DETAIL RICHNESS OF PICTURES DEMONSTRATED BY MEDICAL X-RAY PICTURES*

By

A. BRANDELIK

Budapest. July 10, 1969

Introduction

Knowledge of the information content of pictures may often be needed. In continuous recording or in picture freezing, the adjustment of the independent parameters of picture preparation ensuring maximum information content obtainable in the course of recording, may well be necessary. This is only possible if the actual information content is known.

Here we are not going to investigate pictures constructed of regular geometrical elements, e.g. technical drawings, but so-called natural pictures. We shall call natural picture a picture, whose autocorrelation function is direction-invariant, or its variation stays at least below a predetermined threshold value.

KRETZMER [1] plotted the autocorrelation functions of natural pictures. His experimental arrangement was as follows: a homogeneous parallel light ray transilluminated the negatives of the picture in a tube of approximately diam. 100 mm: the total intensity of the penetrating light could be measured by a photomultiplier with an accuracy of 2%. Two transparent negatives of the picture to be investigated were prepared. The intensity of the penetrated light was measured first by arranging them in perfect coverage, then by shifting them in relation to each other. The light intensity variation vs the shift is the autocorrelation function $R(\tau)$ of the picture in the direction of the shift. Let us note some properties of the autocorrelation function $R(\tau)$ of the monovariable stochastic time function $U(t)$:

$$R(0) = U^2 \tag{1}$$

$$R(\tau) = R(-\tau) \tag{2}$$

$$R(\tau) \leq R(0) \tag{3}$$

Let us select from among the optical autocorrelation measurements of KRETZMER, those we are interested in:

Three pictures are given with identical dimensions, average brightness and $R(0)$ values:

* Extract from the Doctor Thesis.

A) A crowded tribune of a sports ground on a distant snapshot. It is unequivocal for everybody. This is the most pied picture of all three, the richest in details.

B) A lady of good looks with a colourful, variegated background. This picture is obviously poorer in details than picture A.

C) A simple half length portrait with a homogeneous background without details. This is the poorest in details of all three pictures.

The autocorrelation functions belonging to the pictures are shown in Fig. 1. Note the validity of Equations (2) and (3).

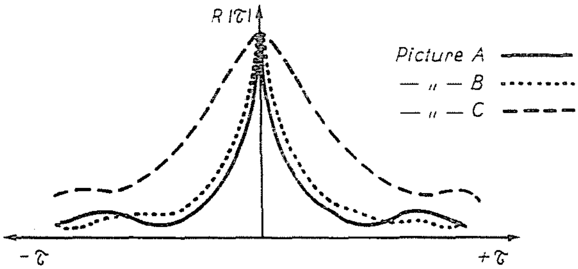

Fig. 1

KRETZMER carried out his measurements not only in one direction, he also studied the dependence of function $R(\tau)$ on the direction. He found that the slight variation of function $R(\tau)$ of the pictures in vertical direction may be explained by the force of gravity. To his opinion this direction dependence is practically unimportant, so function $R(\tau)$ may be said to be direction-independent generally.

The criterion of the natural picture, when using polar coordinates and taking the central point of the picture as origin, with radius r, may be expressed as

$$\frac{\mathrm{d}R(r)}{\mathrm{d}\varphi} \leq \varepsilon . \tag{4}$$

The value of ε should be arbitrarily chosen to allow for the slight modifying effect of the vertical direction observed by KRETZMER. We suggest the value of ε to be determined as 5%.

Some fields where the knowledge of the information content of natural pictures may be of importance are:

a) The TV-, or film camera-man must collect the greatest possible information quantity into the picture by varying the visual angle and the focus depth.

b) Information content is one of the most important aspects with cartographical airial photographs as well.

c) In nuclear physical experiments a few photos, in which the expected phenomenon is recorded, must be selected from among many thousands. The selection, or at least the preliminary selection, could be carried out automatically if the information content of the single pictures could be measured.

d) The success of X-ray diagnosis depends on the information content of the X-ray pictures. Rendering maximum possible information by the outer parameters — in case of X-rays by their intensity and quality (spectrum) — must be aimed at by all means.

Because of the high importance of picture quality and information content in X-ray techniques, it seems justifiable to devote the whole paper to the discussion of this special field.

An X-ray picture of the human body responds to the criterion determined by Equation (4), i.e. it is a natural picture.

The results hold of course not only for X-ray pictures, but for all natural pictures as well.

II. Characteristics of pictures and picture creating devices

Pictures are conventionally qualified by individual picture characteristics. The most important of these are:

1) Average brightness.

2) Contrast span, i.e. the ratio between the brightnesses of the lightest and darkest details in the picture.

3) Contrast resolution.

   In case of graded illumination the reciprocal value of the maximum number of distinguishable grades.

4) Resolution.

   The dimension of the smallest, still distinguishable details in the picture, i.e. pair of lines/cm.

A group of the picture characteristics serves for characterizing not the picture itself, but the picture creating, transfer and reproduction devices. These are as follows:

5) Gamma value.

   The slope of the logarithmic conversion function of the picture converter.

6) Tone.

   The curvature of the gamma function. The tone is uniform or linear, when the gamma function is straight; it is hard, when in case of an identical interpretation range and set of values, the gamma function passes below the linear in the first part of the interpretation range, and above the linear in the second part of it; the tone is soft, when the gamma function passes above the linear in the first part of the intrepertation range and below the linear in the second part of it.

The definition of the tone seems to be a loose conception, as it expresses a characteristic tendency. It does not qualify the gamma functions of one single curvature or those with more than two curvatures. When the characteristic is not unequivocal, the observer has to rely on his subjective judgement.

