SOME REMARKS ON PROCESS IDENTIFICATION BY THE STOCHASTIC APPROXIMATION METHOD

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Introduction

The aim of identification is to determine the structure and the parameters of the studied, *a priori* unknown system. As generally a certain structure is assumed, the task simplifies into the determination of the parameters.

A possible identification method applies the principle of stochastic approximation. The essential description of the method has been published after Tsypkin [1] in a previous paper [2].

In the course of the investigation both the system and the model were excited by the same input signal x. The parameters c_i of the model had to be varied in a way to minimize the expected value of the quadratic deviation between the outputs y and $\hat{y}$ of the system and the model, respectively.

$$I(\mathbf{c}) = M[(y - \hat{y})^2] = M[Q(x, \mathbf{c})] = \min \tag{1}$$

where $\mathbf{c}$ — vector of the parameters, and

Q — performance index.

We note that Q may be an absolute value, a time-weighted quadratic or absolute value, or some other criterion as well.

According to the stochastic approximation the optimum $\mathbf{c}^*$ parameter vector can be determined by the following algorithm.

In the discrete form, the expression of the parameter vector $\mathbf{c}$ in the n-th step is

$$\mathbf{c}[n] = \mathbf{c}[n-1] - \mathbf{\Gamma}[n]\nabla_{\mathbf{c}}\, Q(x[n], \mathbf{c}[n-1]) \tag{2}$$

The continuous form of the algorithm is

$$\frac{\mathrm{d}\mathbf{c}(t)}{\mathrm{d}t} = -\mathbf{\Gamma}(t)\nabla_{\mathbf{c}}\, Q[x(t), \mathbf{c}(t)] \tag{3}$$

Here $\nabla_{\mathbf{c}}$ denotes the partial derivation according to the parameters c_i; and $\mathbf{\Gamma}$ is the convergence coefficient, a diagonal matrix with γ_i elements.

We note that the form of the algorithm becomes simpler if the model structure $\hat{y}$ is assumed to be the sum of forms linearly depending on the individual c_i parameters.

Application

Three parameters of an element with a second-order lag were to be identified and the convergence coefficients γ_i ensuring the convergence of the process were sought for. (As some more complex elements can be approximated by a second-order structure, the validity of the method can be extended.) The transfer function of the system is

$$W(s) = \frac{Y(s)}{X(s)} = \frac{A}{1 + 2\zeta Ts + T^2 s^2} \tag{4}$$

i.e. the differential equation in the time domain is

$$T^2 \frac{d^2 y}{dt^2} + 2\zeta T \frac{dy}{dt} + y = Ax \tag{5}$$

This equation divided by T^2, then integrated twice and arranged for y, results in the following relationship:

$$y = -\frac{1}{T^2} \int_0^t \int_0^\tau y \, d\vartheta \, d\tau - \frac{2\zeta}{T} \int_0^t y \, d\tau + \frac{A}{T^2} \int_0^t \int_0^\tau x \, d\vartheta \, d\tau \tag{6}$$

The model is assumed in a similar form:

$$\hat{y} = c_1 \int_0^t \int_0^\tau y \, d\vartheta \, d\tau + c_2 \int_0^t y \, d\tau + c_3 \int_0^t \int_0^\tau x \, d\vartheta \, d\tau \tag{7}$$

or

$$\hat{y} = c_1 \Theta_1 + c_2 \Theta_2 + c_3 \Theta_3 , \tag{8}$$

where

$$c_1^* = -\frac{1}{T^2} ; \qquad c_2^* = -\frac{2\zeta}{T} ; \qquad c_3^* = \frac{A}{T^2} ;$$

and

$$\Theta_1 = \int_0^t \int_0^\tau y \, d\vartheta \, d\tau ; \qquad \Theta_2 = \int_0^t y \, d\tau ; \qquad \Theta_3 = \int_0^t \int_0^\tau x \, d\vartheta \, d\tau .$$

