ON THE POSSIBILITY OF UNIFIED DESCRIPTION OF MODULATION SYSTEMS

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1. Introduction

Up to now, there is no uniform description for the various kinds of modulation, widely used in communication. In what follows, possibilities will be investigated and a solution suggested. The concept of modulation is extended to every procedure to convert the signal at the transmitter side, to convey this converted signal to the transmission medium, and to reconvert the received signal by a procedure complementer to that used at the transmitter side. The transmission path studied here is shown in Fig. 1. Thus a unified description is sought for which applies, apart from the conventional modulations (such as AM, AM-SSB/SC, FM, etc.), also to time and frequency multiplexing (TDM and FDM, resp.), compander, pre- and deemphasis, etc.

The purpose of communication is, in general, true signal transmission, but in most cases transmission paths do not allow a true transmission. Transmission paths produce disturbances, in the signal, consisting of distortions (linear and nonlinear ones) and noise. When a converted signal passes through a non-ideal inside transmission path, the disturbances produced on the converted signal disturb also the re-converted signal. The disturbances of the re-converted signal depend not only on the disturbances of the inside transmission path but also greatly on the conversion systems. *It seems that one of the most important properties of a conversion system is just how it transforms the disturbances of the inside transmission path.* By "transformation of disturbances" a procedure is meant which yields, in terms of the disturbances of the inside transmission path, the disturbances of an outside transmission path exhibiting without conversion the same transmission properties as the whole system consisting of converter, inside transmission path and re-converter.

An attempt is made to unify the treatment of the transformation of disturbances. It is suggested to describe the transformation of disturbances by means of a matrix. This conversion-matrix can be constructed so as to apply to every conversion system. Besides the transformation of disturbances, *the conver-*

sion-matrix takes into consideration the disturbances caused by the conversion system itself.

Thus, conversion systems may be uniformly described by their conversion-matrices, constructed according to certain principles.

In Sec. 2 the properties of transmission paths are established. The exact characterization of transmission paths is given in the Appendix. In Sec. 3 the principles of the construction of conversion-matrices are given. According to the detailed investigation it turns out that for certain conversion systems the

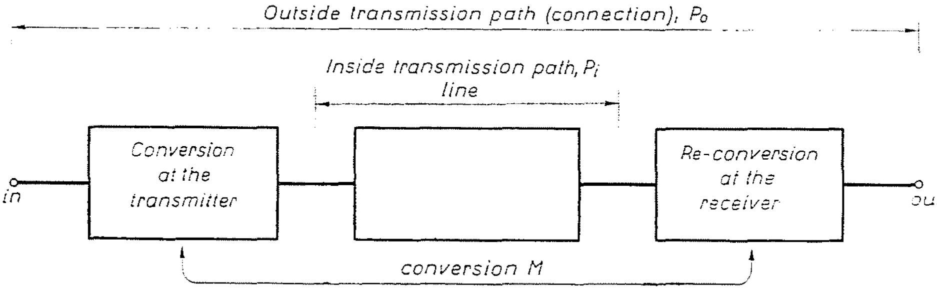

Fig. 1

exact form of the conversion-matrix is rather complicated. Therefore the disturbance-*indication-matrix will be introduced* which lends itself for qualitative rather than quantitative description. Finally, in Sec. 4 and 5. some examples illustrate the construction and application of conversion- and indication-matrices.

2. Properties of transmission paths

In general, a number of disturbances of the transmission path occur simultaneously (for instance linear and nonlinear distortions). A fairly good model of the phenomenon is obtained by assuming that in every transmission path a section with memory and another one without memory are cascaded and that the external noises are additively superimposed to the signal. This concept is shown in Fig. 2.

Both inside and outside transmission paths are characterized by the same properties, only the functions or numbers representing one or another property may differ.

The transformation of disturbances has, of course, significance for such transmission paths which differ from the ideal. The definition of ideal transmission will be given in Appendix 1. Based on this, Appendix 2 deals with the exact determination of the disturbances of a non-ideal transmission path.

Let us now designate properties of the transmission path to be used for the description of conversion systems.

a) Amplitude function, $A(f)$, where $f =$ frequency

b) Phase function, $b(f)$

c) Frequency distortion, Δf

(Theoretically, also frequency-distortion may be a function of frequency, thus, if needed, the symbol $\Delta f(f)$ may be used.)

d) Nonlinear distortion

Nonlinear distortion is regarded to be the effect of the memory-less section. It will be characterized by the distortion factor defined in the conventional way. (The distortion factor k is the ratio of the square root of the squared and summarized amplitudes of the over-tones to the amplitude of the fundamental tone, each amplitude measured at the output of the memory-less section).

