THE ANALYSIS OF THE WAVEGUIDE Y-CIRCULATOR

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Introduction

Nonreciprocal elements, particulary circulators are often used in modern microwave circuits. They can be utilized in filter-multiplexers, before parametric and paramagnetic amplifiers, etc. A circulator can be made of waveguides (Fig. 1) or strip lines. The ideal circulator is also an important element from the point of view of network theory [10].

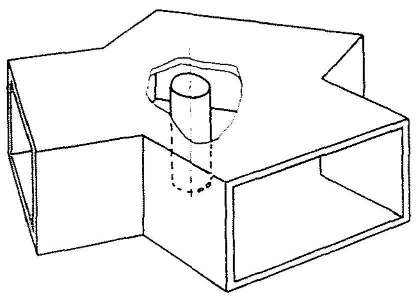


Fig. 1

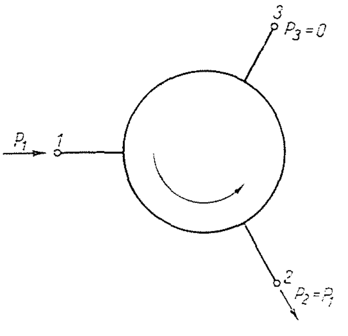


Fig. 2

The ideal circulator is a nonreciprocal element having three or more terminal pairs (ports). The total input power entering any port leaves perfectly through the terminal pair in turn. In the case of three terminal pairs the power entering the port 1 leaves on the terminal pair 2, in accordance with the direction of circulation, while no power will reach the terminal pair 3 (Fig. 2). Practically, the ideal circulator can only be approximated, since losses and reflections occur. Its characteristics:

insertion loss (forward attenuation): 0.2—0.5 dB
isolation (backward attenuation): 20—30 dB

The effect of circulation is mostly produced by circular or triangular based ferrite cylinders, pre-magnetized by a permanent magnetic field (Fig. 1).

The exact calculation of waveguide Y-circulators requires the solution of Maxwell-equations with inhomogeneous boundary conditions. The calculation encounters significant difficulties due to the complicated geometrical arrangement. In order to simplify the problem, some authors [1—4] set out from theories reflecting the fundamental physical properties, but they did not strive for completeness.

In this paper the analysis of waveguide Y-circulator is done on the basis of [5], in the following main steps:

As a first step, through the exact functional analysis of the idealized electrodynamical problem, the general lumped equivalent circuit of the network is determined. (By idealization is meant here the assumption of linearity and exemptness of losses.)

As a second step, this non-reciprocal network of infinite dimensions is approximated in an arbitrary $[\omega_a, \omega_b]$ $(0 \leq \omega_a \leq \omega_b < \infty)$ frequency region by a network of finite dimensions. The approximate network will be the more complicated, the wider the observed frequency region and the closer the original network is to be approximated.

As a conclusion, the general results are applied to analyse numerically the cavity arrangement in Fig. 1. With the knowledge of the given geometrical arrangement and the characteristics of the materials we determine the parameters of the equivalent circuit and compare them with results of measurements.

The difference between the measured and calculated values will appear to be below 10%.

1. The general lumped equivalent of microwave networks

Determination of the general lumped equivalent circuit is suitably set out from the general cavity arrangement in Fig. 3.

The closed cavity V is bounded by a continuous and sectionally smooth surface F.

Let the permittivity tensor $\varepsilon_0 \boldsymbol{\epsilon}$ and the permeability tensor $\mu_0 \boldsymbol{\mu}$ be sectionally smooth functions of place coordinates.

Be the tensors $\boldsymbol{\epsilon}$ and $\boldsymbol{\mu}$ linear, hermitian and positively definite ones. (Conditions of linearity, exemptness of losses and positive field energy.)

These conditions are closely approximated in the cases of most practically used cavity arrangements.

Electric and magnetic fields of the cavity are described by suitably chosen complete ortho-normalized function systems. Each of these function systems can be combined of two subsystems, by uniting a solenoidal and an irrotational one.

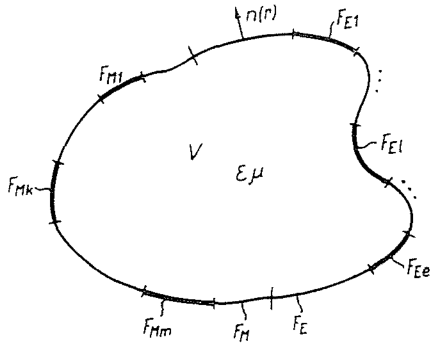


Fig. 3

The $\mathbf{E}_a$, $\mathbf{H}_a$ solenoidal functions of series-expansion are defined by the following system of differential equations:

$$\begin{aligned} \mathbf{rot}\,\mathbf{H}_a(\mathbf{r}) &= k_a\,\epsilon(\mathbf{r})\,\mathbf{E}_a(\mathbf{r}) \qquad && \mathbf{r}\in V \\ \mathbf{rot}\,\mathbf{E}_a(\mathbf{r}) &= k_a\mu(\mathbf{r})\,\mathbf{H}_a(\mathbf{r}) && \mathbf{r}\in V \\ \mathbf{E}_a(\mathbf{r})\,x\mathbf{n}(\mathbf{r}) &= \mathbf{0} && \mathbf{r}\in F_E \\ \mathbf{H}_a(\mathbf{r})\,x\mathbf{n}(\mathbf{r}) &= \mathbf{0} && \mathbf{r}\in F_M\,. \end{aligned} \tag{1}$$

