A METHOD FOR COMPUTING RADIAL NETWORK SYSTEM RELIABILITY

By

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Theoretical methods and practically applicable relationships of distribution network system reliability calculations are regularly presented in the technical literature [1, 2]. After exposing the principles of reliability calculation, relevant relationships of a simple form will be presented to ease manual calculation. Use of a computer increases accuracy and eliminates the need for simplifications. Consequently, suggested relationships are slightly different from those in [2].

1. Applied fundamental relations

Concepts of reliability and unreliability

Service reliability is determined by two factors: by the frequency and by the duration of outages. They can jointly be plotted in a distribution curve, expressing the probability of network or network component a to have an outage time longer than t:

$$P[\bar{A}(\tau > t)] = Q_A(t) = F(t),$$

where

$\bar{A}(\tau > t)$ event where component a has an outage time $\tau > t$;
$Q_A(t)$ unreliability of component a as a function of τ;
$F(t)$ distribution function.

Hence, reliability is the probability of component a to have no outage time longer than t in examined time:

$$R_A(t) = 1 - Q_A(t),$$

where

$R_A(t)$ reliability of component a as a function of τ.

Distribution curve $Q_A(t)$ is likely to be approximated by an exponential distribution curve. The feasibility of this approximation is to be examined on Hungarian statistical data. These examinations are not finished as yet.

The resultant distribution curve of a complex network system ought to be composed of the distribution curves of single network components. This method, however, is rather unsuitable for the practice. The resultant distribution curve may be replaced instead of unreliabilities corresponding to its discrete t values. From the resultant network system unreliability corresponding to these discrete values, conclusions may be drawn on the distribution curve of the resultant network system, T denoting the chosen time intervals:

$$P[\bar{A}(\tau > t)] = Q_{AT}$$

and

$$P[A(\tau > t)] = R_{AT}.$$

Network component outages may be either

— forced outages caused by component failures $\bar{A}$; or

— scheduled outages for maintenance ($\bar{A}^*$).

Outage conditions considered as sets, these sets are not independent of each other [1].

Also maintenance causes outages of various durations similar to accidental failures. Consequently, this can be considered in the same way as forced outages:

$$P[\bar{A}^*(\tau > T)] = Q^*_{AT}$$

$$P[A^*(\tau > T)] = R^*_{AT},$$

where

Q^*_{AT} probability for component a to be out of service for maintenance,

R^*_{AT} maintenance reliability for component a.

Consideration of maintenance

The unreliability of component a is to be calculated under the simulteneous consideration of maintenance and forced outages on the basis of [1] by means of an imaginary component connected in series with the examined network or component a (Fig. 1.1).

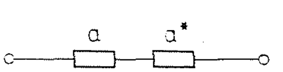

Fig. 1.1

The unreliability of component or network a is:

$$Q_{ATe} = Q_{AT} + Q^*_{AT}.$$

Components in series arrangement

In case of maintenance of components in series arrangement, organization of the scheduled outages is to be taken into consideration [1]. Hence, components in series arrangement should be maintained by simultaneous disconnection [1]. Consequently, that one of the components, $a_1, a_2, \ldots, a_n$ with the greatest probability of being disconnected for maintenance should be found and connected in series to the other components as an imaginary component a^* (Fig. 1.2).

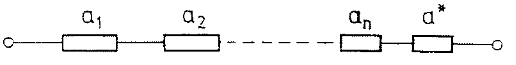

Fig. 1.2

The reliability of components in series arrangement is calculated in two steps:

— the unreliability of components in series arrangement is calculated by ignoring maintenance.

$$R_{ST} = \prod_{i=1}^{n} R_{AiT},$$

where

R_{AiT} reliability of component ai corresponding to time interval T
and

$$Q_{ST} = 1 - R_{ST};$$

— thereafter the effect of the imaginary series component on maintenance will be determined:

$$Q_{STe} = Q_{ST} + Q^*_A$$

where
Q^*_A probability of the imaginary component for maintenance to be disconnected for maintenance.