Some further remarks on the tone:

The tone was formerly an effective means of expression only in the field of artistic or amateur phototechniques. Now it has become an important parameter of the electronic picture conversion and transfer. It has been verified that the amount of lost information with the electronic picture signal transfer devices used in X-ray techniques depends also on the tone of the picture to be transferred.

Recording, picture creating, picture transfer and picture reproduction devices may be qualified by giving the set of values of the above picture characteristics.

However, it is difficult to draw final conclusions as to whether the picture creating device is adequate, as the different characteristics appear with different weights. There are endeavours to find one single characteristic expressing all the rest and their origin as well. Such characteristics are the following:

7) Information storage capacity.

It is easy to see the inadequacy of this characteristic, as it does not contain all the previous characteristics even, yet it is worthwhile to compare some picture storages by it (2).

8 mm film	about	20	kilobit
Diam. 310 mm electronoptical image amplifier	,,	400	,,
0.01 m^2 surface area X-ray screen photo "Odelca'	,,	2000	,,

8) Modulation transfer function.

This is a specific X-ray picture-technical characteristic. A sinusoidal wave profile test body with continuously increasing frequency, but of identical amplitudes, is placed in the way of the X-ray and the intensity vs the densifying signals is measured on the picture [3]. Such functions are shown in Fig. 2.

Qualifying the picture creating devices represented by the single functions is difficult, therefore the equivalent band width has been defined as

$$B = \int I^2(f) \cdot \mathrm{d}f$$

However, this is not an unequivocal characteristic either.

The main disadvantage of both of the latter characteristics, is that they indicate the picture creating capacity, which is merely a possibility, but no indication for the picture of a particular object.

Reference was made in the introduction to our trying to use, instead of all these characteristics, one single parameter, the information content of the picture. This offers itself, as for the X-ray physician, the values of the single parameters are indifferent. and he is only interested in information which can be processed by the eye. Should then the information content of the picture be the only parameter? Yes, if it can be measured! We know the information

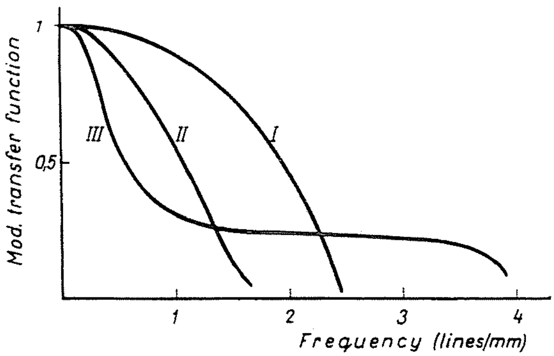

Fig. 2

storage capacities of the single techniques but according to KATIS [4] only 3—6% of these are utilized by the general natural pictures, because of surface exploitation and the correlations. This holds conceivably for the case of X-ray pictures as well. The effective information content is further reduced if the physician issues, after examination e.g. the finding: "Chest finding negative". The word "negative" represents one bit, and for the localization of the chest an information of a few bits is also sufficient. So from the possibility of 2000 kilobit e.g. only an effective information of a few bits is realized. Another physician would not share this opinion without reservations and would describe a non-pathological, so-called anomaly originating from a deviation in development, at which the finding — from a pathological point of view — stays negative. But the information content of the finding changed significantly, depending on the frequency of the indicated anomaly. Did one and the same picture yield then two different information quantities? Is this possible? Rather than the information being an objective material characteristic, is it not then a subjective one? In order to clarify this question, let us start from Shannon's definition of the information. According to Shannon the information is the negative logarithm of the event's occurrence probability. The occurrence probability of an event is an objective reality by the law of the great numbers. Consequently: information is an objective existing conception, which is independent of our mind.

Let us find the cause of the above contradiction by a closer investigation.

The information content of a communication signal from an information source may be determined if the total set of news of the information source and the frequency of occurrence of the individual news are known. In this case, the information quantity in a given picture can also be calculated, as the surface area of the picture can be divided with the help of an appropriately fine tool in parts ensuring a homogeneous intensity of the individual picture elements. Following this, the possible number of intensities and their probability of occurrence may be determined, but only if a number great enough of individuals, for rendering the probability objective, is available.

In simple object lessons, for example dice throwing, card drawing, etc., these conditions are present, but even in these cases only for intelligent observers with memory images, as throwing up a dice or a coin would not mean anything for a primitive man living away from civilisation. He is lacking the a-priori knowledges.

Does the word information, as used in X-ray medical practice, correspond to the above strict criterions? We can state that it does not. To prove this, let us examine the preparation of X-ray pictures from the standpoint of information theory.

The information sources are the patients. They are individuals also in an anatomical sense. Their sizes, and forms are different, even the ratios between the bone-, fat-, and muscle tissues vary in persons of identical sizes. If we also consider the alterations by diseases and the anomalies, an infinite number of possible configurations result in spite of the anatomical correlations. Our task does not correspond to SHANNON's criteria yet, however, let us go on proving.

The frequency of the individual cases is not known either, as statistics are kept only of the frequency of diseases (incidentally), and not of the frequency of configurations referring to diseases. In addition, the examiner is in no possession of the memory images necessary for the classification of the events, as he often sees configurations which are new for him even in cases of the same known disease.

Therefore it can be seen that there exists a confusion in terminology. The term information, as it was generally used before SHANNON, is being used for a strictly defined conception, although this is permissible only in a narrow circle of cases.

We can state that the X-ray picture has an objective information content, but this cannot be determined because the information source, the decoder and the encoder, are partly in the brain of the examining physician.

