boldsymbol{E}_{PL}\,\mathbf{i}_L)\,. \end{aligned} \tag{13}$$

Branch rules (4) and (7) can be written in the following form:

$$\mathbf{u}_R = \boldsymbol{R}\mathbf{i}_R\,, \qquad \mathbf{i}_G = \boldsymbol{G}\mathbf{u}_G\,, \tag{14a}$$

$$\begin{bmatrix} \mathbf{u}_P \\ \mathbf{u}_Q \\ \mathbf{i}_P \\ \mathbf{i}_Q \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{PS} & \boldsymbol{H}_{PT} & \boldsymbol{K}_{PS} & \boldsymbol{K}_{PT} \\ \boldsymbol{H}_{QS} & \boldsymbol{H}_{QT} & \boldsymbol{K}_{QS} & \boldsymbol{K}_{QT} \\ \boldsymbol{M}_{PS} & \boldsymbol{M}_{PT} & \boldsymbol{N}_{PS} & \boldsymbol{N}_{PT} \\ \boldsymbol{M}_{QS} & \boldsymbol{M}_{QT} & \boldsymbol{N}_{QS} & \boldsymbol{N}_{QT} \end{bmatrix} \begin{bmatrix} \mathbf{u}_S \\ \mathbf{u}_T \\ \mathbf{i}_S \\ \mathbf{i}_T \end{bmatrix}. \tag{14b}$$

Matrices $\boldsymbol{R}$ and $\boldsymbol{G}$ are diagonal, while blocks $\boldsymbol{H}, \boldsymbol{K}, \boldsymbol{M}$, and $\boldsymbol{N}$ contain only one non-zero element in each row and column.

Upon substituting Eqs (14) into (13) we obtain $(r + p + s)$ equations for the same number of unknown quatities $\mathbf{i}_R, \mathbf{u}_G, \mathbf{u}_S, \mathbf{u}_T, \mathbf{i}_S, \mathbf{i}_T$, thus these can be expressed by means of the given excitations $(\mathbf{u}_V, \mathbf{i}_A)$ and state variables $(\mathbf{u}_C, \mathbf{i}_L)$. These linear functions, further the linear functions which can be produced by substituting the former into (12), represent the linear function given under (2).

The equations on loops L and cut-sets C are

$$\begin{aligned} \mathbf{u}_L &= -(\boldsymbol{F}_{LV}\,\mathbf{u}_V + \boldsymbol{F}_{LC}\,\mathbf{u}_C + \boldsymbol{F}_{LR}\,\mathbf{u}_R + \boldsymbol{F}_{LS}\,\mathbf{u}_S + \boldsymbol{F}_{LP}\,\mathbf{u}_P)\,, \\ \mathbf{i}_C &= -(\boldsymbol{E}_{CA}\,\mathbf{i}_A + \boldsymbol{E}_{CL}\,\mathbf{i}_L + \boldsymbol{E}_{CQ}\,\mathbf{i}_Q + \boldsymbol{E}_{CT}\,\mathbf{i}_T + \boldsymbol{E}_{CG}\,\mathbf{i}_G)\,. \end{aligned} \tag{15}$$

The branch rules (5) and (6) are

$$\mathbf{u}_L = \boldsymbol{L}\dot{\mathbf{i}}_L, \quad \mathbf{i}_C = \boldsymbol{C}\dot{\mathbf{u}}_C, \tag{16}$$

where $\boldsymbol{C}$ is diagonal, but $\boldsymbol{L}$ is diagonal only in the case where there are no coupled inductors. If $\boldsymbol{L}$ is not singular then on the basis of (16) and (15)

$$\begin{aligned} \dot{\mathbf{i}}_L &= -\boldsymbol{L}^{-1}(\boldsymbol{F}_{LV}\,\mathbf{u}_V + \boldsymbol{F}_{LC}\,\mathbf{u}_C + \boldsymbol{F}_{LR}\,\mathbf{u}_R + \boldsymbol{F}_{LS}\,\mathbf{u}_S + \boldsymbol{F}_{LP}\,\mathbf{u}_P), \\ \dot{\mathbf{u}}_C &= -\boldsymbol{C}^{-1}(\boldsymbol{E}_{CA}\,\mathbf{i}_A + \boldsymbol{E}_{CL}\,\mathbf{i}_L + \boldsymbol{E}_{CQ}\,\mathbf{i}_Q + \boldsymbol{E}_{CT}\,\mathbf{i}_T + \boldsymbol{E}_{CG}\,\mathbf{i}_G). \end{aligned} \tag{17}$$

Substituting the linear functions obtained by solving (13) and (14), state equation (1) has been produced.

