THE CALCULATION OF MATRICES CHARACTERIZING DOUBLE-CIRCUIT THREE-PHASE TRANSMISSION LINES

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Introduction

In previous publications [1, 2], the theory of multiphase transmission lines was established on the basis of the field theory. In practice, phase compensation is usually employed in multiphase transmission lines. Publications exist on the integration of transmission lines with transposition [3], and of the calculation of the influence of the ground wire [4] into the general theory of transmission lines. In the above papers the calculation of single and double-circuit three-phase transmission lines is described for the case of a general compensation. General compensation means in the case of double-circuit transmission lines that the mutual impedance and admittance of any of the two leads in either system are identical. Phase compensation, is, however, limited in many cases to ensuring identical mutual imittances only between the leads of the two systems having identical serial numbers. In the present paper the question is examined how the calculation of such transmission lines can be integrated into the general theory of transmission lines.

Starting equations

In the followings the theory of transmission line systems consisting of n leads arranged over-ground, parallel with each other and with the ground is summarized briefly. The ground is supposed to be limited by a homogeneous, and lossy plane. The electromagnetic fields of the currents in the leads are in coupling. For such a coupled transmission line system the system of differential equations

$$\frac{\mathrm{d}\boldsymbol{i}}{\mathrm{d}z} = \mathbf{Y}_p \boldsymbol{u}$$
$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}z} = \mathbf{Z}_s \boldsymbol{i} \qquad (1)$$

is valid, where z is the place coordinate in the direction of the transmission line, $\boldsymbol{i}$ and $\boldsymbol{u}$ are the column vectors formed of the currents of the leads, and of the voltages between the leads and the ground surface, respectively, $\mathbf{Y}_p$ is the parallel admittance matrix related to unit length, and $\mathbf{Z}_s$ the series imped-

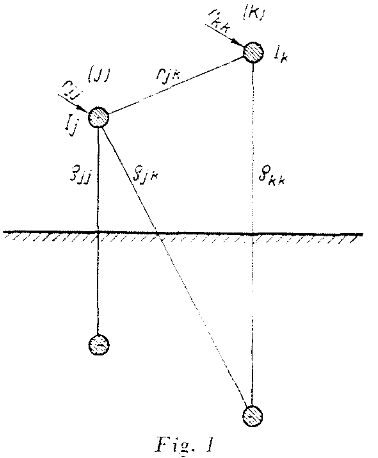

Fig. 1

ance matrix related to unit length. These are quadratic matrices of the n-th order. In the case of double-circuit three-phase transmission lines $n = 2 \times 3 = 6$. $\mathbf{Y}_p$ and $\mathbf{Z}_s$ are calculated on the basis of formulae

$$\begin{aligned} \mathbf{Y}_p &= j\omega\, \varepsilon\pi\, \mathbf{M}^{-1}, \\ \mathbf{Z}_s &= \frac{j\omega\,\mu}{\pi} \mathbf{M} + \mathbf{Z}_b . \end{aligned} \tag{2}$$

Here ε and μ are the permittivity and permeability of the air, respectively. $\mathbf{M}$ is a symmetrical quadratic matrix depending on the geometrical arrangement of the system, in which the k-th element in the j-th row is

$$m_{jk} = \ln \frac{\varrho_{jk}}{r_{jk}} \tag{3}$$

r_{jk} denotes the distance of the j-th lead from the k-th lead, ϱ_{jk} that of the mirror image (Fig. 1). The symmetrical quadratic matrix $\mathbf{Z}_b$ is the sum of two matrices

$$\mathbf{Z}_b = \mathbf{Z}_v + \mathbf{Z}_f \tag{4}$$

$\mathbf{Z}_v$ is a diagonal matrix, the elements of its main diagonal are the skin impedances of the individual leads.

$$\mathbf{Z}_v = \langle Z_{v1}\; Z_{v2}\; \ldots\; Z_{vn} \rangle \tag{5}$$

$\mathbf{Z}_f$ is the ground impedance matrix, its elements can be determined by the help of rows [1], [2].