The $\mathbf{E}_d$, $\mathbf{H}_b$ irrotational functions of series-expansion are defined by the following systems of differential equations:

$$\begin{aligned} \Delta\mu(\mathbf{r})\,\mathbf{H}_b(\mathbf{r}) + k_b^2\,\mathbf{H}_b(\mathbf{r}) &= \mathbf{0} \qquad && \mathbf{r}\in V \\ \mathbf{n}(\mathbf{r})\mu(\mathbf{r})\,\mathbf{H}_b(\mathbf{r}) &= \mathbf{0} && \mathbf{r}\in F_E \\ \mathbf{div}\,\mu(\mathbf{r})\,\mathbf{H}_b(\mathbf{r}) &= \mathbf{0} && \mathbf{r}\in F_M \end{aligned} \tag{2}$$

$$\begin{aligned} \Delta\epsilon(\mathbf{r})\,\mathbf{E}_d(\mathbf{r}) + k_d^2\,\mathbf{E}_d(\mathbf{r}) &= \mathbf{0} \qquad && \mathbf{r}\in V \\ \mathbf{div}\,\epsilon(\mathbf{r})\,\mathbf{E}_d(\mathbf{r}) &= \mathbf{0} && \mathbf{r}\in F_E \\ \mathbf{n}(\mathbf{r})\,\epsilon(\mathbf{r})\,\mathbf{E}_d(\mathbf{r}) &= \mathbf{0} && \mathbf{r}\in F_M \end{aligned} \tag{3}$$

where $\mathbf{n}(\mathbf{r})$ is the outer normal, F_E is an electric wall, and F_M is a magnetic wall. Both latter united give the total bounding surface F.

The eigenfunctions belonging to different k_ν eigenvalues are orthogonal to each other. There being a finite number of linearly independent eigenfunctions having the same eigenvalue k_ν, they can be orthogonalized, for instance by the Schmidt-procedure [14]:

$$k_e \int_v \mathbf{E}_v^* \,\epsilon(\mathbf{r})\, \mathbf{E}_w \, dV = \delta_{vw} \qquad \delta_{vw} = \begin{cases} 0 & v \neq w \\ 1 & v = w \end{cases} \tag{4}$$

$$k_e \int_v \mathbf{H}_p^* \,\mu(\mathbf{r})\, \mathbf{H}_q \, dV = \delta_{pq} \qquad \delta_{pq} = \begin{cases} 0 & p \neq q \\ 1 & p = q \end{cases}$$

$$v, w \in \{a\} \dotplus \{d\}; \quad p, q \in \{a\} \dotplus \{b\}$$

k_e — constant of normalization, with the dimension [l/m].

Let us arrange both the electric and the magnetic functions of series-expansion into column-vectors of infinite dimensions in the same way:

$$\mathcal{E} = \begin{bmatrix} \mathbf{E}_1 \\ \vdots \\ \mathbf{E}_n \\ \vdots \end{bmatrix} \qquad \mathcal{H} = \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_n \\ \vdots \end{bmatrix} \tag{5}$$

The electric and the magnetic field of the cavity can be described by the equations (5) as follows:

$$\mathbf{E} = \mathcal{E}^T \mathbf{U} \qquad \mathbf{H} = \mathcal{H}^T \mathbf{J} \tag{6}$$

$$\mathbf{U} = k_e \int_v \mathcal{E}^{*T} \epsilon \mathbf{E}\, dV \qquad \mathbf{J} = k_e \int_v \mathcal{H}^{*T} \mu\, \mathbf{H}\, dV \tag{7}$$

The correlation between electric and magnetic fields is described by the Maxwell equations. Their form is, in case of harmonic time-dependence:

$$\begin{aligned} \mathbf{rot}\, \mathbf{H}(\mathbf{r}) &= j\omega\varepsilon_0\, \epsilon(\mathbf{r})\, \mathbf{E}(\mathbf{r}) \\ \mathbf{rot}\, \mathbf{E}(\mathbf{r}) &= -\, j\omega\mu_0\, \mu(\mathbf{r})\, \mathbf{H}(\mathbf{r}) \,. \end{aligned} \tag{8}$$

Let us multiply the first equation of (8) by column-vector $\mathcal{E}^*$ and the second one by column-vector $\mathcal{H}^*$, and integrate both equations for the volume V. After appropriate vectoranalytical transformations, and using Eq. (7), the columnvectors formed from the coefficients of series-expansions of electric and magnetic fields, and the column-vectors determined from the excitations are correlated by the following algebraical system of equations:

$$\begin{bmatrix} \mathbf{U} \\ \\ \mathbf{J} \end{bmatrix} = \begin{bmatrix} j\Omega\mathbf{A} & \mathbf{\Omega}\mathbf{A} \\ \\ -\mathbf{\Omega}\mathbf{A} & \Omega\mathbf{A} \end{bmatrix} \begin{bmatrix} \sum_{=1}^{m} \mathbf{J}'_k \\ \\ \sum_{l=1}^{e} \mathbf{U}'_l \end{bmatrix} . \tag{9}$$

The meaning of the given quantities:

m = number of magnetic apertures;
e = number of electric apertures;

$$\mathbf{J}'_k = \int_{F_{Mk}} \mathbf{H}_t\,(\mathcal{E}^* x \mathbf{n})\, df_k \qquad k = 1, 2, \ldots, m \tag{10}$$

$$\mathbf{U}'_l = \int_{F_{El}} \mathbf{E}_t\,(\mathcal{H}^* x \mathbf{n})\, df_l \qquad l = 1, 2, \ldots, e \tag{11}$$

