DIAGRAMS FOR THE OPTIMUM SETTING OF CONTROLS

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(Received May 4, 1971)

The adjustment of the conventional (series connected) PI, PID, etc. controllers of linear, concentrated-parameter, single loop control circuits, i.e. the appropriate choice of the compensation parameters has been amply dealt with by the professional literature.

The applied discussion methods relied in most cases on the frequency and the complex operator domains; so the utilized quality characteristics were also defined there. The illustrative power of the results defined as optimum obtained in this way is very poor—for lack of a simple connection with the time range—as the transition is simple only in principle.

It can be stated generally that the designer with an -engineering attitude can best appreciate the dynamic properties of a control process by the course of the time functions (e.g. the error signal) of a control circuit. This is why relationships between the frequency and the quality characteristics belonging to the time range were sought for. It must be admitted that these methods led also, even at best, to some thumb-rules only. It will be seen later that even one of the most accepted methods for setting the controllers, suggested by Ziegler and Nichols [1], represents only a rather uncertain and sometimes inapplicable process.

From among the methods used for setting the parameters of the P, PI, PID, etc. controllers of conventional construction, the best results were obtained by those adjusting the required quality characteristics in the time range by modelling a control circuit of some structure.

Accounts of such investigations with modelling the control circuit by analogue computers and finding the optimum parameters by adjusting the minimum value of some integral criterion are found among others in the literature [5, 6].

The analogue computer is not suitable for the great number of measurements necessary for the elaboration of a satisfactory setting diagram and for the required accuracy and so the published diagrams suppose a very simple structure in the controlled section and are rather inaccurate.

Because of the problems outlined above, it is advised to rely on a digital computer for the solution of the optimum dimensioning.

At the Chair of Automation of the Technical University Budapest an ALGOL program family, suitable for the optimum dimensioning of the parameters of single-loop, linear, concentrated-parameter control circuits and of those with dead time based on the quadratic integral of some characteristic time function of the system has been developed. The description of the program package and the utilized computation process are found in [2, 3]; the latter are shortly summarized in the Appendix of this paper. Further experiences and the successful tests are described in [4, 9].

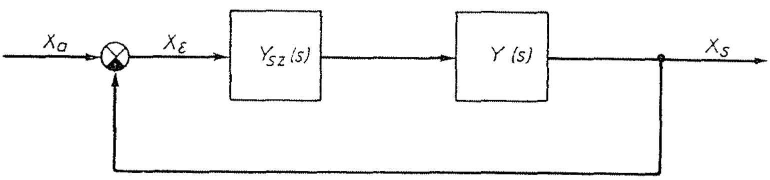


Fig. 1. x_a — reference signal; x_ε — error signal; x_s — controlled signal

In this paper the setting diagrams worked out with the help of the above program package for PI and PID controllers are presented.

In our investigations the simplified control circuit in Fig. 1 was used. A proportional element of three time lags was chosen for the structure of the controlled section. This construction is satisfactory in the majority of the practical cases. It must also be noted that at present the identification of the three highest time constants of a controlled section may be regarded already to be a simple task [6]. So $Y(s)$ had the form of

$$Y(s) = \frac{1}{(1 + sT_1)(1 + sT_2)(2 + sT_3)}$$

The optimum adjustment of a PI and a PID control for the controlled section supposed to have this form has been carried out with the help of the program package for quite a wide range $(0.01 \rightarrow 1.0)$ of the parameters T_2/T_1 and T_3/T_1, respectively.

The transfer function of the applied PI control:

$$Y_{sz}(s) = Y_{PI}(s) = K\left(1 + \frac{1}{sT_I}\right)$$

while that of the PID control:

$$Y_{sz}(s) = Y_{PID}(s) = K\left(1 + \frac{1}{sT_I} + \frac{sT_D}{1 + 0.2sT_D}\right)$$

(K being the loop amplification. T_I the integration time and T_D the differentiation time).

With the latter control the differentiation effect is of an approximative character in order to have it nearer to the controls generally used in practice. (The time constant of the differentiation may be regarded to be a good average.)

The results are summarized in the following. As the compensation depends on the relative values of the time constants, the section has been characterized by two parameters:

$$\frac{T_2}{T_1} = a \qquad \text{and} \qquad \frac{T_3}{T_1} = b$$

The obtained optimum adjustment values were given also in relative values, accordingly in the form of

$$\frac{K_{\text{opt}}}{K_{kr}}; \quad \frac{T_{I\,\text{opt}}}{T_{kr}} \quad \text{and} \quad \frac{T_{D\,\text{opt}}}{T_{kr}}$$

(Here K_{kr}—critical circuit amplification; T_{kr}—critical oscillation period.)

The starting points for the searching algorithms of the programs were chosen as $T_I^0 = T_1$ and $T_D^0 = \max\{T_2, T_3\}$ (supposing the relation $T_1 > T_2, T_3$ to be valid). The initial point of the parameter K was chosen as half of the critical circuit amplification of the system *modified* in this way, i.e.:

$$K^\circ = \frac{0.5T_1}{T_2T_3}(T_i + 5T_j)$$

where $T_i = \max\{T_2, T_3\}$ and $T_j = \min\{T_2, T_3\}$.