5. The generalized network (1st method)

Let us now consider the more general case, namely where the network can contain both capacitive loops and inductive cut-sets.

In choosing the tree, all v-branches are chosen as twigs, all a-branches as links ($v = V$, $a = A$). Hereafter a maximum number of c-branches are chosen as twigs (C-branches), while the rest become links (D-branches). Similarly a maximum number of l-branches are chosen as links (L-branches), while the rest become twigs (Γ-branches). The same is done for r, p and s branches. For the sake of understanding, the symbols of branches are tabulated.

Table 1

Branch	Twig	Link	Equation
v	V	—	$u_V = f(t)$
a	—	A	$i_A = g(t)$
c	C	D	$\begin{cases} i_C = C_{CC}\,\dot{u}_C \\ i_D = C_{DD}\,\dot{u}_D \end{cases}$
l	L	Γ	$\begin{cases} u_L = L_{LL}\,\dot{i}_L + L_{L\Gamma}\,\dot{i}_\Gamma \\ u_\Gamma = L_{\Gamma L}\,\dot{i}_L + L_{\Gamma\Gamma}\,\dot{i}_\Gamma \end{cases}$
r	R	G	(14a)
p	P	Q	(14b)
s	S	T	(14b)

A systematic method of selecting the maximum number of twigs is the following. Consider the partial graphs v, $v + c$, $v + c + r$ one by one. (The other branches are substituted by open circuits.) Branches considered previously as twigs are completed to a forest, thus we obtain branches C and R, while the remaining branches become branches D and G, respectively. The maximum number of links can be formed analogously by examining partial graphs a, $a + l$, $a + l + p$ (but in this case the other branches are to be substituted by a short-circuit).

The structure of matrix $\boldsymbol{F}$ (upon immediately considering the zero blocks) is now the following:

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{F}_{AV} & \boldsymbol{F}_{AC} & \boldsymbol{F}_{AR} & \boldsymbol{F}_{AS} & \boldsymbol{F}_{AP} & \boldsymbol{F}_{A\Gamma} \\ \boldsymbol{F}_{LV} & \boldsymbol{F}_{LC} & \boldsymbol{F}_{LR} & \boldsymbol{F}_{LS} & \boldsymbol{F}_{LP} & \boldsymbol{F}_{L\Gamma} \\ \boldsymbol{F}_{QV} & \boldsymbol{F}_{QC} & \boldsymbol{F}_{QR} & \boldsymbol{F}_{QS} & \boldsymbol{F}_{QP} & \boldsymbol{0} \\ \boldsymbol{F}_{TV} & \boldsymbol{F}_{TC} & \boldsymbol{F}_{TR} & \boldsymbol{F}_{TS} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{F}_{GV} & \boldsymbol{F}_{GC} & \boldsymbol{F}_{GR} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{F}_{DV} & \boldsymbol{F}_{DC} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}. \tag{18}$$

Choose now voltages $\mathbf{u}_C$ and currents $\mathbf{i}_L$ as state variables. The construction of the state equation can be performed in the same way as previously. On the basis of loops A and cut-sets V, $\mathbf{u}_A$ and $\mathbf{i}_V$ can be expressed in terms of the other variables. On the basis of loops Q, T, and G, further of cut-sets R, S, and P, as well as of Eqs (14) the voltages and currents with the indicated subscripts can be expressed by the state variables and excitations. On the basis of loops D and cut-sets Γ, $\mathbf{u}_D$ and $\mathbf{i}_\Gamma$ can be expressed by the state variables and excitations, and so can $\mathbf{i}_D$ and $\mathbf{u}_\Gamma$ by means of relationships given in the table. Finally, on the basis of loops L and cut-sets C, and eliminating the previously expressed variables we obtain the state equation.

