SENSITIVITY INVARIANTS IN THE THEORY OF NETWORK TOLERANCES AND OPTIMIZATION

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1. Introduction

The sensitivity of the network characteristic $y(x_1, \ldots x_i, \ldots x_N)$ is given by

$$S_i = \frac{\partial \ln y}{\partial \ln x_i} = \frac{x_i}{y} \frac{\partial y}{\partial x_i} = S_i(y, x_i) \tag{1}$$

$S_i(y, x_i)$ means that the sensitivity of the network characteristic has been determined as a function of the network parameter x_i. N denotes the number of network parameters. By definition, the tolerance of the network chatacteristic is given by

$$\frac{\Delta y}{y} = \sum_{i=1}^{N} S_i \frac{\Delta x_i}{x_i} = \sum_{i=1}^{N} S_i(y, x_i) \frac{\Delta x_i}{x_i} \tag{2}$$

However, sensitivity may not only be used for tolerance calculation. The adjustment of circuits, i.e. the field of networks with variable parameters may also be treated by methods based on sensitivity. Up-to-date circuit design such as iterative synthesis (optimization) is also strongly related to sensitivity insofar as partial derivatives, i.e. sensitivities are needed for finding the optimum.

The relative sensitivities taken with respect to different circuit parameters are not independent. Notably, interesting relations are found by calculating the summed sensitivity

$$\sum_{i=1}^{N} S_i(y, x_i) \tag{3}$$

which turns out to be an invariant of the networks.

The purpose of this paper is to give a uniform treatment of the basic sensitivity invariants and to show some of the applications in network theory. In the new method to be used in the following, the properties of unit-systems are only made use of, and so the results may be generalized for a large class of physical systems.

2. Generation of sensitivity invariants

As a starting point, the use of relative units in the circuit is presumed. In the case of the circuit shown in Fig. 1, the network elements and the frequency are expressed in relative units. It is known that from this normalized network arbitrary networks may be derived by a proper selection of the corresponding $R_u - L_u - C_u - \omega_u$ relative units. For instance, if the inductance unit is multiplied by a factor λ, the capacitance unit divided by the same

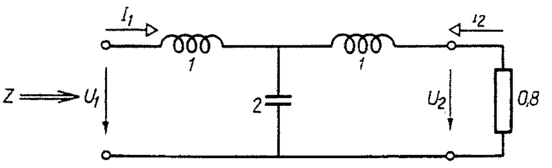


Fig. 1. Circuit to illustrate the consequences of the choice of relative units

factor λ, then the resistance unit is also multiplied by λ and the frequency unit is unchanged. As a result, the input impedance Z is also multiplied by λ. The change of the impedance level does not affect the transfer function $K = U_2/U_1$.

For a straightforward mathematical analysis, let us consider separately the resistance, the inductance and the capacitance in the impedance $Z(x_1, \ldots x_i \ldots x_N, p)$ and let us use the inverse capacitance D. So we have $Z = Z(R_1, \ldots R_{N_R}, L_1, \ldots L_{N_L}, D_1, \ldots D_{N_C}, p)$; where $N_R + N_L + N_C = N$ and $p = \sigma + j\omega$ is the complex frequency. The following relationship holds for the impedances:

$$Z(\lambda R_1, \ldots \lambda R_{N_R}, \lambda L_1, \ldots \lambda L_{N_L}, \lambda D_1, \ldots \lambda D_{N_C}, p) = \lambda Z(R_1, \ldots R_{N_R}, L_1, \ldots L_{N_L}, D_1, \ldots D_{N_C}, p) \tag{4}$$

Differenciating (4) with respect to λ, we have

$$\sum_{i=1}^{N_R} \frac{\partial Z}{\partial \lambda R_i} \frac{\partial \lambda R_i}{\partial \lambda} + \sum_{i=1}^{N_L} \frac{\partial Z}{\partial \lambda L_i} \frac{\partial \lambda L_i}{\partial \lambda} + \sum_{i=1}^{N_C} \frac{\partial Z}{\partial \lambda D_i} \frac{\partial \lambda D_i}{\partial \lambda} = Z \tag{5}$$

and from this expression

$$\sum_{i=1}^{N_R} \frac{R_i}{Z} \frac{\partial Z}{\partial \lambda R_i} + \sum_{i=1}^{N_L} \frac{L_i}{Z} \frac{\partial Z}{\partial \lambda L_i} + \sum_{i=1}^{N_C} \frac{D_i}{Z} \frac{\partial Z}{\partial \lambda D_i} = 1 \tag{6}$$