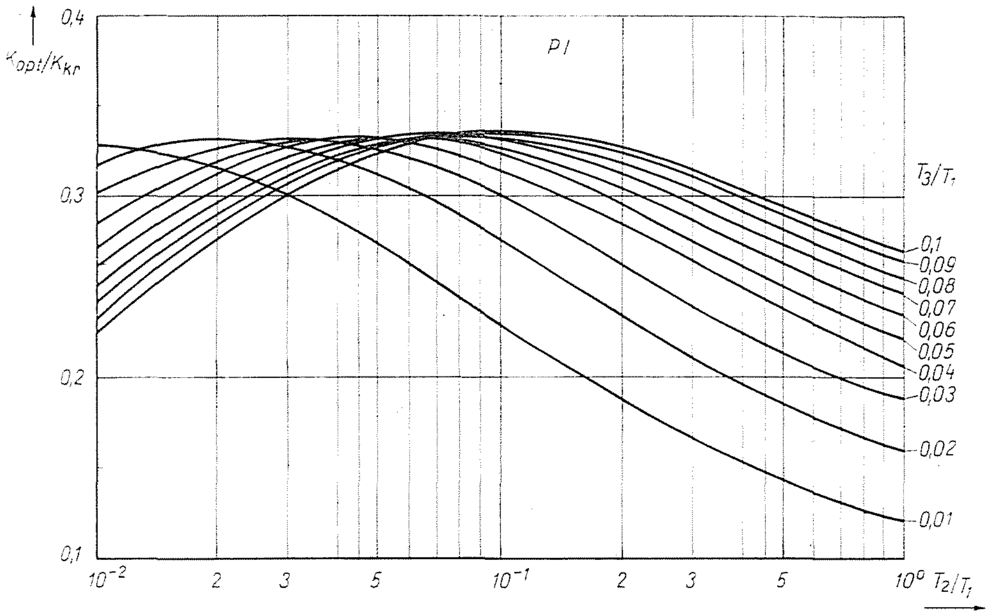
The optimization was carried out on the basis of the quadratic integral criterion

$$I_{20} = \int_0^\infty x_\varepsilon^2 \mathrm{d}t$$

of the error signal x_ε. The minimum function point was determined in the field of the variables at an accuracy of 2.5%.

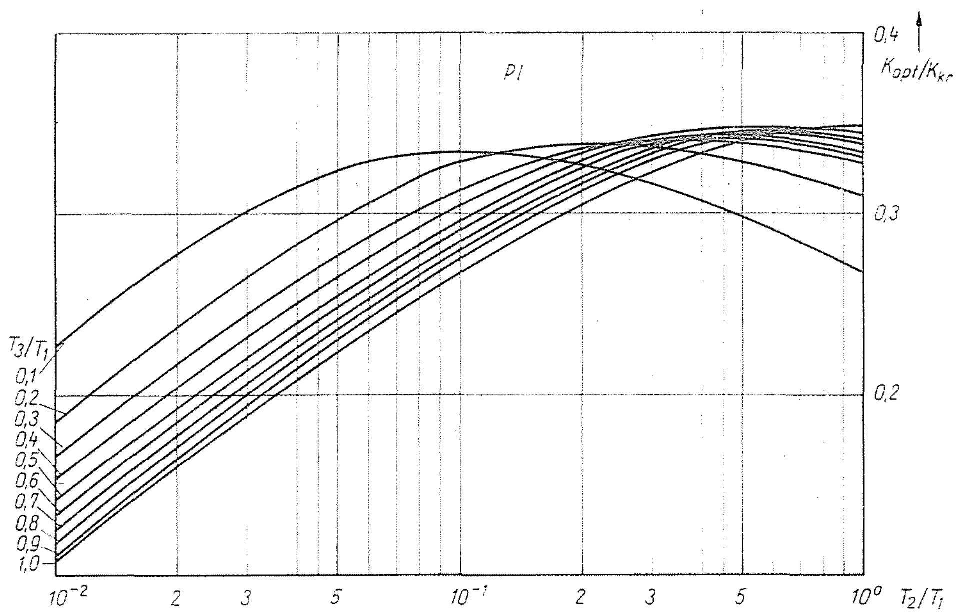
The resulting optimum values are plotted in a diagram. Figs 2, 3, 4 and 5 refer to the PI control and Figs 6, 7, 8, 9, 10 and 11 to the PID control. The diagrams show the optimum relative values of the control parameters versus T_2/T_1 and parametered in T_3/T_1.

