THERMAL BEHAVIOUR OF CLOSED ROOMS EXPOSED TO ARBITRARILY VARYING OUTER AND INNER HEAT LOADS

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1. General

Development of the building industry, introduction of up-to-date building systems of new materials and structures necessitated the study of the building as a whole under dynamic, variable thermal conditions; namely, economically favourable lightweight structures prohibit neglection of instationary thermal flux, though mostly allowed for traditional structures.

Attempts have been made to find laws describing the intricate interactions between structures by introducing thermal characteristics (e.g. winter and summer temperature moduli), these relationships are, however, to be considered as approximative only. Technical and economical considerations call for an expression reckoning with the interaction of structures through closed spaces. Thermal design of buildings, both as to structure and heat demand, is based on empirical prescriptions (of indistinct range of validity). Obviously, however, structures and installations designed for these demands cannot function optimally, since starting data needed for an optimum design are a priori unsatisfactory.

2. The aim of the research

These difficulties required the establishment of a mathematical model likely to describe dynamic thermal behaviour of a building of N rooms, i.e. to determine temperature variation of inner spaces due to an arbitrary outside temperature variation and to energy volumes entering or disengaged in closed spaces, varying according to some time function (for given wall structures) to an accuracy of 10 to 15 per cent. Besides, this model should be convenient for the thermal design of

industrial workshop halls

agricultural buildings:

  buildings for animal keeping

cold storage rooms and icehouses
hothouses
communal establishments:
cultural and sports halls,
commercial establishments,
welfare buildings
living and weekend houses

as well as to select thermally optimum structures and room arrangements and to create economically optimum operating conditions — taking orientation, inner and outer heat loads, character of wall surfaces such as large window panes, lightweight structures and building geometry into consideration.

This quite general objective has two main trends:

2.1 to find the mathematical model describing thermal behaviour of N interconnected rooms under the indicated conditions; 2.2 to apply the model for the listed establishments.

Obviously, if the mathematical model 2.1 has been found and experimentally checked for correctness, then it can be applied to establishments, provided the involved physical assumptions are general enough to be valid for them.

In the light of the above said, establishment of the mathematical model consists of the following steps:

a) to make physical assumptions valid for any building;

b) to find partial differential equations corresponding to the physical assumptions and their solution methods;

c) to find a correction method for errors resulting from physical assumptions and solutions of the differential equation systems;

d) finally, to check experimentally outcomes of the mathematical model on real buildings by constructing the physical-mathematical model of the given building, and confronting results from the partial differential equation system established for recorded outer temperatures and inner heat sources — i.e., inner air temperatures and inner wall temperatures — to real values obtained in the corresponding space.

If the outcome of the mathematical model and our measurements differ by less than the estimated and calculated error, then the physical-mathematical method can be considered to represent truly the real building and permits exact investigation into the thermal processes indicated as objective.

3. Physical assumptions made for the mathematical method

Outer and inner wall and floor structures confining the closed spaces are assumed

3.1 to consist of severa layers, homogeneous in themselves, with different physical constants. If these layers are not homogeneous, they will be substituted by homogeneous layers with average physical parameters ($\bar{\lambda}$, $\bar{\varrho}$, $\bar{c}$);

3.2 their outer and inner surfaces are assumed to follow the Newtionan heat transfer law, and the heat transfer coefficient is assumed to be a function of the temperature and of the angle;

3.3 heat spreads in them by linear conduction;

3.4 there are heat sources on their surfaces (for accounting for absorbed energies from heating, sunshine).

The following assumptions are made to incident sunshine, heating and cooling (arbitrary functions of time):

3.5 the diffuse radiation is assumed to be absorbed on the confining structure surfaces according to the ratio of absorptivities A_i;

3.6 the direct radiation is assumed to be absorbed by the irradiated surface to a given percentage depending on A_i;

3.7 the heating by convection is assumed to transmit all its heat to the air;

3.8 the radiant heating is assumed to transmit a proportion A_i of its heat to the irradiated surface, while the reflected energy is distributed between confining structures according to their A_i ratios;

3.9 the cooling is assumed to absorb heat from the air;

3.10 as to other heating types it is assumed — provided there is also an important contribution of radiant energy — that the transmitted heat can be divided into two parts: radiating heat absorbed by the confining structure surfaces, and conveyed heat transmitted to air;

3.11 heat produced by technical equipment, animals etc., is assumed to be transmitted to air;

3.12 doors and windows are assumed to be of zero heat capacity, and can be replaced by a temperature-dependent thermal resistance;

3.13 infiltration to rooms through doors and windows is assumed with the internationally accepted values;

3.14 any point of the inner space is assumed to be at the same temperature at any instant;

3.15 outer space is assumed to be at a temperature arbitrarily depending on time, just as sunshine-induced heating of outer confining structures follows an arbitrary time function.