Consequently it is more correct to speak not of the information quantity of the picture, but only of the picture content. In this way the error induced by the improper usage may be avoided.

The conventional picture characteristics can be measured, but no conclusion as to the picture content can be drawn from them. The information quantity would be appropriate for this purpose, but it is not measurable, so what is the solution? We must find a measurable physical characteristic between the conventional characteristics and the information quantity.

This characteristic is the detail richness. Therefore our task has changed from the determination of information quantity to that of detail richness.

III. The detail richness of pictures

A primitive definition of detail richness is the following: The pictures may be decomposed into picture elements homogeneous in themselves, but of different sizes and illumination. The smaller the picture elements and the greater the difference in brightness between the adjacent elements, the more detailed the picture appears to be.

In what measure does the conception obtained by the above primitive definition contain the individual picture characteristics already described?

An irreversible relation of it with the average brightness can be found. It is reasonable that if the detail richness of a picture increases in a uniform distribution, then the average light intensity approaches the mean value between the maximum and the minimum brightnesses, as this would mean that all contrast grades are distributed in a uniform ratio. The inverse of this theorem does not hold. Maximum detail richness does not belong to a medium average light intensity unconditionally. The detail richness is in direct proportion with the contrast span. A picture creating device ensuring a greater brightness difference would ensure a greater difference between the adjacent picture elements too. The detail richness is in direct proportion with the contrast resolution also. More brightness grades would create a more diversified picture.

It seems most natural that the fine resolution is in direct proportion with the detail richness. Similarly to the contrast span, a picture converter with a higher gamma value might ensure a greater detail richness. The tone influences the uniform or non-uniform distribution of the contrast grades, so a linear tone would render a greater detail richness. The information storage capacity is a possibility and the detail richness is the measure of the realization of this possibility. The modulation transfer function and the equivalent band width contain the fine resolution and the contrast resolution, therefore their relation to the detail richness is also similar.

An essential conclusion is that the qualification by means of the detail richness conflicts with none of the individual picture characteristics and so its application is permissible.

GOURIET [5] dealt with the conception of the detail richness and its measurement, and his method was adapted also by KATYS. For its definition they used the one described in the first paragraph of this chapter, which we called a primitive definition.

Their measurement method was as follows: The picture is scanned by an electronic picture converter. The electrical signal or signal series of the scanner contains the picture. The diversity of the picture determines the more frequent variation of the signal, and the intensity of the individual picture elements determines the variation of the signal amplitude. In an information transfer formulation there exists an identical correspondence between the picture and the electrical signal series of the scanner. After all this the detail richness equals the average absolute value of the electrical signal variation — as Gouriet defines the conception —

$$G_0 = \frac{1}{T} \int_0^T \left| \frac{\mathrm{d}}{\mathrm{d}t} \cdot U(t) \right| \cdot \mathrm{d}t \tag{5}$$

with G_0 being the detail richness, T the scanning time and $U(t)$ the signal of the scanner. The method assumes a constant scanning rate. To call G_0, — as expressed by Formula (5) — detail richness is an inaccuracy of designation, as it is an average specific unit time richness, as seen by the term $1/T$.

The formula does not contain any physiological laws, i.e. it merely considers the picture in itself though for the evaluation of the picture, the information transferred through the eye to the brain is necessary with consideration to the modifying effect of the eye. The task is not only to count the surface elements in the picture, but also to determine the effect they make on the observer, as the details not processible for the eye do not promote the correct creation of judgement. Therefore the expression of such details should not be our task.

Let us examine test pictures and see what impressions of detail richness they induce in the observers, and let us compare these with the results given by Formula (5).

Because of the picture resolution it suffices to use picture bands varying in one (in the horizontal) direction only for the test pictures. The brightness along the bands should vary according to sinusoidal-, saw tooth- and square functions, but each one at lower and higher illumination levels. The brightness variations of the test pictures along the scanning are shown in Fig. 3.

Ten people of different occupations were asked to rank the individual bands according to "diversity", intentionally avoiding the expression "detail richness", as according to previous experience there arose difficulties concerning the interpretation of the same. The question had to be overexplained and