With the substitution $\lambda = 1$

$$\sum_{i=1}^{N_R} \frac{R_i}{Z} \frac{\partial Z}{\partial R_i} + \sum_{i=1}^{N_L} \frac{L_i}{Z} \frac{\partial Z}{\partial L_i} + \sum_{i=1}^{N_C} \frac{D_i}{Z} \frac{\partial Z}{\partial D_i} = 1 \tag{7}$$

By utilizing the definition (1) of the sensitivity,

$$\sum_{i=1}^{N_R} S_i(Z, R_i) + \sum_{i=1}^{N_L} S_i(Z, L_i) + \sum_{i=1}^{N_C} S_i(Z, D_i) = 1 \tag{8a}$$

or using a single sum notation,

$$\sum_{i=1}^{N} S_i(Z, x_i) = 1 \tag{8b}$$

According to this result, the sum of the impedance sensitivities with respect to R, L and $D = \frac{1}{C}$ is unity. What is now the situation when the frequency unit is changed? In case resistance unit R_u is left unchanged, the change of the frequency unit results in an equal and simultaneous change of the L_u and C_u units. Calling the functions Z, Y, K by a common name as network functions and denoting them by F, this result may be expressed as follows:

$$F\left(R_1, \ldots R_{N_R}, \lambda L_1, \ldots \lambda L_{N_L}, \lambda C_1, \ldots \lambda C_{N_C}, \frac{p}{\lambda}\right) =$$

$$= F(R_1, \ldots R_{N_R}, L_1, \ldots L_{N_L}, C_1, \ldots C_{N_C}, p) \tag{9}$$

Differentiating Eq. (9) with respect to λ, dividing by F, substituting $\lambda = 1$ and rearranging, we have

$$\sum_{i=1}^{N_L} \frac{L_i}{F} \frac{\partial F}{\partial L_i} + \sum_{i=1}^{N_C} \frac{C_i}{F} \frac{\partial F}{\partial C_i} = \frac{p}{F} \frac{\partial F}{\partial p} \tag{10}$$

Introducing the relative sensitivities, we get

$$\sum_{i=1}^{N_L} S_i(F, L_i) + \sum_{i=1}^{N_C} S_i(F, C_i) = S_i(F, p) \tag{11}$$

In case of filters we use the transmission factor Γ instead of F. ($g = \ln \Gamma = a - jb$, where $a(\omega)$ is the attenuation, $b(\omega)$ is the phase and $\tau = \frac{db}{d\omega}$ is the group delay time). Splitting Eq. (11) into real and imaginary parts, we have

$$\sum_{i=1}^{N_L} \mathrm{Re}\, S_i(\Gamma, L_i) + \sum_{i=1}^{N_C} \mathrm{Re} S_i(\Gamma, C_i) = \omega \frac{\mathrm{d}a}{\mathrm{d}\omega} \tag{12a}$$

$$\sum_{i=1}^{N_L} \mathrm{Im}\, S_i(\Gamma, L_i) - \sum_{i=1}^{N_C} \mathrm{Im}\, S_i(\Gamma, C_i) = \omega \frac{\mathrm{d}b}{\mathrm{d}\omega} = \omega\tau . \tag{12b}$$

Thus the sum of the real parts of the sensitivities may be expressed by the derivative of the attenuation with respect to frequency. On the other hand, the sum of the imaginary parts of the sensitivities is related to the group delay time.

In the case where the circuit contains ideal controlled sources, the impedance concept may be extended to include the current controlled voltage sources and the admittance concept, to include voltage controlled current sources. In this way the invariance of the sensitivity sum may be extended, remembering only that the addition has to be performed also for the controlled source parameters.