The values of K_{kr} and T_{kr}, respectively, may be determined by the build-up oscillation of the system. If the oscillations cannot be tolerated by the system, then K_{kr} and T_{kr} may be calculated from the identified highest time



0,4
K_{opt}/K_{kr}
PI
0,3
0,2
0,1
T_3/T_1
0,1
0,09
0,08
0,07
0,06
0,05
0,04
0,03
0,02
0,01
10^{-2}
2
3
5
10^{-1}
2
3
5
10^{0}
T_2/T_1

Fig. 2



0,4
K_{opt}/K_{kr}
PI
0,3
0,2
T_3/T_1
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1,0
10^{-2}
2
3
5
10^{-1}
2
3
5
10^{0}
T_2/T_1

Fig. 3

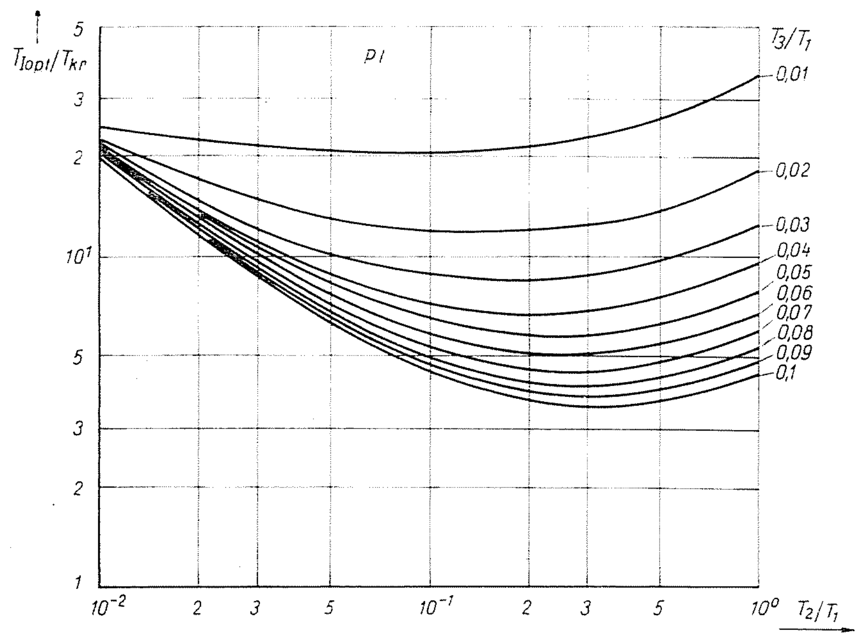

T_{Iopt}/T_{kr}
PI
T_3/T_1
0,01
0,02
0,03
0,04
0,05
0,06
0,07
0,08
0,09
0,1
10^{-2} 2 3 5 10^{-1} 2 3 5 10^{0} T_2/T_1
5 3 2 10^1 5 3 2 1

Fig. 4

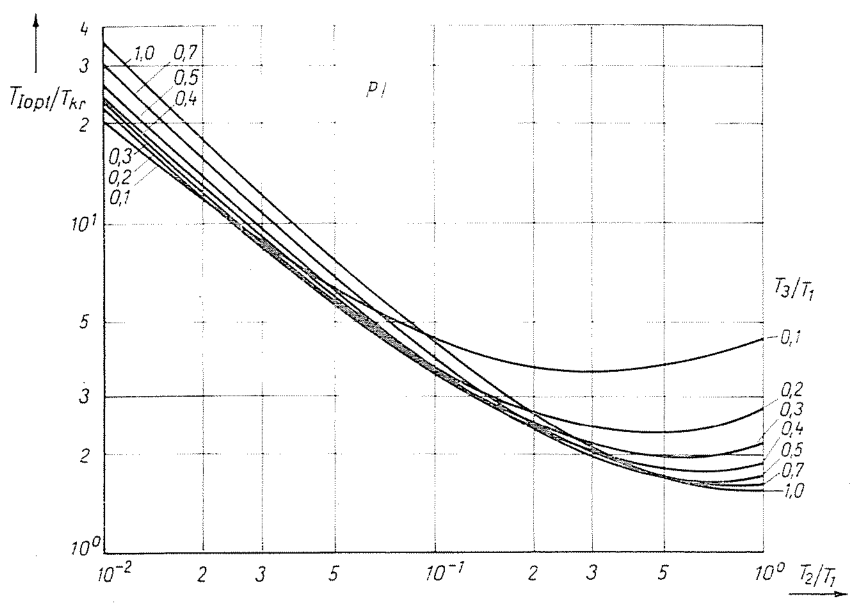

T_{Iopt}/T_{kr}
PI
1,0 0,7
0,5
0,4
0,3
0,2
0,1
T_3/T_1
0,1
0,2
0,3
0,4
0,5
0,7
1,0
10^{-2} 2 3 5 10^{-1} 2 3 5 10^{0} T_2/T_1
4 3 2 10^1 5 3 2 10^0