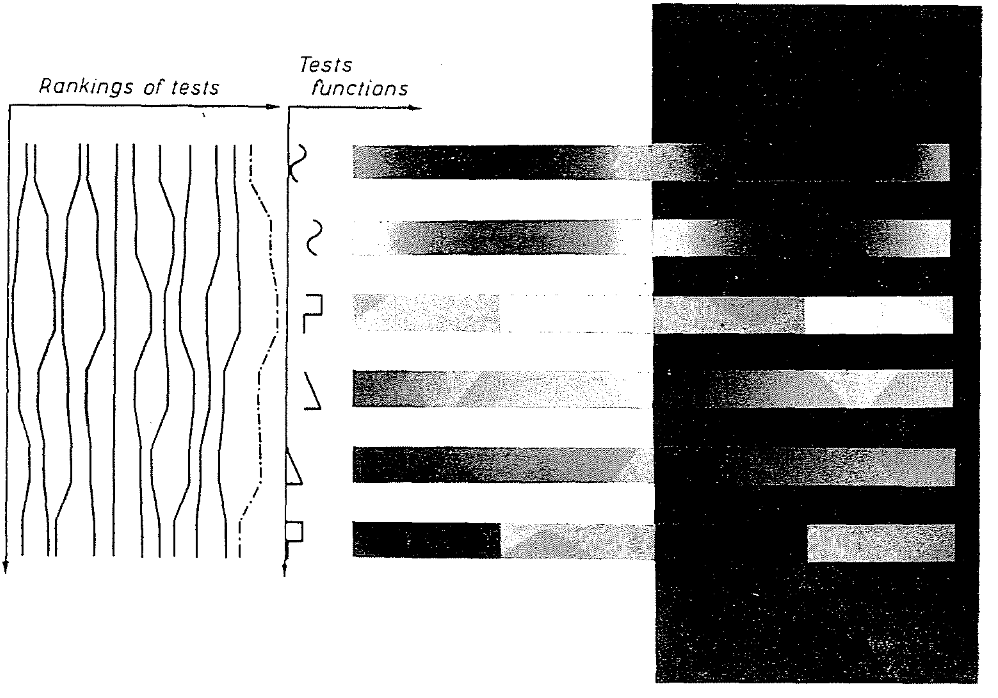

Fig. 3

this influenced the judgement of the subjects. The purpose was not to draw final conclusions from the answers by statistical laws (a greater number of subjects and more ideal experimental conditions would have been necessary for this), but only to gain insight into whether they ranked the pictures at all, and to observe if they separated the identical type bands of different average brightness.

The rankings made by the individual subjects are plotted by proportional heights in the verticals of the functions. The ordinates belonging to the same person are connected vertically. The average result is shown by the dot-and-dash line. It is seen that only one man esteemed the bands to be of identical value, but his opinion was already influenced by the known Formula (5), which states that the same result is obtained for all test functions in consequence of the identical amplitudes and period lengths.

The following observations may be made by this figure: one considers the bands of identical types but of different average brightnesses, to be different. This depends not only on whether the bands are on black or white bases, as this was excluded by supplying the bands with both black and white backgrounds. The bands of identical average brightness, but of different types were also ranked, even if in different rankings.

The possibility of the two following cases may be concluded: Formula (5) is correct and the deemed deviations in Fig. 3 are due to a sensory delusion: or the conception is not defined correctly by the given formula. The latter conclusion seems to be the proper one, as the formula contains no physiological laws.

Physiology teaches [6] that stimulus and sensation are related by the logarithmic Fechner-law:

$$E = \begin{cases} c \cdot \ln \frac{I}{I_0}, & \text{when } I > I_0 \\ 0, & \text{when } I \leq I_0 \end{cases} \tag{6}$$

with I_0 being the stimulus threshold, I the stimulus, c the proportionary factor and E the sensation.

The average stimulus level, in our case the average brightness, is not indifferent, because of the logarithmic sensitivity of the sensory organs. That is why the questioned subjects separated the identical type bands of different average levels in Fig. 3.

Special attention is due to I_0. Its existence is generally known: a stimulus below the stimulus threshold is not being sensed. Formula (5) does not take into consideration this lower limit either.

A second very important physiological law is Weber's law [7]: a change in the sensation can reach consciousness only if the variation exceeds the threshold value characteristic for the responsible sensory organ. This criterion for the eye is 1% in case of average illumination. Mathematically it can be expressed as

$$\frac{dE}{dT} = \begin{cases} \frac{dE}{dT} & \text{when} \quad \frac{dE}{dT} \geq a \\ 0 & \text{when} \quad \frac{dE}{dt} < a \end{cases} \tag{7}$$

with a being the stimulus variation threshold. Weber did not describe the law in the form of Formula (7), but the picture may be analyzed in the following only by picture scanning, therefore — assuming uniform scanning — the variation of the sensation may be conceived to be a time function. This is how the derivation by time was entered into Formula (7).

In the knowledge of the above physiological laws Formula (5), as conceived by GOURIET, is suitable only for very limited tasks. That is why we called Formula (5) primitive. The definition described in the first paragraph of the chapter should be purposefully redefined as follows:

The smaller the elements of the picture as sensed by the eye, and the greater the difference felt between the neighbouring elements, the more detailed the picture appears to be to our senses.

The so defined relation should not be considered to be linear — for cautionary reasons, due to the fact that we did not define the role of the eye exactly.

Let us find now a unit dimension for Formula (5). The position-dependent brightness will have the form of a time-dependent potential — in consequence of the picture scanning —, so the unit of the entire formula, i.e. of the specific detail richness will be Volt/sec. If not the specific but the total scanning dimension is taken into consideration, then we get the unit Volt. On returning we obtain nit, the dimension of brightness, i.e. light current.

This result is rather surprising, as the detail richness is distinguished by nothing — from the viewpoint of unit dimension — from the homogeneous shining surface, which contains no details of identical average brightness and which, for this reason, could not even be called a picture.

It can be seen that the known formula does not suit our purposes. Therefore let us derive a new definition, not losing sight of the above re-worded determination. The new definition should also allow for a fast and simple mode of measurement of the detail richness.

The detail richness, the diversity, the irregularity of the natural picture and the independence of its individual image points of each other, are related conceptions. The measure of independence of the individual image points of each other is given by the autocorrelation function.

In the introductory section we described KRETZMER's basic measurements, of which the following conclusions may be drawn:

a) The picture, judged subjectively as richer in details (picture A) has a steeper sloped autocorrelation function.

b) Detail richness is characterized by the autocorrelation function only up to the first minimum position.

c) The course of the function — with the exception of the immediate vicinity of the maximum — and the first minimum positions, where the effect of the ever-present dispersion makes itself felt, are of an exponential character.

For the assessment of the detail richness by an optical autocorrelator, only a suitable parameter of the function $R(\tau)$, which is characteristic for the detail richness, must be found.

