A NOTE ON THE INFLUENCE
OF SPONTANEOUS FLUCTUATIONS
ON THE LIMITS OF VACUUM MEASUREMENT

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Pressure measurements have a primary importance not only in vacuum science but also in acoustics. Measurement techniques in acoustics arrived already to the point, where (beside other phenomena) spontaneous fluctuations limit the sensitivity [1, 2]. It seems therefore appropriate to examine whether also the sensitivity of ultra-high vacuum measurements is limited by spontaneous fluctuations.

There are essentially two important kinds of spontaneous fluctuations. Communication theory — dealing mainly with their appearance in electric quantities — calls them shot noise and Nyquist's (or Johnson's) noise, respectively. However, they are quite general phenomena. Thus, shot noise is a consequence of Posson's theorem on the average number of random events:

$$\overline{n - \overline{n}^2} \approx \overline{n}$$

Nyquist's theorem is a version of the equipartition law. Its usual Fourier form describes noise power P as the time derivative of fluctuating energy:

$$P = kT\,\Delta f$$

without specifying the kind of energy involved. (The author has shown [3] that making use of the Sampling Theorem of Communication Theory and of the relation $\Delta f = \dfrac{1}{2\Delta t}$ as a definition equation for Δf, one arrives at

$$P = \frac{kT}{2}\,\frac{1}{\Delta t}$$

designating the apparent fluctuation power between two matched two-poles if energy measurements are repeated at time intervals Δt.)

While investigating ultra-high vacuum pressure measurements we naturally exclude all other effects limiting measurement accuracy (X-ray

effect, influence of residual gas composition on ionisation, etc.) and restrict us to the consequences of spontaneous fluctuations.

The basic device is invariably some kind of ionisation gauge, giving in principle continuous readings. This means that to determine the density of molecules we extract a sample quantity which ought to be proportional to the total. Unfortunately, ionisation is a random process and therefore Poisson's theorem makes its appearance. Further, to determine the number of ions, we have to collect and measure their current. Irrespective of its special nature, the measuring device will be a source of Johnson's noise. Usually this second fact will constitute the greater danger.

For the sake of illustration let us consider the problem of measuring a pressure of $p = 10^{-14}$ torr with some kind of total pressure gauge. The gauge has a certain ionisation sensitivity η expressed in A/torr (η may be a function of pressure.) The collector current $I_c = \eta p$ may be measured by some means of D.C. amplification, by the vibrating electrometer principle, etc. In either case the current has to flow across a collector resistance R charging a capacity or stray capacity C. It may be followed from the equipartition law that the capacity has a fluctuating voltage.

Since

$$C \, \overline{V}^2/2 \approx kT/2 ,$$

$$\overline{V} \approx \sqrt{kT/C}$$

Capacity C and resistance R define the time constant $\varDelta t = RC$ which is approximately the minimum measuring time. To have a meaningful reading, the voltage from the D.C. ion current I_c should be at least 10 times the average voltage fluctuation $\overline{V}$ of the device:

$$I_c R = I_c \, \varDelta t/C \geq 10 \sqrt{kT/C}$$

$$I_c \, \varDelta t \geq 10 \sqrt{kTC} \approx 6 \times 10^{-10} \sqrt{C}$$

for room temperature.

Since the stray capacity is surely > 10 pF:

$$I_c \, \varDelta t \geq 2 \times 10^{-15} \text{ Coulomb}$$

which means that independently of the way, how amplification is performed at least 10^4 ions have to be collected. Thus, the sensitivity defined as the minimum measurable pressure is

$$P_{\min} \approx \frac{2 \times 10^{-15}}{\eta \, \varDelta t} \text{ torr}$$

Electrical noise in the amplifying stages of the equipment will at least double this value. It is seen therefore, that high sensitivity against measurement speed must be traded in. For example, the pressure of 10^{-14} torr may be determined with a time constant of 1 second, but only, if an ionisation mechanism of high sensitivity ($\eta > 0.5$ A/torr) like the Penning type, is applied. This again involves a high pumping speed of the gauge, which is rather a drawback.

The only way to get out of this dilemma is to use current multiplication *before* collecting the current. The output current of the multiplier will be large enough to make the Nyquist noise of the collecting RC combination irrelevant, since even a single ion gives rise to a pulse of at least 10^6 electrons or 10^{-13} Coulomb [4]. However, Poisson fluctuations of the primary ion current are not smoothed out by the multiplication process, in fact they are rather emphasized, because secondary emission is a random process by itself.

