CALCULATION FOR THE FALL TIME OF DIODES WITH NON-UNIFORM BASE DOPING

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Introduction

In a previous paper [1] the storage time of the non-uniform-base diodes was discussed. To determine the whole switching time, it is also necessary to know the fall time. Besides, in the case of one of the most important non-uniform-base diodes, the step-recovery diode, the value of the fall time influences its applicability.

The paper gives a semi-exact solution of the fall time, compares the result with the approximate solution, given by MOLL, KRAKAUER and SHEN [2], finally proposes an approximation, fitting better to the exact solution.

Semi-exact solution

At the end of the storage time the hole density in the n side of the diode, at the junction pn reaches zero. After that the decrease of the charge of the accumulated holes is connected to the decrease of the gradient of the hole concentration at the junction pn, thus the reverse current decreases. The fall time is defined as the time from the beginning of the decrease of the reverse current, to the decrease of the reverse current to 1/10th of its original value.

The exact solution could be determined by the following manner, as was done by THORIK [3]. First, the hole distribution should be determined, as a function of space, at the end of the storage time. After that, the continuity equation should be solved, with the mentioned hole distribution as initial condition. This hole distribution itself is a very complicated function. Using it as initial condition makes the continuity equation tedious to solve, and the result difficult to survey.

Instead of taking this way, the method, applied by KÖHLER [4] for homogeneous-base diodes, is used. Fig. 1 shows qualitatively the hole distribution for two cases. In Fig. 1a, the diode is switched off by a non-ideal current generator. Till the end of the storage time the hole concentration at the junction pn is higher than zero, the gradient of the hole concentration is constant. By the end of the storage period, the hole concentration is zero, the gradient decreases at the junction. The current is first constant, then decreases.

In Fig. 1b, the diode is switched off by an ideal voltage generator. The hole concentration at the junction drops immediately to zero. The gradient, and therefore the reverse current is first infinite, then decreases.

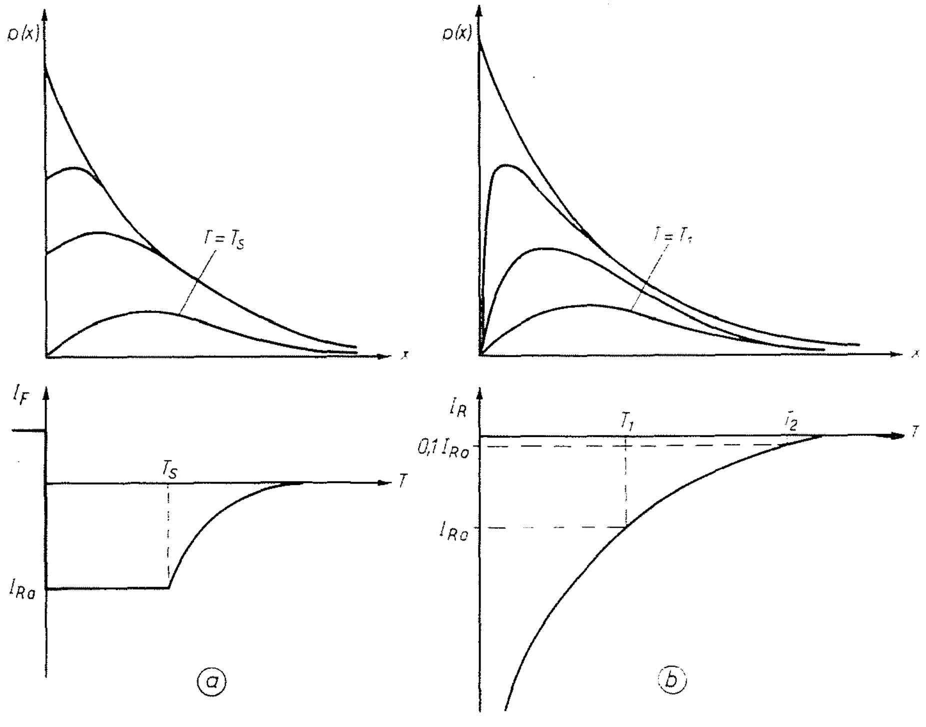

Fig. 1. Switching-off by voltage and current generators

The distributions of holes in Fig. 1b at $I_R = I_{R0}$, and in Fig. 1a, at $t = t_s$ are very similar. So in both cases, after that time, the reverse current is expected to depend on the time in a similar manner. It is easier to calculate the current in the case of a voltage generator driving.