4. Mathematical model of the closed space realized according to the quoted physical assumptions

4.1. According to the *physical model*, for all layers of the i-th wall structure a temperature function $t_{i\beta}(x, \tau)$ is interpreted, meeting the differential equation

$$\frac{1}{\varrho_{i\beta}\, c_{i\beta}} \operatorname{div} \lambda_{i\beta} \operatorname{grad} t_{i\beta} + \frac{f_{i\beta}}{\varrho_{i\beta}\, c_{i\beta}} = \frac{\partial t_{i\beta}}{\partial \tau} \tag{1}$$

Besides, the function $t_{i\beta}$ meets the functions

$$t_{i\beta}\Big|_{\beta}^{+} = t_{i\beta+1}\Big|_{\beta+1}^{-} \tag{2}$$

$$\lambda_{i\beta} \frac{\partial t_{i\beta}}{\partial x}\Big|_{\beta}^{+} = \lambda_{i\beta+1} \frac{\partial t_{i\beta+1}}{\partial x}\Big|_{\beta}^{-} \tag{3}$$

at the boundary of layers β and $\beta + 1$; the function

$$\lambda_{i1} \frac{\partial t_{i1}}{\partial x} = \alpha_{i\beta}(t_{i1} - t_{\beta,n} \tag{4}$$

at the inner wall plane of the i-th wall structure; and function

$$\lambda_{im} \frac{\partial t_{im}}{\partial x} = \alpha_{ik}(t'_k - t_{im}) \tag{5}$$

at the outer wall plane of the i-th wall structure, where $c_{i\beta}$; $\varrho_{i\beta}$; $\lambda_{i\beta}$; $f_{i\beta}$; α_{ik}; $\alpha_{i\beta}$ are specific heat, density, thermal conductivity, source function as well as outer and inner heat transfer coefficients of the β-th layer of the wall structure, resp.

Thermal behaviour of transparent doors and windows of two panes spaced at δ affected by sunshine is expressed by the following equations for the Ω-th window:

$$\varrho_a c_a d_a \frac{dt_\Omega^{1N}}{d\tau} = \alpha_{\Omega\beta}(t)\left(t_\beta^N(\tau) - t_\Omega^{1N}(\tau)\right) + \varepsilon_0 \frac{\lambda}{\delta}\left(t_\Omega^{2N}(\tau) - t_\Omega^{1N}(\tau)\right) + \\ + A_n c_o b\left(t_\Omega^{2N}(\tau) - t_\Omega^{1N}(\tau)\right) + J_{\Omega\beta}(\tau)\, A_1 \tag{6}$$

$$\varrho_a c_a d_a \frac{dt_\Omega^{2N}}{d\tau} = \alpha_{\Omega k}(t)\left(t_k(\tau) - t_\Omega^{2N}(\tau)\right) + \varepsilon_0 \frac{\lambda}{\delta}\left(t_\Omega^{1N}(\tau) - t_\Omega^{2N}(\tau)\right) + \\ + A_n c_o b\left(t_\Omega^{2N}(\tau) - t_\Omega^{1N}(\tau)\right) + J_{\Omega k}(\tau)\, A_2 \tag{7}$$

where ϱ_a; c_a; d_a are density, specific heat and thickness of glass, $\alpha_{\Omega k}$; $\alpha_{\Omega\beta}$ are inner and outer heat transfer coefficients, t_Ω^{2N}; t_Ω^{1N} are temperatures of the outer and inner window pane, resp., ε_0 is the flow coefficient, and λ the thermal conductivity of air, A_0 is the reduced absorptivity of the two window surfaces, C_0 the emissivity of black body, A_i and A_z are absorptivities of each glass surfaces $I_{\Omega,k}(\tau)$ and $I_{\Omega,\beta}(\tau)$ being energy flux density affecting the window panes, and

$$b = \frac{\left(\frac{T_\Omega^{1N}}{100}\right)^4 - \left(\frac{T_\Omega^{2N}}{100}\right)^4}{T_\Omega^{1N} - T_\Omega^{2N}}$$

4.2. For a *mathematical solution*, the differential equation of the heat balance for inner spaces connecting equations (1), (6) and (7), the thermal resistivities representing doors and windows as well as internal sources are needed.

Heat balance of the N-th closed room is

$$\frac{dt_\beta^N}{d\tau} = \frac{1}{V^N \varrho c}\left[\sum_{i=1}^{k} F_{in}^N \alpha_{i\beta}(t)\left(t_{i,1}(\tau) - t_\beta^N(\tau)\right) + \sum_{\Omega=1}^{Z} F_\Omega^N \alpha_{\Omega,\beta}(t)\left(t_\Omega^{1N}(\tau) - t_\beta^N(\tau)\right) + \sum_{\gamma=1}^{Y} \frac{t_\beta^N(\tau) - t_\gamma(\tau)}{R_\gamma^{a,N}} + V_N f^N(\tau)\right] \tag{8}$$

where t_β^N is the air temperature in the N-th closed room, V^n is its volume, c the specific heat of its air, ϱ the air density, F_i^N surface of the i-th $t_{i,1}$ structure confining the N-th closed room, K, Y being the number of structures, doors, windows confining the N-th closed room, t_γ the temperature of the ambience surrounding the γ-th door or window, eventually the adjacent closed room, F_Ω^N the surface of the glazed door or window confining the N-th closed room, Z being the number of glazed doors or windows, $f^N(\tau)$ being the source density in the room.

4.3. Partial differential equations (1) and (6) may be solved either by an *electric analog model* [1—5] or by computer technique if the involved equation systems are converted into an implicit difference equation system. The computer program has been established in ALGOL language for the computer Rasdan 3 of the Computing Centre of the Technical University, Budapest, the equation systems have been solved according to the Gauss—Jordan elimination method. The program has been elaborated by P. SZOMOR, senior research worker at the Crystal Growth Research Group of the Hungarian Academy of Sciences.

A fair agreement appeared between solutions by either method of the difference equation systems representing the model. From practicability aspects, however, the computer method is preferable because of its rapidity. (Running time of the entire program for one or two rooms is 4.2 and 7 minutes, respectively.)