Two definitions offer themselves:

1. Be the detail richness the quotient of $R(0)$ and the integral through τ of the function $R(\tau)$ up to its first minimum position, i.e.

$$G_1 = \frac{R(0)}{\int_0^{\tau_{min}} R(\tau) \cdot d\tau} . \tag{8}$$

This formula states that a greater G_1 detail richness belongs to an $R(\tau)$ of a steeper slope. The dimension of G_1 is 1/m, if τ was given in the dimension of the longitude. If the function $R(\tau)$ was assumed to shift with a constant speed, then τ could be used as time unit and the dimension of G_1 would be 1/sec.

2. Let us express KRETZMER's assumption on the exponential slope of the function $R(\tau)$:

$$R(\tau) = R(0) \cdot e^{-\tau \cdot G_2} \tag{9}$$

It follows that

$$G_2 = \frac{\ln R(0) - \ln R(\tau)}{\tau} . \tag{10}$$

The fact that the dimensions of G_1 and G_2 are coincident, prove the correctness of both chosen definitions.

The autocorrelation function of the picture may be plotted not only by an optical autocorrelator, but also in the form of the autocorrelate of a time function obtained by series development scanning, considering the theorem of identical correspondence.

KRETZMER's statement on the exponential course of the function $R(\tau)$ was related to natural pictures; nevertheless let us compute the values G_1 and G_2 belonging to the autocorrelates of our three sorts of test functions with 1 sec period time. Their autocorrelation functions are shown in Fig. 4.

It is characteristic that the definitions of G_1 and G_2 assess an identical sequence of the test figures, contrary to G_0, the result of which is identical for all three test functions.

			
G_1 (1/sec)	3.6	3.08	2.78
G_2 (1/sec)	2—3.1	1.6—2.7	0.91—1.57

The definition expressed by Formulae (8) and (10), — though it suits the case of the picture transfer by scanning too — does not satisfy the condition of the easy and fast measurability, as for the formation of the autocorrelation function, a storage, capable to accommodate the whole picture, is required.

Kretzmer's measurements proved that the autocorrelation function is suitable for the determination of the detail richness. Therefore it is reasonable to take these as starting points for our further investigations.

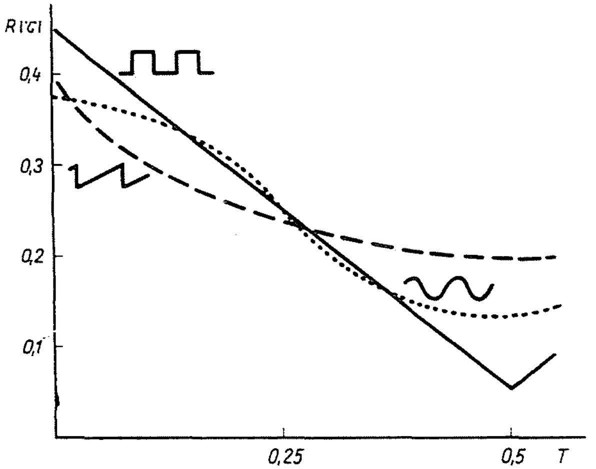

Fig. 4

There is a known relation — for causal time functions — between the correlation function and the output spectrum density:

$$R(\tau) = \int_{-\infty}^{\infty} S(f) \cdot e^{2\pi\tau jf} \cdot df \tag{11}$$

$$S(f) = \int_{-\infty}^{\infty} R(\tau) \cdot e^{-2\pi\tau jf} \cdot d\tau \tag{12}$$

where $S(f)$ is the output spectrum density.

The functions obtained by scanning the pictures are of a random, stochastic type. It is known from the probability calculation (8) that the stochastic functions may be considered as the generalisations of the function concept. This permits not only processes of a random type by their internal characteristics, but often also complex causal phenomenons to be discussed statistically.

The Hintshin—Wiener theorem expressed by Equations (11) and (12) may be equally applied both for ergodic stochastic processes and for causal functions. It is known that the information sources in communication techniques are mostly ergodic. Natural pictures are also ergodic, so we can state that even output spectrum density measurements would be sufficient instead of the autocorrelation measurements, as both are related by the Fourier transformation.

Unfortunately this way is not passable for us either, because plotting the output spectrum density is difficult enough, moreover the operations should be effected in the following with the Fourier transformation of this spectrum. It is known that

$$\int_{-\infty}^{\infty} U^2(t) \cdot dt = 2\pi \int_{-\infty}^{\infty} S(\omega) \cdot d\omega \tag{13}$$

the square integral of the time function, where ω is the circular frequency, is related to the output spectrum density.

Let us summarize the steps taken up to here: The detail richness is related to the autocorrelation, the autocorrelation to the output density spectrum and this latter to the square time function. So in the last analysis the time function contains the detail richness also. Unfortunately this relation cannot be expressed in an explicit form either.

As none of Formulae (5), (8) and (10) involve any biological laws, they should be called physical definitions. We shall distinguish Formula (5) as a primitive causal definition from (8) and (10) which we shall call stochastic physical definitions.

We should remember that the definitions of G_1 or G_2 — if complemented by biological laws, too — would serve our purposes. Let us direct now our investigation towards finding an easily measurable definition.

We observe that in Equations (8) and (10) the square of the intensity function appears! (See also Formula (1))

According to HARRISON [9] the optimum linear predictor is essentially an autocorrelator.

When handling a linear predictor, we subtract the linear combination of the previous values from the expected value, and try to obtain the minimum average square of the difference. It is essential that here too the prediction error is carried by a time function average square. The time function resulted not as the product of two functions, but as the difference of two time functions. By refining the sampling we obtain from the time function difference a time function derivate, which is already similar to the time function appearing in Formula (5).