Components in parallel arrangement

Maintenance of single prallel branches (Fig. 1.3) has to be organized without simulteneous disconnection. The parallel network is calculated in two steps:

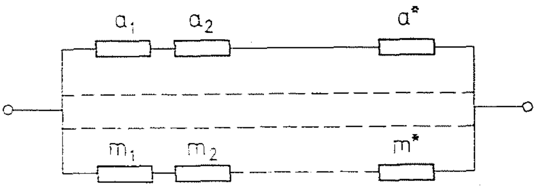

Fig. 1.3

— the unreliability of single branches $Q_{AT}, \ldots, Q_{MT}$ without maintenance is calculated first, then imaginary components for maintenance must be connected in series to each branch;
— the resultant unreliability will be calculated by means of tables of the "complete system of events" in [1].

The 1.1 is an example for the case of two branches.

Consideration of laterals

Between series components, laterals can also be connected (Fig. 1.4). The way considering a lateral depends on the applied disconnection switchgear [2].

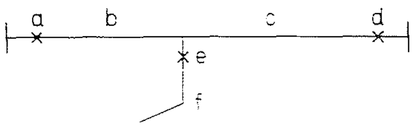

Fig. 1.4

— If the switchgear suits to disconnect outages of the lateral from the feeder (e.g. a circuit-breaker with protection), then components of the lateral are omitted from among the series components of the feeder. Circuit-breaker failures affecting the feeder (e.g. explosions) must be, however, taken into consideration (e') (Fig. 1.5).

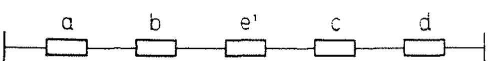

Fig. 1.5

— If the switchgear is unsuitable for disconnecting outages of the lateral, then the feeder and the lateral are out of service simultaneously, i.e. from the viewpoint of the feeder the lateral is a series component (Fig. 1.6).

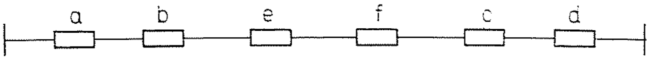

Fig. 1.6

There are various kinds of disconnection switchgears, to be reckoned with differently, depending on the outage time. For example, a pole-type switch is to be considered as a disconnecting component in case of outages of long duration, because it permits the defective lateral to be disconnected on the spot; in case of short time intervals, however, it is a series component, because then a pole-type switch is unsuitable for disconnecting.

Collection of data; statistics

Primary condition of the calculation is to know unreliabilities of single network components corresponding to the chosen discrete time intervals. A wide-spread collection of data is necessary for determining these unreliabilities. In Hungary this survey covered retrospectively several years in respect to every outage in the medium voltage network system of the country.

Fundamental relationship for evaluation:

$$Q_{AT} = \frac{N[\bar{A}(\tau > T)]}{N_v},$$

where

Q_{AT} unreliability of component a,
$N[\bar{A}(\tau > T)]$ number of days where component a is out of service for a time interval longer than T,
N_v period of examination (in days);

$$Q^*_{AT} = \frac{N[\bar{A}^*(\tau > T)]}{N_v},$$

where

Q^*_{AT} probability of component a to be disconnected for maintenance,
$N[\bar{A}^*(\tau > T)]$ number of days with component a to be out of service for a time interval longer than T,
N_v period of examination (in days).

Reliability data of single network components can be stored in matrices.

Let a code number be given every component type, e.g. 20 kV overhead transmission line, EIB-type circuit-breaker, etc. The chosen discrete outage time intervals T shall be indicated similarly by means of code numbers.

Applying these indications, matrices of service reliability data can be written as follows:

— unreliability matrix of components:

$$\mathbf{Q}_e = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ q_{m1} & \cdots & & \end{bmatrix}$$

where

q_{ij} unreliability of a component of the type with code number i, corresponding to the time interval of code number j.

— maintenance unreliability matrix of components:

$$\mathbf{Q}_e^* = \begin{bmatrix} q_{11}^* & q_{12}^* & \cdots & q_{1n}^* \\ q^*_{21} & q^*_{22} & \cdots & \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ q_{m1}^* & \cdots & & \end{bmatrix}$$

where

q_{ij}^* maintenance unreliability of a component of the type with code number i, corresponding to the time interval of code number j.

— reliability matrix of component is: $\mathbf{R}_e$, where

$$r_{ij} = 1 - q_{ij}.$$

— maintenance reliability matrix of components is: $\mathbf{R}_e^*$, where

$$r_{ij}^* = 1 - q_{ij}^*.$$

2. Algorithm of radial network system calculation

Radial networks consist of branches. Points where branches meet are called junction points (Fig. 2.1).

Each of the branches consists of components (Fig. 2.2).