According to the method introduced above, sensitivity invariants can be generated for a number of classes of networks and systems. For some important cases of linear lumped networks the results are tabulated in Table 1. The notations used in the table are as follows:
The elements are:

$R = \frac{1}{G}$: resistance

$L = \frac{1}{L^{-1}}$: inductance

$C = \frac{1}{D}$: capacitance

$R_G = \frac{1}{G_G}$: gyrator resistance (conductance)

n = transformer ratio of the ideal transformer
A: ideal operational amplifier
k: conversion factor of the negative immittance converter
g: transfer conductance of a voltage controlled current source
r: transfer resistance of a current controlled voltage source
μ: voltage gain of a voltage controlled voltage source
β: current gain of a current controlled current source
B: the elements n, k, μ, β, A
F: all the elements mentioned above

Table 1

Summary of sensitivity invariants of linear networks

Class No.	Permissible elements of the network	Type of the sum	The value of the sum (M) in case of K	Y	Z	$p_i z_i$	c_i
1	G, L^{-1}, C, g, G_G: B	$\sum_{N_1}$	0	1	-1	0	0
2	R, L, D, r, R_G: B	$\sum_{N_1}$	0	-1	1	0	0
3	$\forall$	$\sum_L - \sum_C$	$S(F, p)$	$S(F, p)$	$S(F, p)$	-1	i
4	$\forall$ except L	$\sum_C$	$S(F, p)$	$S(F, p)$	$S(F, p)$	-1	i
5	R, C, r, R_G: B	$\sum_{N_1}$	0	1	1	-2	$2i$
			(K + Y + Z: $-2S(F, p)$)				
6	G, D, g, G_G: B	$\sum_{N_1}$	0	1	-1	2	$-2i$
			(K + Y + Z: $-2S(F, p)$)				
7	R, C, r, R_G: B	$\sum_R + \sum_r + \sum_{R_G}$	0	-1	1	-1	i
			(K + Y + Z: $-S(F, p)$)				
8	R, C, r, R_G: B	$\sum_C$	0	0	0	1	i
			(K + Y + Z: $+S(F, p)$)				
9	L, C: B	$\sum_L$	0	-0.5	0.5	-0.5	$0.5i$
			(K + Y + Z: $+0.5S(F, p)$)				
10	L, C: B	$\sum_C$	0	0.5	-0.5	-0.5	$0.5i$
			(K + Y + Z: $+0.5S(F, p)$)				

The $F = F(p)$ are network functions of the following types:

K: voltage and current transfer function

Y: transfer (or driving-point) admittance

Z: transfer (or driving-point) impedance

The p_i and z_i are poles and zeros of the network, respectively; c_i is a coefficient a_i or b_i in the network function

$$F = \frac{\sum_{i=0}^{m} a_i p^i}{\sum_{i=0}^{n} b_i p^i} \tag{13}$$

where the subscript i refers to the power. The third column of Table 1 contains the various types of the sum:

$$\sum_{N_1} = \sum_{i=1}^{N_1} S(y, x_i) \tag{14}$$

where the summation refers to all of the elements in the first rows of the second column containing the permissible elements in the network: $\sum_R$, $\sum_C$, $\sum_L$, etc. It is interesting to note that if the value of an element is a dimensionless number (the elements of B) then the sensitivity related to this element is not contained in the (invariant) sum of the sensitivities.

Summarizing the method of the generation of the summed sensitivity invariants, the following steps are generally important:

a) the formation of the network characteristic as a function of the elements $y(x_i)$;
b) connections among the relative units;
c) the introduction of the λ factor and the determination of its effect;
d) partial differentiation with respect to λ;
e) set $\lambda = 1$.

It must be noted that constants having dimensions (other than a real or complex number) must be considered as elements.

One limitation of the method is the unsuitability for generating the sum of the absolute values of the sensitivities. Though the method using the energy relations in [6] is limited by the passivity condition, the sum of the absolute values (or a limit of it) can be generated. It must be noted furthermore that not all types of sums can be generated. For example in a network containing the elements of the Class No. 1 in Table 1, the sum of the sensitivities related only to the capacitances cannot be generated. This is because if we change the C_i to λC_i fixing thereby the R_i, to ensure the correct relation between the relative units, the L_i must be changed also to λL_i. Hence, beside the sensitivities to the capacitances also the sensitivities to the inductances will occur in the sum. (See Class No. 3 in Table 1)

3. Nonlinear network example and applications

The method can be used also in cases of nonlinear networks. The constants with dimensions must be taken also into account. Let us consider a nonlinear network containing conductances (G), independent voltage sources (E) and nonlinear two poles described by the equation

$$i = cu^2$$

where i and u are current and voltage, resp., of the nonlinear two pole and c is a positive constant. According to this defining equation, c is not a dimensionless number and its relative unit c_u can be defined by

$$c_u = \frac{I_u}{U_u^2} = \frac{G_u}{U_u}$$

where I_u, U_u and G_u are the relative units of the current, voltage and conductance.