5. Estimation of errors

The thermal problems of concern have been treated by solving a difference equation system based on the partial differential equation system describing the behaviour of the systems under test.

Accuracy of the result (in this case, evolution of room temperature under given conditions) is restricted by several sources of error.

Sources of error can be classified according to their nature into those causing *statistical* or *systematical errors.*

Statistical errors may occur in solving the differential equation system (method of finite differences), these being errors either of calculation or of measurement on an analog model.

Sources of error producing *systematic errors* may belong to either of three groups:

1. The first group includes errors inherent in the approximateness of the mathematical model written for physical assumptions made for the thermal system, with the differential equation system describing the thermal system. All but two of physical assumptions are trivial. One of them is that the heat flow through the walls is linear. This is not true for corners and for door and window edges; this can be reckoned with as an error of the model. The other is that of a homogeneous temperature distribution throughout the room resulting again in an error that can be taken into account.

2. The exact solution of the difference equation system approximating the differential equation system is only an approximation of this latter, producing an error depending on the form of the difference equation system, and even more, on the fineness of the scale divisions (x, y, z).

3. Systematic errors result from the faulty adjustment of parameters of the electric circuit applied as analogous model of the difference equation, as well as from systematic errors made at reading off electric quantities. (This source of error is of course absent in case of computer technique.)

The magnitude of errors is inherent in the differential equation system solution.

The mathematical model of a given thermal system is a partial differential equation system, its exact solution (or better, totality of solutions) will be denoted by D. Thus, the partial differential equation system will be called briefly D system.

Difference equation system approximating the D system will be called the Δ system, totality of its exact solutions will be indicated by the symbol Δ. The error δ of the solution equals the deviation between exact solution of system D and numeric solution N of system Δ:

$$\delta = N - D$$

The error can be divided into two parts:

$$\delta = N - D = (N - \Delta) + (\Delta - D)$$

First right-hand side term is the error of the numeric solution of system.

The second one is the deviation between exact solutions of systems Δ and D, exactly equal, for a convergent system Δ, to the division error δ_F.

Calculations [1, 4] may show that random errors of measurement or calculation can be minimized by carefully selecting the proportion between system and division, while division errors δ_F may be reduced to below 10 per cent.

Theoretically, division errors could be reduced arbitrarily by refining the divisions, this would, however, significantly increase the running time, regardless of the practical unimportance of thermal data obtained from finer divisions, since anyhow, physical constants of building materials are "stable" at an accuracy as low as 15 to 20 per cent.

Error is due to "corner effect". Namely, physical assumptions have considered the thermal flows as linear, this is, however, incorrect at corners, doors and windows. Corner effect is a secondary phenomenon to be exactly considered as a model-bound error. According to empirical observations, "corner effect" of two confining structures differing by temperature but similar in thickness and physical characteristics can be ignored since they mutually and equally affect the temperature of each other and leave the room temperature inaffected.

Corner effect of walls of different thickness made of different materials as well as effect of differential temperature around doors and windows can only be considered by developing a spatial model for the room part around the quoted structures. In the spatial model system, at a distance from corners, doors and windows, the wall temperature will differ from that produced by linear heat flow. Their difference will define the percentage of room temperature variation due to the corner effect. This error is not too great, it depends on the number and size of corners, doors and windows. The greater the wall area, the smaller is the error due to corner effect. Calculating with the poorest of practical cases, i.e.,o f living rooms, corner effects distort the room temperature by an error not over 3 per cent.

Error is due to the *assumption of homogeneous room temperature distribution,* as included in the physical assumptions. This assumption obviously fails. Inhomogeneity results from convection currents in rooms, depending on equipment, gaps around doors and windows, wall temperatures. Because of insufficient knowledge of current conditions, errors are assumed at empirically found temperature deviation values between closed rooms of different sizes, such as, for living rooms, ± 0.5 to 1 °C in summer and ± 1 to 1.5 °C in winter.

6. Experimental checking of the mathematical model

As for any theoretically treated problem, the question arises how the model developed under the given physical assumptions is related to reality. Or more exactly, whether there is a practical connection between time-dependent variation of model temperatures and field measurements or not. This problem can only be decided by confronting measurements on erected, eventually on occupied buildings and on models of the same buildings. Two field measurements have been made to check the mathematical model.

6.1. Instrumental checking and modelling classroom No.1, 8th floor, in Building E of the Budapest Technical University

Winter (February 7, 8 and 9, 1970) and summer (July 7, 8 and 9, 1970) thermometries lasted 40 and 76 hours, respectively.

Temperatures were recorded by means of thermopiles connected to a compensograph type BT 12 EN, of 2 mV total deflection, made in the G.D.R. Temperatures were recorded at outer wall surfaces, in the room atmosphere and outside the room at three spots each and averaged. (Values at the same side or in the air differred by max. 1 °C both in summer and in winter.)

Incident solar energy has been picked up at the inner wall surface by a radiometer made in the G.F.R. and recorded by a Multscript 3. Measurement data are presented in Figs 1 to 7.

6.2. Model of classroom No. 1, 8th floor, in Building E

In conformity with items 3 and 4, a complex mathematical model for walls, floors, doors and windows has been developed, taking material constants in Fig. 3, as well as an inner heat source, into consideration.

48 per cent and 52 per cent of radiation transmitted through the windows has been transferred to structure surfaces and to air, respectively. (Our measurements showed diffuse radiation to amount to 48 per cent of total radiation, and since direct radiation affected desks, these were heated and transferred their heat by convection to the air. Thermal capacity of desks equals zero as compared to that of the walls.)

