STABILITY TEST OF LINEAR CONTROL SYSTEMS WITH DEAD TIME COMPENSATED BY PI CONTROLLER

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In a previous paper [1] we have theoretically investigated the stability region variations of a linear control system of second order lag and dead time shown in Fig. 1 with a unite feedback, compensated, generally speaking, by a PID controller. We have derived that the transcendental equation determin-

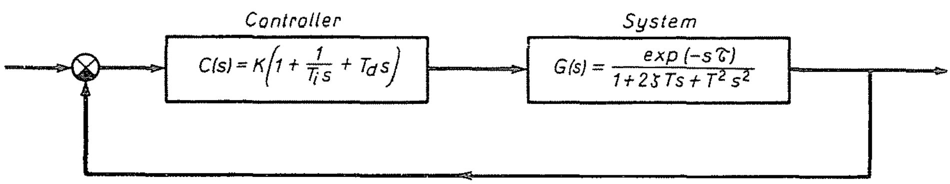


Fig. 1

ing ω_{cr}, the angular frequency belonging to the critical loop gain K_{cr} is:

$$\begin{aligned}(T^2 T_i\omega_{cr}^3 - \omega_{cr} T_i + 2\zeta T\omega_{cr} - 2\zeta T\omega_{cr}^3 T_d T_i)\tan\omega_{cr}\tau = \\ = 2\zeta T T_i\omega^2 - T^2\omega^2 + 1 + \omega^4 T_d T_i T^2 - \omega^2 T_d T_i.\end{aligned} \tag{1}$$

In the knowledge of $\omega = \omega_{cr}$ obtained from (1) by iteration the loop gain may be evaluated:

$$K_{cr} = \frac{1}{|Y_1(j\omega_{cr})|} \tag{2}$$

where $Y_1(s) = Y_1(j\omega)$ is:

$$Y_1 = \left(1 + \frac{1}{T_i s} + T_d s\right)\frac{\exp(-s\tau)}{1 + 2\zeta T s + T^2 s^2}.$$

Control systems with second order lag and dead time compensated by P, I, PI, and PD type controllers may be considered and calculated as special cases of the above control system.

In a following paper [10] we presented graphs showing the stability region variations in function of the dead time and the system time constants for controllers of types P and I with values of K_{cr} obtained by digital computer.

Proportional-plus-integral control

The present paper investigates the stability region variations of the same control system, if it is compensated by PI type controller.

The transcendental equation determining ω_{cr}, the angular frequency belonging to K_{cr} putting $T_d = 0$ into (1) with the assumption of $T_i = 1$ is:

$$(T^2\omega_{cr}^3 - \omega_{cr} + 2\,\zeta T\omega_{cr}) \tan \omega_{cr}\,\tau = 2\,\zeta T\omega_{cr}^2 - T^2\omega_{cr}^2 + 1. \qquad (3)$$

Figs 2—6 show the values of K_{cr} for time-constant ratio $\tau/T = 0.2, 0.6, 1, 4, 10$ vs $0.05 \leq \tau/T_i \leq 10$ with parameter values $\zeta = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.25, 1.5, 1.75, 2$ in log-log scale for the sake of clearness. The figures demonstrate that

a) with increasing ζ, the stability region increases under a given value of τ/T_i, over this value it diminishes. The value of τ/T_i, in the vicinity of which the critical loop gain depends but slighthly on the value of ζ, diminishes with increasing τ/T;

b) with increasing τ/T for very law values of τ/T_i the stability limit more and more approximates the loop gain value $K_{cr} = 1$ independently of the damping factor ζ.

From this last behaviour of the control system follows:

when τ/T_i and τ/T simultaneously approach to 0 and to ∞, respectively, the control system with second order lag and dead time compensated by PI type controller may be substituted by a pure dead time system, for wich the stability limit is $K_{cr} = 1$.