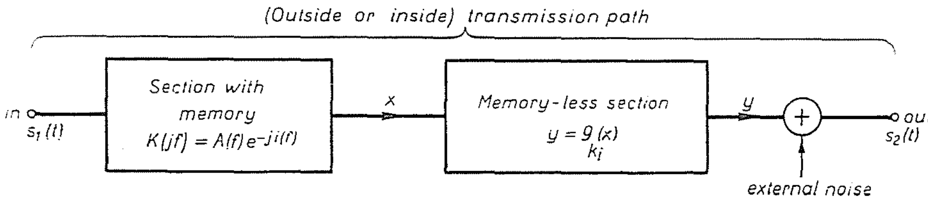

Fig. 2

e) Noise, N

The term "noise", as used here, includes internal and multiplexing noise. The characterization of noise may be of various depths. Here the mean noise power, N, in the frequency-band of the signal will be used. (Thereby the noise is supposed to be stationary.) For a more exact characterization the power spectrum, the amplitude distribution function, or perhaps the autocorrelation function of the noise might be used.

For further purposes it is proposed to characterize every transmission path by a column matrix whose elements are the above properties. Apart from these five properties the column matrix has to contain a "one" too, the role of which will be explained later (in sub-section 3.2.2.) consequently the column matrix of the transmission path is of the form

$$\bar{\mathbf{P}} = \begin{bmatrix} A(f) \\ b(f) \\ \Delta f \\ k \\ N \\ 1 \end{bmatrix} \tag{1}$$

Though the above properties do not characterize the transmission path unambiguously, they are sufficient for practical requirements. Nevertheless it is to be noted that the principles given later for the description of conversion systems are also valid in the case where the transmission path is characterized more in detail. In this case, for example, the column matrix of the transmission path may have the form

$$\bar{\mathbf{P}} = \begin{bmatrix} A(f) \\ b(f) \\ \Delta f \\ g(x) \\ G_n(f) \\ F_n(x) \\ 1 \end{bmatrix} \tag{2}$$

where

$g(x)$ stands for the dynamic characteristic, $y = g(x)$, of the memory-less section of the transmission path;

$G_n(f)$ power spectrum of the noise;

$F_n(x)$ amplitude distribution function of the noise.

If the column matrix is chosen according to Eq. (2) the conversion matrix introduced in subsequent sections has to be modified.

Henceforth transmission path will be characterized by column matrix given in Eq. (1).

3. Conversion-matrix

3.1. Requirements for the description of conversion

As it was seen, transmission paths can be characterized by column matrices. But in many cases the signal cannot be directly transmitted to the transmission medium (denoted by inside transmission path in Fig. 1). Under such circumstances some conversion (e.g. modulation) is required. This creates a new transmission path denoted by outside transmission path in Fig. 1.

By definition, *a conversion is considered to be described if the relation between the properties of inside and outside transmission paths is known.* Mathematically, a conversion is considered as described if the operation $\mathcal{M}$ relating the column matrices of inside and outside transmission paths is known:

$$\bar{\mathbf{P}}_0 = \mathcal{M}\{\bar{\mathbf{P}}_i\} \tag{3}$$

The requirements for the description of conversion [given by Eq. (3)] are as follows:

(i) The relation between the properties of inside and outside transmission paths should be given unambiguously.

(ii) The simultaneous effects of multiple conversions ought to be determined. Representing the first conversion by operation $\mathcal{M}_1$, the second one by $\mathcal{M}_2$, and so on, (Fig. 3), the complex conversion should be described in the form:

$$\overline{\mathbf{P}}_0 = \mathcal{M}_1\{\mathcal{M}_2\{\cdots \mathcal{M}_n\{\overline{\mathbf{P}}_i\}\cdots\}\}. \tag{4}$$

(iii) Uniform description is needed for the procedures applied in communication, such as various modulations (e.g. AM, SSB, FM, PCM, etc.), multiplexing (FDM and TDM), compander, preemphasis and deemphasis, quantization etc.

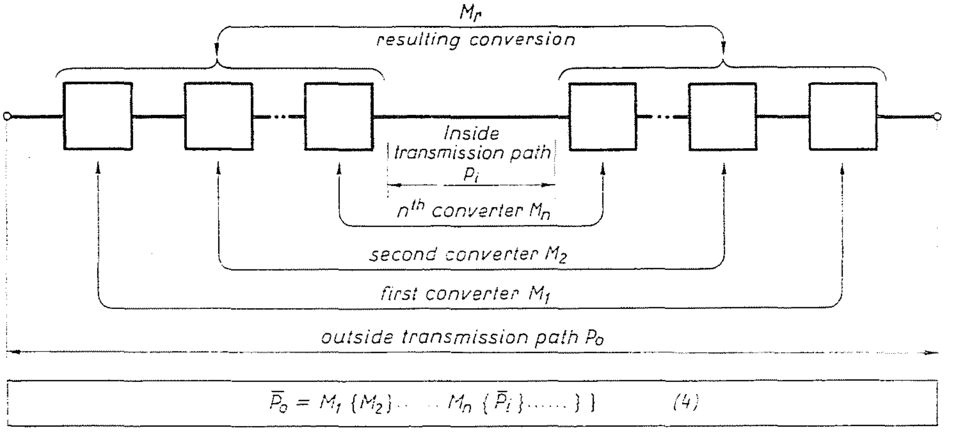

Fig. 3

(iv) A procedure to select the conversion, optimum in certain respect, could be developed for an inside transmission path with given properties.