The solution of the system of differential equations (1) is found to be

$$\begin{aligned} u(z) &= e^{-\Gamma z} U_0^{(+)} + e^{-\Gamma z} U_0^{(-)} \\ i(z) &= \mathbf{Y}_0 [e^{-\Gamma z} U_0^{(+)} - e^{-\Gamma z} U_0^{(-)}] , \end{aligned} \tag{6}$$

where $U_0^{(+)}$ and $U_0^{(-)}$ are the column vectors formed of the values of the voltages propagating in the directions $+z$ and $-z$, respectively, assumed at $z = 0$. $\mathbf{\Gamma}$ is the propagation coefficient matrix, the square of which is

$$\mathbf{\Gamma}^2 = \mathbf{Z}_s \mathbf{Y}_p , \tag{7}$$

and the expression of the wave admittance matrix $\mathbf{Y}_0$ is found to be

$$\mathbf{Y}_0 = \mathbf{Z}_s^{-1} \mathbf{\Gamma} . \tag{8}$$

Matrix functions in (6) can be expressed with the help of the matrix LAGRANGE-polynomials

$$\mathrm{f}(\mathbf{X}) = \sum_{k=1}^{n} f(\lambda_k) \mathbf{L}_k \tag{9}$$

λ_k ($k = 1, 2, \ldots, n$) are the eigenvalues of $\mathbf{X}$, these can be determined from the equation

$$\det |\mathbf{X} - \lambda \mathbf{E}| = 0 \tag{10}$$

where $\mathbf{E}$ is the unit matrix of n-th order. The definition of matrix Lagrange-polynomials is given by

$$\mathbf{L}_k(\mathbf{X}) = \prod_{\substack{j=1 \\ j \neq k}}^{n} \frac{\mathbf{X} - \lambda_k \mathbf{E}}{\lambda_j - \lambda_k} . \tag{11}$$

Accordingly, the relationships (6) can be written also in the following form:

$$\begin{aligned} u(z) &= \sum_{k=1}^{n} \mathbf{L}_k(\mathbf{\Gamma}^2) [U_0^{(+)} e^{-\gamma_k z} + U_0^{(-)} e^{\gamma_k z}] \\ i(z) &= \mathbf{Y}_0 \sum_{k=1}^{n} \mathbf{L}_k(\mathbf{\Gamma}^2) [U_0^{(+)} e^{-\gamma_k z} - U_0^{(-)} e^{\gamma_k z}] \end{aligned} \tag{12}$$

γ_k is the square root drawn from the eigenvalues of $\mathbf{\Gamma}^2$ which falls into the first quarter of the number plane.

On the basis of Eqs (12) the phenomenon taking place on the transmission line system can be interpreted as follows. The solution for both voltage and current consists of two parts. One consists of the generally attenuated waves propagating in the direction $+z$, the other of those in the direction $-z$. The members of the sum correspond to one mode each. One propagation coefficient (γ_k) belongs to the individual modes. The number of modes cannot be higher than the number of leads. If the characteristic equation of $\mathbf{\Gamma}^2$ has coinciding roots too, then the number of modes is lower than the number of leads.

The values $\boldsymbol{U}_0^{(+)}$ and $\boldsymbol{U}_0^{(-)}$ in the equations can be determined from the conditions arising at terminating the system.

Consideration of the compensation

Compensation brought about by phase change can be taken into consideration in the determination of matrices $\mathbf{Y}_p$ and $\mathbf{Z}_s$. For the system without compensation, the matrix $\mathbf{Y}_p$ and $\mathbf{Z}_s$ of the sixth order can be written in the form

$$\mathbf{X}' = \begin{bmatrix} \mathbf{x}'_{11} & \mathbf{x}'_{12} \\ \mathbf{x}'_{21} & \mathbf{x}'_{22} \end{bmatrix} \tag{13}$$

where $\mathbf{x}'_{11}, \mathbf{x}'_{12}, \mathbf{x}'_{21}, \mathbf{x}'_{22}$ are quadratic matrices of the third order, and $\mathbf{x}'_{11}$ contains data pertaining to one of the three-phase systems, $\mathbf{x}'_{22}$ those of the other. In consequence of phase change, the own immittance values occurring in the individual matrix blocks are equal, and the value of the mutual immittance between any two leads can also be taken as equal. This means that in place of the quadratic matrix of the third order, designated in (13) by $\mathbf{x}'_{11}$ and $\mathbf{x}'_{22}$, of the form

$$\mathbf{x}' = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \tag{14}$$

calculations can be carried out by the matrix

$$\mathbf{x}_1 = \begin{bmatrix} \alpha & \beta & \beta \\ \beta & \alpha & \beta \\ \beta & \beta & \alpha \end{bmatrix} \tag{15}$$

where

$$\begin{aligned} \alpha &= \frac{1}{3}(x_{11} + x_{22} + x_{33}) \\ \beta &= \frac{1}{3}(x_{12} + x_{13} + x_{23}) = \frac{1}{3}(x_{21} + x_{31} + x_{32}) . \end{aligned} \tag{16}$$