After determining the reverse current as a function of time, the fall time can be calculated as follows: in Fig. 1b the values of t for $I_R = I_{R0}$ and $I_R = 0.1\ I_{R0}$ (t_1 and t_2 respectively) can be determined if the exact function $I_R = f(t)$ is known. The difference of the two values gives the fall time t_f:

$$t_f = t_1 - t_2 \tag{1}$$

The hole distribution in Fig. 1b can be splitted into two parts: a static distribution, caused by I_F, the forward current, and a transient hole distribution. As the hole concentration at the junction (at $x = 0$) is always zero, the transient distribution must have the same value at the junction as the static distribution p_0. From the transient distribution the transient current I_{Rtr} can be calculated. After that, the reverse current is given by Eq. (2)

$$I_R = I_F - I_{Rtr} \tag{2}$$

It is advantageous to use the transformed form of the continuity equation:

$$\frac{\partial U(X, T)}{\partial T} = \frac{\partial^2 U(X, T)}{\partial X^2} \tag{3}$$

where

$$U(X, T) = p(X, T) \exp\{(1 + E_n^2) T - E_n X\} \tag{4}$$

is the transformed hole distribution.

$$E_n = \frac{q}{kT} \cdot \frac{EL_p}{2} \tag{5}$$

is the normalized field-strength in the base (E is the field-strength, L_p is the diffusion length of the holes), T' is the temperature in Kelvin degrees,

$$X = \frac{x}{L_p} \tag{6}$$

the normalized distance and

$$T = \frac{t}{\tau_p} \tag{7}$$

is the normalized time (τ_p is the lifetime of the holes in the base).

The initial condition is:

$$p_{tr}(X, 0) = 0 \text{ thus } U_{tr}(X, 0) = 0 \tag{8}$$

The boundary conditions are:

$$p_{tr}(0, t) = p_0 \qquad U_{tr}(0, T) = p_0 \exp\{(1 + E_n^2) T\} \tag{9}$$

$$p_{tr}(\infty, t) = 0 \qquad U_{tr}(\infty, T) = 0 \tag{10}$$

The solution of the continuity equation by this initial and boundary conditions is given in Appendix I. To calculate I_{Rtr}, it is necessary to know the gradient of the hole distribution. From Eq. (4)

$$\frac{\partial p_{tr}}{\partial X}\bigg|_{X=0} = \frac{\partial U_{tr}}{\partial X}\bigg|_{X=0} \exp\left\{-(1+E_n^2)T\right\} + E_n U_{tr}\bigg|_{X=0} \exp\left\{-(1+E_n^2)T\right\} \quad (11)$$

$\frac{\partial U_{tr}}{\partial X}$ is given by Eq. (A5), $U_{tr}(0, T)$ by Eq. (9).

After substituting both into Eq. (11), one gets

$$\frac{\partial p_{tr}}{\partial X}\bigg|_{X=0} = p_0\left\{E_n - \frac{\exp\left[-(1+E_n^2)T\right]}{\sqrt{\pi T}} - \sqrt{1+E_n^2}\ \mathrm{erf}\ \sqrt{(1+E_n^2)T}\right\}. \quad (12)$$

The transient reverse current at $X = 0$ is the sum of a diffusion and a drift current.

$$I_{Rtr} = -qAD_p \frac{\partial p_{tr}}{\partial x}\bigg|_{X=0} + qA\mu p_0 E = \frac{qAD_p}{L_p}\left(-\frac{\partial p_{tr}}{\partial X}\bigg|_{X=0} + 2E_n p_0\right). \quad (13)$$