Heat loss by infiltration through closed windows and doors has been assumed at $k = 2$ kcal/m^2 · h. °C. The k value of doors has been assumed to equal that of the windows.

The mathematical model established, as described previously, received outer winter temperatures (Fig. 4), heat source (Fig. 6) as well as outer summer temperatures (Fig. 5) and positive heat source (Fig. 7) as input, to output winter and summer inner wall surface and air temperatures of the classroom, see Figs 8, 9 and 10.

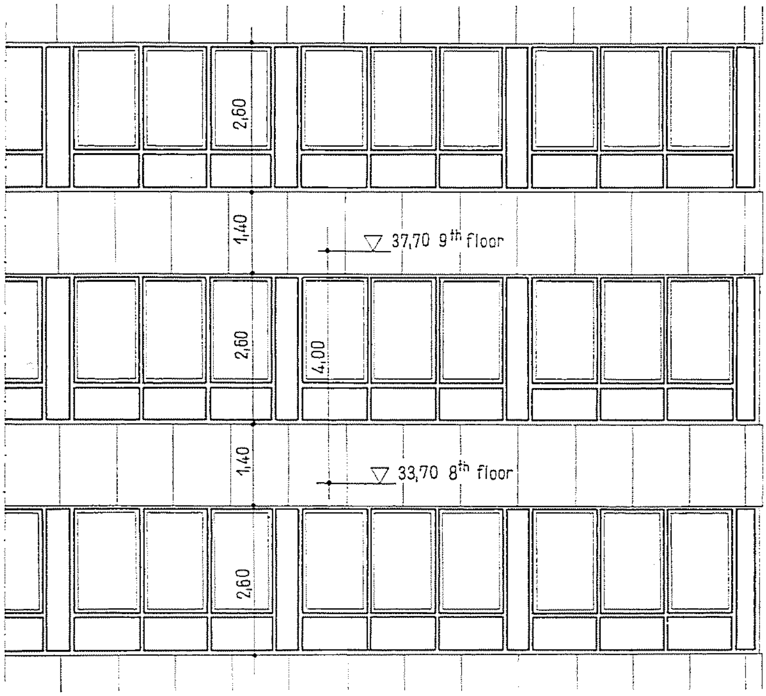


Fig. 1. Façade of Building *E*

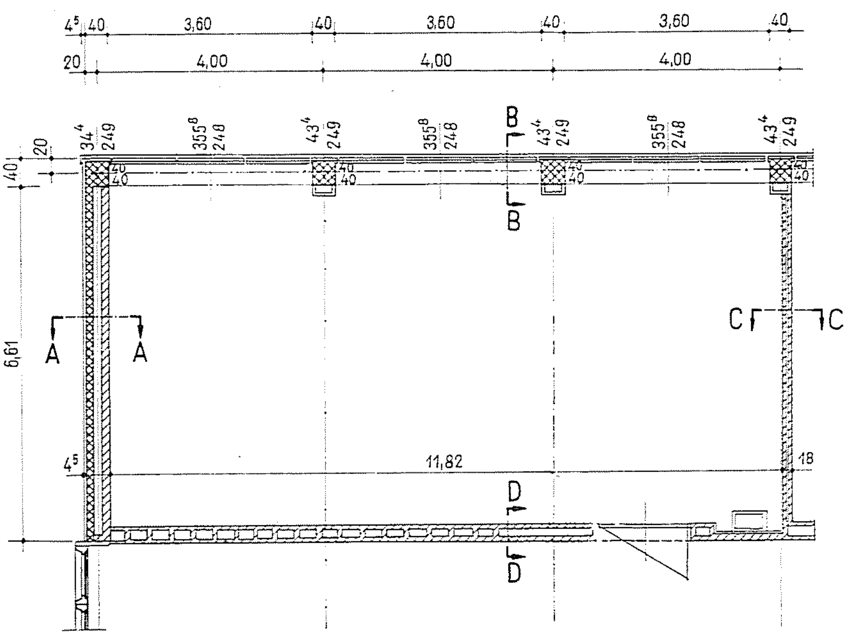
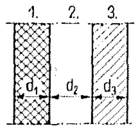


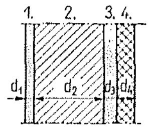
Fig. 2. Floor plan of Building *E*

Section A-A in Fig. 2



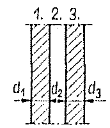
	1. R.C.	2. Air	3. Brick
λ [kcal/m·h·°C]	1,33	0,24	0,3
ρ [kg/m³]	2400	1,3	1100
c [kcal/kg·m]	0,2	0,24	0,21
d [m]	0,12	0,16	0,12

Section B-B in Fig. 2



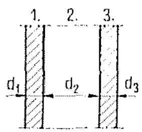
	1. Tuff concrete	3. Tuff concrete	2. Brick	4. R.C.
λ [kcal/m·h·°C]	0,7	0,7	0,3	1,75
ρ [kg/m³]	1600	1600	1100	2400
c [kcal/kg·m]	0,2	0,2	0,21	0,22
d [m]	0,02	0,04	0,25	0,06

Section C-C in Fig. 2



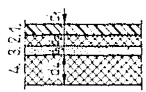
	1. 3. Brick	2. Air
λ [kcal/m·h·°C]	0,3	0,24
ρ [kg/m³]	1100	1,3
c [kcal/kg·m]	0,21	0,24
d [m]	0,06	0,06

Section D-D in Fig. 2



	1. 3. Brick	2. Air
λ [kcal/m·h·°C]	0,3	0,24
ρ [kg/m³]	1100	1,3
c [kcal/kg·m]	0,21	0,24
d [m]	0,06	0,20