The calculation is done in two steps:

— calculation of resultant reliabilities of single branches;

— determination of reliabilities at single points of the radial network system.

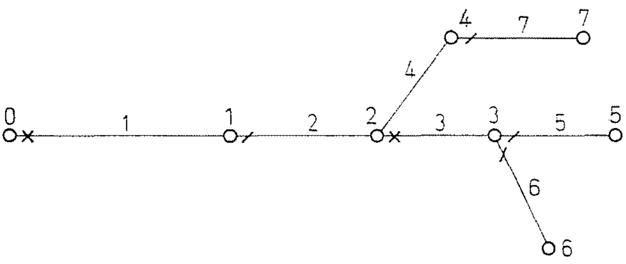

Fig. 2.1

Fig. 2.2

Resultant reliabilities of single branches

Reliability of a branch depends on its components and on their reliability Matrices $\mathbf{R}_e$ and $\mathbf{R}_e^*$ give informations on component reliabilities. Components of the branch are described by a vector giving the quantity of components with single code numbers:

$$d_{\text{branch}} = \begin{bmatrix} d_1 \\ d_2 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

where

d_j quantity of components of code number j of the branch.

Vector of the resultant branch reliability:

$$r_{\text{branch}} = r_{\text{branch}}(R_e, d_{\text{branch}})$$
$$r_{\text{branch } i} = (r_{\text{eij}})^{d_j},$$

where

$r_{\text{branch } i}$ reliability of the branch corresponding to the time interval of code number i.

The resultant maintenance reliability vector of the branch is calculated in two steps. First step is the calculation of an auxiliary matrix:

$$\mathbf{K} = \mathbf{K}(R_e^*, d_{\text{branch}})$$

$$k_{ij} = \begin{cases} r^*_{ij} & \text{with} \quad d_j \neq 0 \\ 1 & \text{with} \quad d_j = 0 \end{cases}$$

The required vector resulting from the second step:

$$r^*_{\text{branch}} = r^*_{\text{branch}}(\mathbf{K})$$

$$r^*_{\text{branch}\,i} = \min_j k_{ij},$$

where

$r^*_{\text{branch}\,i}$ maintenance reliability of the branch corresponding to the time interval of code number i.

Unreliability vector of the branch:

$$q_{\text{branch}} = e - r_{\text{branch}},$$

where

e unit vector.

Maintenance unreliability vector of the branch:

$$q^*_{\text{branch}} = e - r^*_{\text{branch}}.$$

Service reliability of radial network systems

Service reliability of junction points in radial network systems is to be calculated from the full knowledge of network topology by means of reliability data of the individual branches. Informations on topology are contained in topology matrices. The structure of topology matrices is perceptible in the network model, Fig. 2.1. The index number of any branch agrees with that of its junction point off feeding. The network structure is contained in the following table:

junction point i ↓	j → number of the next branch 1	2	3	4	5	6	7
1	0	1	0	0	0	0	0
2	0	0	1	1	0	0	0
3	0	0	0	0	1	1	0
4	0	0	0	0	0	0	1
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0

Hence, fundamental topology matrix:

$$\mathbf{T}_a = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$t_{ij} = \begin{cases} 1 \text{ with branch } j \text{ following branch } i \\ 0 \text{ with branch } j \text{ not following branch } i \end{cases}$$

It is apparent that the rows of matrix $\mathbf{T}_a$ representing end branches (i.e. branches to which no further branches are connected) contain only zero elements, they give no new information on topology. Therefore, these rows are omitted. In the model network of Fig. 2.1, single branches have index numbers growing from the feeding towards the ends, so that the end branches have the highest index numbers. This kind of indication is unsuitable for the practice, therefore, just the system of indices in the data assembly of the prepared programme is arbitrary with the computer producing the required sequence.

After all, topology matrix ordered according to the required topological sequences with zero rows removed is:

$$\mathbf{T} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

In calculating reliability, components connecting branches to the preceding ones must be taken into consideration. Be this component a circuit-breaker, then failures of the lateral do not affect the feeding network. Be it an isolating switch, then short time outages affect the feeding network, i.e. the lateral must be considered as a series component. From the viewpoint of long-time outages, however, isolating switches count as disconnection points; so defective sections of the network can be isolated. Consequently, topology matrices must be of the kind containing this information.

