STABILITY DEGREE ANALYSIS OF A LINEAR CONTROL SYSTEM WITH DEAD TIME COMPENSATED BY A PROPORTIONAL-PLUS-INTEGRAL CONTROLLER

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Our previous papers [4, 5, 6, 7] were concerned with the variation of the stability of the linear control system with dead time shown in Fig. 1. The transfer function of the plant is:

$$G(s) = \frac{\exp(-s\tau)}{1 + 2\zeta Ts + T^2 s^2}, \tag{1}$$

where

τ — dead time,

T — time constant of the second order lag,

ζ — damping factor.

In the general case the controller is a PID element connected in series, for which

$$C(s) = K\left(1 + \frac{1}{T_i s} + T_d s\right), \tag{2}$$

where

T_i — integral time constant,

T_d — derivative time constant,

K — loop gain.

A later paper determined the limit position of the stability region for proportional P and integral I-type compensations when certain spare stability was also required [8]. Diagrams from a digital computer were presented showing the stability region variation, choosing the phase margin value as $\varphi' = 30°$, $45°$, $60°$, $75°$.

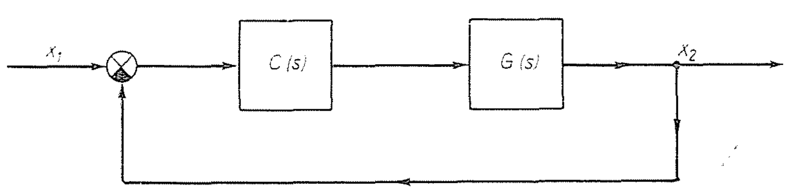

Fig. 1

In many cases it is desirable to choose a proportional-plus-integral element as series connected compensation element. Therefore here the variation of the loop gain of a PI-compensated control—permitting to reach any arbitrary phase margin—in the case of given time constant values will be considered.

Proportional-plus-integral control

With the choice of $T_d = 0$, the transfer function of the controller based on (2) is

$$C(s) = K\left(1 + \frac{1}{T_i s}\right). \tag{3}$$

The transcendent equation determining the loop gain which permits to reach the arbitrary phase margin φ' is:

$$\tan^{-1}\frac{2\zeta T\omega}{1 - T^2\omega^2} + \omega\tau - \tan^{-1}\omega T_i = 90° - \varphi'. \tag{4}$$