Cross-section of floor structure



	1. P.V.C.	2. R.C.	3. Slag wool felt	4. R.C.
λ [kcal/m·h·°C]	0,2	1,3	0,045	1,3
ρ [kg/m³]	200	2400	250	2400
c [kcal/kg·m]	0,3	0,2	0,18	0,2
d [m]	0,003	0,04	0,03	0,10

Fig. 3. Material constants of wall and other structures

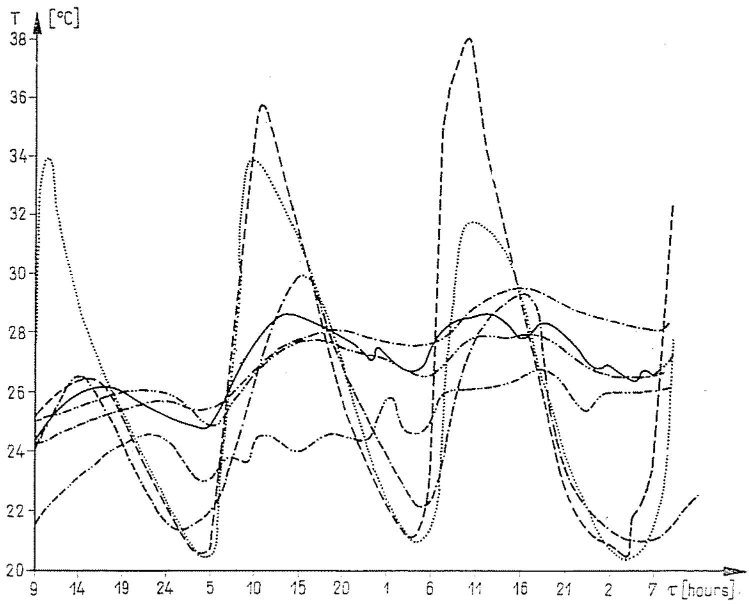


Fig. 4. Outer wall surface temperature of classroom No. 1 in summer. Limiting conditions: – – – – – East wall outer temperature; · · · · · · · South wall outer temperature; – – · – – Gallery wall temperature; ——— Adjacent room wall temperature; – · · – · · – 7th floor temperature; – – · – – · 9th floor temperature; – · – · – · – Free air temperature

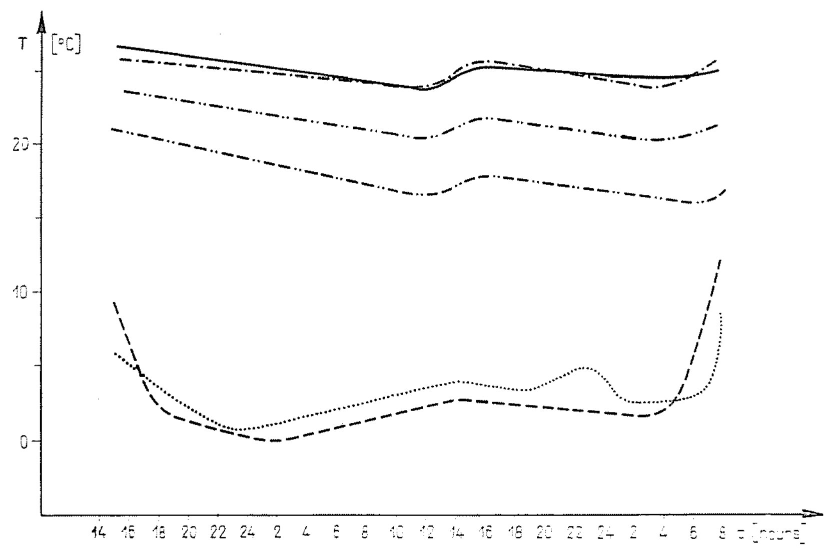


Fig. 5. Oouter wall surface temperatures of classroom No. 1 in winter. Limiting conditions: – – – – – East wall outer temperature; · · · · · · · South wall outer temperature; – – · – – Gallery wall temperature; ——— Adjacent room wall temperature; – · · – · · 7th floor temperature; – · – · – · – 9th floor temperature

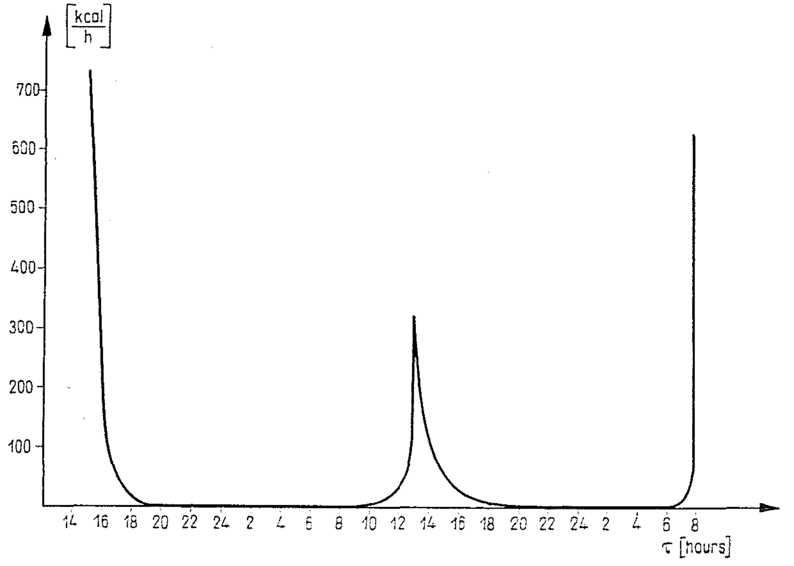


Fig. 6. Solar heat affecting the entire window area (31.2 sq. m) in winter

7. Conclusions

7.1. Experimental checking of mathematical model outputs shows convincingly the model to truly represent realistic conditions, for any kind of outer or inner walls, doors, windows and heat loads.