Fig. 2

$\mathbf{x}'_{12}$ and $\mathbf{x}'_{21}$ contain the mutual characteristics of the two systems. If along the length of the transmission line a phase change is applied in such a way that the leads belonging to the same system have all the three possible positions along the third of the line length (Fig. 2), then by taking the compensation into consideration, matrices $\mathbf{x}'_{21}$ and $\mathbf{x}'_{12}$ of the form (14) will be cyclical, that is, they can be written in the form

$$\mathbf{x}_2 = \begin{bmatrix} \gamma & \delta & \varepsilon \\ \varepsilon & \gamma & \delta \\ \delta & \varepsilon & \gamma \end{bmatrix} \tag{17}$$

where

$$\begin{aligned} \gamma &= \frac{1}{3}(x_{11} + x_{22} + x_{33}) \\ \delta &= \frac{1}{3}(x_{12} + x_{23} + x_{31}) \\ \varepsilon &= \frac{1}{3}(x_{13} + x_{21} + x_{32}) . \end{aligned} \tag{18}$$

On the basis of the aforegoing, the compensation is taken into consideration in such a way that matrices $\mathbf{Y}_p$ and $\mathbf{Z}_s$ are determined for the case without compensation, and replaced in the calculations by a matrix of the form

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_{11} & \mathbf{x}_{12} \\ \mathbf{x}_{21} & \mathbf{x}_{22} \end{bmatrix} \tag{19}$$

where $\mathbf{x}_{11}$ and $\mathbf{x}_{22}$, $\mathbf{x}_{12}$ and $\mathbf{x}_{21}$ resp., can be formed out of the matrices of the form (15) and (17) resp., and of the corresponding block of the matrix of the system without compensation of the form $\mathbf{X}'$, by using (16) and (18).

For the operations performed with matrices of the structure (19) the following rules apply, as can be realized easily. The linear combination of matrices of the examined type is also of this type. The product of two matrices of this type is a matrix in which the quadratic blocks of the third order are cyclical matrices. Accordingly the Lagrange polynomials of matrix (19) as defined in (11), further the matrix functions formed according to (9) are matrices in which the quadratic blocks of the third order are cyclical matrices as given under (17).

It follows from the foregoing that for the double-circuit three-phase system built with phase change the $\mathbf{\Gamma}^2$ under (7), the propagation coefficient $\mathbf{\Gamma}$, and the wave admittance matrix $\mathbf{Y}_0$ under (8) have third order blocks which are cyclical matrices.

Determination of eigenvalues and eigenvectors

In the followings the eigenvalues and the eigenvectors of the matrix (19) are determined.

It is known [6] that the eigenvectors and eigenvalues of the cyclical matrix (17) are the following:

$$\mathbf{s}_0 = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}; \quad \mathbf{s}_1 = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\a^2\\a\end{bmatrix}; \quad \mathbf{s}_2 = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\a\\a^2\end{bmatrix} \tag{20}$$

$$\varphi_0 = \gamma + \delta + \varepsilon; \ \varphi_1 = \gamma + a^2\delta + a\varepsilon; \ \varphi_2 = \gamma + a\delta + a^2\varepsilon \tag{21}$$

where

$$a = e^{j\frac{2\pi}{3}}. \tag{22}$$

Designate some eigenvector by $\mathbf{s}$, the corresponding eigenvalue by φ. Then the eigenvector of the matrix (19) is

$$\boldsymbol{T} = \begin{bmatrix}u\,\mathbf{s}\\v\,\mathbf{s}\end{bmatrix}. \tag{23}$$