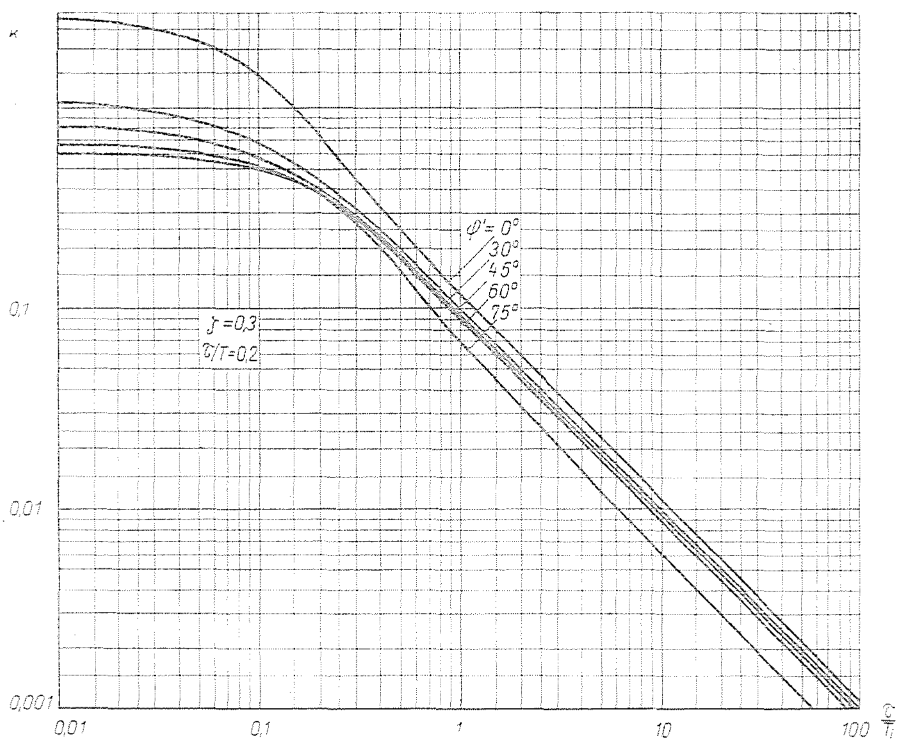

Fig. 2. Proportional-plus-integral control

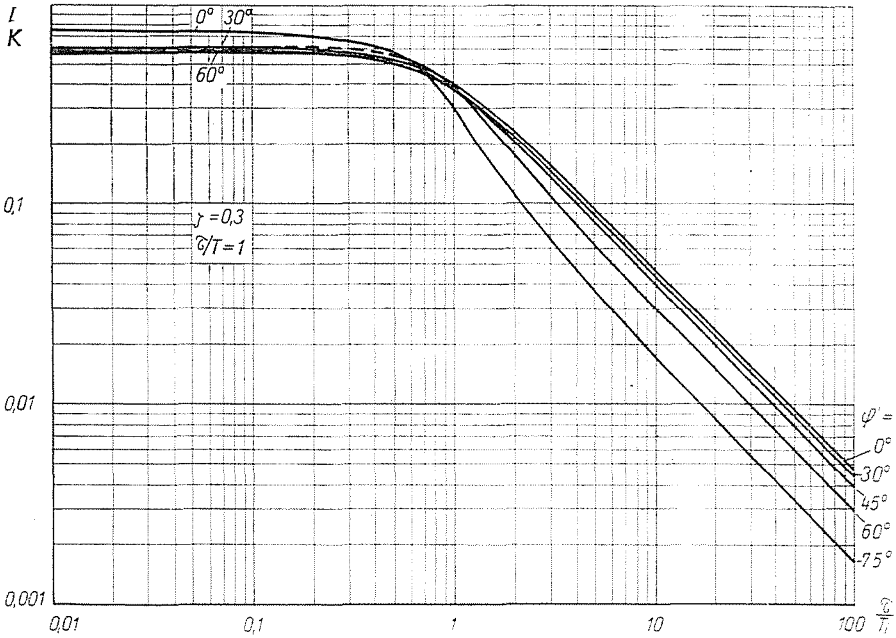

Fig. 3. Proportional-plus-integral control

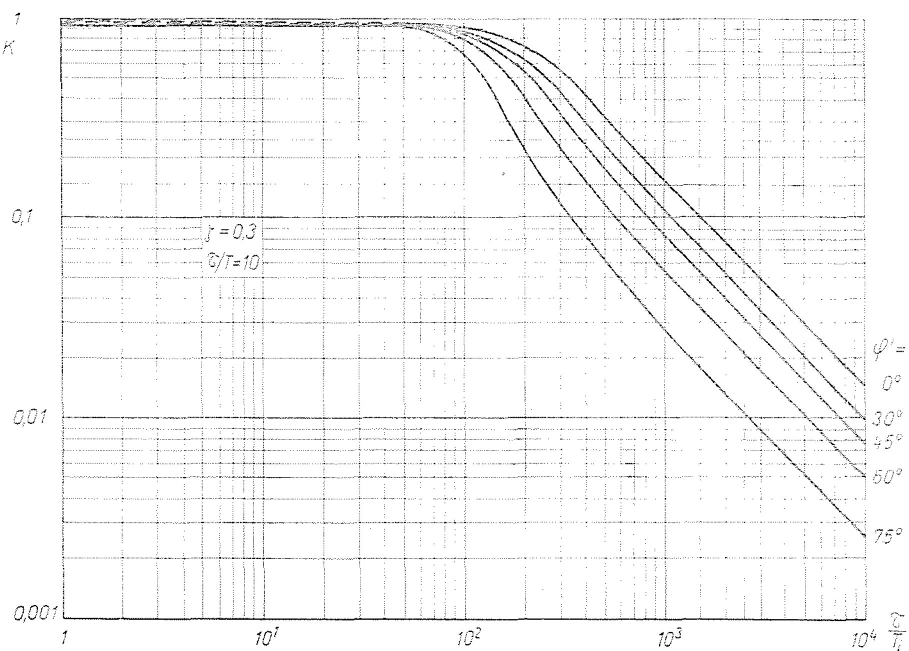

Fig. 4. Proportional-plus-integral control

Solving this transcendent equation by iteration to an arbitrary accuracy the limit position of the stability region is obtained as

$$K = \frac{\omega T_i}{\sqrt{1 + (\omega T_i)^2}} \sqrt{(1 - \tau^2\omega^2)^2 + (2\zeta T\omega)^2}\,. \tag{5}$$