Formula (5) suggests by itself to choose as integrand — instead of the absolute value of the derivative function — the square of the same.

$$G_3 = \int_0^T \left(\frac{\mathrm{d}U(t)}{\mathrm{d}t} \right)^2 \cdot \mathrm{d}t. \tag{14}$$

The value of G_3 of the test function calculated by this new definition is given

— together with the previous one — in the following table:

			
G_0	4	4	4
G_1	3.6	3.08	2.57
G_2	2—3.1	1.6—2.7	0.91—1.57
G_3	25.5	13.4	3.14

It is seen that the test functions are ranked into an identical sequence by G_1, G_2 and G_3 contrary to G_0.

Between G_1 and G_2 a proportionality is observed. Let us express G_1 vs. G_2.

$$G_1 = \frac{G_2}{1-e^{-G_2 \cdot \tau}} \,. \tag{15}$$

When $G_2 \cdot \tau > 3$, the proportionality error is below 5%, so the difference between G_1 and G_2 is insignificant. The proportionality is not followed by G_3, because with the linear predictor not merely the previous value, but the linear combination of several successive values, must be taken into consideration. Therefore the substitution by the square of the derivative function may be regarded only as an approximation. For the more exact calculation of the detail richness, a storage unit must be utilized by all means, either as a predictor or as a correlator.

That is why the definition of G_3 cannot be called a stochastic physical definition. It is better to call it a modified causal physical detail richness.

From the above table it may be seen that G_3 permits the possibility of ranking, in spite of the rough approximation and finding the maximum in the case of detail richness measurement.

The advantage of employing G_3 lies in its measuring method. It does not require any considerable storage, merely an elementary storage effecting the derivation, a capacitor or an inductivity.

The described test function could allow only a rough comparison. The suitability of the established definition of G_3 was also checked by X-ray pictures made of a chest simulating phantom satisfying the criterion described by Equation (4).

There was no possibility of measuring the values G_1 and G_2 in the exposures made from the phantom.

For a better understanding of the results, some outlining of the phantom is necessary. The phantom is the property of the Budapest Medical University. The exposures were made at the Radiological Clinic of the Medical University. The phantom contains real human bones embedded in plastic materials simulating the softer tissues. The schematized windpipe consists of an intubation tube made of rubber. The picture shows also the iron wires supporting the bones. The parameters of the optimum picture obtainable from the phantom are known. With these parameters kept constant, six exposures were made by the slight variation of the high voltage affecting the picture content. According to the co-workers of the clinic, an optimum picture could be obtained with an X-ray tube high voltage of 72—75 kV, so the exposures were taken within the range of 64—82 kV.

The pictures were so similar to each other that only X-ray specialists with a great practice, were able to spot any difference between them, so it would be pointless to reproduce them in this paper, as the existing minute differences could not be accurately rendered typographically anyway.

The phantom exposures were projected by the TV-set Mark RTV of the Medicor Works. The constant average brightness of the pictures were controlled by a Nitometer. The synchronous signals of the video-signal, branched off the last video stage of the television, were cut off and the image signal obtained in this way had a peak voltage of 0.8 V. A device realizing the operations corresponding to formula (5) was built, and the relative G_0 and G_3 values of the exposures were measured.

High voltage values during exposures (kV):	64	68	72	75	78	82
G_0	0.95	1	0.97	0.97	1	0.94
G_3	0.73	1	0.8	0.8	0.95	0.65

It is seen by the table that the difference, G_0 shows, between the individual exposures can hardly be indicated at all, whereas a well observable ranking is established by G_3.

Both G_0 and G_3, as expressed versus the high voltage, have two maxima. The absolute maximum belongs to the exposure made at 68 kV, the second local maximum to that made at 78 kV. In the range between 72—75 kV, where the optimum high voltage is found, according to the co-workers of the clinic, just a local minimum appeared during our measurements. Is there a contradiction between the measurement and the experience?

Although no biological laws were taken into consideration at our measurements, the apparent contradiction may be explained. The local minimum in case of G_3 is 80% of the maximum, i.e. it is not too deep. The phantom consists decisively of two kinds of material: of bone and of plastic. By tables constructed for the use of X-ray physicians, it is known that the optimum high voltage for examining the bones of the spinal column lies in reality in the vicinity of the second maximum, around 75—80 kV, whereas the position of the first maximum is found at a high voltage value characteristic of plastic, the second construction material of the phantom. Therefore there exists no contradiction; the greatest number of details is seen in effect in the range 72—

75 kV, both for the bones and the plastic. Their separation, with regard to the insignificant role of the phantom, was not necessary so far.

The physical detail richness can be perceived and forwarded to the brain only in a limited measure by the human eye. The physical detail richness is being varied by known and as yet, less known, physiological laws.

The detail richness corrected by the known physiological laws is called the physiological detail richness.

Let us consider first the law of Fechner described by Eq. (6). The stimulus is given by the brightness, represented by the function $U(t)$ during the scanning. The stimulus threshold determines a minimum voltage U_0, so the correlation function assumes the modified form of

$$R^{+}(\tau) = \int_0^T C^2 \cdot \ln \frac{U(t)}{U_0} \cdot \ln \frac{U(t-\tau)}{U_0} \cdot dt \tag{16}$$

i.e. $R^{+}(\tau)$ is the autocorrelation function of the change of the sensation.