The effect of these additional fluctuations may be eliminated if the multiplier is cooled to liquid N_2 temperature and individual ion pulses are counted. There is a finite probability, however, that an ion will not start an avalanche and will not be counted [5]. Anyway, at least 100 pulses are needed, according to Poisson's theorem, to achieve a reproducibility of 10%. In terms of primary ion current this would mean

$$I_c \Delta t \approx 1.6 \times 10^{-17} \text{ Coulomb}$$

or a gain of two orders of magnitude over the case treated before, and so:

$$P_{\min} \approx \frac{1.6 \times 10^{-17}}{\eta \Delta t} \text{ torr}$$

This relation has the greatest importance for partial pressure gauges, where Δt is the time needed to determine the partial *pressure* of a single component (or of unit mass number). The total scanning time T_t for the entire mass spectrum is about $T_t \approx 100 \, \Delta t$. It should be remembered that the sensitivity of most partial pressure gauges is rather low, typically $\eta \approx 10^{-4}$ A/torr. Thus, a partial pressure of 10^{-14} torr corresponds to only 10^{-18} A or 6 ions/second! This would mean $\Delta t = 15$ seconds or a total scanning time of nearly half an hour. This is a very unrealistic requirement.

Considering that the main scope is the continuous observation of changes in the composition, we cannot dispense with a visual display. The inertia of our eye allows us a scanning time of 0.1 second. The time interval for unit mass number is then 1 msec. 100 ions needed for a 10% inaccuracy would mean a collector current of 10^{-14} A or 10^{-10} torr, as a *limit to partial pressure measurement* of fast scanning mass spectrometers.

As an illustration of this calculation we may review the famous apparatus of DAVIS and VANDERSLICE [6]. Here the authors claim a minimum sweep time

of 1 microsecond per unit mass number. This is made possible through the application of a multiplier, the output resistance of which is limited to 10,000 ohms. (This limit is due to the time constant and it means that the output capacitance is about 100 pF.) The multiplication factor is given as $10^{+6}-10^{+7}$, corresponding to a charge of 10^{-12} Coulomb. The smallest meaningful output signal is 1 V corresponding to 100 ions. Taking the sweep time of 1 microsecond into account, the primary ion current is 10^{-11} A and thus the partial pressure sensitivity is 10^{-7} torr. The extreme sensitivity limit of the apparatus itself, which is claimed to be 10^{-16} torr, is incompatible with large scanning speeds, since it would need 1500 seconds measurement time per unit mass number. This is of course out of question for scanning; the sensitivity of 10^{-16} torr may be used eventually to detect a single component if time of measurement is unimportant. Thus, even extreme amplification with counting individual ion pulses will not help to avoid the fluctuation limits.

The general relationship:

$$P_{\min} T_t \eta \approx 10^{-15} \text{ Coulomb}$$

(with T_t for total scanning time) sets a severe limit to the use of scanning mass spectrometers as neither the scanning time nor the efficiency may be much raised. One may think that some gain in total scanning time could be achieved by making the scanning speed a function of the gas composition itself; with $\dfrac{\Delta m}{\Delta t}$ large, when ion current is high, and $\dfrac{\Delta m}{\Delta t}$ small, when it is low. (This may be performed by letting the output current control the quantity responsible for scanning, e.g. frequency of the H.F. voltage in an omegatron.)

Unfortunately, this would not work, since only a few components have relatively large partial pressures. However, the same idea might be applied to a total pressure gauge. Instead of ion current, or charge during a predetermined time, one could measure the time needed for a certain charge (e.g. 10^{-15} Coulomb). This would mean equal measurement accuracy for all values. The time indicated is in principle inversely proportional to pressure in this case. Digital display is easily performed.

Summary

A theoretical limit is set by noise to either minimum measurable pressure or scanning speed of present day partial pressure analysers. A total pressure gauge is suggested with constant accuracy and reduced measurement time.

References

1. ZAALBERG VAN ZELST: Philips Techn. Rev. **9**, 357 (1948).
2. VALKÓ, I. P.: MTA Műsz. Tud. Közl. **27**, 313 (1960).
3. VALKÓ, I. P.: Periodica Polytechnica, El. Eng. **15**, 77 (1971).
4. HUBER, W. K.: Vacuum **13**, 399, 469 (1963).
5. SAUTER, F.: Z. für Naturforschung **4a**, 682 (1949).
6. DAVIS, W. D.—VANDERSLICE, T. A.: Nat. Symp. on Vacuum Techn. Trans. 417 (1960).

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