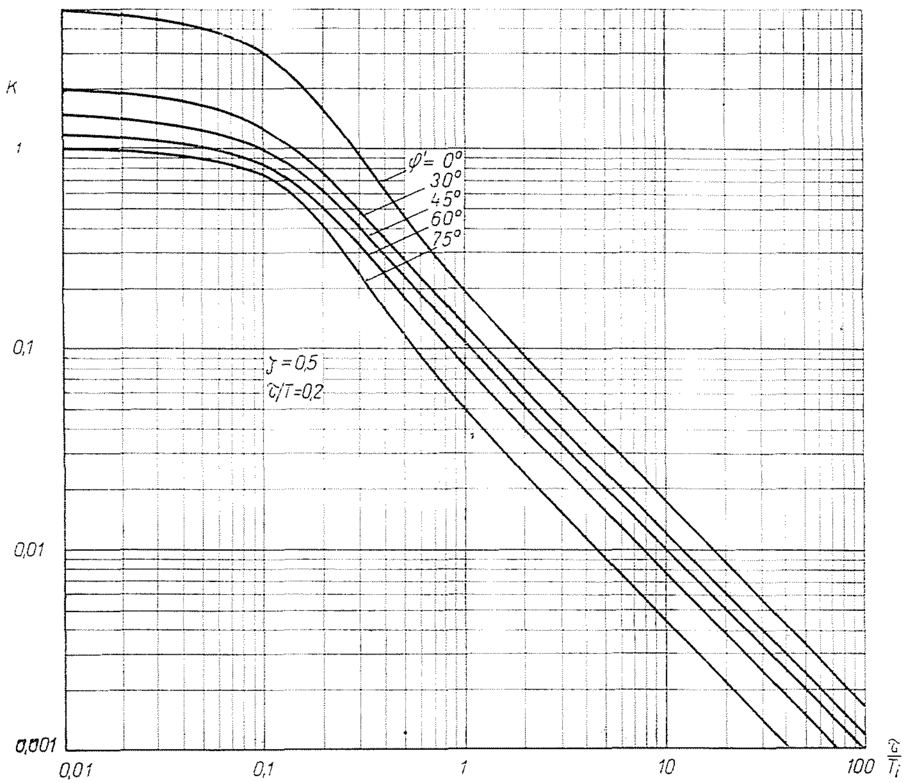

Fig. 5. Proportional-plus-integral control

The variation of the stability region is shown in Figs 2 through 16 in the region of $0.01 \leq \tau/T \leq 100$, when the values of the individual parameters are:

$$\xi = 0.3,\ 0.5,\ 0.7,\ 1,\ 2;$$

$$\tau/T = 0.2,\ 1,\ 10;$$

$$\varphi' = 0°,\ 30°,\ 45°,\ 60°,\ 75°.$$

Reading the diagrams reveals that the control behaves

a) in the case of $\tau/T_i \to 0$ like a control with second order lag and dead time, with a unit feedback and a proportional compensation element connected in series,