Fig. 5

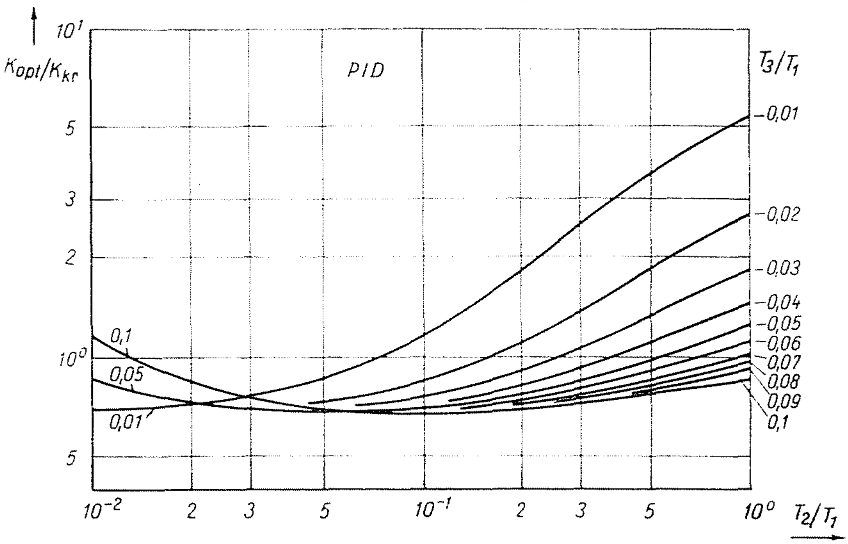


Fig. 6

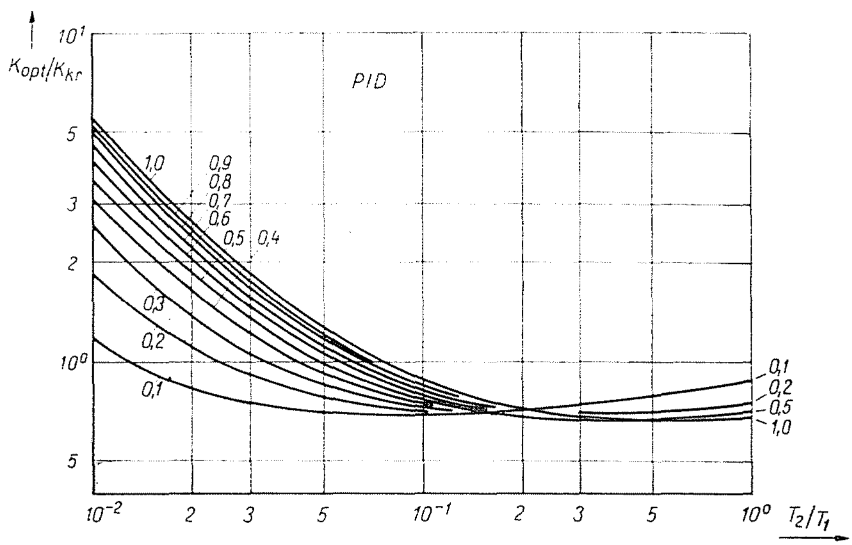


Fig. 7

constants T_1, T_2 and T_3 as follows:

$$K_{kr} = 2 + \frac{a^2 + a^2 b + a + b + ab^2 + b^2}{ab}$$

and

$$T_{kr} = 2\pi \sqrt{\frac{T_1 T_2 T_3}{T_1 + T_2 + T_3}}$$

The diagrams corresponding to these relationships are seen in Figs 12, 13 and 14. Naturally only an approximative solution is given by these formulae, as the controlled sections have in reality several time lags.

Let us consider an example for the utilization of the diagrams. Be the controlled section, indeed, a proportional element of three time lags. Be the time constants:

$$T_1 = 100 \text{ sec}; \quad T_2 = 8 \text{ sec}; \quad T_3 = 10 \text{ sec}.$$