$\mathbf{\Omega} = \langle \Omega_i \rangle$ diagonal matrix;

$\Omega_i = \begin{cases} 0 & \text{for solenoidal functions of series-expansion;} \\ k_i/k_e & \text{for irrotational functions of series-expansion;} \end{cases}$

$\Omega = \omega \sqrt{\mu_0 \varepsilon_0}/k_e$ relative frequency;

$\sqrt{\mu_0/\varepsilon_0}$ impedance unit chosen

$$\mathbf{A} = [\mathbf{\Omega}^2 - \Omega^2 \mathbf{E}]^{-1}. \tag{9b}$$

The network, given by the system of equations (9) disintegrates to an infinite number of mutually independent two terminal pairs, $\mathbf{\Omega}$, $\mathbf{A}$ being diagonal. One of the two terminal pairs is shown on the framed part of Fig. 5.

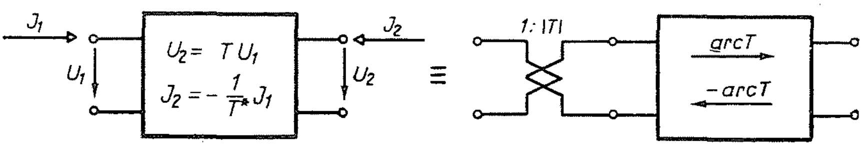


Fig. 4

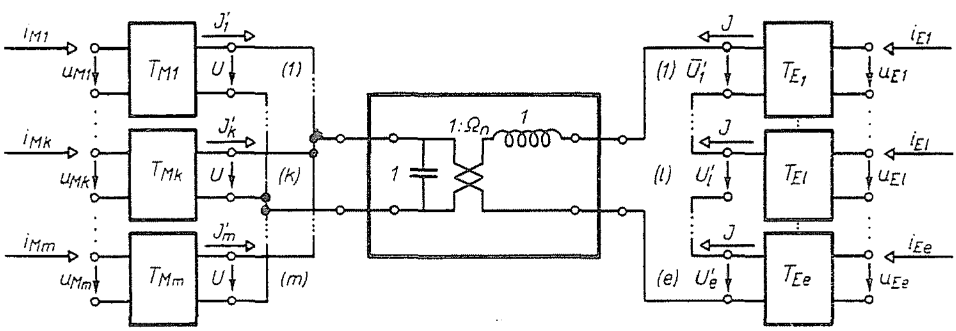


Fig. 5

The so-called internal cavity-terminal pairs of the different cavities defined by the equations (7), (10) and (11) connected by a common aperture cannot be connected directly. For this reason by generalizing REITER's [9] procedure, we expand in a series the tangential electric and magnetic field of each aperture with the aid of a real, complete, ortho-normalized surface function system:

In case of electric aperture:

$$\mathbf{E}_{tl} = \mathbf{u}_{El}^T \, \boldsymbol{e}_l \qquad \mathbf{u}_{El} = \int_{F_{El}} \boldsymbol{e}_l \, \mathbf{E}_{tl} \, df_l \tag{12}$$

$$\mathbf{H}_{tl} \, x\mathbf{n} = - \, \mathbf{i}_{El}^T \, \boldsymbol{e}_l \qquad \mathbf{i}_{El} = \int_{F_{El}} \boldsymbol{e}_l \, (\mathbf{H}_{tl} x\mathbf{n}) \, df_l \tag{13}$$

In case of magnetic aperture:

$$\mathbf{H}_{tk} = \mathbf{i}_{Mk}^T \, \boldsymbol{h}_k \qquad \mathbf{i}_{Mk} = \int_{F_{Mk}} \boldsymbol{h}_k \, \mathbf{H}_{tk} \, df_k \tag{14}$$

$$\mathbf{E}_{tk} \, x\mathbf{n} = \mathbf{u}_{Mk}^l \, \boldsymbol{h}_k \qquad \mathbf{u}_{Mk} = \int_{F_{Mk}} \boldsymbol{h}_k \, (\mathbf{E}_{tk} \, x\mathbf{n}) \, df_k \tag{15}$$

The network between the internal cavity-terminal pairs and the so-called aperture terminal pairs defined by the equations (12)—(15) is obtained by expanding into series the column-vectors $\mathcal{E}$ and $\mathcal{H}$ with respect to the function-systems $\boldsymbol{h}_k$ and $\boldsymbol{e}_l$, respectively.

$$\mathcal{H}(\mathbf{r}) \, x\mathbf{n}(\mathbf{r}) = - \, \mathbf{T}_{El} \, \boldsymbol{e}_l(\mathbf{r}) \tag{16}$$

$$\mathbf{T}_{El} = - \int_{F_{El}} \boldsymbol{e}_l(\mathbf{r}) \left(\mathcal{H}^T(\mathbf{r}) \, xn(r) \right) df_l \tag{17}$$

$$\mathbf{i}_{El} = - \, \mathbf{T}_{El}(-\mathbf{J}) \qquad \mathbf{U}_l' = \mathbf{T}_{El}^{*T} \, \mathbf{u}_{El}$$

$$\mathcal{E}(\mathbf{r}) \, x\mathbf{n}(\mathbf{r}) = \mathbf{T}_{Mk} \, \boldsymbol{h}_k(\mathbf{r}) \tag{18}$$

$$\mathbf{T}_{Mk} = \int_{F_{Mk}} \boldsymbol{h}_k(\mathbf{r}) \left(\mathcal{E}^T(\mathbf{r}) \, x\mathbf{n}(\mathbf{r}) \right) df_k \tag{19}$$

$$\mathbf{u}_{Mk} = \mathbf{T}_{Mk} \, \mathbf{U} \qquad -\mathbf{J}_k' = - \, \mathbf{T}_{Mk}^{*T} \, \mathbf{i}_{Mk} \, .$$