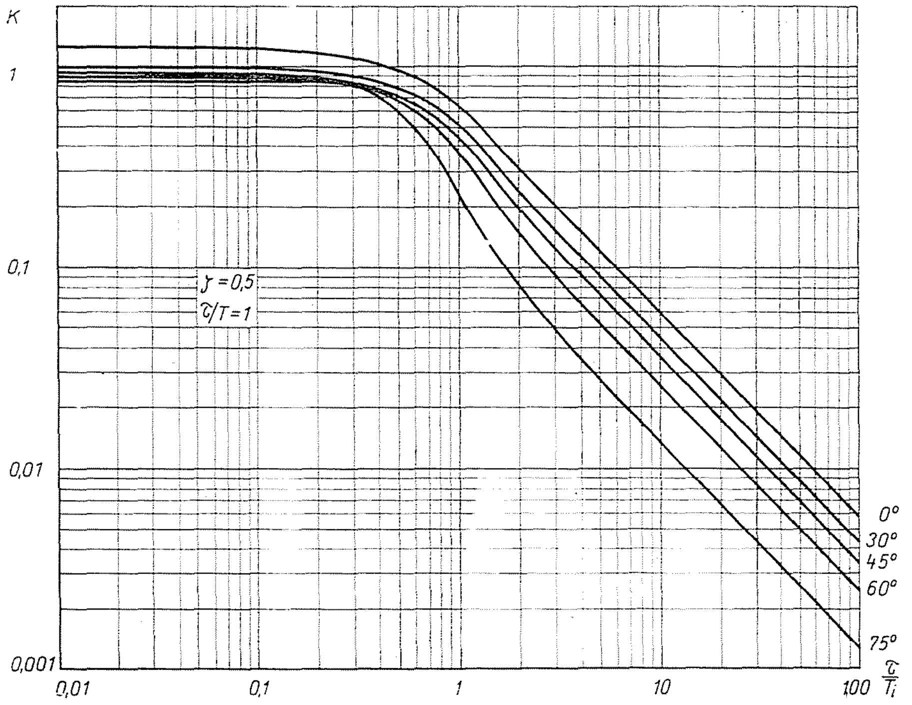

Fig. 6. Proportional-plus-integral control

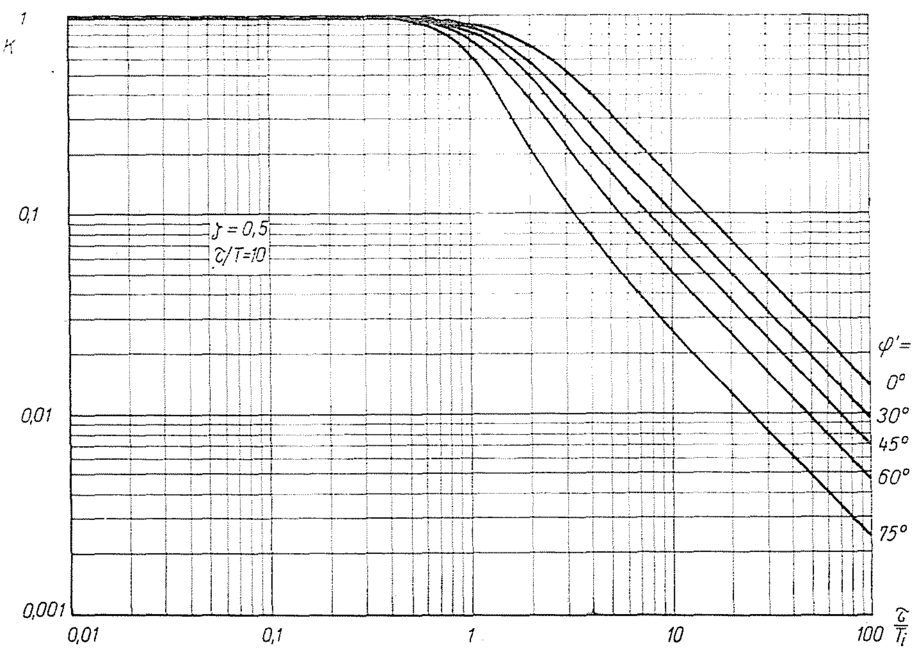

Fig. 7. Proportional-plus-integral control

b) in the case of $\tau/T_i \to \infty$ it behaves like a control with second order lag and dead time, with a unit feedback and an integral compensation element connected in series.

The above two statements satisfy the expectations and are easily appreciated. The frequency function of the opened loop is obtained after certain

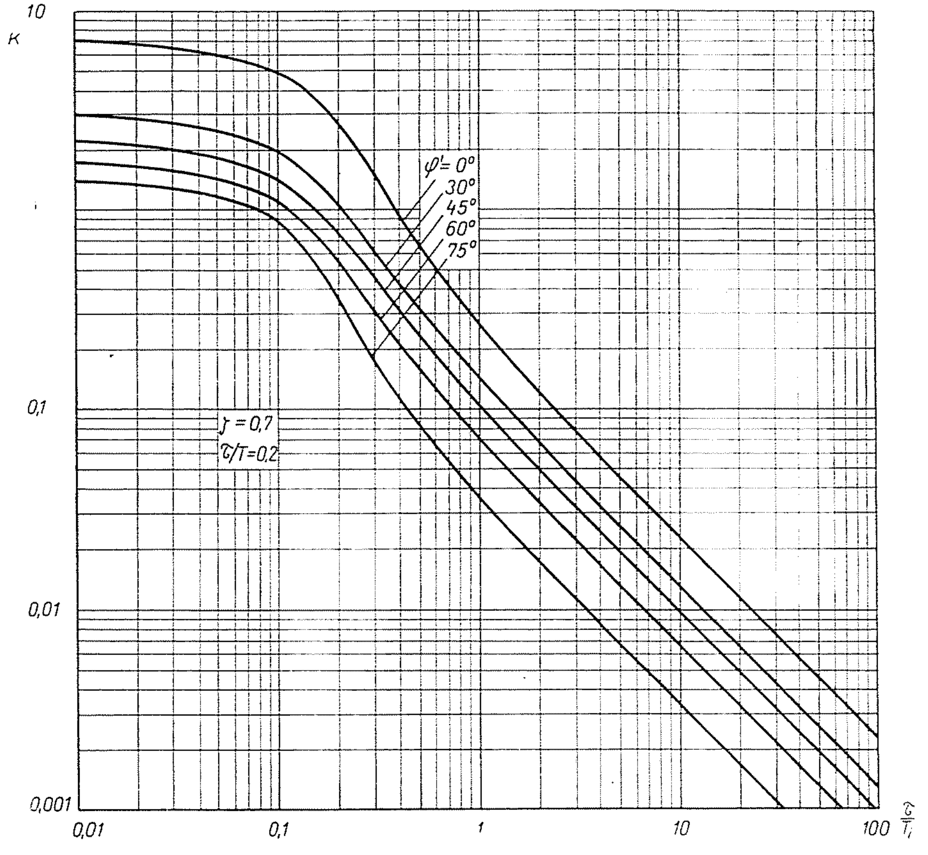

Fig. 8. Proportional-plus-integral control

transformations as

$$Y(j\omega) = K\,\frac{1 + j(\omega\tau)(T_i/\tau)}{j(\omega\tau)(T_i/\tau)}\;\frac{\exp(-j\omega\tau)}{1 - T^2\omega^2 + j2\zeta T\omega}\,.$$

If $\tau/T_i \to 0$, then the frequency function is:

$$Y(j\omega) \approx K\,\frac{\exp(-j\omega\tau)}{1 - T^2\omega^2 + j2\zeta T\omega}$$

in agreement with our first statement.