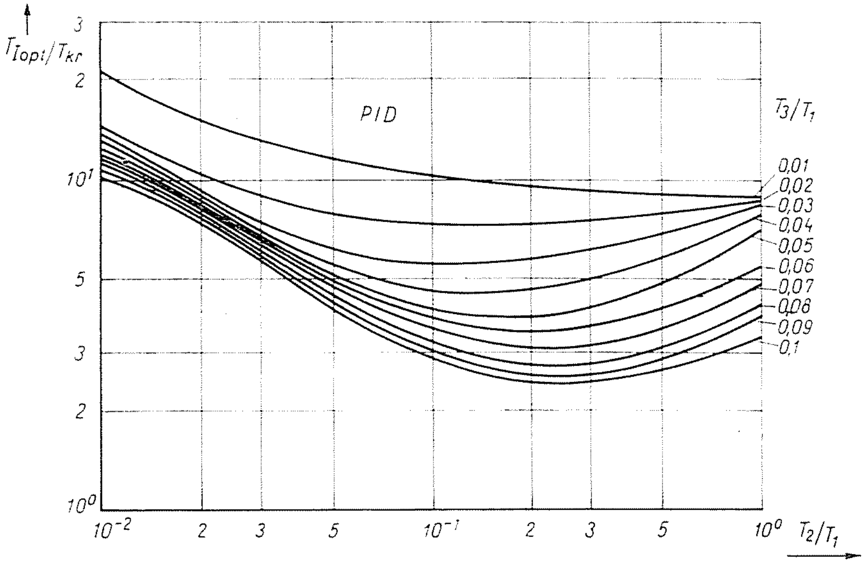


Fig. 8

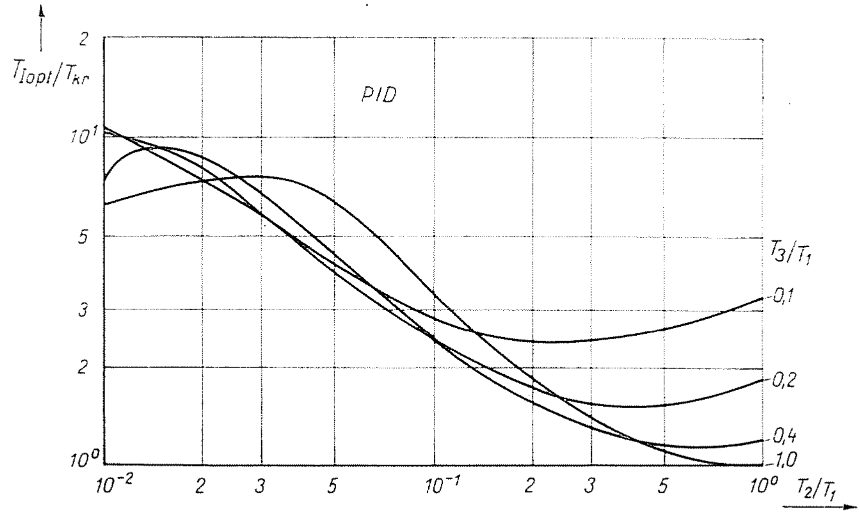


Fig. 9

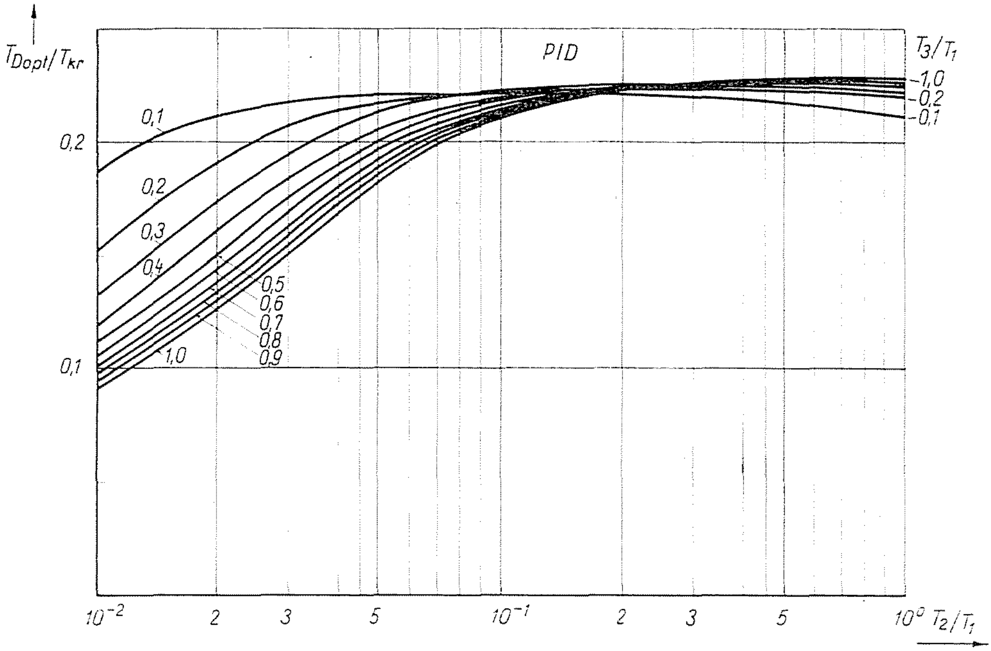

T_{Dopt}/T_{kr}
PID
T_3/T_1
1,0
0,2
0,1
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1,0
0,2
0,1
10^{-2}
2
3
5
10^{-1}
2
3
5
10^{0}
T_2/T_1