The correctness of the last statements is easy to see. The stability limit from the over-all transfer function of the open loop system is:

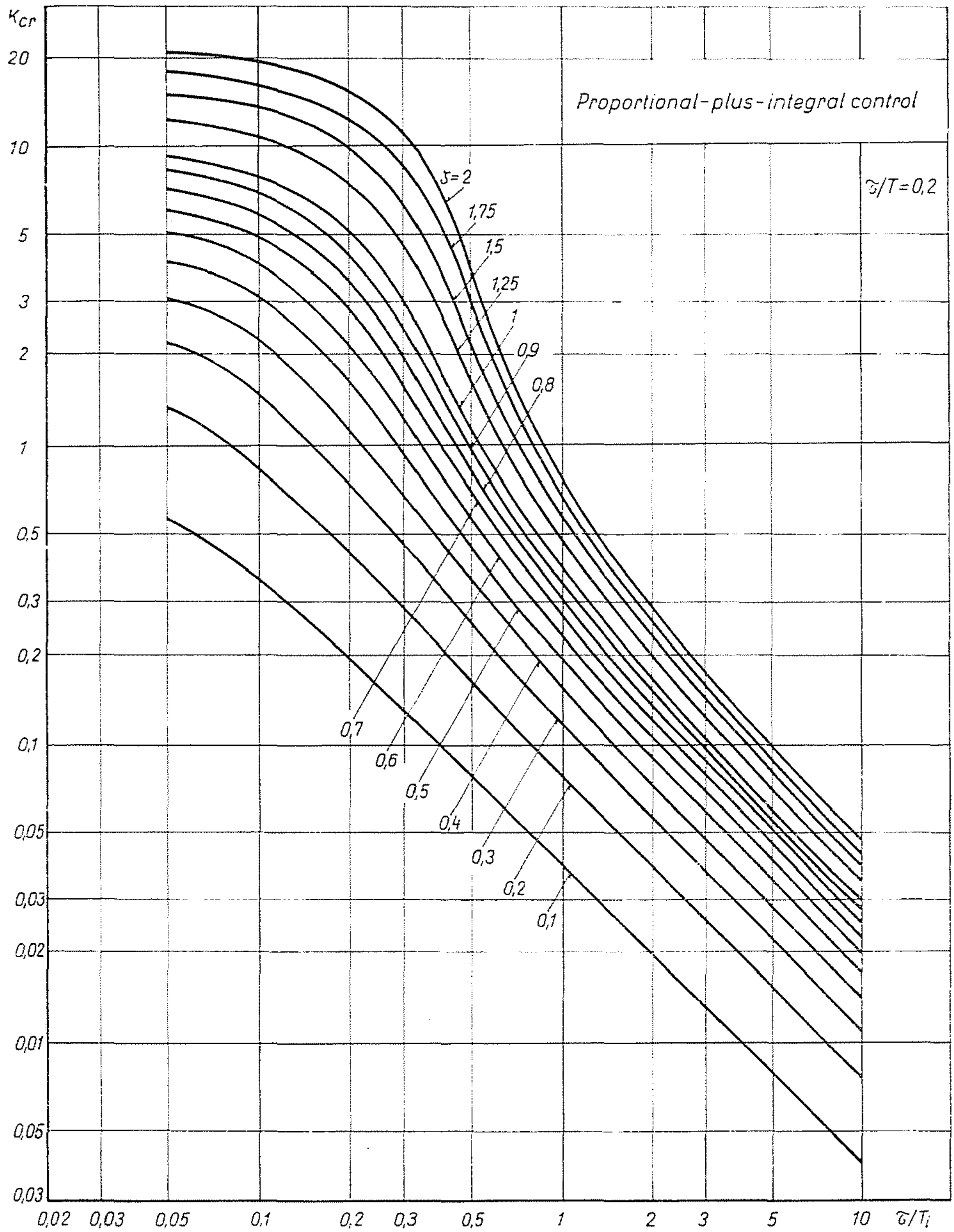
$$Y(s) = K_{cr}\left(1 + \frac{1}{T_i s}\right)\frac{\exp(-s\tau)}{1 + 2\,\zeta Ts + T^2 s^2} = -1\,.$$

After some arrangements we have:

$$K_{cr} = -\frac{T_i/\tau(s\tau)}{1 + T_i/\tau(s\tau)}\;\frac{1 + 2\,\zeta T/\tau(s\tau) + (T/\tau)^2(s\tau)^2}{\exp(-s\tau)}\,. \qquad (4)$$