7.2. In addition to room temperature, the model indicates internal and surface temperatures of partitions and floors, permitting thereby to realize the objective in item 2, that is, to investigate the effect of factors decisive for thermal conditions.

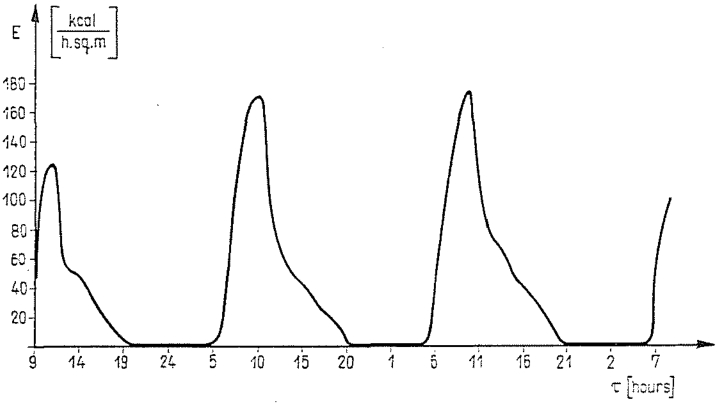


Fig. 7. Incident solar heat in summer (recorded)

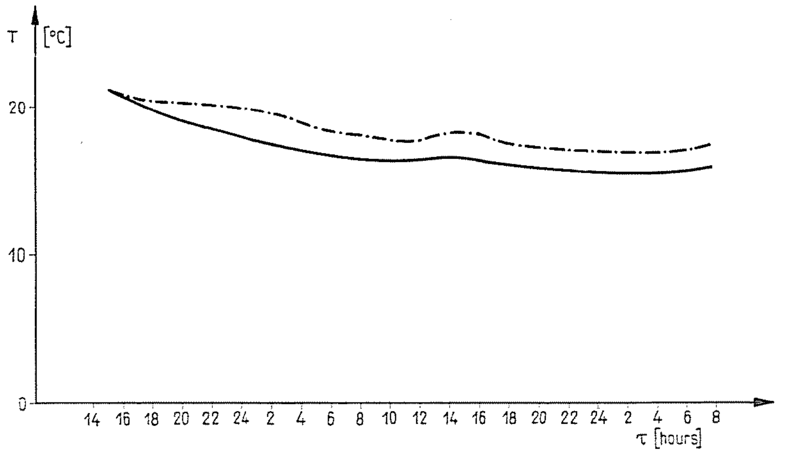


Fig. 8. Calculated and recorded winter room air temperatures. —·—·—·— Recorded; ——— Calculated

7.3. The same facility is open to analyse thermal effects of room arrangement and building geometry of premises of the quoted type.

7.4. There is a possibility of analyzing a single one among several parameters affecting temperature inside buildings such as: screening, confinedness, orientation, one outer or inner structure, one window size, intermittent, continuous or nightly ventilation etc.

7.5. From the knowledge of inner wall temperatures, thermal comfort can be predicted.

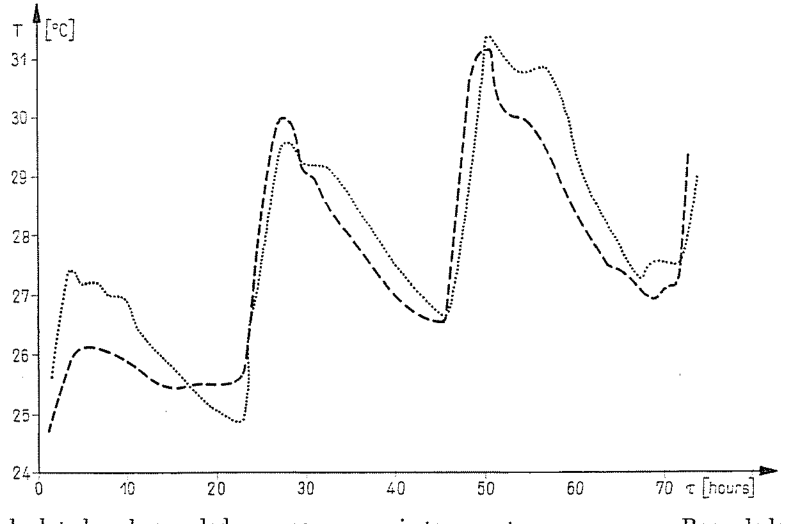


Fig. 9. Calculated and recorded summer room air temperatures. – – – – – – Recorded; · · · · · · Calculated