Obviously, the conversion operation $\mathcal{M}$ describes the whole conversion system including the effects of the converter at the transmitter side and the re-converter at the receiver side as well.

3.2. *Use of a matrix for describing a conversion*

The proposed use of a matrix to describe a conversion is supported by the following considerations:

— $\overline{\mathbf{P}}_0$ and $\overline{\mathbf{P}}_i$, Eq. (3), being column matrices, a matrix can relate them by simple multiplication:

$$\overline{\mathbf{P}}_0 = \mathbf{M} \cdot \overline{\mathbf{P}}_i \tag{5}$$

— it is easy to survey a matrix

— the product of a matrix and a column matrix is also a column matrix, thus the requirement of Eq. (4) is fulfilled:

$$\overline{\mathbf{P}}_0 = \mathbf{M}_1 \cdot \mathbf{M}_2 \cdots \mathbf{M}_n \cdot \overline{\mathbf{P}}_i \tag{6}$$

3.3. *Use of a matrix to describe multiple conversion*

Practical communication systems usually apply a number of conversions on the signal to be transmitted. Re-interpretation of Eq. (6) simplifies handling of systems consisting of multiple conversions. Let us introduce the matrix of the resulting conversion by the equation

$$\mathbf{M}_r = \mathbf{M}_1 \cdot \mathbf{M}_2 \cdots \mathbf{M}_n \tag{7}$$

(see Fig. 3). Thus Eq. (6) can be rewritten into the form

$$\overline{\mathbf{P}}_0 = \mathbf{M}_r \cdot \overline{\mathbf{P}}_i \,. \tag{8}$$

This equation implies the overfulfilment of requirement (ii) imposed on the description of conversion in sub-section 3.1. Namely, using a matrix for the description statement (ii) will be the following:

(ii) The product of the matrices of the conversion stages, cascaded one after another, yields the matrix of the resulting conversion. (The matrices of each conversion stage have to be written in the same order as the signal, to be transmittedthrough these stages in the transmitter.)

For example, in a multichannel telephone equipment there may be involved first a single-sideband (SSB) modulation, then frequency-multiplexing (FDM) and finally perhaps preemphasis. According to the common terminology the totality of these steps is termed "modulation". To generalize treatment, these three conversions will be separated to compose the matrix of the conversion used in multichannel telephone equipments according to the equation:

$$\mathbf{M}_{multich.\,tel.\,equ.} = \mathbf{M}_{SSB} \cdot \mathbf{M}_{FDM} \cdot \mathbf{M}_{pre} \,.$$

3.4. *Elements of conversion matrix*

The conversion-matrix $\mathbf{M}$ connecting column matrices, each of 6 elements, must be a square (6×6) matrix of the form

$$\underbrace{\begin{matrix} A_0(f) \\ b_0(f) \\ \Delta f_0 \\ k_0 \\ N_0 \\ 1 \end{matrix}}_{effect} \quad \mathbf{M} = \overbrace{\begin{matrix} A_i(f) & b_i(f) & \Delta f_i & k_i & N_i & S \end{matrix}}^{cause} \begin{bmatrix} m_{AA} & m_{Ab} & m_{A\Delta f} & m_{Ak} & m_{AN} & m_{AS} \\ m_{bA} & m_{bb} & m_{b\Delta f} & m_{bk} & w_{bN} & m_{bS} \\ m_{\Delta fA} & m_{\Delta fb} & m_{\Delta f\Delta f} & m_{\Delta fk} & m_{\Delta fN} & m_{\Delta fS} \\ m_{kA} & m_{kb} & m_{k\Delta f} & m_{kk} & m_{kN} & m_{kS} \\ m_{NA} & m_{Nb} & m_{N\Delta f} & m_{Nk} & m_{NN} & m_{NS} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} . \tag{9}$$

This matrix has been created on the basis of Eq. (5), but its architecture has been only formally verified so far. Now we turn to determine its meaning.

The physical interpretation of the matrix-elements unambiguously follows from the multiplication rule of matrix and column matrix. It is expedient to divide the matrix elements into three groups.

3.4.1. Interpretation of the (5 ×5) principal-minor matrix

Interpretation of the (5 ×5) principal-minor-matrix must start from the principle that, physically, the conversion-matrix corresponds to a table [Eq. (9)]. Columns represent the properties of the inside transmission path while rows represent those of the outside transmission path. The m_{ij} element of the matrix shows how the j-th property of the inside transmission path affects the i-th property of the outside transmission path. Let us see as illustration some examples:

a) For AM-SSB/SC the amplitude function of the outside transmission path is known to be obtained by shifting the amplitude function of the inside transmission path along the frequency axis by an amount equivalent to the carrier frequency f_v of the demodulator (see Sec. 4, Fig. 4). Mathematically

$$A_0(f) = A_i(f + f_v)$$

This effect is simply described using a shifting operator:

$$\mathscr{T}_{-f_v}\{A_i(f)\} = A_i(f + f_v) \tag{11}$$

where

$\mathscr{T}_{-f_v}$ denotes the shifting operator.