Let the network function be the voltage between any two nodes of the network. We can write the network characteristic U

$$U = U(G_i, E_i, c_i)$$

Because

$$c_u = \frac{\lambda G_u}{\lambda E_u} = \frac{G_u}{E_u} = \frac{G_u}{U_u}$$

so after the introduction of the λ factor we can get

$$U(\lambda G_i, \lambda E_i, c_i) = \lambda U(G_i, E_i, c_i)$$

Differentiating this equation with respect to λ the result is

$$\sum_G \frac{\partial U}{\partial \lambda G_i} \frac{\partial \lambda G_i}{\partial \lambda} + \sum_E \frac{\partial U}{\partial \lambda E_i} \frac{\partial \lambda E_i}{\partial \lambda} = U$$

and dividing this equation by U and setting $\lambda = 1$

$$\sum_G S(U, G_i) + \sum_E S(U, E_i) = 1 \tag{15a}$$

Eq. (15a) shows that the sum of the sensitivities related to the conductances and independent voltage sources is invariant over the class of nonlinear networks specified above. The sensitivity to the constants does not occur in Eq. (15a) because this unit remains unchanged by changing G_u and U_u by λ. However, if we change G_u and c_u by λ and so U_u remains unchanged, then

$$U(\lambda G_i, E_i, \lambda c_i) = U(G_i, E_i, c_i)$$

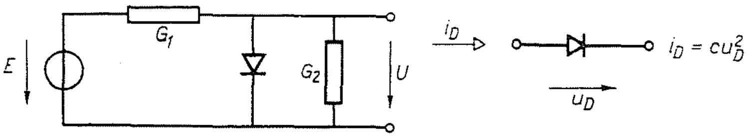
and so

$$\sum_G S(U, G_i) + \sum_C S(U, c_i) = 0 \tag{15b}$$

An example illustrating the invariant of Eq. (15a) is shown in Fig. 2.

If the term $\frac{\Delta x_i}{x_i}$ in Eq. (2) is the same value for all of the network parameters, then:

$$\frac{\Delta y}{y} = \frac{\Delta x_i}{x} \sum_{i=1}^{N} S_i(y, x_i) \tag{16}$$



$$U = \frac{G_1}{2c}\left[-1 - \frac{G_2}{G_1} + \sqrt{\left(1 - \frac{G_2}{G_1}\right)^2 + \frac{4cE}{G_1}}\right] = \frac{G_1}{2c}\left(-1 - \frac{G_2}{G_1} + \sqrt{A_1}\right) = \frac{G_1}{2c} A_2$$

$$S(U, G_1) = \frac{\frac{G_2}{G_1} - \frac{1}{\sqrt{A_1}}\left(1 - \frac{G_2}{G_1}\right)\frac{G_2}{G_1} - \frac{2cE}{\sqrt{A_1}G_1}}{A_2} + 1$$

$$S(U, G_2) = \frac{-\frac{G_2}{G_1} + \frac{1}{\sqrt{A_1}}\left(1 + \frac{G_2}{G_1}\right)\frac{G_2}{G_1}}{A_2}$$

$$S(U, E) = \frac{\frac{2cE}{\sqrt{A_1}\, G_1}}{A_2}$$

Fig. 2. Sensitivity invariant in a nonlinear circuit

Since the summed sensitivity is invariant, in this case the tolerance of the impedance, transfer function and admittance is as follows:

$$\frac{\Delta y}{y} = \begin{cases} \frac{\Delta x_i}{x_i} \\ 0 \\ -\frac{\Delta x_i}{x_0} \end{cases} \tag{17}$$

Eq. (17) shows that as a consequence of the summed sensitivity invariance, the tolerance of the network function depends only on the relative tolerance of the circuit parameters.