G_2, corrected by the law of Fechner, is modified into

$$G_2^{+} = \begin{cases} \dfrac{\ln R^{+}(0) - \ln R^{+}(\tau)}{\tau}, & \text{when } U > U_0 \\ 0, & \text{when } U \leq U_0 . \end{cases} \tag{17}$$

Let us consider the law of Weber also. In case of the autocorrelation function the preliminary condition may be expressed as

$$\left| \frac{dR(\tau)}{d\tau} \right| > b . \tag{18}$$

Let us correct G_2 with this condition as well:

$$G_2^{++} = \begin{cases} \dfrac{\ln R^{+}(0) - \ln R^{+}(\tau)}{\tau}, & \text{when } U > U_0 \text{ and } \left| \dfrac{dR(\tau)}{d\tau} \right| \geq b \\ 0, & \text{when } U \leq U_0 \\ 0, & \text{when } \left| \dfrac{dR(\tau)}{d\tau} \right| < b . \end{cases} \tag{19}$$

G_1 was related to G_2 by Eq. (15), so we will not discuss them any further.

The rest of the known physiological laws have no significance for the detail richness, so they are disregarded here.

The exact denomination of G_2^{++} is: stochastic physiological detail richness.

Let us correct the modified expression of the causal detail richness expressed by Eq. (14) as well:

$$G_3^{++} = \begin{cases} C^2 \cdot \int\limits_0^T \left(\frac{d}{dt} \ln \frac{U(t)}{U_0} \right)^2 \cdot dt, & \text{when } U > U_0 \text{ and } \frac{dU}{dt} \geq h \\ 0 & \text{, when } U \leq U_0 \\ 0 & \text{, when } \frac{dU}{dt} < h \end{cases} \tag{20}$$

with h being the threshold corresponding to the law of Weber.

Let us compare the unit dimensions of G_2^{++} and G_3^{++}. As the proportionality factor C is dimensionless in the law of Fechner, the unit dimensions of expressions (19) and (20) are coincident in case of scanning and equal 1/sec, while without series development they equal the inverse of the unit area (e.g. $1/\text{m}^2$).

U_0, as described by the Fechner and Weber laws, and the threshold value may be established always experimentally with consideration to the actual conditions. According to the teachings of physiology the value of the stimulus threshold is determined mainly by the eye's adaptation. The adaptation of the eye to the ambient average illumination is a complicated process. Attention is drawn here to one important fact, i.e. the eye adapting itself to darkness attains its maximum sensitivity in approximately one hour. Therefore the determination of I_0 or U_0 versus the average illumination is not sufficient, but the adaptation time of the examining eye must also be allowed for, or the time-dependence of the adaptation must be taken into consideration. The Weberian threshold value also depends on the average illumination, but its variation may be neglected in our present work [7].

G_3, as defined by Eq. (14), gives identical results for identical type test functions of identical amplitudes, but of different average brightness, as it takes only the variation into account. This does not correspond to the experience mentioned previously. Let us examine, therefore, whether the result of G_3^{++} agrees with the experience.

			
Calculated value of G_3^{++} at low average brightness	121	63.7	8.8
at a threefold average brightness	2.96	1.55	0.355

It is seen that the definition marked G_3^{++} shows a difference not only between the individual types of test functions, but also within the range of an identical type, if the average brightness varies.

This result does agree with the experience, already!

The values G_1, G_2, G_3, G_3^{++} of the test functions were calculated by a type Minsk 22 digital computer.

Let us check further the correctness of the square relation appearing in Formula (14).

It is known [7] that a series of electrical pulses of a frequency corresponding to the intensity of the light stimulus, is conducted from the eye to the brain. Hartlin plotted the relation between the light intensity and the frequency (Fig. 5).

Two curves appear in this figure; the upper one shows the frequency measured at the instance of switching on the light, whereas the lower one shows

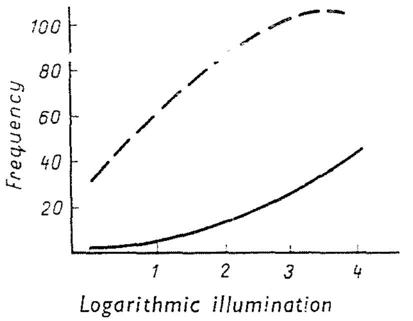

Fig. 5

the frequency belonging to the steady condition 3.5 seconds after switching on the light. Two essential conclusions drawn from the above figure are:

1. The switching on is accompanied by a transient phenomenon.
2. The curve characterizing the steady condition is — with a good approximation — of a square character, i.e.

$$(\ln I)^2 = k \cdot f \tag{21}$$

with I being the light intensity, f the frequency of the pulse series measured on the stimulus conductor and k the proportionality factor.

The relation shown in Fig. 5 verifies the correctness of the square type relation assumed in the definition of G_3, but only for the steady case, i.e. for standing, or quasi-standing pictures. The fast changing pictures require further study. In this paper we do not intend to discuss these.

A further proof for the correctness of the square character is given as follows: It is known from the physiology of the eye that the eye possesses two sorts of light-sensitive receptors: cones and rods, functioning in different ways in the cases of high and low illumination. Some receptors step into action at low, others at medium and again others at higher illumination. HECHT [7] established the number of the functioning receptors depending on light intensity by calculations based on the photochemical properties of the eye (Fig. 6).

Note that the functions are of a square character up to the beginning of saturation in both ranges of the cones and the rods. A deviation from the square character is shown only in the range where both curves meet. The ranges of both cones and rods are rather wide, of 2—3 order of magnitude.

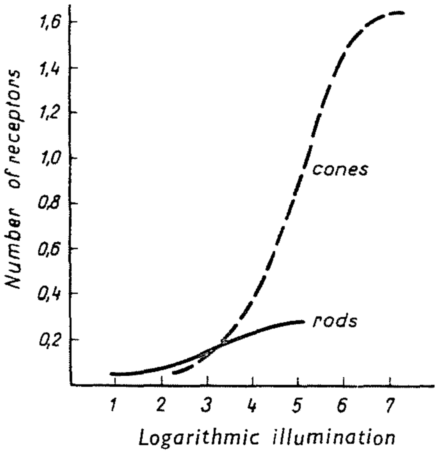

Fig. 6

According to Weber's law, two light stimuli with intensities of different order of magnitude can become consciously felt, so it is sufficient to work either in the range of the cones, or in that of the rods. In this case the square character is fully verified.