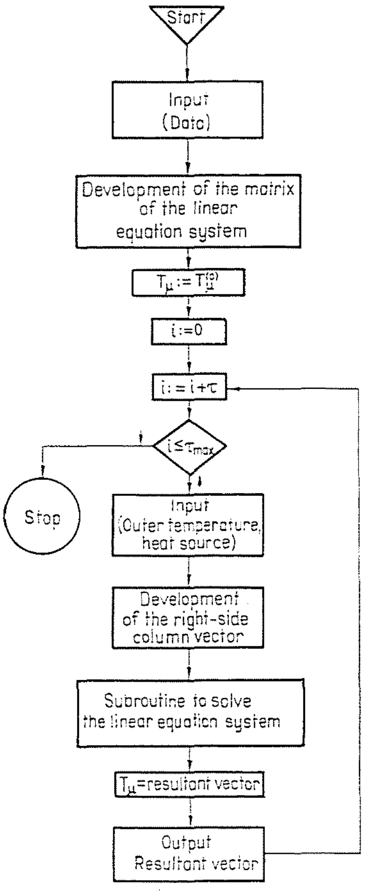


Fig. 10. Flow chart of the model

7.6. The model lends itself to optimation design of icehouses, cold storage rooms or any building to be air-conditioned.

7.7. The model helps to establish exact functions of thermal conditions for various kinds of buildings in dependence on structures, orientation, glass areas and room arrangement.

7.8. Analyses 7.2 through 7.7 may be based on daily mean temperature and insolation data as delivered by the National Meteorological Office.

7.9. By this time, solution of the partial differential equation systems corresponding to the mathematical model has been programmed for two adjacent rooms only. Computer of the Technical University, Budapest, lends itself to the complex analysis of twelve rooms. Number of rooms to be handled depends on the storage capacity of the given computer.

8. Appendix

8.1. Solution of the involved differential equation systems

Obviously, analytic solution of differential equation systems (1), (6), (7) and (8) — for given initial and limiting conditions — could not be undertaken, since a solution seemed a priori hopeless to be found, and no universally valid method for solving this kind of equations could be expected. Hence, the given differential equation systems were converted into a difference equation system accessible either for analogous model or for computer. Applying the method of finite differences to differential equations (6), (7), (8) these assume the form:

$$\varrho_a c_a d_a \frac{t^{1N}_{\Omega,n} - t^{1N}_{\Omega,n-1}}{\Delta\tau} = \alpha_{\Omega,\beta}(t^{N}_{\beta,n} - t^{1N}_{\Omega,n}) + \varepsilon_0 \frac{\lambda}{\delta}(t^{2N}_{\Omega,n} - t^{1N}_{\Omega,n}) + + A_o c_o b(t^{2N}_{\Omega n} - t^{1N}_{\Omega,n}) + J_{\Omega,\beta,n} A_1 \tag{9}$$

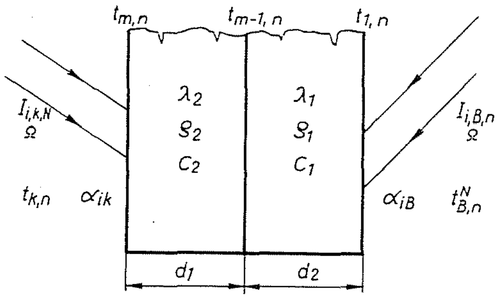
$$\varrho_a c_a d_a \frac{t^{2N}_{\Omega,n} - t^{2N}_{\Omega,n-1}}{\Delta\tau} = \alpha_{\Omega,k}(t^{N}_{k,n} - t^{2N}_{\Omega,n}) + \varepsilon_0 \frac{\lambda}{\delta}(t^{1N}_{\Omega,n} - t^{2N}_{\Omega,n}) + + A_o c_o b(t^{1N}_{\Omega n} - t^{2N}_{\Omega,n}) + J_{\Omega,k,n} A_2 \tag{10}$$

$$\frac{t_{\beta,n} - t_{\beta,n-1}}{\Delta\tau} = \frac{1}{V^N \varrho c}\left[\sum_{i=1}^{k} F_i^N \alpha_{i\beta}(t^N_{i,1,n} - t^N_{\beta,n}) + \sum_{\Omega=1}^{Z} F^N_\Omega \alpha_{\Omega,\beta}(t^{1N}_{\Omega,n} - t^N_{\beta,n}) + + \sum_{\gamma=1}^{Y} \frac{t^N_{\beta,n} - t_{\gamma,n}}{R^{Q,N}_\gamma} + V^N f^N_n\right] \tag{11}$$

with the same notations as for (6) except for the complementary subscript n indicating that variables assume the given or calculated discrete values at time $\tau = n\Delta\tau$.

For the given examples, it is rather simple to convert the differential equation systems (1), for finite differences, but here not all equations occurring in our examples will be written down, only illustrated on a generalizable example.

For the sake of simplicity let us take a wall structure of two layers, — each being homogeneous in itself — and write down the difference equation system at a minimum of intervals, applying heat balances so as to include heat transfer, heat gain by radiation and thermal sources. Assuming a one-way linear heat flow, temperatures will be indicated by two subscripts, one being for the plane where the temperature is to be determined, the other is for the temperature at time $n\Delta\tau = \tau$. α_{ik}, $\alpha_{i\beta}$, are outer and inner heat transfer coefficients, resp. depending on the temperature and the room size; t_{kn} and $t^N_{\beta,n}$ are outer and inner air temperature, resp., $I_{\Omega}^{i,k,n}$ and $I_{\Omega}^{i,\beta,n}$ are radiation energy flow densities; A_1' and A_2'' being surface absorptions; and f_{in}' an eventual inner heat source.