It is known that the tolerance of the attenuation of a reactance filter can be expressed by the real parts of the sensitivities:

$$\Delta a = \sum_{i=1}^{N_L+N_C} \operatorname{Re} S_i(\Gamma, x_i) \frac{\Delta x_i}{x_i} \tag{18}$$

In the case of identical relative network parameter tolerances:

$$a = \frac{\Delta x_i}{x_i} \sum_{i=1}^{N_L+N_C} \operatorname{Re} S_i(\Gamma, x_i) \tag{19}$$

According to Eq. (12a) our final result is

$$\Delta a = \frac{\Delta x_i}{x_i} \omega \frac{\mathrm{d}a}{\mathrm{d}\omega} \tag{20}$$

We conclude that the $\frac{\mathrm{d}a}{\mathrm{d}\omega}$ partial derivative has a paramount importance in the calculation of tolerances. From the point of the tolerances the attenuation poles at finite frequencies and the transition from pass band to stop band (the so called no man's land) are critical.

4. Sensitivity optimization

Using the summed sensitivity invariants, a lot of useful results can be derived relating to the problem of optimization, specifically to the design of minimum sensitivity networks.

A network has the minimum sensitivity property if the value

$$P = \sum_{i=1}^{N} |S_i|^2 \tag{21}$$

is minimized, where N is the total number of elements.

After the basic publication of Schoeffler a lot of works were dealt with the problem of minimizing P in various classes of networks.

Leeds and Ugron published the following conjectures relating to the continuously equivalent networks.

a) If a network minimizes P at a given frequency then P is minimized at every frequency.

b) $P_{\min}$ can be reduced by increasing the number of elements.

c) The sum of the sensitivities is invariant over the continuously equivalent networks.

The last conjecture is a special case of the summed sensitivity invariants in Table 1.

To clear the problem, the subsidiary constraints at the minimization are classified as follows.

A. There is no subsidiary constraint.

B. The network function $F(p)$ = constant, the structure of the network is fixed, all possible elements of zero value (within the given structure) are allowed.

C. As in B, but the zero (or infinite) value of the elements is not allowed.

Case A

Let us consider a $GL^{-1}C$ network and optimize it by minimizing the sum of the squares of the sensitivities related to all the elements, that is, P must be minimized and

$$\sum_{i=1}^{N} S_i = M \tag{22}$$

Eq. (22) is a subsidiary constraint. Now, using the Lagrange method, the function

$$P_1 = P + \lambda \left(\sum_{i=1}^{N} S_i - M \right) \tag{23}$$

must be minimized without constraints.

Let be introduced the real and imaginary parts of S_i and M, then introducing the Lagrange multiplicators for the real and imaginary parts we have for Eq. (23)

$$P_1 = \sum_{i=1}^{N} (\mathrm{Re}\, S_i)^2 + \sum_{i=1}^{N} (\mathrm{Im}\, S_i)^2 + \lambda_1 \left(\sum_{i=1}^{N} \mathrm{Re}\, S_i - \mathrm{Re}\, M \right)$$
$$+ \lambda_2 \left(\sum_{i=1}^{N} \mathrm{Im}\, S_i - \mathrm{Im}\, M \right) \tag{24}$$

Differentiating Eq. (24) and using Eq. (22) it can be shown that at the minimum

$$\mathrm{Re}\, S_i = \frac{\mathrm{Re}\, M}{N}; \quad \mathrm{Im}\, S_i = \frac{\mathrm{Im}\, M}{N}; \quad P_{\min} = \frac{(\mathrm{Re}\, M)^2 + (\mathrm{Im}\, M)^2}{N} \tag{25}$$

In the case of a $GL^{-1}C$ network $\mathrm{Im}\, M = 0$ and so

$$\mathrm{Re}\, S_i = \frac{M}{N}; \quad \mathrm{Im}\, S_i = 0; \quad P_{\min} = \frac{M^2}{N} \tag{26}$$

In these computations the possible relation between the real and imaginary parts was not taken into account and $F(p)$ was not fixed, so Eq. (26) refers to the so called absolute minimum. Now, according to this eqaution, the absolute minimum is where all the sensitivities related to the $GL^{-1}C$ elements are equal and real and have the value of Eq. (26). This is more than the equality of the absolute values. According to Eq. (25) the value of the absolute minimum decreases if N increases (see conjecture b).

Because of $|S(F, x_i)|^2 = |S(F, 1/x_i)|^2$, our results are valid for any *RLC* network. The way of thinking can be applied to any class of networks of Table 1.

Case B

Eqs (25) and (26) refer to the absolute minimum, so in case *B*, P_{min} is the lower limit of P. But there is no guarantee that this can be reached.