Thus, if the conversion-matrix of AM-SSB/SC systems is to be determined, the element m_{AA} takes the form of a shifting operator as defined in Eq. (11):

$$m_{AA} \doteq \mathscr{T}_{-f_v} \tag{12}$$

b) In FM systems with small frequency deviation and ideal limiter the amplitude function of the inside transmission path does not effect that of the outside one. In this case m_{AA} is to be replaced by an operator changing every function into that of "uniformly one".

c) Another example will be the element $m_{N\Delta f}$ of the conversion-matrix of frequency-multiplexing. If the inside transmission path has a frequency distortion, Δf, the neighbouring channels crosstalk each other which results in a noise proportional to the frequency off-set (supposing a small off-set). In this case $m_{N\Delta f}$ is a constant multiplier. Making use of it, the first approximation of the noise resulting from frequency distortion can be given in the form

$$N_0 = m_{N\Delta f} \cdot \Delta f_i . \tag{13}$$

d) The element m_{NN} shows how noise in the inside transmission path is transformed to the output of the outside transmission path. For example, for AM-SSB/SC systems the equation $N_0 = N_i$ is well-known, consequently for these systems $m_{NN} = 1$.

e) Nonlinear distortions of the inside transmission path can also produce noise (e.g. frequency-multiplexing systems). These effects are expressed by the element m_{Nk}. If nonlinear distortion does not cause noise (e.g. FM systems), then, of course, $m_{Nk} = 0$.

From the above it can be established that the (5×5) principal-minor-matrix transforms the properties of the inside transmission path into those of the outside one.

3.4.2. First five elements in the last column

The outside transmission path may exhibit disturbances also in case of an *ideal* inside transmission path. These disturbances have their origin in the conversion system itself, so it is reasonable to consider them as the own disturbances of the conversion. Typical example is quantization noise in PCM systems.

The first five elements in the last column of the conversion-matrix are chosen to account for the own disturbances of conversion. Namely, according to the multiplication rule of matrices these elements are multiplied by the last element of the column matrix $\overline{\mathbf{P}}_i$, which is compulsorily "one", independently of the properties of the inside transmission path [see Eq. (1)]. This also explains for the role of the last element in column matrices $\overline{\mathbf{P}}$ (see Eq. 1 and 2).

3.4.3. The last row

In case of multiple conversions the description must take into consideration the own disturbances of all conversions. Using, for example, the notations in Fig. 3 the product $\mathbf{M}_n \cdot \overline{\mathbf{P}}_i$ must be a column matrix whose last element is "one". As the last element of $\overline{\mathbf{P}}_i$ is also "one" this requirement can only be fulfilled if the last row of the conversion-matrix $\mathbf{M}_n$ is of the form (0; 0; 0; 0; 0; 1). As any product of the form $(\mathbf{M}_1, \mathbf{M}_2 \ldots \mathbf{M}_n)$ must also yield a matrix whose last row is (0; 0; 0 ;0; 0; 1), the following statement generally holds:

the last row of every conversion-matrix contains zeros except the last element which is always "one".

Thus, the interpretation of the elements of the conversion matrix has been accomplished.

3.5. Difficulties inherent in the description

1. The first difficulty is that of principle. The equation $\overline{\mathbf{P}}_0 = \mathbf{M} \cdot \overline{\mathbf{P}}_i$ involves that various disturbances of the inside transmission path produce additive effects on the outside transmission path. (The addition follows from the multiplication rule of matrices.) This imposes serious difficulties upon the construction of the matrix $\mathbf{M}$ in cases where one or more properties of the outside transmission path are to be given in the form of certain functions [e.g. $A_0(f)$]. It is probable that the matrix cannot even be exactly constructed for certain systems. In such cases two ways can be followed:

— If the disturbances of the inside transmission path are of small order, their effects are nearly independent and therefore also additive. (Such a theorem has not been proved yet in general, but the statement can be justified in many individual cases.)

— In many practical cases an effect produced by a certain cause much surpasses the similar-type effects produced by any other causes. (E.g. in FM systems the phase-function $b_0(f)$ of the outside transmission path greatly depends on $b_i(f)$, but is nearly independent of $A_i(f)$; Δf_i, k_i etc.) Thus the m_{ij} elements, representing negligeable causes, can be approximated by zero.

2. The second difficulty is of practical nature. For certain conversion systems the operators or functions replacing some of the m_{ij} are rather complicated. Nevertheless, it may be stated that for conversion systems of practical importance conversion-matrices could be formed on the bases found in the literature (e.g. [1]).

3. The third difficulty is of aesthetic nature. Namely, in sub-section 3.2.3 it was shown that the mathematical nature of the individual elements of the conversion-matrix was very different. They may be operators, functions and constants and the distinction among them requires ingenuity in notation technics.