The continuous algorithm for the variation of the parameters is:

$$
\begin{aligned}
\frac{dc_1}{dt} &= \gamma_1(y - \hat{y}) \int_0^t \int_0^\tau y d\vartheta d\tau \\
\frac{dc_2}{dt} &= \gamma_2(y - \hat{y}) \int_0^t y d\tau \\
\frac{dc_3}{dt} &= \gamma_3(y - \hat{y}) \int_0^t \int_0^\tau x d\vartheta d\tau .
\end{aligned}
\tag{9}
$$

The scheme of the solution is shown in Fig. 1. The convergence was investigated for constant, hyperbolic and optimum γ coefficients with a deterministic $\cos \omega t$ course of the input signals, for various ω values, with the help of the BOCS-program developed for a MINSK-22 type digital computer.

For constant γ values the convergence is rather occasional, good values were obtained up to $\omega T \approx 1$ in the range of $\gamma = 20 - 50$. But the transients of the adjustment process showed considerable oscillations; adjustment was reached after 8—10 periods approximately (Fig. 2). In the case of higher

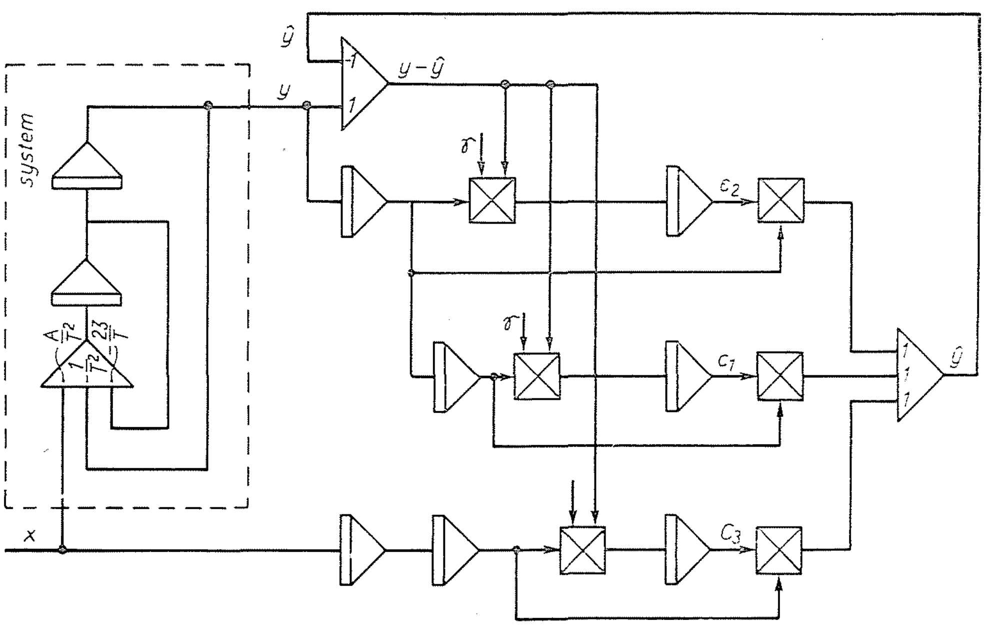

Fig. 1. Identification scheme of a second order lag element

ωT values, γ had to be increased (e.g. for $\omega T = 1.5$, $\gamma = 250$ proved to be a fair value). The accuracy of the adjustment is good.

In investigating the hyperbolic γ, the system parameters $A = 2$, $T = 2$, $\zeta = 0.2$ were considered. In the vicinity of the break-point frequency, a good convergence was given by the values $\gamma = C/(t + D)$; $C = 60 — 100$; $D = 0.1 — 1$ (Fig. 3). Assuming initial parameter values the convergence may be somewhat improved.