A problem exists in the transition range. It was proved by practical observations that the clear sight is limited in the transition range by other disturbing circumstances; that is why the microscopes and other similar means of optical observation were formerly constructed in a way that their average brightness should be either below, but if possible, above the transition range [7]. This proves the correctness of the square character also by the photoelectrical and photochemical effects of the eye.

The proof must be complemented by two conditions: the square character holds for standing pictures, either in the range of the cones or in that of the rods.

What does the square character mean?

It means only that we choose a function relation form approximating the experience best.

Does the square character have a deeper sense? Is there a concept difference between the expressions

$$\int_0^T \left| \frac{dU(t)}{dt} \right| \cdot dt \quad \text{and} \quad \int_0 \left(\frac{dU(t)}{dt} \right)^2 \cdot dt .$$

According to the following train of thought there is. We call to help the electrical phenomenons of the eye [7]. The physiological corrections are — for the sake of simplicity-disregarded, as they should have been figuring in both expressions anyway.

Under the effect of the illumination, a series of uniform pulses passes the stimulus conductor. The pulses arise in consequence of a variation of the electrical charge. We may assume the existence of a constant capacitor, which — on reaching the threshold voltage — discharges and renders the pulse. The capacitor is charged in the interval between the pulses by an electrical current. The current flows under the influence of voltage. The current and the voltage are in direct proportion, at least up to the first term incl. of the series development of the function relation existing between them. A phenomenon referring to non-linearity is observed only from the instant of switching on the light up to the fading away of the transient phenomenons. Linearity may be assumed at static standing pictures. Voltage is only the primary cause, it might have arisen under the influence of the illumination.

Let us reverse the reasoning! The illumination evokes an electrical potential, the potential forces current to flow through a resistance to a capacitor, which discharges at a given voltage. The voltage U is proportional to the illumination B:

$$U = k \cdot B$$

A current I proportional to the voltage flows through the conductor S:

$$I = S \cdot k \cdot B$$

The product of voltage and current gives power:

$$W = k \cdot B \cdot S \cdot k \cdot B = q \cdot B^2$$

Therefore the square of the illumination is proportional to the power, i.e. formula (14) represents an energy character, in correspondence with the common knowledge that for fixing information-, or detail richness, an energy is necessary.

According to the above considerations: in formula (5) only intensity variations, whereas in formula (14) quantities proportional to the power variations are summarised. Finding the proportionality factor is not necessary, as relative results can also be utilized.

Conclusions

We have established that formula (5) gives a definition suitable only for the solution of limited tasks; it does not reflect an energy character and takes no biological laws into consideration.

Utilizing the properties of the autocorrelation functions, two new definitions were formulated, which are — in case of sufficiently high detail richness — proportional to each other. These definitions express the energy character (see Eq. (1)) and may be complemented by the physiological laws. They take the weighting of the independence of the adjacent image points into consideration. Their disadvantage is that they require a complicated and expensive storage.

A more modest (but e.g. for the adjustment of the X-ray parameters equally suitable) definition has also been established (see Eq.(20)), which could be regarded as the best one; it differs from that expressed by Eq. (19) only by its failing to consider the independence of the adjacent image points, but it expresses the energy character and contains the most important biological laws. Its advantage is that it does not require any storage, so a simpler instrument can be prepared for its measurement and the detail richness of the pictures, as projected by television, may be easily determined from the time-continuous signal of the videosignal.

Summary

The paper defines the conception of the natural picture. A natural picture is a picture whose autocorrelation function is direction-invariant. Often, e.g. in preparing X-ray pictures, the knowledge of the picture information content might be necessary. This is, in most cases, indeterminable. Instead of it the detail richness may be determined. The paper criticizes the formerly known definition of the detail richness. It establishes two new definitions considering the independence of the adjacent image points, the biological laws and the fact that energy is necessary for fixing the detail richness. These two new definitions are — in case of a sufficiently high detail richness — proportional with each other.

The paper gives also a third definition for more modest requirements, e.g. for the optimization of X-ray picture preparation. This definition takes into consideration not only the independence of the adjacent image points, but it can be measured by a far simpler equipment than the former two.

The correctness of the new definitions are verified by physiological experimental results.

References

1. KRETZMER, E. R.: Statistics of TV signals. Bell S. T. J. VI, 1952.
2. BOWERS, A.: Der Informationsinhalt des Rtgbildes. Röntgen, Blättr. **16**, 1963.
3. GEBAUER, A.: Das Röntgenfernsehen. Springer V. 1965.
4. Катыс, Е.: Информационные сканирующие системы. Москва, 1962.
5. GOURIET, G.: A method of measuring television picture detail. Electronic Engineering, **24**, (1952).
6. WENT, I.: Élettan. Medicina, 1958.
7. ADLER, F. H.: Physiology of the eye. St. Louis., 1959.
8. PRÉKOPA, A.: Valószínűség elmélet. Műszaki K., Budapest 1962.
9. HARRISON, C. W.: Experiments with linear prediction in TV. Bell S. T. J. VI. (1952).
10. HOWEL, H.: Textbook of physiology. Philadelphia, 1949.

Dr. Sándor BRANDELIK, Budapest, V., Vármegye u. 15. Hungary