These networks can be built up with the aid of transformers of complex transformation ratio [5]. A transformer of complex T ratio can be established by a cascade connection of an ideal transformer of ratio $|T|$ and a nonreciprocal phase shifter (Fig. 4).

Fig. 5 shows the general lumped equivalent circuit of the cavity.

The corresponding aperture-terminal pairs of the cavities coupled through a mutual aperture can be interconnected directly in case of electric aperture. and through a transformer of ratio 1 : —1 in case of magnetic aperture. due to the continuity of $\mathbf{E}_t$ and $\mathbf{H}_t$.

2. The simplified equivalent circuit of the coupled cavity-system

The general equivalent circuit of the coupled cavity-system with respect to Fig. 5 is too complicated for practical calculations. For this reason this network will be approximated by a network of finite dimensions. For sake of simplification the function system of series-expansion $(\{\mathsf{E}_r\}, \{\mathsf{H}_{r}\})$ are divided in the same way into two groups, t and r, respectively.

To the group t belong a finite N number of modes. The number and type of selected modes are partly determined by the frequency-region, and partly by the accuracy desired. The wider the frequency-region observed, and the less the error permitted, the more modes must be taken into account for the group t. *All the other modes belong to group r.*

The simplified network of cavities α and β, coupled with each through a mutual aperture, is composed by cascade connecting three partial networks (Fig. 6). The first and the third part consist of the internal cavity-network of

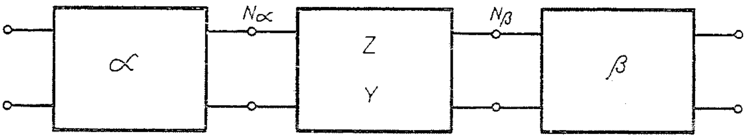


Fig. 6

dimensions N_α and N_β belonging to the suitably chosen functions of series-expansion of coupled cavities α and β, while the second part joins the N_α terminal pairs of the cavity α, and N_β terminal pairs of the cavity β, by a network, given by its Y or Z matrix [5].

In case of electric aperture:

$$Z_{tt'} = -j\,\frac{\left[\int_{F_{El}} (\mathbf{H}_{t'} \times \mathbf{n})\,\mathbf{\Phi}_t^*\,df_e\right]\left[\int_{F_{El}} (\mathbf{H}_t^* \times \mathbf{n})\,\mathbf{\Phi}_{t'}\,df_e\right]}{\int_{F_{El}}\int_{F_{El}} \mathbf{\Phi}_t^*(\mathbf{s})\,\eta^e_{N\alpha+\beta}(\mathbf{s},\mathbf{r})\,\mathbf{\Phi}_{t'}(\mathbf{r})\,df_{ls}\,df_{lr}} \tag{20}$$

where t, t' run through each of the modes N_α and N_β.

$$t, t' \in N_\alpha \times N_\beta$$

$\mathbf{\Phi}_t$, $\mathbf{\Phi}_{t'}$ are the solutions of the following integral equations:

$$\mathbf{H}_t(\mathbf{r}) \times \mathbf{n}(\mathbf{r}) = -\int_{F_{El}} \eta^e_{N\alpha+\beta}(\mathbf{r},\mathbf{s})\,\mathbf{\Phi}_t\,df_{ls}\,, \quad t \in N_\alpha \cup N_\beta \tag{21}$$

where $\eta^e_{N\alpha+\beta}$ — the tensor kernel of the integral equations:

$$\eta^e_{N\alpha+\beta}(\mathbf{r},\mathbf{s}) = \Omega \sum_{r_\alpha, r_\beta}^{\infty} A_r \left(\mathbf{H}_r(\mathbf{r}) \times \mathbf{n}(\mathbf{r})\right) \circ \left(\mathbf{H}_r(\mathbf{r}) \times \mathbf{n}(\mathbf{r})\right)^*. \tag{22}$$

It can be proved that the variation of (20) vanishes if $\mathbf{\Phi}_t$, $\mathbf{\Phi}_{t'}$ are solutions of integral equations (21) [11].