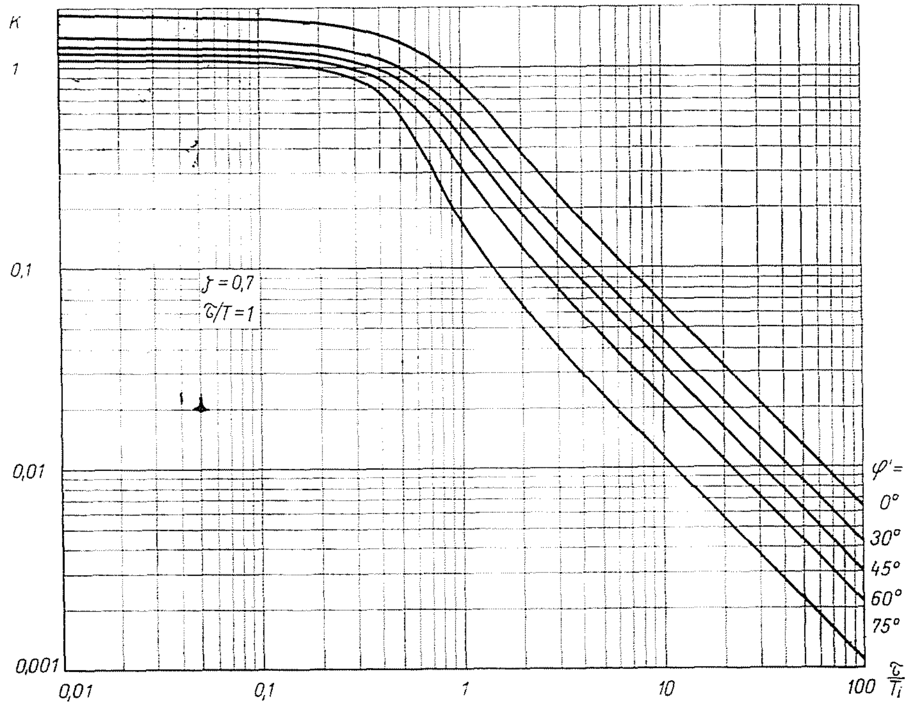

Fig. 9. Proportional-plus-integral control

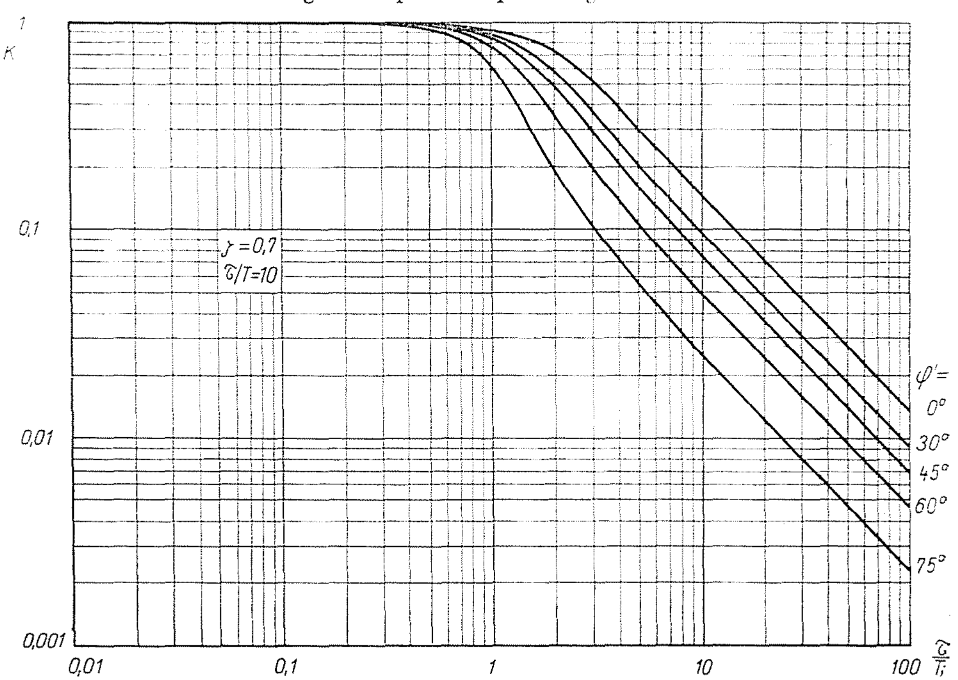

Fig. 10. Proportional-plus-integral control

If $\tau/T_i \to \infty$, then we have

$$Y(j\omega) \approx K \frac{1}{j(\omega\tau)(T_i/\tau)} \frac{\exp(-j\omega\tau)}{1 - T^2\omega^2 + j2\zeta T\omega},$$

in agreement with our second statement.