Fig. 10

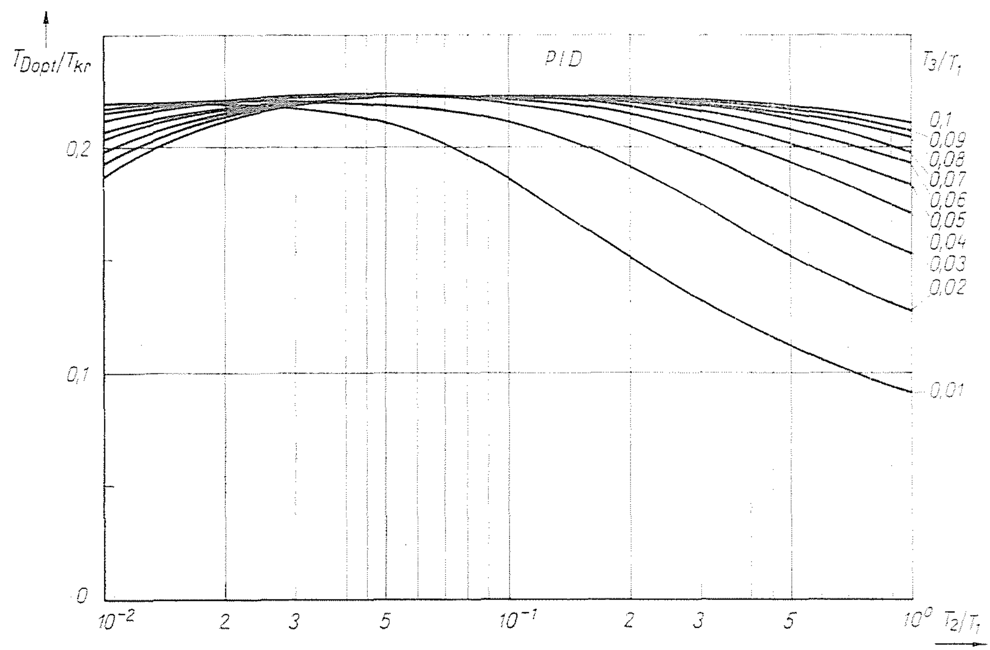

T_{Dopt}/T_{kr}
PID
T_3/T_1
0,1
0,09
0,08
0,07
0,06
0,05
0,04
0,03
0,02
0,01
0,2
0,1
0
10^{-2}
2
3
5
10^{-1}
2
3
5
10^{0}
T_2/T_1

Fig. 11

the relative parameters:

$$a = \frac{T_2}{T_1} = 0.08 \qquad \text{and} \qquad b = \frac{T_3}{T_1} = 0.1$$

Then

$$K_{kr} = 26.73 \qquad \text{and} \qquad T_{kr} = 51.71 \text{ sec}$$

as calculated from the parameters of the controlled section. The optimum relative values as read off the diagrams are:

PI:

$$K_{\text{opt}}/K_{kr} = 0.33 \qquad \text{and} \qquad T_{I\,\text{opt}}/T_{kr} = 4.92,$$

PID:

$$K_{\text{opt}}/K_{kr} = 0.69; \quad T_{I\,\text{opt}}/T_{kr} = 3.18 \qquad \text{and} \qquad T_{D\,\text{opt}}/T_{kr} = 0.221.$$

Hence, the optimum values of the control parameters:

PI:

$$K_{\text{opt}} = 8.87; \quad T_{I\,\text{opt}} = 254.6 \text{ sec}$$

PID:

$$K_{\text{opt}} = 18.47; \quad T_{I\,\text{opt}} = 164.5 \text{ sec}; \quad T_{D\,\text{opt}} = 11.43 \text{ sec}.$$

The optimum error unit step responses $X_a(t) = 1(t)$ for both controls are seen in Fig. 15, where the transient process which is optimum on the basis of the quadratic integral criterion is seen to permit quite a high overshot (of about 15%). This fact was demonstrated in our investigations [2] as well. This overshot can be prevented with the help of some other optimization criteria but only at the cost of the control period.

In connection with the prepared diagrams we note: the digital computer delivered great many points for the various combinations of parameters a and b (for two decades in the values of both) and the diagrams were plotted corresponding to the accuracy of graphical readability by these points. In some cases the identification of the curves (e.g. in the PID case) was disregarded as their course versus the parameters appeared to be extremely entangled.

From the course of the diagrams some valuable relationships may be deduced.

At the beginning of our investigations we have chosen the conditions of Ziegler and Nichols [8] for the initial points of the searching algorithms, but the results have shown that in part of the cases, these conditions resulted instable systems (e.g. $T_1 = 100$ sec; $T_2 = 100$ sec; $t_3 = 1$ sec). The use of the widely accepted $Z - N$ adjustment method is much reduced by this observation.

This result and the course of some of the diagrams (e.g. of K_{opt}/K_{kr} in the PID case) may be explained by the phase conditions of the system being

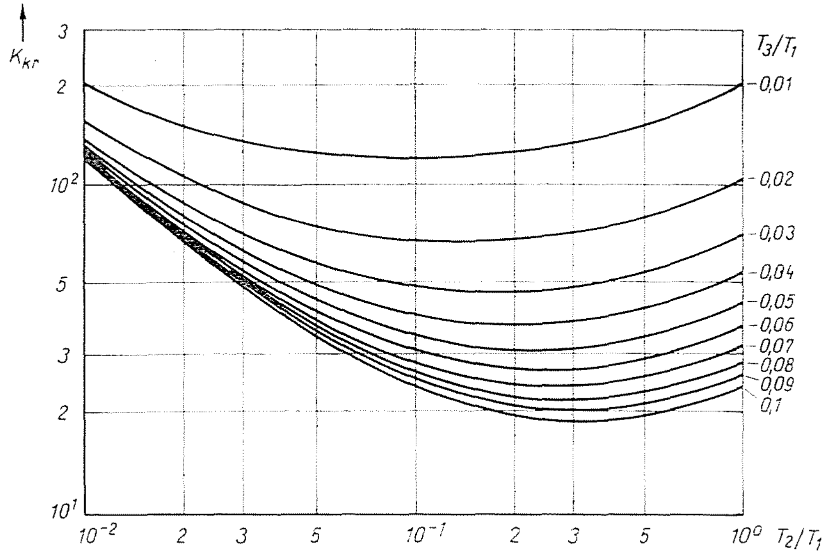


Fig. 12

fundamentally changed by the compensation. Therefore conclusions concerning the compensated system on the basis of the initial one may be drawn only with a rather limited safety. An unexpected result is also that in the case of the PID compensation a value higher than that of the initial circuit amplification