For a magnetic aperture we get a similar expression:

$$Y_{tt'} = -j\,\frac{\left[\int_{F_{Mk}} (\mathbf{E}_{t'} \times \mathbf{n})\,\mathbf{\Psi}_t^*\,df_k\right]\left[_{F_{Mk}} (\mathbf{E}_t^* \times \mathbf{n})\,\mathbf{\Psi}_{t'}\,df_k\right]}{\int_{F_{Mk}}\int_{F_{Mk}} \mathbf{\Psi}_t^*(\mathbf{s})\,\eta^m_{N\alpha+\beta}(\mathbf{s},\mathbf{r})\,\mathbf{\Psi}_{t'}\,df_{ks}\,df_{kr}}\,. \tag{23}$$

The variation of (23) vanishes if $\mathbf{\Psi}_t$, $\mathbf{\Psi}_{t'}$ are solutions of the following integral equations:

$$\mathbf{E}_t(\mathbf{r})\, x\mathbf{n}(\mathbf{r}) = \int_{F_{Mk}} \eta^{m}_{N\alpha+\beta}(\mathbf{r}, \mathbf{s})\, \mathbf{\Psi}_t\, df_k,\ t \in N_\alpha\ U\ N_\beta \tag{24}$$

where

$$\eta^{m}_{N\alpha+\beta} = \Omega \sum_{\beta r_\alpha, r}^{\infty} A_r\big(\mathbf{E}_r(\mathbf{r})\, x\mathbf{n}(\mathbf{r})\big)\ \mathbf{o}\ \big(\mathbf{E}_r(\mathbf{r})\, x\mathbf{n}(\mathbf{r})\big)^* . \tag{25}$$

A_r is the corresponding element of diagonal matrix (9b); the mark **o** means the tensor multiplication.

3. Numerical analysis of the waveguide Y-circulator

A. Determination of the expounding function-systems

Now, the analysis described above, will be applied to a given geometrical arrangement. The cavity-arrangements seen in Figs 1 and 7 will be examined.

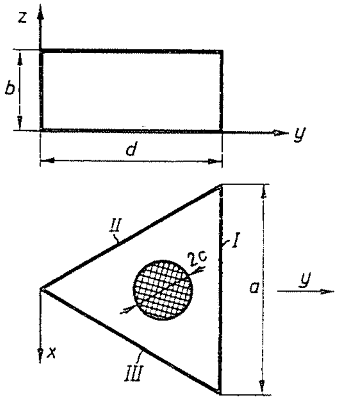


Fig. 7

Let one rectangular waveguide carrying TE_{10} mode join each side of the triangular based cylindrical cavity, bounded by electric wall. So the planes I, II and III are electric aperture surfaces. Since the magnetic field of the mode TE_{10} does not vary in direction of the z axis, it is sufficient to examine only the so-called zero-type eigenfunctions, constant along the z axis. (From the solenoidal group only TM-types exist).

The solenoidal TM oscillations of the hollow cavity resonator are [12]:

$$\begin{aligned} E^{\pm}_{az} &= R\,[e^{\pm j\alpha x} \sin \alpha' y + e^{\mp j\beta x} \sin \beta' y - e^{\pm j\gamma x} \sin \gamma' y] \\ H^{\pm}_{ay} &= \pm j \frac{R}{k_a} [\alpha e^{\pm j\alpha x} \sin \alpha' y - \beta e^{\mp j\beta x} \sin \beta' y - \gamma e^{\pm j\gamma x} \sin \gamma' y] \\ H^{\pm}_{ax} &= \frac{R}{k_a} [\alpha' e^{\pm j\alpha x} \cos \alpha' y + \beta' e^{\mp j\beta x} \cos \beta' y - \gamma' e^{\pm j\gamma x} \cos \gamma' y] . \end{aligned} \tag{26}$$

The irrotational oscillations of the hollow cavity resonator:

$$H_{py}^{\pm} = \frac{R}{k_p}\left[\alpha' e^{\pm j\alpha x} \sin \alpha' y + \beta' e^{\mp j\beta x} \sin \beta' y + \gamma' e^{\pm j\gamma x} \sin \gamma' y\right]$$

$$H_{px}^{\pm} = \mp j \frac{R}{k_p}\left[\alpha e^{\pm j\alpha x} \cos \alpha' y - \beta e^{\mp j\beta x} \cos \beta' y + \gamma e^{\pm j\gamma x} \cos \gamma' y\right] \quad (27)$$

The values of the constants in the formulae:

$$\alpha' = \pi \frac{m}{d} \qquad \beta' = \pi \frac{n}{d} \qquad \gamma' = \pi \frac{m+n}{d}$$

$$\alpha = 2\pi \frac{m+2n}{3a} \qquad \beta = 2\pi \frac{2m+n}{3a} \qquad \gamma = 2\pi \frac{m-n}{3a} \quad (28)$$

$$k_p^2, k_a^2 = \frac{4\pi^2}{3d^2}(m^2 + n^2 + mn); \quad k_e = \frac{1}{a}$$

m, n are positive integers labelling the modes:

$$R = \sqrt{\frac{2}{k_e \sqrt{3}\, d^2\, b}}\,. \quad (29)$$

The magnetic fields of each aperture are seen to be connected by the following relations:

$$H_{mn}^{\pm}(x_{II}, y_{II} = b) = e^{\pm j\, 2/3\, \pi (n-m)} H_{mn}^{\pm}(x_I, y_I = b) \quad (30)$$

$$H_{mn}^{\pm}(x_{III}, y_{III} = b) = e^{\pm j\, 2/3\, \pi (n-m)} H_{mn}^{\pm}(x_I, y_I = b)$$

By introducing the ferrite rod, the field-structure and the natural frequency change. Various methods exist to approximate the natural fields and natural frequencies at a given accuracy [6, 7].

In case of a sufficiently small cylinder diameter, the magnetic field of the aperture remains constant, only the natural frequency alters. The effects of the ferrite dielectric constant and of the permeability tensor are considered separately [6]:

$$\frac{k_{EM}}{k_0} \cong \frac{k_E}{k_0} \cdot \frac{k_M}{k_0} \quad (31)$$

where k_E is the natural frequency of the cavity for pure dielectric influence, k_M that for only magnetic perturbance. k_{EM} is the approximative natural frequency of the jointly loaded cavity, k_0 is the natural frequency of the unloaded cavity.