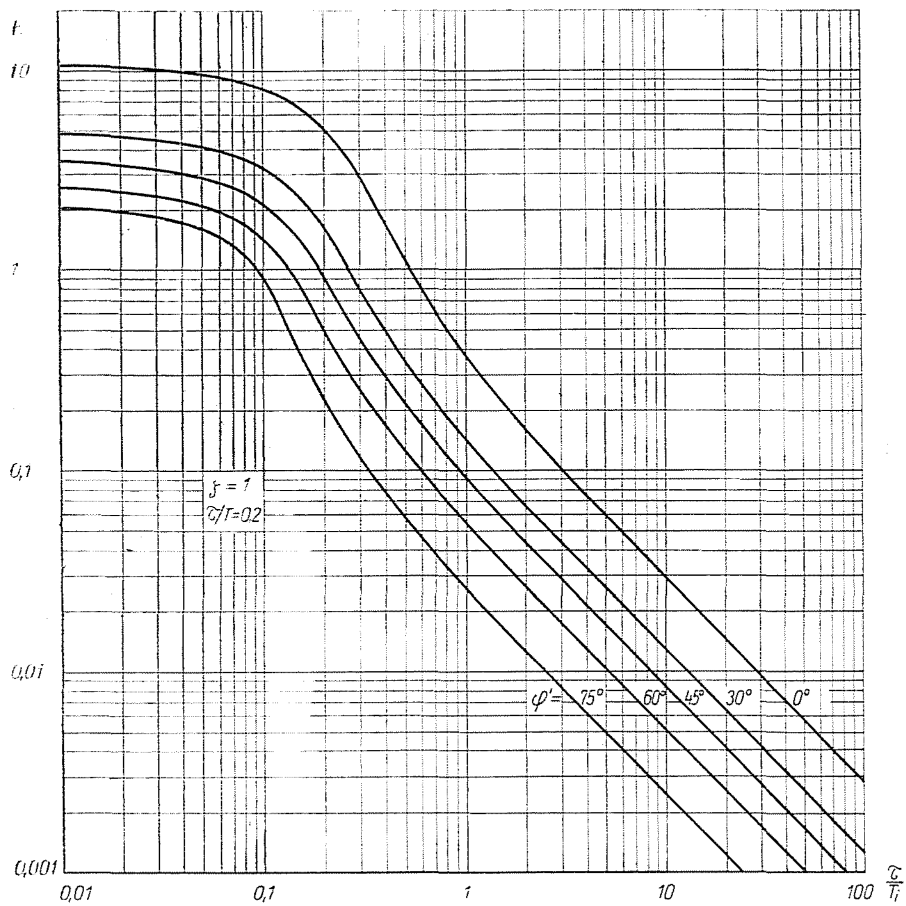

Fig. 11. Proportional-plus-integral control

We note that the dashed sections shown in the individual diagrams for the damping factors of $\zeta = 0.3$ and 0.5 refer to controls of an anormal behaviour (see 1 in [8]). In such cases no conclusions concerning the qualitative characteristics can be drawn from the value of the phase margin.