3.6. Description by means of indication-matrix

In many practical cases it suffices to know whether the conversion reduces, does not affect or increases the disturbances of the inside transmission path. This demand can easily be fulfilled by means of the following simplifications:

1. Let the elements of the column matrix of the transmission path be real numbers indicating the seriousness of the disturbances. (The last element is, of course, maintained to be "one".)

2. Let the elements of the conversion-matrix be also numbers. If the conversion increases, does not alter or reduces the i-th type of disturbances in the inside transmission path, the corresponding m_{ii} will be denoted by v_{ii}, 1 or

ε_{ii}, respectively. If the effect of the conversion system may be to reduce or to increase, depending on the circumstances, or if the type of the cause and effect are different, the notation for the corresponding element remains m, its actual value, however, is always a number. The new matrix will be called "indication-matrix", because it indicates the direction in which the conversion transforms the disturbances of the inside transmission path.

The indication-matrix will be denoted by **m** instead of **M**. The conversion-matrix and the indication-matrix describe the transformation of disturbances in conversion systems quantitatively and qualitatively, respectively.

Practically, often modulations or multiple modulations are sought which, for a given inside transmission path, keep at a low rate some of the disturbances of the outside transmission path. In synthesis problems like this, the application of the indication-matrix may prove to be very useful.

4. Examples of matrices of simple systems

The purpose of the presented examples of conversion- and indication-matrices is illustration, therefore derivations are omitted. The use of matrices to describe conversion systems is seen to yield results agreeing with the practice. (For further details see [2].)

4.1. Single sideband AM (AM-SSB/SC)

Let the single-sideband, suppressed carrier amplitude modulation utilize the upper sideband. The modulator translates the spectrum of the modulating signal upwards by the value of the modulating carrier frequency f_v, while the demodulator re-translates it by the value of the demodulating carrier frequency $f_v' = f_v - \Delta f_v$.

Assume the initial phase of the modulating and demodulating carrier to be φ_v and $\varphi_v' = \varphi_v - \Delta\varphi_v$, respectively. The frequency difference Δf_v and the phase difference $\Delta\varphi_v$ indicate an operation declining from the ideal. These disturbances are assumed to show how the effects of the own disturbances of the conversion system are taken into consideration. The modulator and demodulator are supposed to be free of further disturbances.

The conversion matrix can be given in the form

$$\mathbf{M}_{\mathrm{AM-SSB/SC}} = \begin{bmatrix} \mathcal{T}_{-f_v} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{T}_{-f_v} & 0 & 0 & 0 & \Delta\varphi_v \\ 0 & 0 & 1 & 0 & 0 & \Delta f_v \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{14}$$

Here $\mathcal{T}_{-f_v}$ denotes the shifting-operator introduced in Eq. (11).

The meaning of $\mathcal{T}_{-f_v}$ in the first row is simple: the AM-SSB/SC system treats an input signal of frequency f_1 with the properties, at the frequency $f_v + f_1$, of the inside transmission path. Fig. 4 illustrates the amplitude functions of the inside and outside transmission paths.

The second row can similarly be interpreted except its last element. As a consequence of the phase-difference between modulating and demodulating carriers, the transmitted signal has a phase-shift $\Delta\varphi_v$ at any frequency.

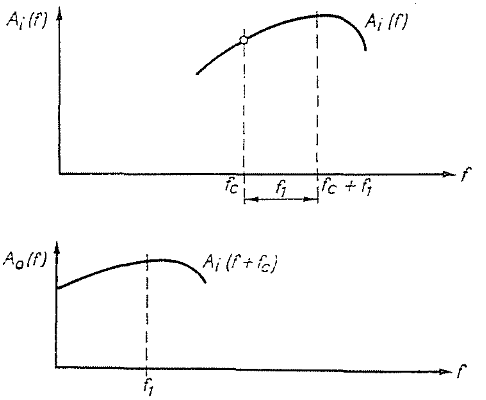

Fig. 4

The "1" 's in the rows 3 to 5 on the principal diagonal represent the fact that single-sideband modulation does not affect the frequency distortion, nonlinear distortion and noise of the transmission medium (i.e. inside transmission path) after demodulation.

The last element in the third row represents the frequency-distortion caused by shortcomings of the conversion system. Whatever frequency f_1 the modulating signal has, the demodulated signal has a frequency $(f_1 + \Delta f_1)$.

The indication matrix of the AM-SSB/SC system is of the form

$$\mathbf{m}_{\text{AM—SSB/SC}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} . \qquad (15)$$

4.2. *Frequency modulation*

The indication matrix of a frequency modulation system with ideal modulator and ideal discriminator-tape demodulator has the form

$$\mathbf{m}_{FM} = \begin{bmatrix} 0 & m_{Ab} & 0 & 0 & 0 & 0 \\ 0 & m_{bb} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{kb} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon_{NN} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{16}$$

This matrix clearly indicates that an ideal FM system generally reduces the noise of the inside transmission path and that only phase-distortion produces disturbances on the outside transmission path.