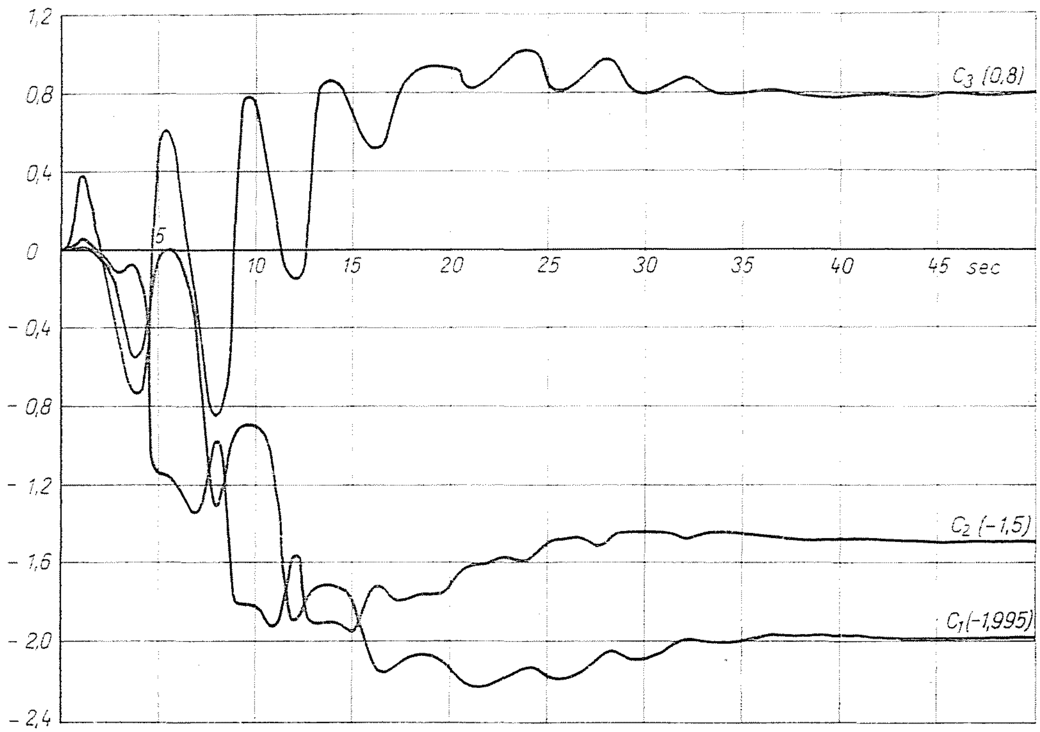

Fig. 2. The dynamics of the identification process with constant γ values. $A = 0.4$; $T = \sqrt{0.5}$; $\zeta = 0.53$; $f_x = 0.25$ c/s; $\gamma = 50$; $c_1^* = -2$; $c_2^* = -1.5$; $c_3^* = 0.8$

The effect of the initial oscillations may be considerably increased by decreasing the values of the circuit frequency and D, and the process may lose its convergence.

By assuming optimum convergence coefficients, the convergence may be accelerated and the uncertainty of the process considerably reduced.

According to TSYPKIN [1], the optimum convergence coefficients are given by the following relationship:

$$\mathbf{\Gamma}_{\text{opt}} = \frac{1}{\int\limits_0^t \nabla_c^2 Q[x(\tau), \mathbf{c}(t)] d\tau} \tag{10}$$