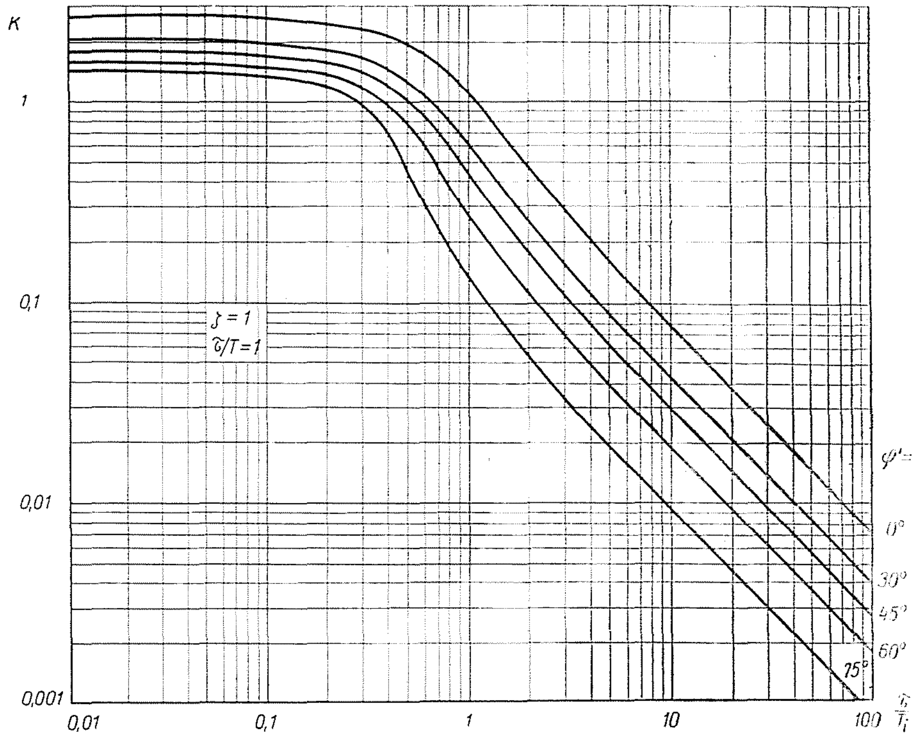

Fig. 12. Proportional-plus-integral control

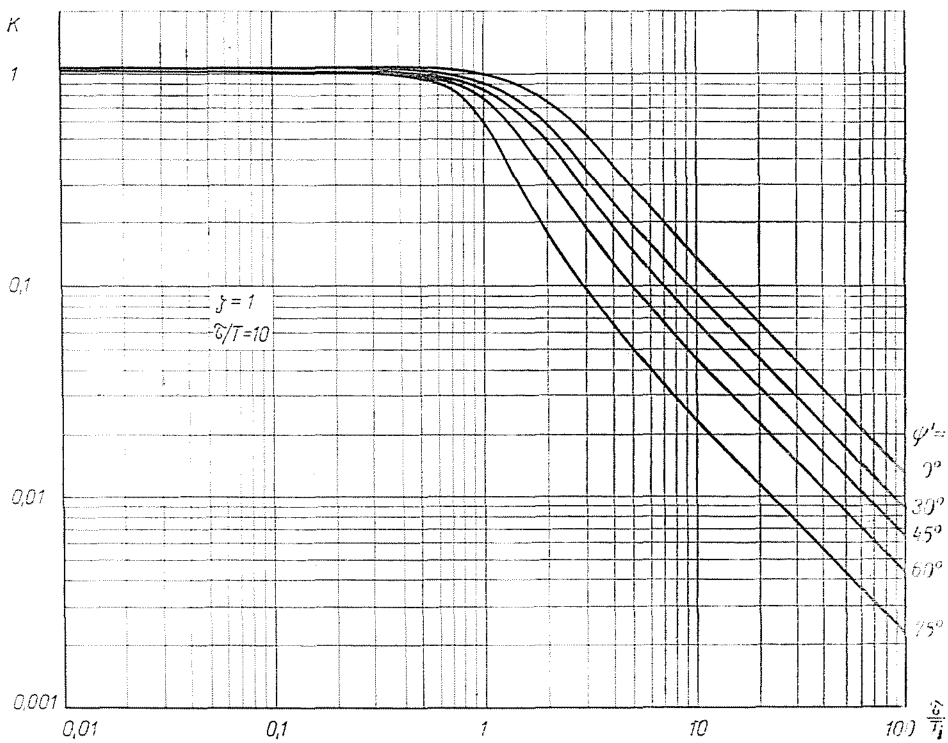

Fig. 13. Proportional-plus-integral control

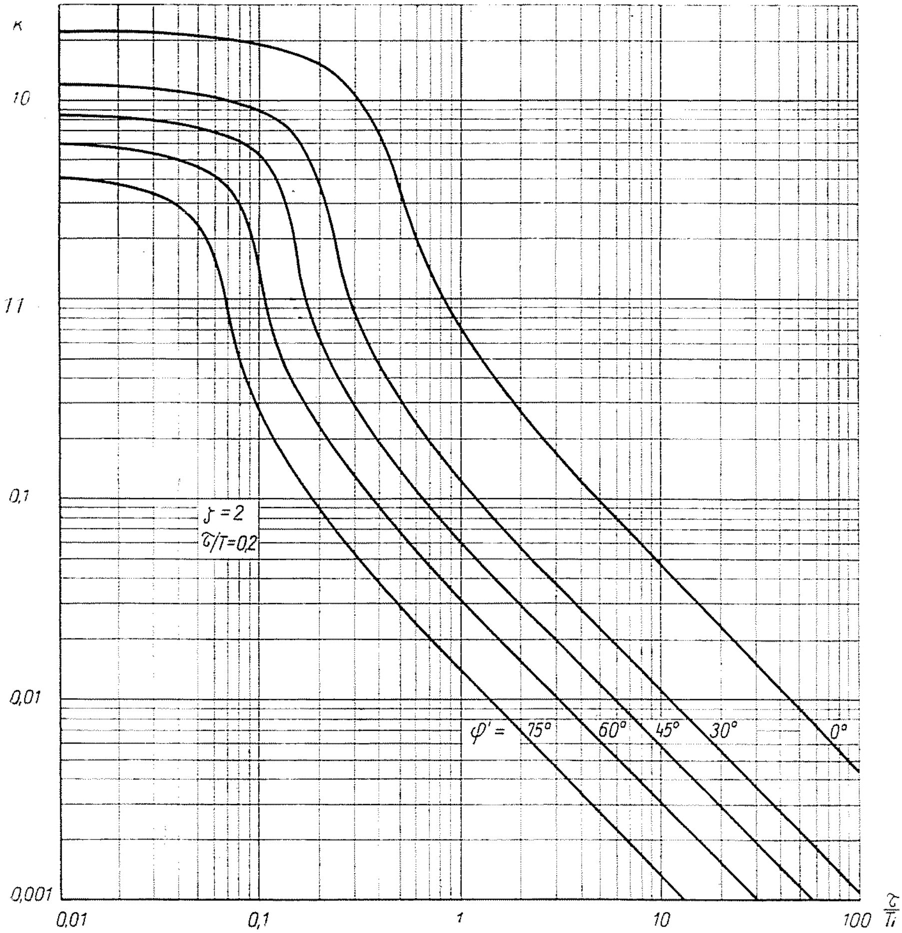

Fig. 14. Proportional-plus-integral control

Final conclusions

A previous paper [8] studied the stability region variation of a control with second order lag and dead time, with a unit feedback compensated by a proportional-plus-integral element connected in series, when a given phase