Topology matrices $\mathbf{T}_a$ or $\mathbf{T}$ should therefore indicate the kinds of disconnecting components applied at the junction points. Let the following indication be introduced:

circuit-breaker 1 (disconnection in case of either short or long time outages)

isolating switch 2 (disconnection in case of long time outages)
no disconnecting
component 3 (no disconnection).

In this way from matrix $\mathbf{T}_a$ a new matrix, the so called network matrix is obtained, with the following structure in case of the network in Fig. 2.1:

$$\mathbf{H} = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$h_{ij} = \begin{cases} 1 & \text{in case of laterals with circuit-breakers,} \\ 2 & \text{in case of laterals with isolating switches.} \\ 3 & \text{in case of laterals without disconnecting component.} \end{cases}$$

This matrix indicates which of the branches in the radial network system is to be considered as a series component determinant for junction point reliabilities. Since branches must be judged differently whether short or long time intervals are implied, it is advisable to describe these by independent network matrices.

The network matrix for short time intervals designates the branches connected to the junction point in question to be considered as a series component for short- time outages. For example: branch 3 is connected by a circuit-breaker to junction point 2; consequently, only circuit-breaker failures affecting the feeding network must be taken into consideration. Branches 4 and 7 have no disconnecting device at junction points 2 and 4, resp.; consequently, they must be considered as components arranged in series to junction point 2. Thus, the row corresponding to junction point 2 in the network matrix is:

$$h_2 = \begin{bmatrix} 0 & 0 & 1 & 3 & 0 & 0 & 2 \end{bmatrix}$$

The short-time network matrix referring to the model network:

$$\mathbf{H}_R = \begin{bmatrix} 0 & 2 & 1 & 3 & 0 & 0 & 2 \\ 0 & 0 & 1 & 3 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

For long times, only branches with no disconnecting device are to be considered as series components. Thus, the network matrix for long times:

$$\mathbf{H}_H = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

These matrices can be deduced mechanically from network matrices by means of the following simple algorithm:

Matrices $\mathbf{H}_R$ and $\mathbf{H}_H$ must be completed by zero rows of matrix $\mathbf{T}_a$ omitted when producing matrix $\mathbf{T}$. Then, elements of the matrix must be handled one by one, beginning with the last element of the last row of the matrix and proceeding backwards. On producing matrix $\mathbf{H}_R$:

$$h_{Ri} = \begin{cases} h_{Ri} + h_{Rj} & \text{with } h_{ij} > 1 \\ h_{Ri} & \text{with } h_{ij} \leqq 1 \end{cases}$$

On producing matrix $\mathbf{H}_H$:

$$h_{Hi} = \begin{cases} h_{Ri} + h_{Rj} & \text{with } h_{ij} = 3 \\ h_{Ri} & \text{with } h_{ij} \neq 3 \end{cases}$$

With the knowledge of short-time and long-time network matrices, service reliability of the junction points can be calculated. For this purpose resultant reliability and maintenance reliability vectors of the branches are applied.

The following matrices and vectors are known respectively:

$$\mathbf{H}_R,\ \mathbf{H}_H,\ r_{\text{branch}} \text{ and } r^*_{\text{branch}}\,.$$

Short-time and long-time reliabilities and maintenance reliabilities of the junction points are required, represented by reliability vector r_{cs} and maintenance reliability vector r^*_{cs} of the junction points, respectively.

Short times shall be calculated first.

According to Chapter 1, vector r_{branch} contains reliability of the examined branch corresponding to the single discrete outage durations. The branch reliability matrix

$$\mathbf{R}_{\text{branch}} = [r_{1\,\text{branch}}, r_{2\,\text{branch}}, \ldots, r_{n\,\text{branch}}] =$$

$$= \begin{bmatrix} r_{11} & r_{12} & \dots & \dots \\ r_{21} & r_{22} & \dots & \dots \\ \dots & \dots & \dots & r_{ij} \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

containing all branches of the network can be produced from these vectors; here

r_{ij} reliability of branch with subscript j, corresponding to the time interval of code number i.

The branch maintenance reliability matrix can be produced in a similar way:

$$\mathbf{R}^*_{\text{branch}} = [r^*_{1\text{ branch}}, r^*_{2\text{ branch}}, \dots, r^*_{n\text{ branch}}]$$

Junction point reliability

As was pointed out above, separate calculations are needed for short and long times. Matrix $\mathbf{R}_{\text{branch}}$ is divided into two:

$$\mathbf{R}_{\text{branch}} = \begin{bmatrix} \mathbf{R}_{\text{branch short}} \\ \mathbf{R}_{\text{branch long}} \end{bmatrix}$$

where

$\mathbf{R}_{\text{branch short}}$ contains branch reliabilities corresponding to short time intervals;

$\mathbf{R}_{\text{branch long}}$ contains branch reliabilities corresponding to long time intervals.