When $\tau/T_i \to 0$ the critical loop gain approximately is:

$$K_{cr} \approx \frac{1 + 2\,\zeta T/\tau(s\tau) + (T/\tau)^2(s\tau)}{\exp(-s\tau)}\,.$$



K_{cr}
20
10
5
3
2
1
0,5
0,3
0,2
0,1
0,05
0,03
0,02
0,01
0,05
0,03
Proportional-plus-integral control
$\tau/T=0,2$
$\delta=2$
1,75
1,5
1,25
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,02
0,03
0,05
0,1
0,2
0,3
0,5
1
2
3
5
10
τ/T_i

Fig. 2

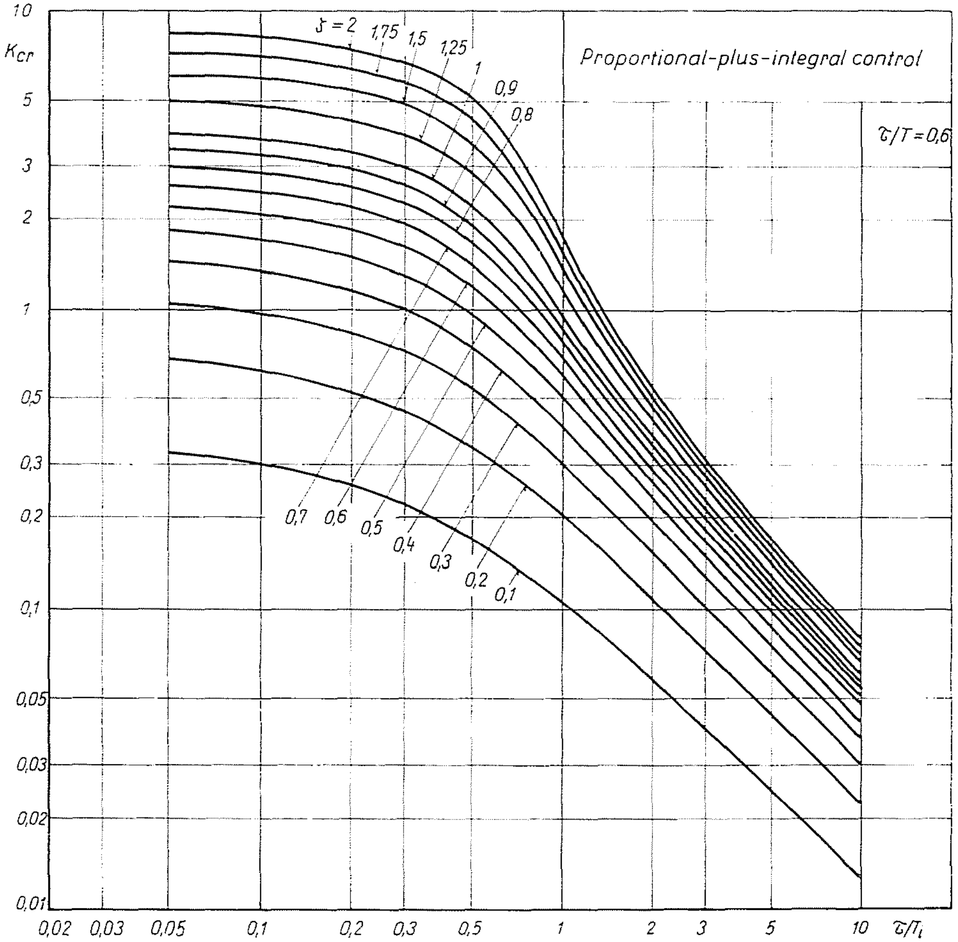


Fig. 3

From the last equation we can see, in the case of small value of τ/T the second order lag character dominates still strongly.

With increasing τ/T the second order lag character may be more and more neglected in comparison to the dead time, as we can see from Figs 2—6.