6. The generalized network (2nd method)

Let us choose the tree and the individual branches in the manner described previously. Let the state variables be

$$\begin{aligned} \mathbf{u} &\triangleq \mathbf{u}_C + \boldsymbol{C}_{CC}^{-1} \boldsymbol{E}_{CD} \boldsymbol{C}_{DD} \mathbf{u}_D , \\ \mathbf{i} &\triangleq \left(\boldsymbol{1} + \boldsymbol{L}_{LL}^{-1} \boldsymbol{F}_{L\Gamma} \boldsymbol{L}_{\Gamma L}\right) \mathbf{i}_L + \boldsymbol{L}_{LL}^{-1} + \left(\boldsymbol{L}_{L\Gamma} + \boldsymbol{F}_{L\Gamma} \boldsymbol{L}_{\Gamma\Gamma}\right) \mathbf{i}_\Gamma . \end{aligned} \tag{19}$$

(We have assumed that $\boldsymbol{L}_{LL}$ is not singular.) It can be easily conceived that $\boldsymbol{C}_{CC}\mathbf{u}$ is the total charge of cut-sets generated by branches C, and $\boldsymbol{L}_{LL}\mathbf{i}$ is the total flux of loops generated by branches L. Cut-set charge and loop flux are continuous in the case of bounded excitation, thus $\mathbf{u}$ and $\mathbf{i}$ are themselves continuous.

The equations on loops D and cut-sets Γ are

$$\begin{aligned} \mathbf{u}_D + \boldsymbol{F}_{DV} \mathbf{u}_V + \boldsymbol{F}_{DC} \mathbf{u}_C &= 0 \\ \mathbf{i}_\Gamma + \boldsymbol{E}_{\Gamma A} \mathbf{i}_A + \boldsymbol{E}_{\Gamma L} \mathbf{i}_L &= 0 . \end{aligned} \tag{20}$$

Substituting the expressions for $\mathbf{u}_C$ and $\mathbf{i}_L$ from (19), we have:

$$\begin{aligned} \mathbf{u}_D &= -\left(\mathbf{1} - \boldsymbol{F}_{DC}\,\boldsymbol{C}_{CC}^{-1}\,\boldsymbol{E}_{CD}\,\boldsymbol{C}_{DD}\right)^{-1} \cdot \left(\boldsymbol{F}_{DV}\,\mathbf{u}_V + \boldsymbol{F}_{DC}\,\mathbf{u}\right), \\ \mathbf{i}_\Gamma &= -\left(\mathbf{1} - \overline{\boldsymbol{E}}_{\Gamma L}\,\boldsymbol{L}_{LL}^{-1}\,(\boldsymbol{L}_{L\Gamma} + \boldsymbol{F}_{L\Gamma}\,\boldsymbol{L}_{\Gamma\Gamma})\right)^{-1} \cdot \left(\boldsymbol{E}_{\Gamma A}\,\mathbf{i}_A + \overline{\boldsymbol{E}}_{L\Gamma}\,\mathbf{i}\right) \end{aligned} \tag{21}$$

where

$$\overline{\boldsymbol{E}}_{\Gamma L} \triangleq \boldsymbol{E}_{\Gamma L}\left(\mathbf{1} + \boldsymbol{L}_{LL}^{-1}\,\boldsymbol{F}_{L\Gamma}\,\boldsymbol{L}_{\Gamma L}\right)^{-1}. \tag{22}$$

(If there are no coupled coiles, then $\overline{\boldsymbol{E}}_{\Gamma L} = \boldsymbol{E}_{\Gamma L}$ and $\boldsymbol{L}_{L\Gamma} = \boldsymbol{L}_{\Gamma L} = \boldsymbol{0}$.) Substituting variables $\mathbf{u}_D$ and $\mathbf{i}_\Gamma$ from (20) into (19), we obtain:

$$\begin{aligned} \mathbf{u}_C &= \left(\mathbf{1} - \overline{\boldsymbol{E}}_{CD}\,\boldsymbol{F}_{DC}\right)^{-1} \cdot \left(\overline{\boldsymbol{E}}_{CD}\,\boldsymbol{F}_{DV}\,\mathbf{u}_V + \mathbf{u}\right), \\ \mathbf{i}_L &= \left(\mathbf{1} + \boldsymbol{L}_{LL}^{-1}\,\boldsymbol{F}_{L\Gamma}\,\boldsymbol{L}_{\Gamma L} - \overline{\boldsymbol{F}}_{L\Gamma}\,\overline{\boldsymbol{E}}_{\Gamma L}\right)^{-1} \cdot \left(\overline{\boldsymbol{F}}_{L\Gamma}\,\boldsymbol{E}_{\Gamma A}\,\mathbf{i}_A + \mathbf{i}\right) \end{aligned} \tag{23}$$