The value of k_E can be determined by NIKOLSKI's method [6]. If the dielectric constant $\varepsilon = 9$, the decrease of natural frequency is shown in Fig. 8 for the modes TM_{110} and $TM_{210}^{\pm}$.

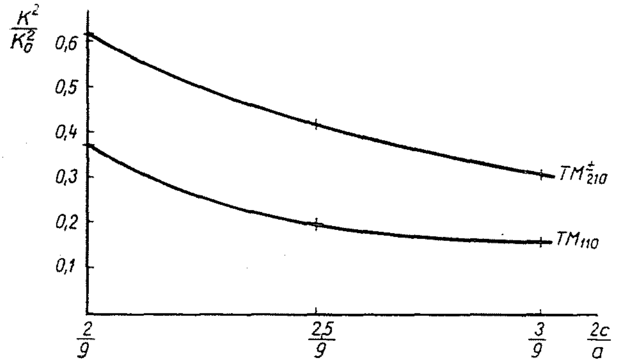


Fig. 8

The value of k_M can be determined by perturbance calculus:

$$\frac{k_{M210}^{\pm} - k_{0210}^{\pm}}{k_{M210}^{\pm}} = \pi \frac{\sqrt{3}\,27}{7} \left(\frac{c}{a}\right)^2 \frac{\mu^2 - (\varkappa \mp 1)^2}{(\mu + 1)^2 - \varkappa^2} \tag{32}$$

where μ and $\varkappa$ are elements of permeability tensor:

$$\mu = \begin{bmatrix} \mu & -j\varkappa & 0 \\ j\varkappa & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{33}$$

The change of natural frequency can be determined similarly for different ferrite shapes.

B. *Determination of the simplified equivalent circuit of the aperture*

Let the examined frequency region approach the natural frequency of mode $TM_{210}^{\pm}$. Then the cavity-modes $TM_{210}^{\pm}$ and P_{110} fall into the group t, all others belong to the group r. The waveguide modes are obtained by uniting the appropriate cavity modes [9]. The propagating TE_{10} mode falls into group t, while the other TE_{n0} and TE_{m0} modes are ranged into group r.

The equations describing the field of the TE waveguide modes:

$$
\begin{aligned}
H_{xn} &= \sqrt{\frac{2}{ab}} \cos (2n-1)\frac{\pi}{2}\frac{2x}{a} \\
H_{xm} &= \sqrt{\frac{2}{ab}} \sin m\pi \frac{2x}{a} \\
Z_{m0}^{-1} &= Y_{m0} = \sqrt{1-\lambda_0^2/\lambda_m^2}\,;\; Z_{n0}^{-1} = Y_{n0} = \sqrt{1-\lambda_0^2/\lambda_n^2} \\
\lambda_0 &= (f\sqrt{\varepsilon_0\mu_0})^{-1}\,;\; \lambda_n = \frac{2a}{2n-1}\,;\; \lambda_m = \frac{a}{m}\,.
\end{aligned}
\tag{34}
$$

In order to determine the topological setting up of the network, the functions of series expansion are expressed by new functions at the apertures:

$$
\begin{aligned}
H'_x &= \frac{1}{2}[H_x(x) + H_x(-x)] \\
H''_x &= \frac{1}{2j}[H_x(x) - H_x(-x)]
\end{aligned}
\qquad x \in \left[-\frac{a}{2}, \frac{a}{2}\right].
\tag{35}
$$

$H'_x(x)$ and $H''_x(x)$ being even and odd functions of x, resp. thus, for an arbitary index:

$$
\int_{F0}(H'_k xn)\;(H''_e xn)df_0 = 0 \qquad l, k = 1, 2, \ldots . \tag{36}
$$

With these functions we introduced new terminal pairs, related to the previous terminal pairs by the equations (37) and (38):

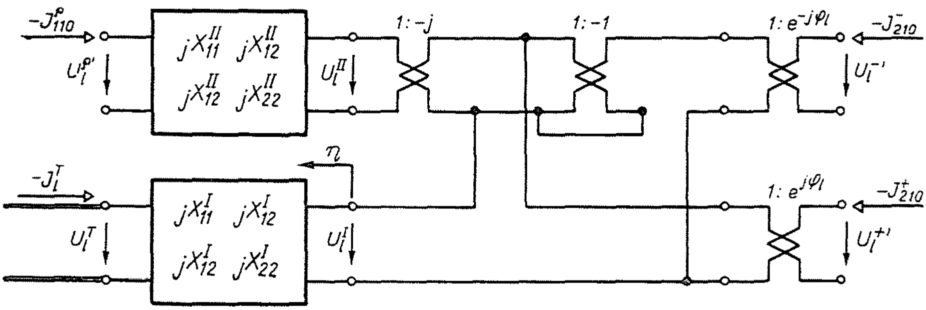
$$
H_x^{\pm} = H_x^{\mathrm{I}} \pm jH_x^{\mathrm{II}}
$$

$$
I^{\mathrm{I}} = I^{+} + I^{-} \qquad U^{+\prime} = U^{\mathrm{I}\prime} - jU^{\mathrm{II}\prime} \tag{37}
$$

$$
I^{\mathrm{II}} = j(I^{+} - I^{-}) \qquad U^{-\prime} = U^{\mathrm{I}\prime} + jU^{\mathrm{II}\prime} \tag{38}
$$