$$\frac{d_1\varrho_1 c_1}{2\Delta\tau}(t_{1n}-t_{1n-1}) = \frac{\lambda_1}{\delta_1}(t_{m-1,n}-t_{1,n}) + \alpha_{i\beta}(t_{\beta,n}^N - t_{1,n}) + J_{i,\beta,n}A_1 + f_{i,n}\frac{d_1}{2} \quad (12)$$

$$\frac{d_1 c_1\varrho_1 + d_2 c_2\varrho_2}{2\Delta\tau}(t_{m-1,n}-t_{m-1,n-1}) = \frac{\lambda_2}{d_2}(t_{m,n}-t_{m-1,n}) + \frac{\lambda_1}{d_1}(t_{1,n}-t_{m-1,n}) \quad (13)$$

$$\frac{d_2 c_2\varrho_2}{2\Delta\tau}(t_{m,n}-t_{m,n-1}) = \alpha_{ik}(t_{k,n}-t_{m,n}) + \lambda_2\frac{t_{m-1,n}-t_{m,n}}{d_2} + J_{i,k,n}A_2' \quad (14)$$

Eqs (9) through (14) constitute a linear equation system with 21 unknowns [since Eqs (12) to (14) are to be written six times with different material constants and initial conditions if a room of arbitrary size with six sides, one window and any number of doors is to be represented] likely to be solved for any time $t_{\beta,0}^N$; $t_{1,0} \ldots t_{\Omega,0}^N$ if the pertinent energy flow densities, sources and outer limiting conditions are given as discrete values, together with initial temperature values $n\Delta\tau = \tau$.

8.2. Temperature and angle dependence of heat transfer coefficients

Outer heat transfer coefficients are composed of two parts: those for radiation and for forced flow convection.

$$\alpha_{\Omega,k} = \alpha_{i,k} = \alpha_{\text{rad.}} + \alpha_{\text{conv.}}$$

$$\alpha_{\text{rad.}} = \varepsilon_0 c_0 \frac{\left(\frac{T_{i,m,n}}{100}\right)^4 - \left(\frac{T_{k,n}}{100}\right)^4}{T_{i,m,n} - T_{k,n}} \quad (15)$$

$$\alpha_{\text{conv.}} = 0{,}032\,\frac{\lambda}{l}\left(\frac{w_0\, l}{\nu}\right)^{0{,}8} \quad (16)$$

(see p. 104 in [7])

where ε surface blackness degree
$C_0 = 4.9$ kcal/m² · h. °K⁴
T_{imn} = external surface temperature
T_{kn} = environmental temperature at time $n\Delta\tau = \tau$
λ = thermal conductivity coefficient of air
l = size along the flow
w_0 = air flow rate
ν = kinetic viscosity.

Inner heat transfer coefficient consists again of two parts: those for radiation and for free flow convection, but only this latter will be taken into account, the absorptivity of air being negligible.

$$\alpha_{\Omega,\beta} = \alpha_{i\beta} = \alpha_{\text{conv.}}$$

$$\alpha_{\text{conv.}} = 1{,}36(t_{i,1,n} - t^N_{\beta,n})^{\frac{1}{3}} \tag{17}$$

(see p. 69 in [7]).

Of course, the i-th structure is in radiational interaction with all other limiting structures, thus, I_{ibn}, will also consist of two parts; sum of solar radiation and of other radiations due to heating as well as a radiation term due to this interaction with other surfaces:

$$J'_{\underset{\Omega}{i,\beta},n} = c_0 \varepsilon_i \sum_k \varepsilon_k \, \varphi_{\underset{\Omega}{i,k}} \left[\left(\frac{T_{\underset{\Omega}{i,1,n}}}{100} \right)^4 - \left(\frac{T_{k,1,n}}{100} \right)^4 \right] \tag{18}$$

(see p. 148 in [7])

where ε_i; ε_k are blackness degrees of the i-th and k-th structure, resp.;
$\varphi_{\underset{\Omega}{i,k}}$; $T_{\underset{\Omega}{i,1,n}}$ are irradiation coefficients and absolute surface temperatures of the i-th structure and the Ω-th window, respectively, at time
$T_{k,1,n}$ $n\Delta\tau = \tau$.

8.3. Determination b and ε_0 values for windows and glazed doors

ε_0 can be determined in closed form according to the relationship (see p. 72 in [8]):

$$\varepsilon_0 = 18\sqrt{\delta^3(t^{1N}_{\Omega,n} - t^{2N}_{\Omega,n})} \tag{19}$$

Also b is easily found by the method of finite differences:

$$b = \frac{\left(\frac{T^{1N}_{\Omega,n}}{100}\right)^4 - \left(\frac{T^{2N}_{\Omega,n}}{100}\right)^4}{T^{1N}_{\Omega,n} - T^{2N}_{\Omega,n}} \tag{20}$$

8.4. *Practical computation of the coefficients in items 8.2 and 8.3*

Starting arbitrary values assumed for α; ε_0; $J'_{i,\beta,n}$ and b are applied for solving difference equation systems (9) to 6 times [(12) to (14)] for time and the resulting temperature values are substituted into Eqs (15) through (20) (special complementary computer program), then the outputs for α, ε_0, b, $J'_{i,\beta,n}$ are resubstituted into the difference equation system to be solved in turn for time $\tau = 2\Delta\tau$ and this process is continued to the desired time $\tau = n\Delta\tau$.

Summary

Possibility to establish a mathematical model, likely to describe the dynamic thermal behaviour of a building of N rooms, at an accuracy of 10 to 15 per cent, is presented. It lends itself to thermal design, selection of thermally optimum structures and room arrangements as well as development of the economically most favourable service conditions in view of orientation, inner and outer heat loads, wall surface nature (e.g. large vitreous surfaces), lightweight structures and building geometry.

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* In Hungarian