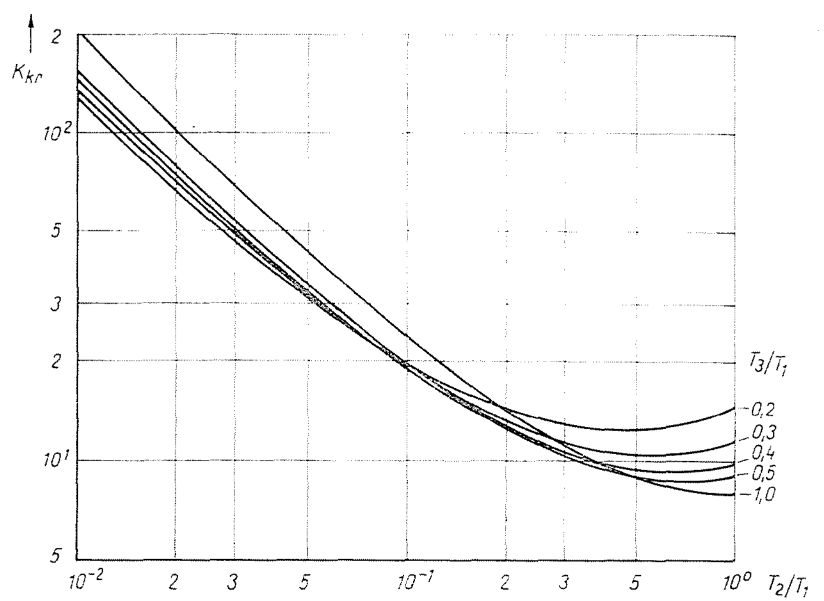
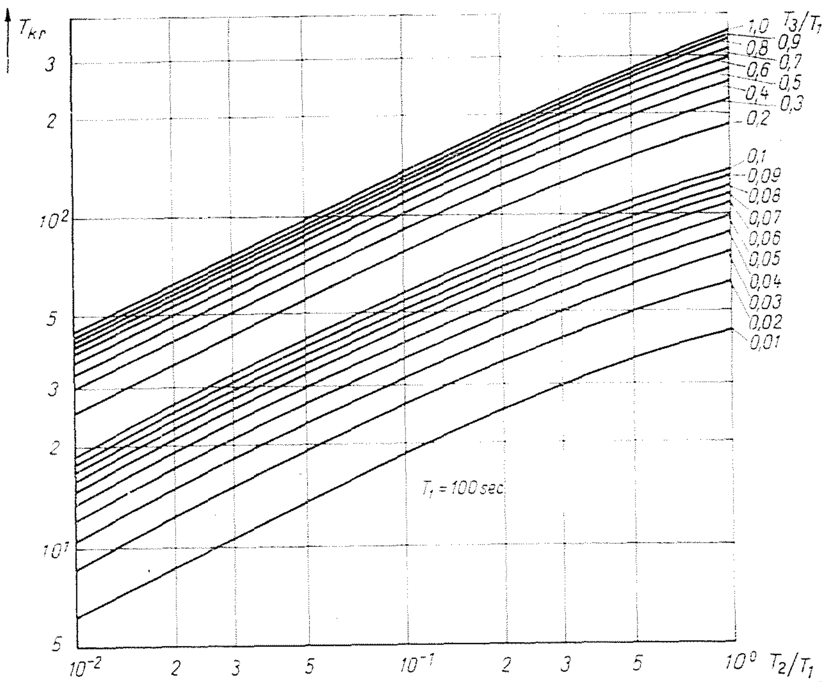
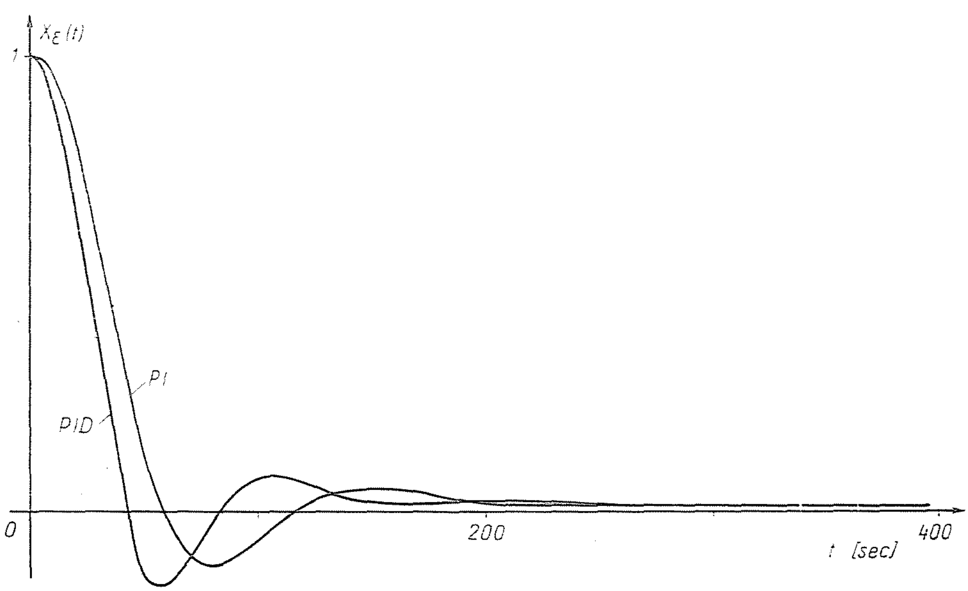