4.3. *Time-multiplexing (TDM)*

Time multiplexing is accomplished by sampling the signals of different information sources and sending the samples onto the transmission path in a proper sequence. It is to be noted that the information is carried by the amplitude of the sample. (For a quantized transmission, time-multiplexing must be preceded or followed by a quantizer which has to be taken into consideration by another matrix.)

The indication-matrix of time-multiplexing is

$$\mathbf{m}_{\mathrm{TDM}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ m_{NA} & m_{Nb} & m_{N\Delta f} & 0 & m_{NN} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{17}$$

It is to be emphasized that every disturbance, except nonlinear distortion, of the inside transmission path may cause noise in the outside transmission path. It is due to the dispersive nature of transmission paths producing intersymbol-interference among the samples of the various information sources.

4.4. *Compander*

Transmission equipments often make use of a compander consisting of a compressor at the transmitter side and an expander at the receiving side.

From the indication-matrix of a compander

$$\mathbf{m}_{\mathrm{komp.}} = \begin{bmatrix} \nu_{AA} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon_{NN} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

it is seen that it increases the imperfection of the amplitude-function in the inside transmission path but reduces the noise introduced there, according to its intended purpose.

5. Examples for matrices of complex systems

The treatment of matrices of some complex conversion systems, each containing two individual conversions, will be restricted to the indication-matrix.

5.1. Double frequency modulation

The indication matrix of the FM + FM system can be obtained from Eq. (7) and (16). Assuming, for sake of simplicity, the two FM systems to be of the same features:

$$\underset{\text{Eq. (7)}\quad\text{Eq. (16)}}{\mathbf{m}_{\mathrm{FM+MF}} = \mathbf{m}_{\mathrm{FM}} \cdot \mathbf{m}_{\mathrm{FM}}} = \begin{bmatrix} 0 & m_{Ab} \cdot m_{bb} & 0 & 0 & 0 & 0 \\ 0 & m_{bb} \cdot m_{bb} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{kb} \cdot m_{bb} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon_{NN} \cdot \varepsilon_{NN} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} . \tag{19}$$

This matrix is of the same type as that of the single FM given in Eq. (16), but the values of the corresponding elements are different. For usual disturbances and frequency deviations $m_{ij} < 1$, hence the elements of the indication-matrix for double FM are smaller than the elements corresponding to single FM. This statement is especially valid for noises supposing usual (large enough) signal-to-noise ratios.

Consequently, the properties of transmission medium can greatly be improved by the use of double FM.

5.2. Simultaneous use of compander and FM

Let us investigate the system, to transmit the signal first into a dynamic-compressor and then into an FM modulator, in the receiver of which a dynamic-expander succeeds the FM demodulator.

The resulting matrix can be found by using Eqs (7), (16) and (18).

$$\mathbf{m}_{\mathrm{komp+FM}} = \mathbf{m}_{\mathrm{komp}} \cdot \mathbf{m}_{\mathrm{FM}} = \begin{bmatrix} 0 & \nu'_{AA} \cdot m''_{Ab} & 0 & 0 & 0 & 0 \\ 0 & m''_{bb} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m'_{kb} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon'_{NN} \cdot \varepsilon''_{NN} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} . \tag{20}$$

Here ′ and ″ denote compander and FM conversion, respectively.

From this matrix it is seen that the transmission system consisting of compander and FM combines the advantages of the component conversions without their disadvantages. Compander reduces the effect of the noise of the inside transmission path (see ε'_{NN}) but it increases the effect of the amplitude-function (see ν'_{AA}). The latter deleterious effect is eliminated by the use of FM conversion. Namely, the multiplier m''_{Ab}, usually less than one, appears in the term $(\nu'_{AA}\, m''_{Ab})$ which, in turn, expresses the resulting distortion of the amplitude-function. As against this, compander doesn't impair the advantage of FM systems that only the phase-distortions of the inside transmission path are transformed onto the outside one, and the effect of noise is reduced.

6. Conclusions

The way of description outlined thus far offers possibility for the mathematical treatment of complex modulation systems.

The quantitative treatment by the indication-matrix can readily be carried out and checked manually. Neither is the quantitative treatment by the conversion-matrix, in principle, more complicated, though the assembly of conversion-matrices as well as the development of the computer-aided analysis to accelerate results are rather tedious.

Appendix: Properties of transmission paths

In the investigation of conversion systems the quantities used to characterize transmission paths are of great importance.

Therefore it is necessary to exactly define the disturbances and thus the properties of the transmission paths. The concept of disturbance is best understood from the concept of ideal transmission (called also true transmission).

A.1. True transmission and its conditions

The transmission path has to convey the signal from its input to its output. A transmission path is said to be free of disturbances (i.e. ideal) if the signal at its input and output are connected by the relationship

$$s_2(t) = A_0 \cdot s_1(t - T_0) \tag{A.1}$$

Here A_0 and T_0 are arbitrary constants, $s_1(t)$ and $s_2(t)$ denote input and output signals, respectively.

Eq. (A.1) is illustrated in Fig. A.1. It is seen that the phenomenon is properly described by the term "true transmission".