where

$$\overline{\boldsymbol{E}}_{CD} \triangleq \boldsymbol{C}_{CC}^{-1}\,\boldsymbol{E}_{CD}\,\boldsymbol{C}_{DD}, \quad \overline{\boldsymbol{F}}_{L\Gamma} = \boldsymbol{L}_{LL}^{-1}\,\boldsymbol{L}_{L\Gamma} + \boldsymbol{F}_{L\Gamma}\,\boldsymbol{L}_{\Gamma\Gamma}. \tag{24}$$

The state equation can be produced in the way described under item 5, since $\mathbf{u}_C$ and $\mathbf{i}_L$ are expressed by the state variables and excitations according to (23).

In writing the state equation, the terms $\mathbf{u}_L + \boldsymbol{F}_{L\Gamma}\,\mathbf{u}_\Gamma$ and $\mathbf{i}_C + \boldsymbol{E}_{CD}\,\mathbf{i}_D$ occur in the equations pertaining to loops L and cut-sets C, respectively. These can be expressed simply in terms of the state variables, according to (19) in the forms $\boldsymbol{L}_{LL}\,\dot{\mathbf{i}}$ and $\boldsymbol{C}_{CC}\,\dot{\mathbf{u}}$, respectively. Accordingly, the derivatives of excitations are not figuring here and just this ensures that $\mathbf{u}(t)$ and $\mathbf{i}(t)$ are continuous in the case of bounded excitation. With the 1st method this is not ensured.

7. On the solvability of the equations

For the sake of completeness it should be mentioned that our process for choosing the state variables and for constructing the state equations takes only the topology of the network into consideration. In the case of degenerate two-ports, however, it may happen that the independence of the designated state variables and the solvability of equations (the invertibility of matrices) is not ensured.

A problem may arise if both a primary and a secondary quantity of some of the two-ports are state variables, and the two-port itself is degenerate. It is easy to prove that if some of the two-port parameters are zero, then some termination pairs are not permitted, in this case one of the state variables

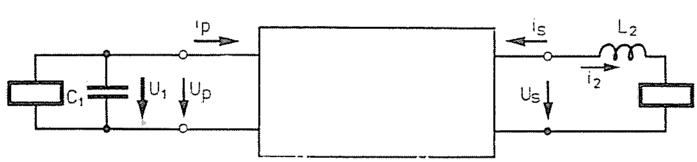

Fig. 1

would determine the other, hence only one of them can be regarded as state variable. The *forbidden* terminations are the following (Fig. 1):

Parameter	Primary	Secondary	Relationship
$H = 0$	C_1	L_2	$u_1 = -Ki_2$
$K = 0$	C_1	C_2	$u_1 = Hu_2$
$M = 0$	L_1	L_2	$i_1 = Ni_2$
$N = 0$	L_1	C_2	$i_1 = -Mu_2$

In the following the *permitted* termination pairs are summarized for the typical degenerate two-ports:

Two-port	Primary	Secondary
Nullor ($H = 0,\ K = 0,\ M = 0,\ N = 0$)	—	—
Voltage controlled voltage source ($H \neq 0$)	C	L
Voltage controlled current source ($K \neq 0$)	C	C
Current controlled voltage source ($M \neq 0$)	L	L
Current controlled current source ($N \neq 0$)	L	C
Ideal transformer } ($H \neq 0,\ N \neq 0$)	L	C
Negative converter } ($H \neq 0,\ N \neq 0$)	C	L
Gyrator ($K \neq 0,\ M \neq 0$)	L	L
	C	C

The construction of the state equation by the described method is ensured only if the given conditions are satisfied.

Summary

A method is given for the systematic construction of the state equation of linear networks containing sources, condensers, uncoupled and coupled inductors, resistors, and resistive two-ports. The two-ports can be degenerate too. The cases where there are, or are not capacitive loops and inductive cut-sets in the network, are examined separately.

Reference

1. FODOR, G.: The Analysis of Linear Networks Containing Two-ports and Coupled Two-poles· Periodica Polytechnica, Electr. Eng., **17**, 321—332 (1973).

Further references see there.

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