Fig. 13



T_{kr}
3
2
10^2
5
3
2
10^1
5
10^{-2}
2
3
5
10^{-1}
2
3
5
10^0
T_2/T_1
T_3/T_1
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,09
0,08
0,07
0,06
0,05
0,04
0,03
0,02
0,01
$T_1 = 100$ sec

Fig. 14



$x_\varepsilon(t)$
1
PI
PID
0
200
400
t [sec]

Fig. 15

may result as optimum value (see Figs 6 and 7), although the phase-increasing influence of the D-effect is generally known.

The behaviour of the curves can be explained in nearly all the cases. Let us consider e.g. the case in Fig. 5. For $T_1 = T_2$ and $T_3 \ll T_1$, or $T_2 = T_3 \ll T_1$, the phase characteristic of the system intersects the $-180°$ straight line at a very little slope, which means that the cutting frequency belonging to a given phase margin falls very far from the critical circuit frequency and accordingly the optimum T_I/T_{kr} value will also be very high; whereas for $T_1 \sim T_2 \sim T_3$, the phase curve will intersect the $-180°$ straight line at a great slope and so the cutting frequency belonging to the same phase margin will be near to the critical value. So the optimum T_I/T_{kr} value is about 1 in this case.

The analysis of the diagrams is likely to show that the optimum parameters of the controls vary within quite a wide range, so it is difficult to contract the relationships in some simple thumb-rule.

On the basis of the diagrams a nearly optimum setting may be suggested only for the PID control, namely when T_1, T_2 and T_3 are within one decade, i.e.

$$0.1 \leq \frac{T_2}{T_1} \leq 1 \qquad \text{and} \qquad 0.1 \leq \frac{T_3}{T_1} \leq 1\,;$$

then the following approximative relationships hold:

$$T_{D\,\mathrm{opt}} \approx 0.225\; T_{kr}; \quad K_{\mathrm{opt}} \approx 0.7\; K_{kr}$$

and

$$T_{I\,\mathrm{opt}} \approx \frac{1}{\sqrt{x}}\, T_{kr}\,;$$

where $x = \min\,(T_2, T_3)/T_1$.

This latter relationship is acceptable for other time constant ratios as well.

Appendix

Let us suppose that the L-transform of the error signal can be expressed in the form of

$$X_\varepsilon(s) = \frac{c_{n-1}s^{n-1} + \ldots + c_1 s + c_0}{\mathrm{d}_n s^n + \mathrm{d}_{n-1}s^{n-1} + \ldots + \mathrm{d}_1 s + \mathrm{d}_0} = \frac{C(s)}{D(s)}.$$

Then the integral value

$$I_{20} = \int_0^\infty x_\varepsilon^2(t)\,\mathrm{d}t$$

may be calculated by the formula [1]

$$I_{20} = (-1)^{n+1}\frac{B}{2\mathrm{d}_n H_n}$$

Here H_n is the n-th Hurwitz-determinant belonging to the equation $D(s) = 0$, while B is also an n-order determinant produced by substituting the first line of the determinant H_n:

$$d_{n-1}, \; d_{n-3}, \; d_{n-5}, \; \ldots, \; 0, \; \ldots, \; 0$$

by the line

$$b_{2n-2}, \; b_{2n-4}, \; b_{2n-6}, \; \ldots, \; b_2, \; b_0$$

The coefficients appearing here are those of the polynomial

$$C(s)C(-s) = b_{2n-2}s^{2n-2} + \ldots + b_2s^2 + b_0.$$

Thus, the described calculation method corresponds to the solution of a linear equation system. Also the value I_{20} is produced by our computer program as the solution of an equation system.

The minimum of I_{20} as a multivariable function is determined by our extreme value searching algorithm operating according to the sequential simplex method [7].

Summary

Our investigations proved again that the digital computers represent a very useful help in many fields of control technics, even for the solution of problems considered today as classical ones.

The analysis of the obtained results supplied many valuable experiences concerning the optimum setting of the PI and PID control parameters. The diagrams are of a great help in the design of single loop, linear, concentrated-parameter control circuits.

Of course, no optimum results are given for the real systems by diagrams elaborated similarly, supposing the controlled section to have a preliminarily given structure. Yet digital computer outputs have shown that even so they are far better (apart from a few extreme cases) than those of the most widely accepted thumb rules for the setting of the controls.

As to the future: we intend to elaborate similar diagrams for systems containing dead time as well.

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