Two remarks, however, should be made.

a) By "transmission path" a simple or complex system is meant, intended to transmit signals faithfully. Thus, definition (A.1) cannot be applied to stages (e.g. modulators) *intended* to change the physical parameters (e.g. bandwidth, dynamic range, frequency, etc.) of the signal. But it does apply to pairs of such stages (e.g. modulator and demodulator.)

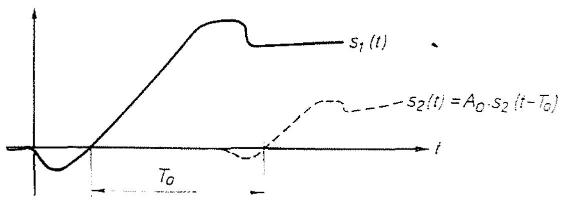

Fig. A.1

b) In general, transmission path is not required to transmit *every* signal faithfully. For example filters needn't fulfil Eq. (A.1) in handling signals with spectra partly or entirely within their stop-band.

Taking into consideration remarks *a)* and *b)* the following definition is valid:

The transmission of a transmission path is called true if Eq. (A.1) holds for every signal to be transmitted.

This definition has been formulated in the time domain as Eq. (A.1) contains functions of time. Consequently, this definition is not suitable to check the transmission for truth. To indicate this, we mention how difficult the production of "every signal to be transmitted" is. Eq. (A.1) is usually transformed using Laplace-transformation; thus yielding the frequency-domain-condition of true transmission. To write this condition let us define the transfer function in a usual way

$$K(jf) = \frac{\text{complex amplitude of the output sinusoidal signal, with frequency } f\text{, in the steady-state}}{\text{complex amplitude of the input sinusoidal signal, with frequency } f\text{, in the steady-state}} \tag{A.2}$$

Using the common notation

$$K(jf) = A(f) \cdot e^{-jb(f)} \tag{A.3}$$

where

$A(f)$ amplitude-function
$b(f)$ phase-function

the frequency-domain-condition of true transmission can be written in the form

$$K(jf) = A_0 \cdot e^{-j2\pi T_0 f} \quad \text{for } f \in F \tag{A.4}$$

or more precisely

$$A(f) = A_0 \quad \text{for } f \in F \tag{A.5.a}$$

$$b(f) = 2\pi T_0 f \quad \text{for } f \in F \tag{A.5.b}$$

where F denotes the set of frequences to be transmitted.

Condition A.5. is illustrated in Fig. A.2.

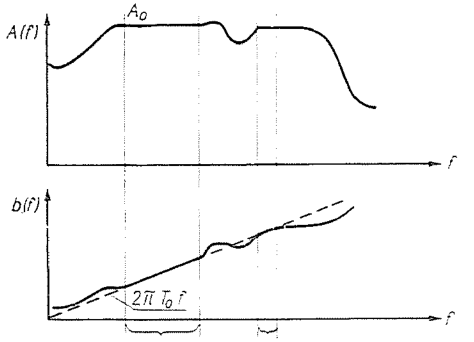

Fig. A.2

According to the remark *b)*, Eqs (A.4) and (A.5) hold only at frequencies to be transmitted. Conditions (A.4) and (A.5) are easy to check by sinusoidal generators, sinusoidal voltmeters and sinusoidal phase-meters.

A.2. Disturbances of the transmission path

Most of the transmission paths fall short of the condition of true transmission given by the equation

$$s_2(t) = A_0 \cdot s_1(t - T_0) \tag{A.1}$$

Disturbances are defined by

$$e(t) = s_2(t) - A_0 \cdot s_1(t - T_0) \tag{A.6}$$

Thus, disturbances may be interpreted as errors of transmission. In a real case only $s_1(t)$ and $s_2(t)$ being given but A_0 and T_0 being undetermined, Eq. (A.6) must be completed by the remark that A_0 and T_0 are to be chosen so as to minimize the error in a certain sense (e.g. mean square).

It is reasonable to decompose disturbances into two parts

$$e(t) = d(t) + n(t) \tag{A.7}$$

where $d(t)$ and $n(t)$ denote distortions and noise, respectively. In particular:

1. Distortions $d(t)$ are those parts of disturbances which are present at the output of the transmission path only if, at its input, there is a signal to be transmitted. Thus, distortion is a function of the input signal

$$d(t) = f[s_1(t)]\,. \tag{A.8}$$

Distortions may be divided into sub-groups as follows:

a) **Linear distortions** are attributed to a transmission path if the transmission is not faithful, but proportional changing of the input signal causes proportional changing of the output signal by the same ratio. According to the latter statement the transmission path is featured by linearity.

Instead of the above time-domain-definition alternative frequency-domain-definition is widely used:

linear distortions are attributed to a transmission path if its transfer function differs from the ideal one given in Eq. (A.4) but is independent of the level of the input signal. (The latter statement refers to linearity.)

The various deviations from ideal transfer function yield different types of linear distortions:

α) Amplitude (or attenuation) *distortion* is attributed to a transmission path if condition $A(f) =$ const doesn't hold for the set of frequencies to be transmitted.