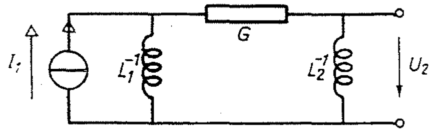


Fig. 3. A circuit having no absolute minimum state

Furthermore, according to [15], a slight difference was found in P_{min} at various frequencies which is in contradiction with conjecture *a*.

Case C

In this case it is very important to note that if the $GL^{-1}C$ network to be optimized has a fixed structure (no element of zero value can be allowed), then in some cases Eq. (25) does not hold at the minimum of P.

Let us consider for instance the circuit of Fig. 3 where

$$F = Z_T(p) = \frac{U_2}{I_1} = \frac{Gp^2}{pG(L_1^{-1} + L_2^{-1}) + L_1^{-1}L_2^{-1}} = \frac{Kp^2}{pb_1 + b_0} \tag{27}$$

and b_1, b_0 are prescribed.

Minimizing

$$P = |S(F, G)|^2 + |S(F, L_1^{-1})|^2 + |S(F, L_2^{-1})|^2 \tag{28}$$

the element values at the minimum are

$$L_1^{-1} = L_2^{-1} = \sqrt{b_0}\,; \quad G = \frac{b_1}{2\sqrt{b_0}} \tag{29}$$

and with these elements:

$$S(F,G)=\frac{b_0}{pb_1+b_0}; \quad S(F,L_1^{-1})=-\frac{p\frac{b_1}{2}+b_0}{pb_1+b_0}.$$

so

$$S(F,G)\neq S(F,L_1^{-1}) \tag{30}$$

even

$$|S(F,G)|\neq|S(F,L_1^{-1})| \tag{31}$$

which means that at the local minimum, Eq. (25) does not hold. Other examples and the detailed discussion of the various results presented in the literature can be found in [12].

A theorem

In a $GL^{-1}C$ network, if F is a voltage ratio or current ratio then no absolute minimum of the sum of the squares of the sensitivities exists, except the pathological case when all the sensitivities are of zero value.

This is because in this case Re M = Im $M = 0$ and so, according to Eq. (25):

$$P_{\min}=0=\sum_{i=1}^{N}|S_i|^2 \tag{32}$$

Eq. (32) holds only when $S_i = 0$ for all elements. These networks are — using the term of Holt and Fiedler — not potentially optimally sensitive. Similar results can be got at the optimization of other types of networks.

It is interesting to note that as a special case of this theorem, in case of the passive RC networks realizing voltage transfer functions the absolute minimum does not exist. In [6] two networks were investigated. Consider the first of them (Fig. 4). If $F(p)$ is not fixed, then $P_{\min} = 0$ ($M = 0$) and this can be realized when $S(F, G_1) = S(F, C_1) = 0$, that is for $G_1 = \infty$ or $C_1 = 0$. If $F(p)$ is fixed, (G_1/C_1 is fixed) then $S(F, G_1) = -S(F, C_1)$ is of constant value (there is no minimization process).

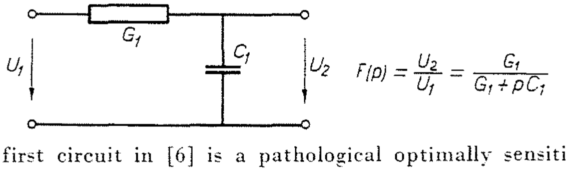


Fig. 4. The first circuit in [6] is a pathological optimally sensitive network

Generally, in networks in which the network function is a bilinear function of the elements, according to [2], $S(F, x_i) = 0$ at all frequencies for $x_i = 0$ or $x_i = \infty$.

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Summary

A new treatment of the summed sensitivity invariants was given on the theoretical basis of unit systems. The results were illustrated by numerous examples, including nonlinear circuits. The sensitivity invariants were applied to the analysis of the tolerances especially for identical parameter variations and filter networks. Optimization questions related to the absolute minimum of the sum of squares of the sensitivities were dealt with.

The absolute minimum of the sum of the squared sensitivities and the conditions for it can be determined using Eqs (25) and (26). This requires more than the equality of the absolute values of the sensitivities.

In caseses when the network has a fixed structure then at the local minimum the absolute values of the sensitivities are different.

There are classes of networks having no absolute minimum state except the pathological case $\forall S_i = 0$

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