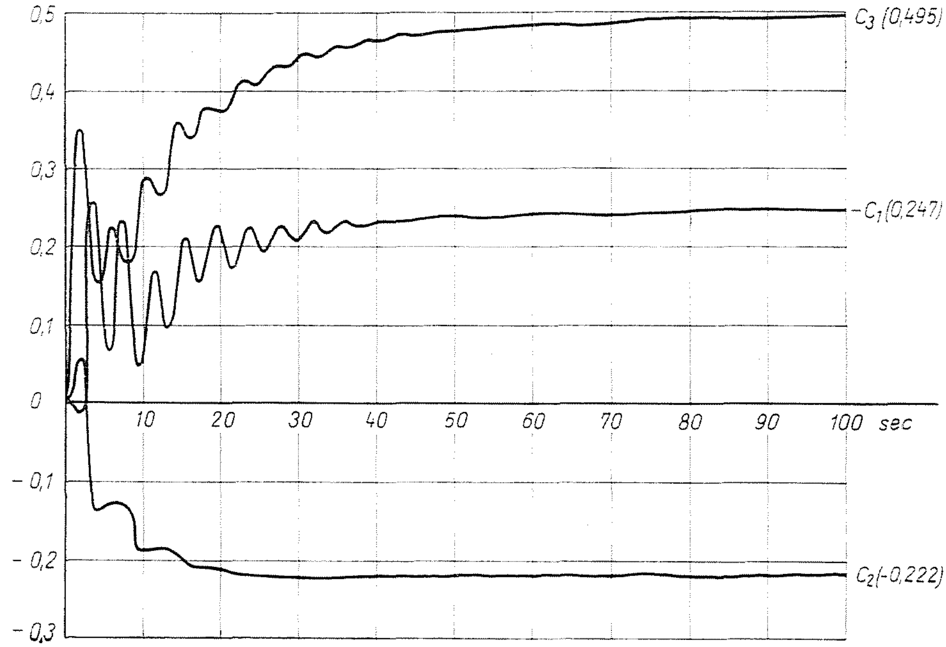

Fig. 3. The dynamics of the identification process with hyperbolic γ values. $A = 2$; $T = 2$; $\zeta = 0.2$; $f_x = 0.25$ c/s; $\gamma = 100/(t + 1)$; $c_1^* = -0.25$; $c_3^* = -0.2$; $c^* = 0.5$

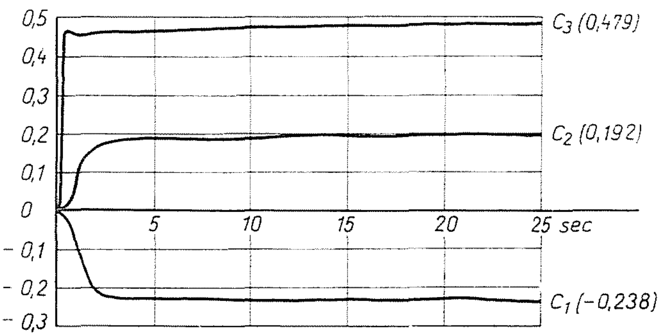

Fig. 4. The dynamics of the identification process with optimum γ. $A = 2$; $T = 2$; $\zeta = 0.2$; $f_x = 0.25$ c/s; $k_1 = k_2 = k_3 = 1$; $c_1^* = -0.25$; $c_2^* = -0.2$; $c_3^* = 0.5$

In our case

$$\gamma_{1\,\mathrm{opt}} = \frac{k_1}{\int\limits_0^t \Theta_1^2 \mathrm{d}\tau}; \qquad \gamma_{2\,\mathrm{opt}} = \frac{k_2}{\int\limits_0^t \Theta_2^2 \mathrm{d}\tau}; \qquad \gamma_{3\,\mathrm{opt}} = \frac{k_3}{\int\limits_0^t \Theta_3^2 \mathrm{d}\tau}. \tag{11}$$

The value of the constants k_i is 1 according to (10), but their values may be varied within a small range in the interest of further improving the convergence. The dynamics of the identification process with the previous values ($A = 2$, $T = 2$, $\zeta = 0.2$) is shown in Fig. 4. It is seen that no oscillations occur in the transients and the adjustment is fast, within one period it is