On the other hand, in case of $\tau/T_i \gg 10$, i.e. when $\tau/T_i \to \infty$, the stability limit from the resultant transfer function of the open loop system (4) is:

$$K_{cr} \approx T_i/\tau(s\tau)\,\frac{1 + 2\,\zeta(T/\tau)\,(s\tau) + (T/\tau)^2\,(s\tau)^2}{\exp\,(-s\tau)}\,.$$

Consequently, when $\tau/T_i \to \infty$, the system may be substituted by a system with second order lag and dead time compensated by I type controller.

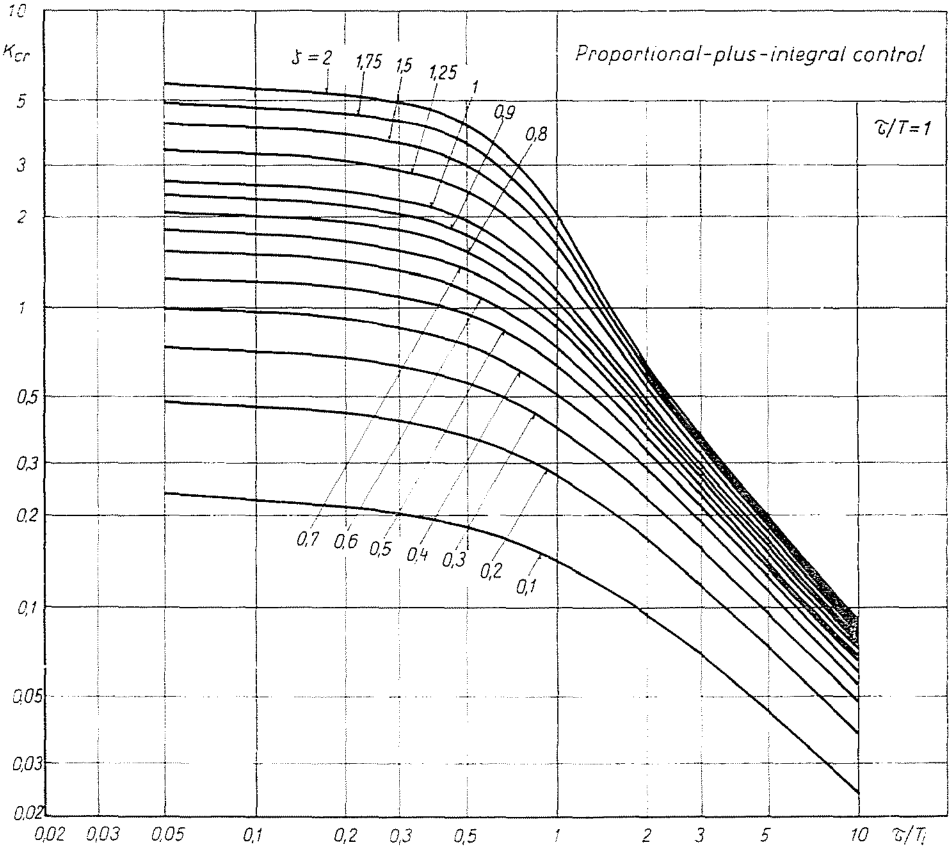

Proportional-plus-integral control
τ/T=1
K_{cr}
ζ = 2 1,75 1,5 1,25 1 0,9 0,8
0,7 0,6 0,5 0,4 0,3 0,2 0,1
τ/T_i