Designate its eigenvalue by λ. In this case namely

$$\begin{bmatrix}\mathbf{x}_{11} & \mathbf{x}_{12}\\ \mathbf{x}_{21} & \mathbf{x}_{22}\end{bmatrix}\begin{bmatrix}u\,\mathbf{s}\\v\,\mathbf{s}\end{bmatrix} = \lambda\begin{bmatrix}u\,\mathbf{s}\\v\,\mathbf{s}\end{bmatrix} \tag{24}$$

hence

$$u\,\mathbf{x}_{11}\,\mathbf{s} + v\,\mathbf{x}_{12}\,\mathbf{s} = \lambda\,u\,\mathbf{s} \tag{25}$$

$$u\,\mathbf{x}_{21}\,\mathbf{s} + v\,\mathbf{x}_{22}\,\mathbf{s} = \lambda\,v\,\mathbf{s}\,. \tag{26}$$

Designate the eigenvalues of matrices $\mathbf{x}_{11}, \mathbf{x}_{12}, \mathbf{x}_{21}, \mathbf{x}_{22}$ pertaining to $\mathbf{s}$ by $\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}$, respectively. Thus

$$\begin{aligned} u\,\varphi_{11}\,\mathbf{s} + v\,\varphi_{12}\,\mathbf{s} &= \lambda\,u\,\mathbf{s}\\ u\,\varphi_{21}\,\mathbf{s} + v\,\varphi_{22}\,\mathbf{s} &= \lambda\,v\,\mathbf{s}\,. \end{aligned} \tag{27}$$

From these

$$\begin{aligned} u(\varphi_{11} - \lambda) + v\,\varphi_{12} &= 0\\ u\,\varphi_{21} + v(\varphi_{22} - \lambda) &= 0\,. \end{aligned} \tag{28}$$

The obtained relationships form a homogeneous system of equations for u and v. This has a not trivial solution, if

$$\begin{vmatrix} \varphi_{11} - \lambda & \varphi_{12} \\ \varphi_{21} & \varphi_{22} - \lambda \end{vmatrix} = 0 \tag{29}$$

hence:

$$\lambda = \frac{\varphi_{11} + \varphi_{22} \pm \sqrt{(\varphi_{11} - \varphi_{22})^2 + 4\varphi_{12}\varphi_{21}}}{2} . \tag{30}$$

The quotient u/v can be determined from (28).

$$\frac{u}{v} = \frac{\varphi_{11} - \varphi_{22} \pm \sqrt{(\varphi_{11} - \varphi_{22})^2 + 4\varphi_{12}\varphi_{21}}}{2\varphi_{21}} \tag{31}$$

For u and v an equation can be obtained also from that condition that

$$\boldsymbol{T}^* \tilde{\boldsymbol{T}} = 1 \tag{32}$$

where * designates the transpose, and ~ the conjugate. Thus

$$[u\,\mathbf{s}^* \quad v\,\mathbf{s}^*] \begin{bmatrix} u\,\tilde{\mathbf{s}} \\ v\,\tilde{\mathbf{s}} \end{bmatrix} = 1 \tag{33}$$

hence

$$u^2 + v^2 = 1 \tag{34}$$

where it was taken into consideration that

$$\mathbf{s}^* \tilde{\mathbf{s}} = 1 . \tag{35}$$

Thus

$$u = \frac{k}{\sqrt{1 + k^2}} \quad \text{and} \quad v = \frac{k}{\sqrt{1 - k^2}} . \tag{36}$$

In the foregoing we obtained six eigenvectors and six eigenvalues. Namely $\mathbf{s}$ may denote any one of $\mathbf{s}_0$, $\mathbf{s}_1$, $\mathbf{s}_2$, and accordingly on the basis of (21) any one of φ_{11}, φ_{12}, φ_{21}, φ_{22} may assume three different values. Since (31) supplies two solutions for u, and (30) two for λ, thus, upon substituting the corresponding φ values we obtain six values for λ, and six values each for u and v, that is we obtain the complete eigenvector and eigenvalue system of the sixth order matrix $\mathbf{X}$.