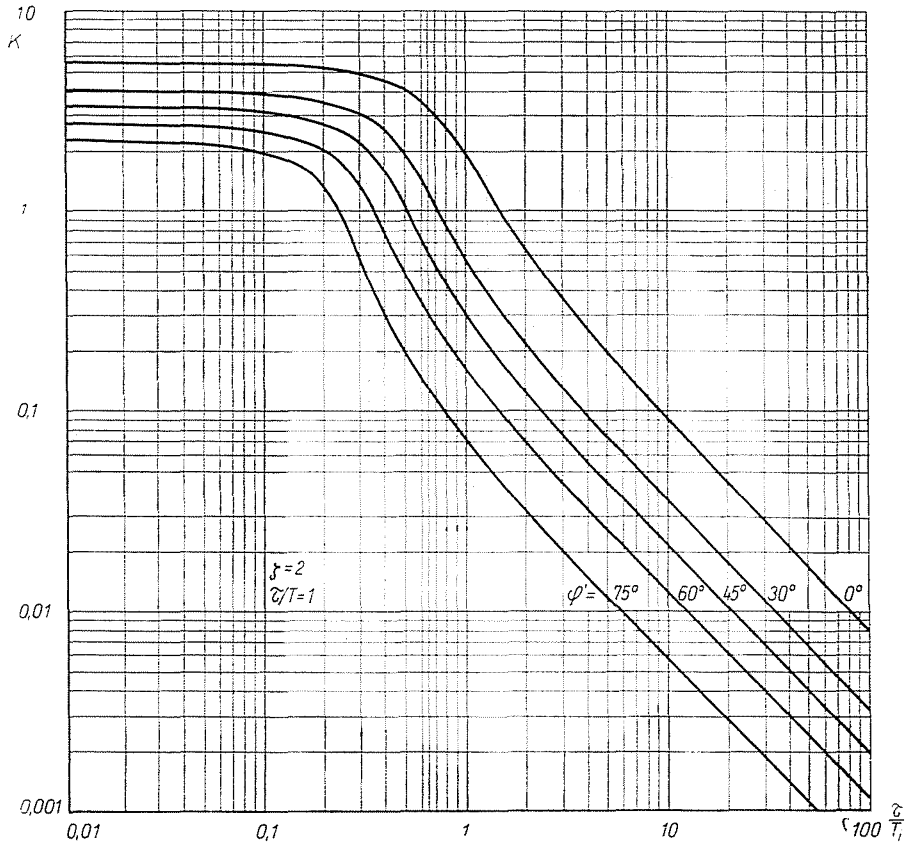

Fig. 15. Proportional-plus-integral control

margin value—referring to the qualitative characteristics—was required. It was also pointed out that for low and high dead time values a proportional-, and an integral-type controller, respectively, is advised. But for medium dead time values another type of compensation, e.g. a P-, or an I-compensation may be more advantageous.

On the basis of the above consideration, the variation of the stability region—permitting to reach an arbitrary phase margin—of a control compensated by a proportional-plus-integral element connected in series has been determined.

Next the type of variation in the course of the stability region brought about by the introduction of a differential element into the controller, will be investigated.

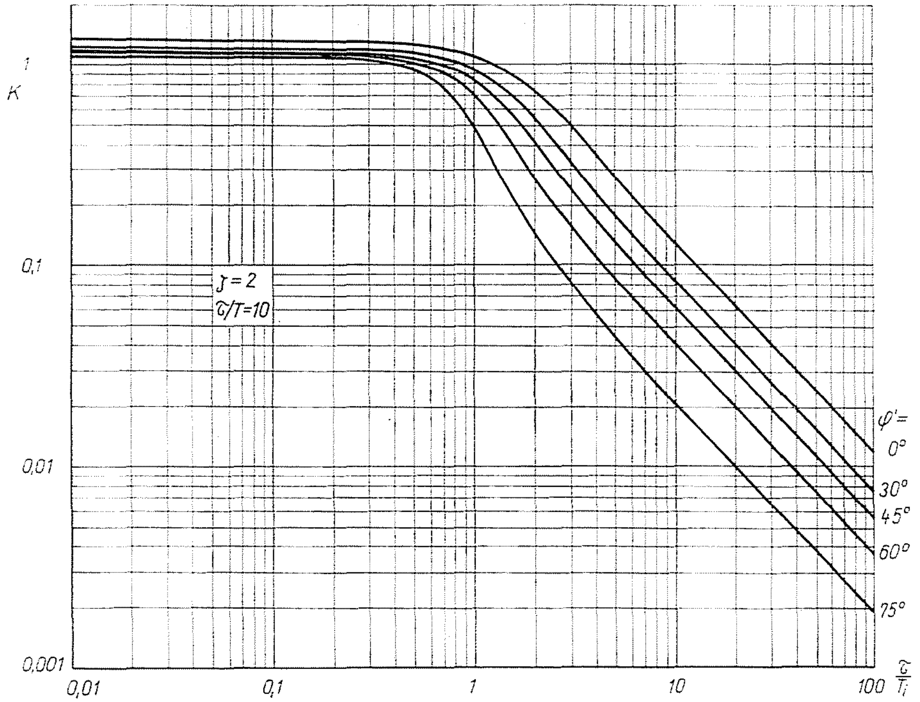

Fig. 16. Proportional-plus-integral control

Summary

Diagrams are presented for the variation of the loop gain—permitting to reach an arbitrary phase margin—of a linear control systems with second order lag and dead time, with a unit feedback and a proportional-plus-integral compensation element connected in series for the dead time pro integral time constant values of $0.01 \leq \tau/T_i \leq 100$. The values of the damping factor were chosen as $\zeta = 0.2, 0.5, 0.7, 1, 2$. Further parameters: $\tau/T = 0.2, 1, 10$, where T is the time constant of the second order lag and the phase margin: $\varphi' = 0°, 30°, 45°, 60°, 75°$. The diagrams were plotted using a digital computer.

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