For the mode P_{110} $H'_x \equiv 0$ from Eq. (27), while for the TE_{10} waveguide mode $H''_x \equiv 0$. Both H'_x and H''_x exist for the modes $TM_{210}^{\pm}$. Hence the network can be decomposed to the partial networks shown in Fig. 9. The elements of the impedance matrices are to be determined by variational expressions again:

$$
Z_{ik}^{\mathrm{I,II}} = -j\,\frac{\left[\int_{F0}(H_i^{\mathrm{I,II}} xn)\,\Phi_k^{\mathrm{I,II}}\,df_0\right]\left[\int_{F0}(H_k^{\mathrm{I,II}} xn)\,\Phi_i^{\mathrm{I,II}}\,df_0\right.}{\sum\limits_{r_\alpha, r_\beta}^{\infty} A_r \int_{F0}(H_r^{\mathrm{I,II}} xn)\,\Phi_k^{\mathrm{I,II}}\,df_0 \int_{F0}(H_r^{\mathrm{I,II}} xn)\,\Phi_i^{\mathrm{I,II}}\,df_0}\,. \tag{39}
$$



$\varphi_l = 0$ on the aperture I $(l = 1)$

$\varphi_l = -j\frac{2}{3}$ on the aperture II $(l = 2)$

$\varphi_l = j\frac{2}{3}$ on the aperture III $(l = 3)$

Fig. 9

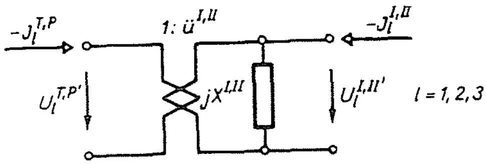


Fig. 10

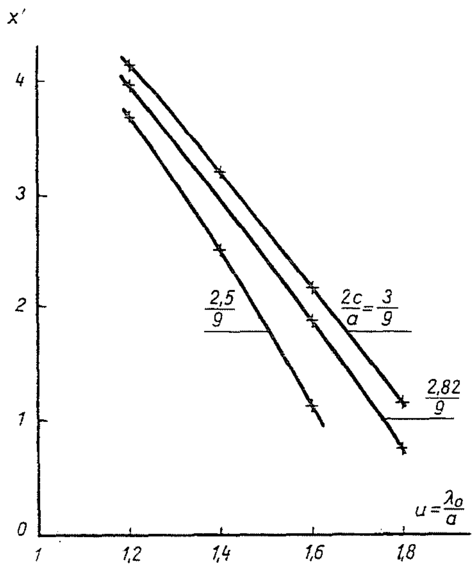


Fig. 11

By the numerical evaluation of (39) the networks Z^{I} and Z^{II} turn to be closely approximated by a cascade connection of a shunting reactance and an ideal transformer of ratio $\ddot{u}$ (Fig. 10).

For network I $\ddot{u} = 1.07$

For network II $\ddot{u} = —2.32$

The values of shunting reactances for networks I and II are shown in Figs. 11 and 12, respectively.

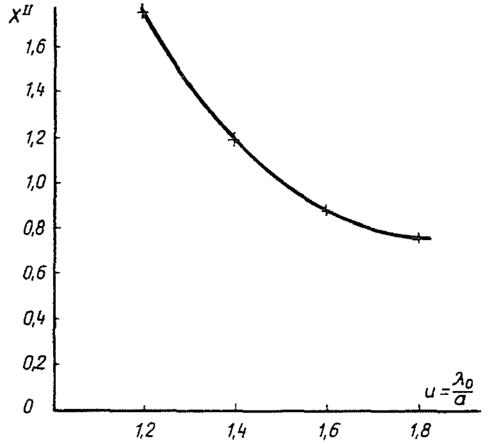


Fig. 12

C. The analysis of the resultant network

The individual elements of the simplified aperture network determined the detailed study of the complete network can be begun with. According to the conclusions of chapter 2, the terminal pairs of the aperture network are joined to the corresponding terminal pairs of the internal cavity network. So a three terminal pair is obtained, with waveguide terminal pairs as appropriate terminal pairs.

According to Eq. (9), the internal cavity network belonging to cavity modes $TM^{\pm}_{210}$ and P_{110} is described by the following equations:

$$J^{\pm}_{210} = \frac{j\Omega}{(\Omega^{\pm}_{210})^2 - \Omega^2} (U^{\pm\prime}_1 + U^{\pm\prime}_2 + U^{\pm\prime}_3) \tag{40}$$

$$J^{p}_{110} = \frac{1}{j\Omega} (U^{p\prime}_1 + U^{p\prime}_2 + U^{p\prime}_3) . \tag{41}$$