Determine also the Lagrange polynomials belonging to the individual eigenvectors. These can be calculated from the eigenvectors $\boldsymbol{T}$ on the basis of relationship

$$\mathbf{L} = \boldsymbol{T} \tilde{\boldsymbol{T}}^* . \tag{37}$$

Thus

$$\mathbf{L} = \begin{bmatrix} u\,\mathbf{s} \\ \\ v\,\mathbf{s} \end{bmatrix} [u\,\tilde{\mathbf{s}}^* \qquad v\,\tilde{\mathbf{s}}^*] = \begin{bmatrix} u^2\,\mathbf{s}\,\tilde{\mathbf{s}}^* & u\,v\,\mathbf{s}\,\tilde{\mathbf{s}}^* \\ \\ u\,v\,\mathbf{s}\,\tilde{\mathbf{s}}^* & v^2\,\mathbf{s}\,\tilde{\mathbf{s}}^* \end{bmatrix} = \begin{bmatrix} u^2\,\mathbf{l} & u\,v\,\mathbf{l} \\ \\ u\,v\,\mathbf{l} & v^2\,\mathbf{l} \end{bmatrix} \tag{38}$$

where

$$\mathbf{l} = \mathbf{s}\,\tilde{\mathbf{s}}^* \tag{39}$$

that is

$$\mathbf{l}_0 = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}; \quad \mathbf{l}_1 = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{bmatrix}; \quad \mathbf{l}_2 = \frac{1}{3}\begin{bmatrix} 1 & a^2 & a \\ a & 1 & a^2 \\ a^2 & a & 1 \end{bmatrix}. \tag{40}$$

According to the six eigenvectors, six Lagrange polynomials can be obtained with the help of the above expression.

Decomposition of voltages and currents according to the eigenvectors

On the basis of (21) and (30) the eigenvalues of matrices $\mathbf{\Gamma}^2$ and $\mathbf{Y}_0$ can be determined. Denote these by the indexed symbols $\gamma_{01}^2, \gamma_{02}^2, \gamma_{11}^2, \gamma_{12}^2, \gamma_{21}^2, \gamma_{22}^2$, and $Y_{01}, Y_{02}, Y_{11}, Y_{12}, Y_{21}, Y_{22}$, respectively. The part of column vector $\boldsymbol{u}(z)$ in (12) related to the wave propagating in the direction $+z$, can be expressed in terms of these as follows.

$$\boldsymbol{u}^{(+)}(z) = \sum_{n=0}^{2} \sum_{m=1}^{2} e^{\gamma_{nm} z}\, \mathbf{L}_{nm}\, \boldsymbol{U}_0^{(+)} \tag{41}$$

where $\boldsymbol{U}_0^{(+)}$ is the column vector formed of voltages connected to the leads of the transmission line at the point $z = 0$.

The column vector of current waves propagating in the direction $+z$ is found to be, on the basis of (12):

$$\boldsymbol{i}^{(+)}(z) = \sum_{n=0}^{2} \sum_{m=1}^{2} Y_{nm}\, e^{\gamma_{nm} z}\, \mathbf{L}_{nm}\, \boldsymbol{U}_0^{(+)}. \tag{42}$$

It is evident from the obtained results that in consequence of the interaction of the two three-phase systems in general two components of zero order, two of positive order, and two of negative order arise.

Symmetrical arrangement

Examine the frequently occurring case where the two three-phase systems are of symmetrical arrangement relative to each other (Fig. 3). In this case

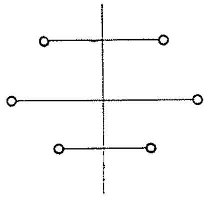

Fig. 3

$$\mathbf{\Gamma}^2 = \begin{bmatrix} \mathbf{\Gamma}_1^2 & \mathbf{\Gamma}_2^2 \\ \mathbf{\Gamma}_2^2 & \mathbf{\Gamma}_1^2 \end{bmatrix} \tag{43}$$

and

$$\mathbf{Y}_0 = \begin{bmatrix} \mathbf{Y}_1 & \mathbf{Y}_2 \\ \mathbf{Y}_2 & \mathbf{Y}_1 \end{bmatrix} \tag{44}$$

where $\mathbf{\Gamma}_1^2, \mathbf{\Gamma}_2^2, \mathbf{Y}_1, \mathbf{Y}_2$ are symmetrical matrices of the third order. Consequently, on account of symmetry arising in the matrices, $\varphi_{11} = \varphi_{22}$ and $\varphi_{12} = \varphi_{21}$, that is, according to (30) and (31)