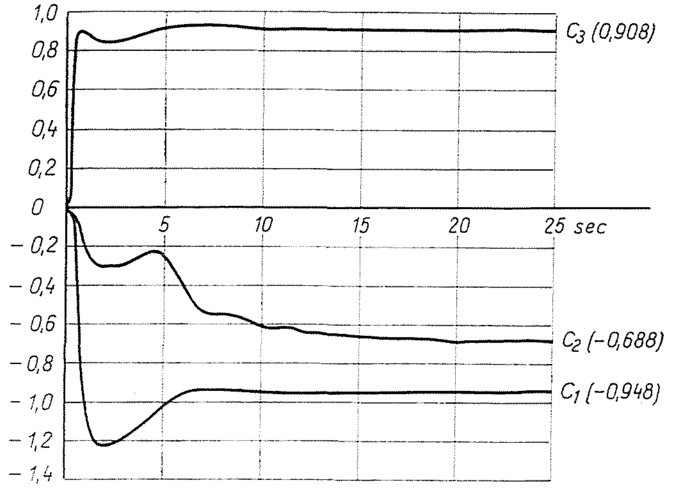

Fig. 5. The dynamics of the identification process with optimum γ. $A = 1$; $T = 1$; $\zeta = 0.4$; $f_x = 0.158$ c/s; $k = 1$; $c_1^* = -1$; $c_2^* = -0.8$; $c_3^* = 1$

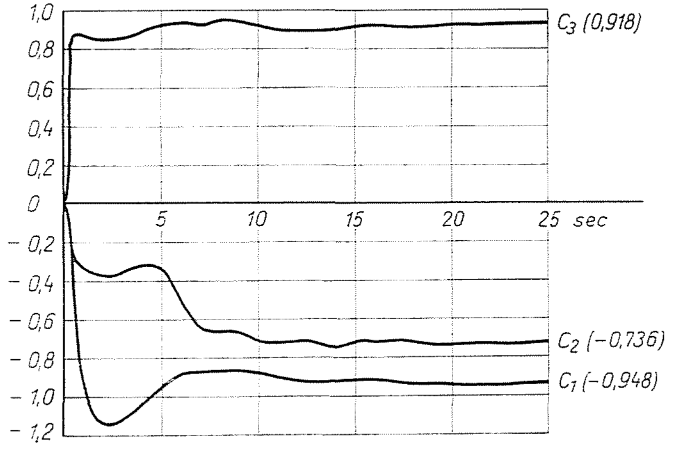

Fig. 6. The dynamics of the identification process with optimum γ. $A = 1$; $T = 1$; $\zeta = 0.4$; $f_x = 0.158$ c/s; $k = 2$; $c_1^* = -1$; $c_2^* = -0.8$; $c_3^* = 1$

essentially completed. With the system parameters assumed as $A = 1$, $T = 1$ and $\zeta = 0.4$, the variation of the model parameters c_i is shown in Figs 5 through 8. The convergence proved to be satisfactory from the break-point $1/T$ of the element's Bode-diagram for circuit frequency input signals assumed in the range of ± 1 decade. With a circuit frequency higher than 10-times that of the break-point, no convergence can be ensured because of the great attenuation of the element. The parameters c_i approach their steady values within