After substituting Eq. (12) into Eq. (13):

$$I_{Rtr} = \frac{p_0 qAD_p}{L_p}\left\{\frac{\exp\left[-(1+E_n^2)T\right]}{\sqrt{\pi T}} + \sqrt{1+E_n^2}\ \mathrm{erf}\ \sqrt{(1+E_n^2)T} + E_n\right\}. \quad (14)$$

p_0 was calculated from I_F in [1], supposing a constant forward current. For the diode switched into the forward direction for a long time, p_0 is given by Eq. (15):

$$p_0 = \frac{I_f L_q}{qAD_p}\left(\sqrt{1+E_n^2} - E_n\right). \quad (15)$$

From Eqs (14), (15) and (2)

$$\frac{I_R}{I_F} = 1 - \left(\sqrt{1+E_n^2} + E_n\right)\left(\frac{\exp\left[-(1+E_n^2)T\right]}{\sqrt{\pi T}} + \sqrt{1+E_n^2}\ \mathrm{erf}\ \sqrt{(1+E_n^2)T}\right) \quad (16)$$

For a final switching-in impulse, the current is calculated in Appendix II.

If the base is homogeneous, $E_n = 0$, and Eq. (16) yields:

$$\frac{I_R}{I_F} = 1 - \mathrm{erf}\ \sqrt{T} - \frac{e^{-T}}{\sqrt{\pi T}} \quad (17)$$

an expression given by KINGSTON [5].

The reverse current, calculated from Eq. (16) is shown in Fig. 2, for different normalized field-strengths. From this curves the fall times can be determined, applying the method mentioned above. The results are plotted in Fig. 3 by continuous line. Increasing the field-strength, the fall time decreases.