$$\lambda = \varphi_{11} \pm \varphi_{12} \tag{45}$$

and

$$\frac{u}{v} = k = \pm 1\,. \tag{46}$$

Thus the Lagrange matrices are

$$\mathbf{L}_{n1} = \begin{bmatrix} \mathbf{1}_n & \mathbf{1}_n \\ \mathbf{1}_n & \mathbf{1}_n \end{bmatrix}; \qquad \mathbf{L}_{n2} = \begin{bmatrix} \mathbf{1}_n & -\mathbf{1}_n \\ -\mathbf{1}_n & \mathbf{1}_n \end{bmatrix} \tag{47}$$
$$(n = 0,1,2)\,.$$

If the two systems are connected in parallel at $z = 0$, then

$$\boldsymbol{U}_0^{(+)} = \begin{bmatrix} \boldsymbol{U}^{(+)} \\ \boldsymbol{U}^{(+)} \end{bmatrix} \tag{48}$$

where

$$\boldsymbol{U}^{(+)} = \begin{bmatrix} U_1^{(+)} \\ U_2^{(+)} \\ U_3^{(+)} \end{bmatrix} \tag{49}$$

$U_1^{(+)}, U_2^{(+)}, U_3^{(+)}$ are the voltages connected to the individual leads at $z = 0$. Then the voltage wave propagating in direction $+z$, belonging to the matrices $\mathbf{L}_{01}, \mathbf{L}_{11}, \mathbf{L}_{21}$ is found to be

$$\mathbf{L}_{n1} e^{\gamma_{n1} z} \boldsymbol{U}_0^{(+)} = 2e^{\gamma_{n1} z} \begin{bmatrix} \mathbf{1}_n \\ \mathbf{1}_n \end{bmatrix} \boldsymbol{U}^{(+)} \qquad (50)$$
$$(n = 0, 1, 2.)$$

and the voltage pertaining to the matrix $\mathbf{L}_{02}, \mathbf{L}_{12}, \mathbf{L}_{22}$ is expressed as:

$$\mathbf{L}_{n2} e^{\gamma_{n2} z} \boldsymbol{U}_0^{(+)} = \begin{bmatrix} \boldsymbol{O} \\ \boldsymbol{O} \end{bmatrix} \qquad (51)$$
$$(n = 0, 1, 2.)$$

That is, in the case of symmetrical arrangement and parallel circuit, voltages pertaining to three eigenvectors are equal to zero, the voltages pertaining to the further three eigenvectors correspond to the symmetrical components. The eigenvalues pertaining to the latter can be calculated according to (45) on the basis of relationship

$$\lambda = \varphi_{11} + \varphi_{12}, \qquad (52)$$

that is, the eigenvalues of matrices $\mathbf{\Gamma}^2$ and $\mathbf{Y}_0$ are given as the sum of the corresponding eigenvalues of the different blocks of matrices (43) and (44), respectively.

Summary

In previous publications conditions in multiphase transmission lines were set out on the basis of the field theory, considering a complete compensation for the effect of phase change. These results are further developed in the present paper for the practical case where in consequence of compensation, mutual immittance between two leads of identical serial number of two three-phase systems is identical.

References

1. VÁGÓ, I.: Over-ground power transmission line systems. Periodica Polytechnica El. **9**, 67–96 (1965).
2. FODOR, GY., SIMONYI, K., VÁGÓ, I.: Elméleti villamosságtan példatár. Tankönyvkiadó, Budapest, 1967.
3. VÁGÓ, I.: Three-phase transmission line systems with transposition. Periodica Polytechnica El. **14**, 243–257 (1970).
4. VÁGÓ, I.: Influence of the ground wire on the electromagnetic waves propagating in transmission lines. Periodica Polytechnica El. **14**, 259–268 (1970).
5. GESZTI, O. P., RÓZSA, L.: Electrical parameters of double-circuit three-phase high voltage aerial lines. Periodica Polytechnica El. **12**, 285–299 (1968).
6. VÁGÓ, I., IVÁNYI, A.: Theoretical foundations of the reduction of triphase networks to components. Periodica Polytechnica El. **11**, 13–27 (1967).

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