Fig. 4

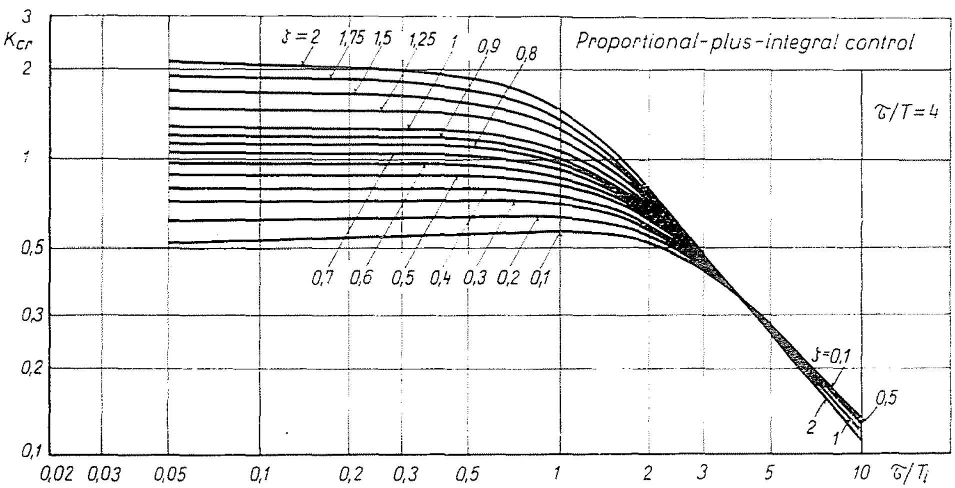

Proportional-plus-integral control
τ/T=4
K_{cr}
ζ = 2 1,75 1,5 1,25 1 0,9 0,8
0,7 0,6 0,5 0,4 0,3 0,2 0,1
ζ=0,1
0,5
2
1
τ/T_i

Fig. 5

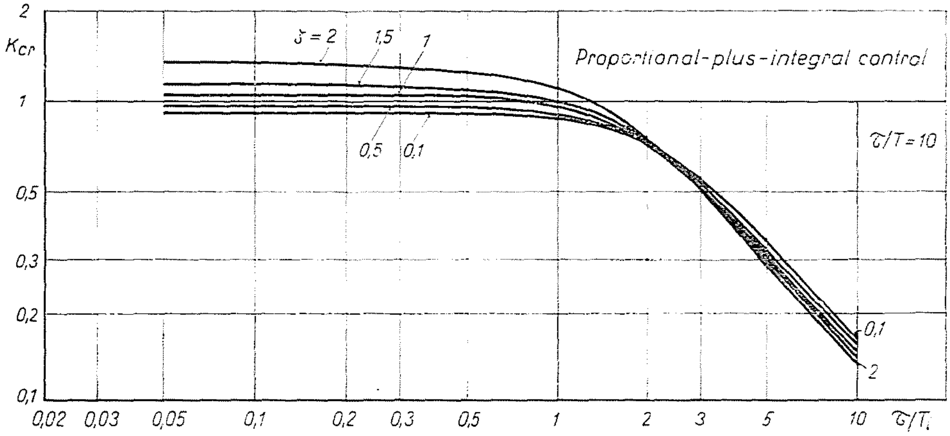


Fig. 6

According to the last statements in case of $\tau/T_i \gg 10$ the critical loop gain values extrapolated from Figs 2—6 agree with the values K_{cr} obtained for I type compensation.

For instance let be:

$$\tau/T = 0.6$$
$$\zeta = 0.5$$
$$\tau/T_i = 30.$$

On the basis of Fig. 3 the extrapolated value is $K_{crPI} = 1.4 \cdot 10^{-2}$. On the other hand using Fig. 7 of Ref. [10] the corresponding value is:

$$K_{cr} \frac{\tau}{T_i} \Big|_I = 0{,}42 \,.$$

Hence $K_{crI} = 0.42/30 = 1.4 \cdot 10^{-2}$ is identical with K_{crPI}.

Conclusion

In the knowledge of the stability region belonging to a given linear continuous control system with second order lag and dead time compensated by P, I and PI controllers we are already able to choose the most advantageous type of compensation.

We found out [10] that for low dead time values a proportional compensation is preferable. For high dead time values the integral type control proves to be best. On the basis of [10] and this paper the most suitable controller may be calculated for the intermediate dead time values as well, with the help of the given system constants.

For the sake of completeness the investigation of PID type compensation will be dealt with in a coming paper.

Summary

The present paper gives the stability region variations of a linear continuous control system with second order lag and dead time compensated by proportional-plus-integral action controller. The critical loop gain values were evaluated by digital computer. The diagrams representing K_{cr} in function of the dead time for $0.05 \leq \tau/T_i \leq 10$ are plotted in log-log scale for the sake of clearness with the system time constants as parameter.

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