For calculation purposes transposed matrices

$$\mathbf{R}_{\text{branch } t} = [\mathbf{R}_{\text{branch short } t}, \mathbf{R}_{\text{branch long } t}]$$

are needed.

Let the following simplification be introduced:

$$\mathbf{R}_{\text{branch short } t} = \mathbf{M}_r$$

$$\mathbf{R}_{\text{branch long } t} = \mathbf{M}_h$$

and

$$\mathbf{R}_{\text{branch } t} = [\mathbf{M}_r, \mathbf{M}_h]$$

Similarly, maintenance reliability matrix:

$$\mathbf{R}^*_{\text{branch } t} = [\mathbf{M}^*_r, \mathbf{M}^*_h]$$

The calculation consists of two steps:

1. calculation of a three-dimensional auxiliary matrix $\mathbf{S}$;

2. use of this auxiliary matrix to calculate junction reliability matrix $\mathbf{R}_{cs}$ representing reliabilities of single junction points.

Auxiliary matrix for short-time calculations:

$$\mathbf{S}' = \mathbf{S}'(\mathbf{M}_r, \mathbf{H}_r)$$

$$s'_{ijk} = \begin{cases} 1 & \text{with } h_{ij} = 0 \\ r_m & \text{with } h_{ij} = 1 \\ m_{jk} & \text{with } h_{ij} > 1 \end{cases}$$

where

$\mathbf{S}'$ three-dimensional auxiliary matrix;

s'_{ijk} element of the auxiliary matrix corresponding to junction point i, branch j and time interval of code number k.

Elements of matrix $\mathbf{M}_r$:

m_{jk} reliability corresponding to branch j and to time interval of code number k.

Elements of matrix $\mathbf{H}_r$:

h_{ij} element corresponding to junction point i and to branch j;

r_m reliability deduced from the probability of a circuit-breaker failure affecting the feeding network:

$$r_m = 1 - q_m.$$

Similarly, the auxiliary matrix $\mathbf{S}''$ for long-time calculations:

$$\mathbf{S}'' = \mathbf{S}''(\mathbf{M}_h, \mathbf{H}_h)$$

$$s''_{ijk} = \begin{cases} 1 & \text{for } h_{ij} = 0 \\ r_m & \text{for } h_{ij} = 1 \\ r_{sz} & \text{for } h_{ij} = 2 \\ m_{jk} & \text{for } h_{ij} = 3 \end{cases}$$

where

r_{sz} reliability deduced from the probability of an isolating switch failure affecting the feeding network:

$$r_{sz} = 1 - q_{sz}.$$

For junction point reliability calculations, it matters, which is the component feeding the tested junction point, i.e. to which preceding compo-

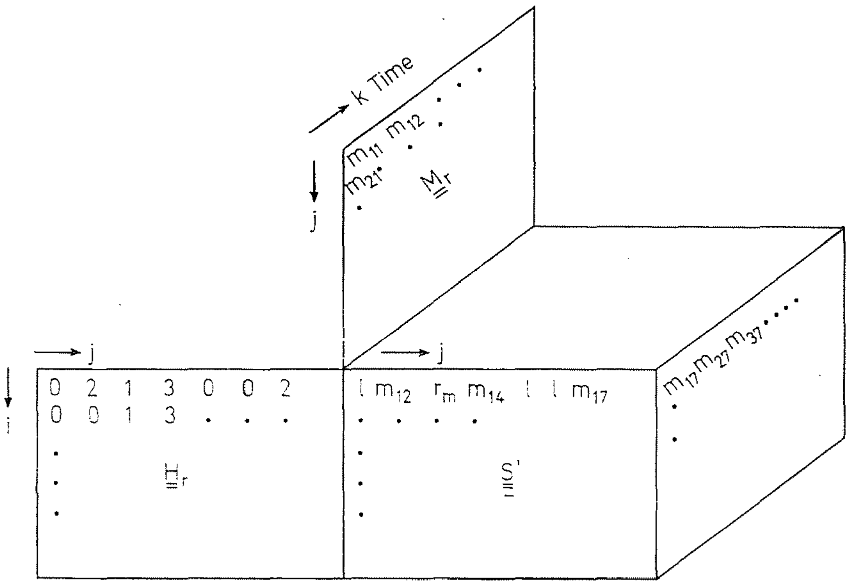

Fig. 2.3. 1 junction point; 2 time; 3 branch

nents the branch feeding the new junction point is connected in series. The network matrix informing on this question will be obtained by transpositon for the junction point preceding the examined one:

$$\mathbf{H}_t = \overset{i \rightarrow \text{junction point}}{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \end{bmatrix}} \quad j \text{ next branch} \downarrow$$

Junction point i is connected to the junction point at the end of branch j for $h_{t\,ij}$ 1.

Junction points remaining connected during operations at the disconnection point are working or breaking down simultaneously, hence their reliabilities are equal. These conditions can be expressed by the following matrices:

Junction point reliability matrix for short times:

$$\mathbf{R}'_{cs} = \mathbf{R}'_{cs}(\mathbf{S}', \mathbf{H}_t)$$

$$r'_{cs\ ik} = \begin{cases} \prod\limits_j s_{ijk} & \text{with } i = 1 \\ (\prod\limits_j s_{ijk}) r'_{cs\ jk} & \text{with } h_{t\ ji} = 1 \\ r'_{cs\ jk} & \text{with } h_{t\ ji} > 1 \end{cases}$$

where

$r''_{cs\ ik}$ reliability of junction point i corresponding to time interval of code number k.

The operation above is not interpreted for values $h_{t\ ji} = 0$.

Junction point reliability matrix for long times:

$$\mathbf{R}_{cs} = \mathbf{R}'_{cs}(\mathbf{S}''_{cs}, \mathbf{H}_t)$$

$$r''_{cs\ ik} = \begin{cases} \prod\limits_j s_{ijk} & \text{with } i = 1 \\ (\prod\limits_j s_{ijk}) r''_{cs\ jk} & \text{with } 1 \leq h_{t\ ji} \leq 2 \\ r''_{cs\ jk} & \text{with } h_{t\ ji} = 3 \end{cases}$$

where

$r_{cs\ ik}$ reliability of junction point i corresponding to time interval of code number k.

The operation above is not interpreted for values $h_{t\ ji} = 0$.

Junction point reliability matrix can be obtained by uniting short-time and long-time matrices:

$$\mathbf{R}_{cs} = [\mathbf{R}'_{cs}, \mathbf{R}''_{cs}]$$

where

$r_{cs\ ik}$ reliability for junction point i corresponding to time interval of code number k.

Junction point unreliability matrix is:

$$\mathbf{Q}_{cs} = \mathbf{E} - \mathbf{R}_{cs}$$

where

E is unit matrix.

Junction point maintenance reliability

For this calculation — similarly to that of reliability — an auxiliary matrix **T** is applied.

Auxiliary matrix for short-time calculations is:

$$\mathbf{T}' = \mathbf{T}'(\mathbf{M}_r^*, \mathbf{H}_r)$$

$$t'_{ijk} = \begin{cases} 1 & \text{with } h_{ij} < 2 \\ m_{jk}^* & \text{with } h_{ij} \geq 2 \end{cases}$$

Auxiliary matrix for lont-time calculations is:

$$\mathbf{T}'' = \mathbf{T}''(\mathbf{M}_h^*, \mathbf{H}_h)$$

$$t'_{ijk} = \begin{cases} 1 & \text{with } h_{ij} < 3 \\ m_{jk}^* & \text{with } h_{ij} = 3 \end{cases}$$

Junction point maintenance reliability matrix can be calculated with auxiliary matrices and the transposed network matrix.

Junction point maintenance reliability matrix for short times:

$$\mathbf{R}_{cs}^{*\prime} = \mathbf{R}_{cs}^{*\prime}(\mathbf{T}', \mathbf{H}_t)$$

$$r_{cs\ ik}^{*\prime} = \begin{cases} \min\limits_j t_{ijk}^* & \text{with } i = 1 \\ \min[(\min\limits_j t_{ijk}^*), r_{cs\ jk}^*] & \text{with } h_{t\ ji} = 1 \\ r_{cs\ jk}^* & \text{with } h_{t\ ji} > 1 \end{cases}$$

The operation above is not interpreted for values $h_{t\ ji} = 0$.