β) Phase or delay distortion is attributed to a transmission path if condition $b(f) = 2\pi T_0 f$ or $\tau(f) = T_0$ doesn't hold for the set of frequencies to be transmitted. (τ denotes group delay.)

γ) Frequency distortion is attributed to a transmission path if it changes the frequency of sinusoidal signals passing through it. In this case the Fourier-transforms of the input and output signals are related by the equation

$$\mathbf{F}_2(jf) = \mathbf{F}_1[j(f - \Delta f)].$$

Here Δf denotes frequency offset which in certain cases may be a function of frequency. Frequency-distortion may be encountered when transmission path is led through a frequency-multiplexing carrier system whose modulator and demodulator operate with carriers of different frequencies.

δ) Modulation-distortion is attributed to a transmission path if its amplitude, phase, or group delay function varies with time.

b) Nonlinear distortion is attributed to a transmission path if proportional change of the input signal of *any shape* doesn't proportionally change the output signal. This is illustrated in Fig. A.3 where signals marked with the same type of comma (e.g. single or double) correspond to an input-output pair.

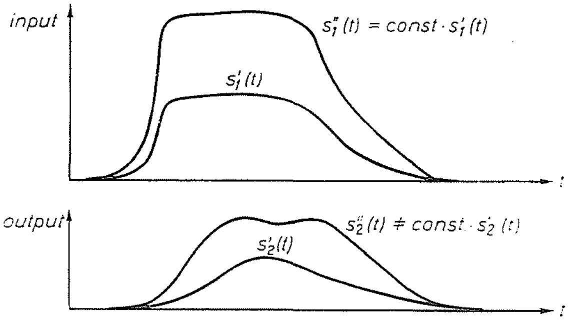

Fig. A.3

Definition of nonlinear distortion is described mathematically as follows:

Suppose that input signal $s_1'(t)$ corresponds to output signal $s_2'(t)$ and the same holds for $s_1''(t)$ and $s_2''(t)$. Suppose further that $s_1'(t)$ and $s_2''(t)$ are proportional, i.e.

$$\frac{s_1''(t)}{s_1'(t)} = c = constant\,.$$

If the transmission path is nonlinear, we may write

$$\frac{s_2''(t)}{s_1'(t)} \neq c\ .$$

This inequality not only means that the output ratio differs from the input one, but also that it may be a function of time. Such a general case is shown in Fig. A.3.

2. Noise $n(t)$ is that part of the disturbances which occurs at the output of the transmission path also in the case where there is no signal at its input, i.e. $s_1(t) = 0$.

From the point of view of noise, two kinds of transmission paths must be distinguished. The first is called "isolated" transmission path. An isolated transmission path uses all the stages of the transmission equipment alone. To the contrary, there exists such a transmission path, which is one out of many, the others using partly or entirely the same transmission equipment. This will be called "one-out-of-many" transmission path.

a) Internal noise denotes first the entire noise in an isolated transmission path and secondly that part of the noise in an one-out-of-many transmission path which doesn't depend on the signals of the other transmission paths in the same multiplexing bunch. Internal noise may originate from

α) the circuitry itself (then it is called *noise of zero order*, e.g. thermal noise), or from

β) sources outside the transmission path (then it is called *noise of first order*, e.g. interference, power supply hum, ionospheric noise, etc.).

b) Multiplexing noise originates from the signals of transmission paths using the same equipment as the transmission path in question. There is no multiplexing noise in an isolated transmission path. Multiplexing noise may be of first or higher order.

α) First order multiplexing noise originates from linear distortions of transmission equipment. Linear crosstalk in frequency-multiplexing as well as cross-talk from intersymbol-interference in time-multiplexing belong to this sub-group.

β) Higher order multiplexing noise originates from nonlinear distortions of transmission equipment. Characteristic for this sub-group is intermodulation noise in frequency-multiplexing.

Summary

In this paper a unified description of modulation systems is proposed. By means of this description

(i) the post-demodulation-effects of the disturbances (like linear and nonlinear distortions as well as noise) of the transmission medium can easily be determined (in other terms the determination of the transformation of disturbances is simple);

(ii) a common principle can be found for the treatment of modulation (where modulation is meant in a narrow sense) and for other conversions, employed in the transmission, like time-multiplexing, frequency-multiplexing, preemphasis and deemphasis, compander, quantization e.t.c.

(iii) the features of complex modulation systems can easily be determined from the features of the component modulations and other conversions.

The description is performed by a matrix. The "conversion-matrix" and "indication-matrix" are introduced for the purpose of quantitative and qualitative description, respectively. After the definition of the matrix some examples concerning simple and complex modulation systems are given. The examples cover AM-SSB/SC, FM, time-multiplexing, compander, FM+FM, compander ÷ FM.

The description offers possibility for the mathematical characterization of modulation systems, and thereby also for the selection of a modulation system which is optimum for given transmission medium and for given specifications concerning distortions as well as noise.

In order to construct the conversion matrix the characterization of transmission paths is necessary. This characterization is given in the Appendix.

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