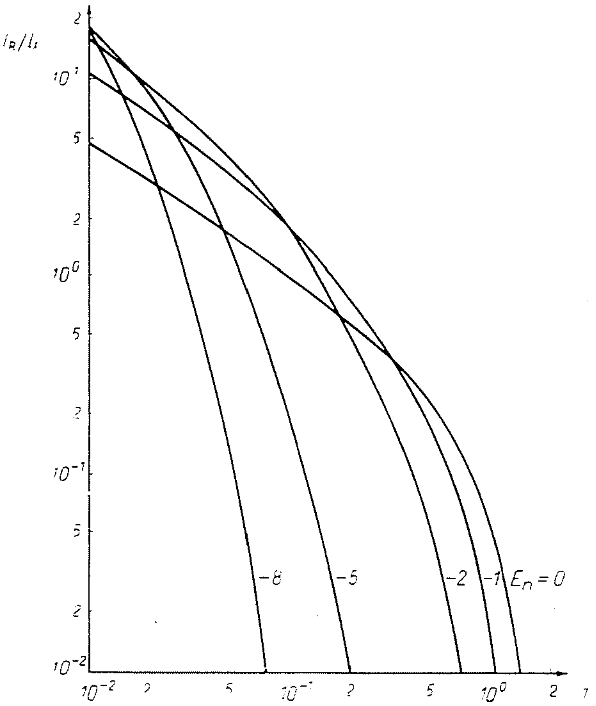

Fig. 2. Current waveforms, when the diode is switched off by a voltage generator

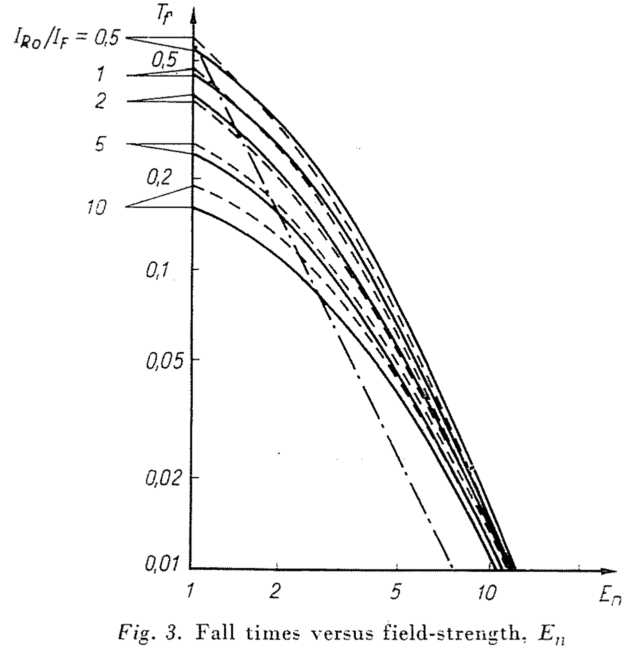

Fig. 3. Fall times versus field-strength, E_n

The fall time depends upon the value of I_R/I_F, although increasing the field-strength, this dependence decreases.

Approximations

In [2] an approximative calculation is presented. Assuming an exponential relationship between the reverse current and the time, using an approximative function for the distribution of holes at the end of the storage period and neglecting the recombination, a solution was obtained for the reverse current:

$$I_R(t) = I_{R0} e^{-\frac{t}{t_r}} \tag{18}$$

where

$$t_r = \frac{1}{D_p} \left(\frac{kT'}{qE} \right)^2 . \tag{19}$$

Using the definition of t_f, one gets:

$$t_f = t_r \ln 10 = \frac{2.3}{D_p} \left(\frac{kT'}{qE} \right)^2 . \tag{20}$$

Introducing the normalized field-strength and time:

$$T_f = \frac{0.575}{E_n^2} . \tag{21}$$

The values given by Eq. (21) are plotted in Fig. 3 by dotted line.

At higher field-strengths the approximate curve runs roughly parallel with the curves giving the exact solution. That means that the character of the function $\left(T_f \sim \frac{1}{E_n^2} \right)$ is correct. The coefficient, on the other hand, should be fitted better.

To have a better fitting and to express the current-dependence of the fall time, the approximate expression, given by Eq. (22) is proposed:

$$T_f = \frac{1.4}{2\sqrt{I_R/I_F} + E_n^2} . \tag{22}$$

This expression has no physical background, and was constructed only to possibly fit the exact result, and at the same time to have a form, not much more complicated than that of Eq. (21). The values, given by Eq. (22) are indicated with dashed line in Fig. 3.

Effect of junction capacitance

In general, the outer circuit consists of a non-ideal generator, which can be substituted by an ideal voltage generator and a series resistance. As the reverse current varies, the reverse voltage on the diode and therefore the width of the depletion layer at the junction varies too. This causes an additional current. The whole process can be discussed with the help of the circuit diagram given in Fig. 4.

The current of the current generator, as given in Fig. 2, is a function of time. The junction capacitance depends on the reverse voltage of the diode.

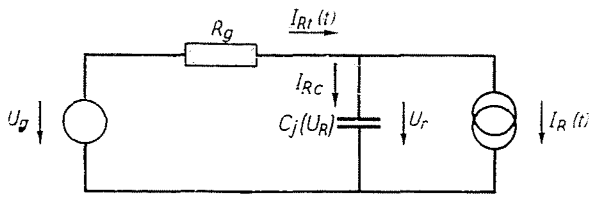

Fig. 4. The effect of junction capacitance

I_{Rt}, the total reverse current, can only be determined by numerical methods. The function $C_j(U_R)$ depends on the doping profile of the junction. The values of U_g and R_g can be different in many cases.

With rough approximation an estimation can be given for T_{ft}, the total fall time. $C_j(U_R)$ can be replaced by an average capacitance:

$$\bar{C}_j = \frac{1}{U_{R\max}} \int_0^{U_{R\max}} C_j(U)\, dU \tag{23}$$

this capacitance and the series resistance gives a time constant,

$$\tau = R_g \bar{C}_j \, . \tag{24}$$

Without any minority-carrier charge storage, the fall time would be determined only by R_g and $\bar{C}_j$.