Junction point maintenance reliability matrix for long times:

$$\mathbf{R}_{cs}^{*\prime\prime} = \mathbf{R}_{cs}^{*\prime\prime}(\mathbf{T}'', \mathbf{H}_t)$$

$$r_{cs\ ik}^{*\prime} = \begin{cases} \min\limits_j t''_{ijk} & \text{with } i = 1 \\ \min[(\min\limits_j t''_{ijk}), r_{cs\ jk}^{*\prime\prime}] & \text{with } 1 \leq h_{t\ ji} \leq 2 \\ r_{cs\ jk}^{*\prime\prime} & \text{with } h_{t\ ji} = 3 \end{cases}$$

The operation above is not interpreted for values $h_{t\ ji} = 0$.

Junction point maintenance reliability matrix:

$$\mathbf{R}_{cs}^* = [\mathbf{R}_{cs}^{*\prime}, \mathbf{R}_{cs}^{*\prime\prime}]$$

where

$r_{cs\ ik}^*$ maintenance reliability of junction point i, corresponding to time interval of code number k.

And finally, junction point maintenance unreliability matrix:

$$\mathbf{Q}_{cs}^* = \mathbf{E} - \mathbf{R}_{cs}^*.$$

Radial network system service reliability is unambiguously determined by matrices $\mathbf{R}_{cs}$ and $\mathbf{R}^*_{cs}$ or $\mathbf{Q}_{cs}$ and $\mathbf{Q}^*_{cs}$.

3. Computer program for radial network system service reliability

A computer program using the presented algorithm developed.

— The program is suitable for the examination of radial network systems.

In its present form the program is unsuitable for loop-type and arc-type network systems. An independent program based on the following considerations is in preparation for this purpose.

— Several parallel branches can be connected between two junction points of a radial network system. Then, however, the number of these parallel branches to be in service to provide availability of the resultant radial network system has to be indicated.
— A junction point must be attached to every disconnection point, the latter arranged off feeding.
— Parallel branches must be connected to the feeding-side junction point through equivalent disconnecting devices.
— Network components indicated by code numbers must be recorded on the data tapes, with the following restrictions:
code number 11. . .19 are reserved for circuit-breakers,
code numbers 20. . .26 are reserved for isolating switches.
— Also discrete outage time intervals must be indicated by code numbers and together with the number of discrete time intervals to be considered short or long ones.
— The program is suitable for radial network systems with not more than 100 junction points.

By way of illustration, let network system in Fig. 2.1 be examined by means of the program. Drawing in branch components, Fig. 3.1 is obtained.

For the given network the program resulted in Fig. 3.2.

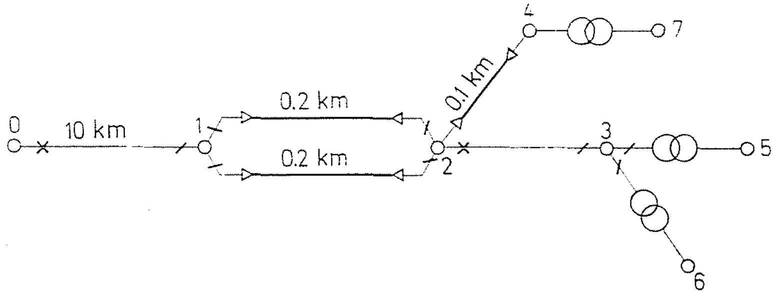

Fig. 3.1

1. Radial Network System Reliability Calculation
Distinctive mark of the programme: HMF 116R-E
Identification number: Radial network
Branch reliability data:

Branch index	Time interval code number	Unreliability	Reliability	Maintenance unreliability	Maintenance reliability
1	1	4624.79	+.9953752110/+00	5878.15	+.9941218500/+00
1	2	1618.15	+.9983818461/+00	5250.68	+.9947493200/+00
1	3	472.35	+.9995276481/+00	2926.56	+.9970734400/+00
2	1	0.35	+.9999996467/+00	0.00	+.1000000000/+01
2	2	0.28	+.9999997201/+00	0.00	+.1000000000/+01
2	3	0.05	+.9999999538/+00	0.00	+.1000000000/+01
4	1	43.01	+.9999569891/+00	160.57	+.9998394280/+00
4	2	32.29	+.9999677092/+00	132.03	+.9998679710/+00
4	3	13.98	+.9999860161/+00	66.78	+.9999332168/+00
7	1	160.19	+.9998398126/+00	1514.76	+.9984852400/+00
7	2	136.40	+.9998635952/+00	1440.61	+.9985593900/+00
7	3	62.19	+.9999378109/+00	412.70	+.9995872990/+00
3	1	4834.16	+.9951658393/+00	5878.15	+.9941218500/+00
3	2	1778.14	+.9982218638/+00	5250.68	+.9947493200/+00
3	3	534.45	+.9994655548/+00	2926.56	+.9970734400/+00
5	1	124.51	+.9998754888/+00	1514.76	+.9984852400/+00
5	2	104.17	+.9998958329/+00	1440.61	+.9985593900/+00
5	3	49.88	+.9999501246/+00	412.70	+.9995872990/+00
6	1	124.51	+.9998754888/+00	1514.76	+.9984852400/+00
6	2	104.17	+.9998958329/+00	1440.61	+.9985593900/+00
6	3	49.88	+.9999501246/+00	412.70	+.9995872990/+00

Junction point reliability data

Junction point index	Time interval code number	Unreliability	Reliability	Maintenance unreliability	Maintenance reliability
1	1	4857.19	+.9951428074/+00	7543.39	+.9924566077/+00
1	2	1810.34	+.9981896568/+00	6814.87	+.9931851276/+00
1	3	484.76	+.9995153396/+00	2926.56	+.9970734400/+00
2	1	4857.19	+.9951428074/+00	7543.39	+.9924566077/+00
2	2	1810.34	+.9981896568/+00	6814.87	+.9931851276/+00
2	3	506.51	+.9994934894/+00	2926.56	+.9970734400/+00
3	1	9884.82	+.9901151764/+00	8887.58	+.9911124190/+00
3	2	3769.39	+.9962306082/+00	8114.71	+.9918852928/+00
3	3	1057.47	+.9989425346/+00	2926.56	+.9970734400/+00
4	1	4857.19	+.9951428074/+00	7543.39	+.9924566077/+00
4	2	1810.34	+.9981896568/+00	6814.87	+.9931851276/+00
4	3	506.51	+.9994934894/+00	2926.56	+.9970734400/+00
7	1	4857.19	+.9951428074/+00	7543.39	+.9924566077/+00
7	2	1810.34	+.9981896568/+00	6814.87	+.9931851276/+00
7	3	556.36	+.9994436392/+00	2926.56	+.9970734400/+00
5	1	9884.82	+.9901151764/+00	8887.58	+.9911124190/+00
5	2	3769.39	+.9962306082/+00	8114.71	+.9918852928/+00
5	3	1094.99	+.9989050127/+00	2926.56	+.9970734400/×00
6	1	9884.82	+.9901151764/+00	8887.58	+.9911124190/+00
6	2	3769.39	+.9962306082/+00	8114.71	+.9918852928/+00
6	3	1094.99	+.9989050127/+00	2926.56	+.9970734400/+00

Fig. 3.2

Table 1

Serial number	Service conditions	Probabilities	Resultant network with 1 branch	Resultant network with 2 branches
			required for carrying the full load	
1	$\overline{A^*} \cap B$	$Q_A^* R_B$	available	half load
2	$\overline{A^*} \cap \overline{B}$	$Q_A^* Q_B$	unavailable	unavailable
3	$\overline{B^*} \cap A$	$Q_B^* R_A$	available	half load
4	$\overline{B^*} \cap \overline{A}$	$Q_B^* Q_A$	unavailable	unavailable
5	$A^* \cap B^* \cap A \cap B$	$R_A R_B - Q_A^* R_B - Q_B^* R_A$	available	available
6	$A^* \cap A \cap \overline{B}$	$R_A Q_B - Q_A^* Q_B$	available	half load
7	$B^* \cap \overline{A} \cap B$	$Q_A R_B - Q_B^* Q_A$	available	half load
8	$\overline{A} \cap \overline{B}$	$Q_A Q_B$	unavailable	unavailable

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Summary

Theoretical fundaments of network system reliability calculations published earlier are valid for radial, loop-type, and grid-type networks. The volume of required numerical calculation, however, imposed to elaborate a computer method.

Algorithm steps made use of different reliability, topology and auxiliary matrices. The computer programme was written in RAZDAN-3 ALGOL language, presented in an example.

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