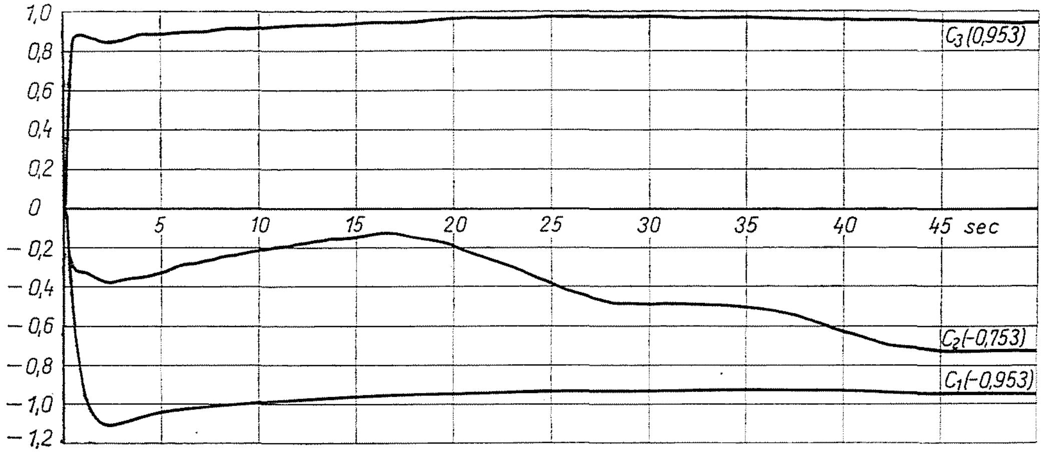

Fig. 7. The dynamics of the identification process with optimum γ. $A = 1$; $T = 1$; $\zeta = 0.4$; $f_X = 0.0316$ c/s; $k = 2$; $c_1^* = -1$; $c_2^* = -0.8$; $c_3^* = 1$

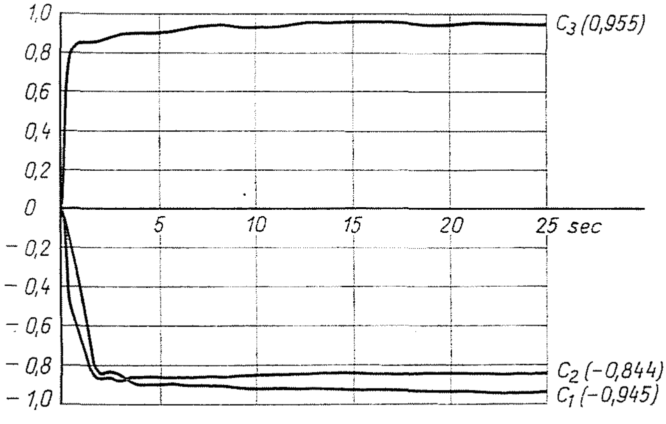

Fig. 8. The dynamics of the identification process with optimum γ. $A = 1$; $T = 1$; $\zeta = 0.4$; $f_X = 1.58$ c/s; $k = 2$; $c_1^* = -1$; $c^* = -0.8$; $c_3^* = 1$

2—3 periods. According to the experiments, the k_i values must be assumed as identical or else the convergence will deteriorate considerably, or oscillations appear. The convergence of the parameter c_2 containing ζ is more sensitive to the variation of both the k and the ω values than that of both other parameters. E.g., for $k = 0.5$, the convergences of c_1 and c_3 were good, while c_2 attained only about 60% of its required value. Assuming $k = 2$, an improvement was

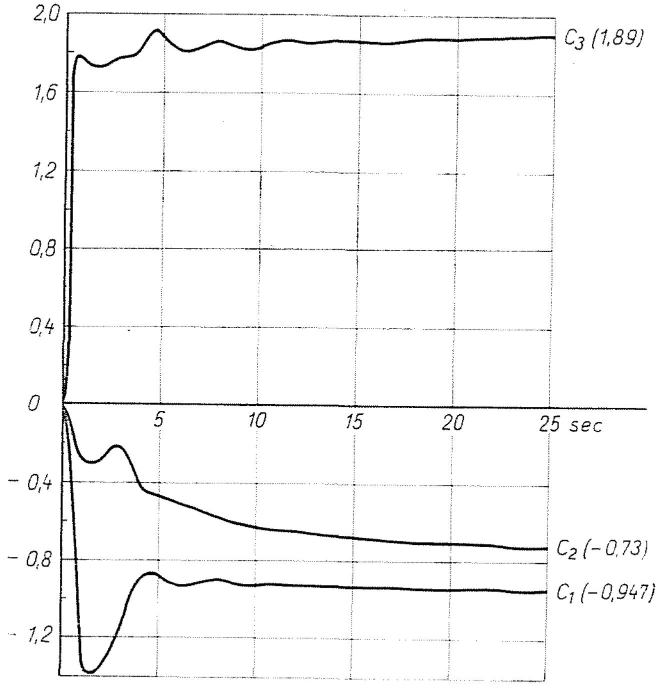

Fig. 9. The dynamics of the identification process with optimum γ. $A = 2$; $T = 1$; $\zeta = 0.4$; $f_x = 0.316$ c/s; $k = 2$; $c_1^* = -1$; $c_2^* = -0.8$; $c_3^* = 2$