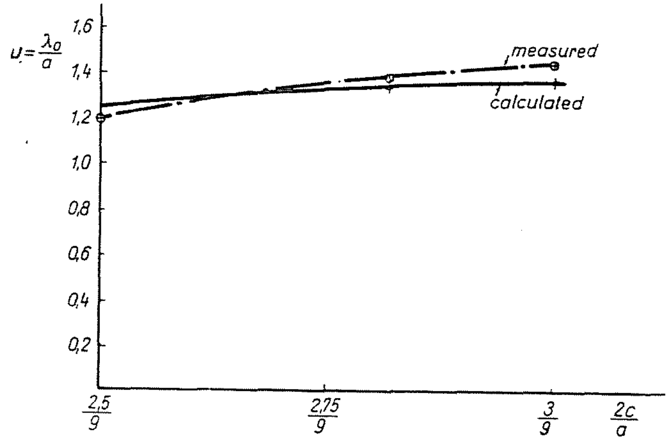


Fig. 13

By comparing the equations (30), (38), (40) and (41) the admittance matrix of the three terminal pair between the terminal pairs I is as follows:

$$\begin{bmatrix} I_1' \\ I_2' \\ I_3' \end{bmatrix} = \begin{bmatrix} j\xi & \Phi & -\Phi^* \\ -\Phi^* & j\xi & \Phi \\ \Phi & -\Phi^* & j\xi \end{bmatrix} \begin{bmatrix} U_1' \\ U_2' \\ U_3' \end{bmatrix} \tag{42}$$

where

$$\begin{aligned} \xi &= \left[\frac{(\Omega_{210}^{+})^2}{\Omega} - \Omega - 3X''\right]^{-1} + \left[\frac{(\Omega_{210}^{-})^2}{\Omega} - \Omega - 3X''\right]^{-1} \\ \Phi &= je^{-j\frac{2}{3}\pi}\left[\frac{(\Omega_{210}^{+})^2}{\Omega} - \Omega - 3X''\right]^{-1} + je^{j\frac{2}{3}\pi}\left[\frac{(\Omega_{210}^{-})^2}{\Omega} - \Omega - 3X''\right]^{-1}. \end{aligned} \tag{43}$$

From a study of Eqs (42), (43) it is seen that if $\Omega_{210}^{+} = \Omega_{210}^{-}$, the three terminal pair is not symmetrical, thus the condition of reciprocity is not satisfied. It is known from the literature that a three terminal pair likely to have a matched termination defines a circulator [3]. To determine the matching admittance, let us set out of the scatter matrix of the three terminal pair:

$$\mathbf{S} = \begin{bmatrix} S_1 & S_3 & S_2 \\ S_2 & S_1 & S_3 \\ S_3 & S_2 & S_1 \end{bmatrix} \tag{44}$$

The relation between admittance and scatter matrices is defined by Eq. (45):

$$\mathbf{S} = -[\mathbf{Y} - \zeta\mathbf{E}][\mathbf{Y} + \zeta\mathbf{E}]^{-1}. \tag{45}$$

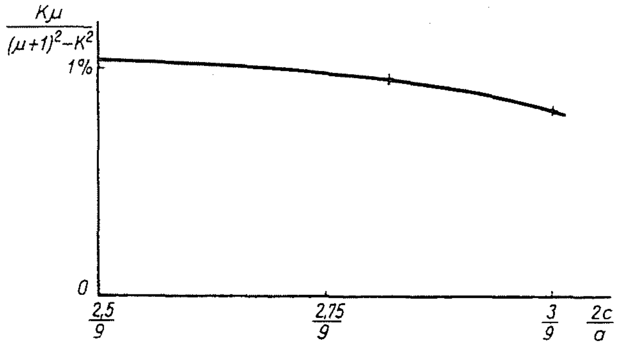


Fig. 14

Here $\zeta = \eta^{-1}$ is the terminal admittance, E is the unity matrix. In order to get a counter-clockwise circulation it is necessary, that

$$S_1 = 0 \qquad S_2 = 0 \qquad |S_3| = 1 \tag{46}$$

be satisfied. From these conditions the matching admittance can unambiguously be determined:

$$\eta = \zeta^{-1} = \frac{2}{3}\left\{j\left[\frac{(\Omega_{210}^{-})^2 + (\Omega_{210}^{+})^2}{\Omega} - 2\Omega - 6X''\right] + \sqrt{3}\left[\frac{(\Omega_{210}^{-})^2 - (\Omega_{210}^{+})^2}{\Omega}\right]\right\}. \tag{47}$$

By equalizing the imaginary and real parts the frequency and magnetic field strength of matching can be determined. (Figs 13 and 14) From a comparison with records in [13] the error of calculation is seen to be below 7%.

There is no exact matching but at one frequency. Deviating from the frequency (wave length) of matching, reflection will occur. From the admissible value of reflection the circulator band width can be determined. By considering the frequency-dependence of reactive elements, expansion into series holds the reflection coefficient:

$$r \cong 1 + 24\frac{\Delta\lambda}{\lambda_0}. \tag{48}$$

Records in (13) lead to the following expression for r:

$$r \cong 1 + 22\frac{\Delta\lambda}{\lambda_0}. \tag{49}$$

Thus, the error is 10%. The measured and calculated values are seen to be in a good agreement. The approximation can be further refined by adding many more cavity modes to the group t. The case of triangular based ferrite is to be calculated in the same way. Then only the reactance X' and the frequencies $\Omega_{210}^{\pm}$ alter.

Summary

The article contains the analysis of a waveguide Y-circulator. By generalizing G. Reiter's procedure, the lumped equivalent circuit is established for arbitrary shaped coupled cavity-system filled by linear lossless nonreciprocal media of tensor permeability and permittivity. This nonreciprocal network of infinite dimensions, composed of inductances, capacitances and transformers of complex transformation ratio is approximated in an arbitray frequency region by a network of finite dimensions.

In the second part of the paper the general results are applied to the analysis of a waveguide Y-circulator. The parameters of equivalent circuit are established and composed to the results of measurements.

The difference between measured and calculated values has probed to be less then 10%.

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