$$T_{fj} = \tau \ln 10 = 2.3\, \tau \, . \tag{25}$$

In fact, the sum of the charges, represented by the minority carriers and by the junction capacitance, has to be removed. Therefore the total fall time, T_{ft} can be calculated as the sum of the ideal fall time, T_f, and the additional time, T_{fj}, caused by the junction capacitance:

$$T_{ft} = T_f + T_{fj} \, . \tag{26}$$

Appendix I

Denote the Laplace-transformed, normalized time by s. The transformed form of Eq. (3), taking the initial condition, given by Eq. (8) into account:

$$\frac{\partial^2 U_{tr}}{\partial X^2} = s U_{tr}(X, s)\,. \tag{A1}$$

For wide-base diodes $U_{tr}(\infty, s) = 0$, therefore the solution of Eq. (A1) is:

$$U_{tr}(X, s) = C e^{-\sqrt{s}\,X}\,. \tag{A2}$$

As $U_{tr}(0, T) = p_0 \exp{(1 + E_n^2)\,T}$, thus $U_{tr}(0, s) = \dfrac{p_0}{s - (1 + E_n^2)}$ and $C = U_{tr}\,(0, s)$, the solution is:

$$U_{tr}(X, s) = \frac{p_0}{s - (1 + E_n^2)}\, e^{-\sqrt{s}\,X}\,. \tag{A3}$$

The gradient at the junction, from Eq. (A3), is

$$\left.\frac{\partial U_{tr}}{\partial X}\right|_{X=0} = \frac{-p_0\sqrt{s}}{s - (1 + E_n^2)}\,. \tag{A4}$$

After transforming back

$$\left.\frac{\partial U_{tr}(X, T)}{\partial X}\right|_{X=0} = -\,p_0 \left\{\frac{1}{\sqrt{\pi T}} + \sqrt{1 + E_n^2}\, \exp\,[(1 + E_n^2)\,T]\; \mathrm{erf}\, \sqrt{(1 + E_n^2)\,T}\right\} \tag{A5}$$

Appendix II

A. The diode is driven in forward direction by a current generator. The hole concentration at the junction by a final switch-in time is given in [1]

$$p_0 = \frac{I_F L_p}{q D_p} \{\sqrt{1 + E_n^2}\; \mathrm{erf}\; \sqrt{(1 + E_n^2)\,T_{fo}} - E_n[1 - e^{-T_{fo}}] - E_n e^{-T_{fo}}\, \mathrm{erf}\; (E_n \sqrt{T_{fo}})\} \tag{A6}$$

From Eqs (12), (A6) and (2) I_R is:

$$\frac{I_R}{I_F} = 1 - \{\sqrt{1 + E_n^2}\; \mathrm{erf}\; \sqrt{(1 - E_n^2)\,T_{fo}} - E_n[1 - e^{-T_{fo}}] - E_n\, e^{-T_{fo}}\, \mathrm{erf}\; (E_n \sqrt{T_{fo}})\}$$
$$\left\{\frac{\exp\,[-\,(1 + E_n^2)\,T]}{\sqrt{\pi\, T}} + \sqrt{1 + E_n^2}\;\; \mathrm{erf}\;\; \sqrt{(1 + E_n^2)\,T}\right\}. \tag{A7}$$

B. The diode is driven in forward direction by a voltage generator. Both the forward current and the transient reverse current can be calculated from Eq. (14). replacing T. by $(T + T_{f0})$ for the forward current:

$$I_R = I_F - I_{Rtr} = \frac{qp_0 AD_p}{L_p} \left\{ \frac{e^{-(1+E_n^2)(T+T_{fo})}}{\sqrt{\pi(T+T_{fo})}} - \frac{e^{-(1+E_n^2)T}}{\sqrt{\pi T}} + \right.$$
$$\left. + \sqrt{1+E_n^2} \left[\operatorname{erf} \sqrt{(1+E_n^2)(T+T_{fo})} - \operatorname{erf} \sqrt{(1+E_n^2)T} \right] \right\} \tag{A8}$$

where

$$p_0 = p_n e^{\frac{qU_{fo}}{kT}}. \tag{A9}$$

Summary

The fall time of diodes with nonuniform base is discussed. The exact solution is compared with the approximate one found in the literature. A new approximation, fitting better to the exact solution, is proposed.

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