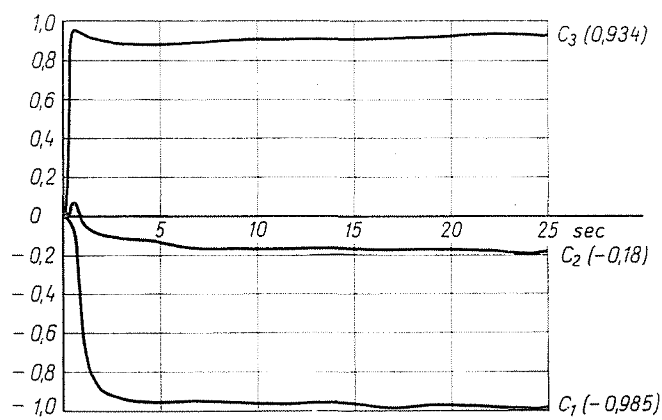

Fig. 10. The dynamics of the identification process with optimum γ. $A = 1$; $T = 1$; $\zeta = 0.1$; $f_x = 0.158$ c/s; $k = 1$; $c_1^* = -1$; $c_2^* = -0.2$; $c_3^* = 1$

noted in comparison with the results for $k = 1$. But by increasing the value of k further, the process became unstable.

The dynamics of the identification process for the variation of the parameters A, ζ and T, are shown in Figs. 9, 10 and 11, respectively.

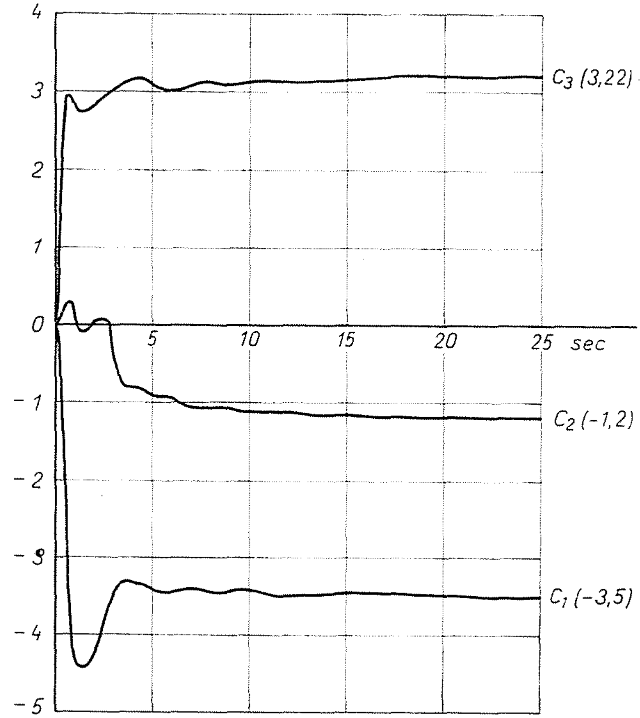

Fig. 11. The dynamics of the identification process with optimum γ. $A = 1$; $T = 0.5$; $\zeta = 0.4$; $f_x = 0.316$ *c/s*; $k = 1$; $c^* = -4$; $c_2^* = -1.6$; $c_3^* = 4$

Summary

The application of the principle of stochastic approximation for the identification of an element with a second order lag is presented. The convergence of the process for constant, hyperbolic and optimum convergence coefficients with deterministic input signals of a cos ωt course is investigated. For the identification certain *a priori* informations are necessary, concerning the order of magnitude of the system parameters for the proper selection of the input signal. Some guidance for the assumption of the convergence coefficients is given.

References

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