SOME REMARKS ON ABSOLUTE STABILITY

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In 1946, M. A. AIZERMAN, Soviet researcher, studied the system shown in Fig. 1—1. This system is a oneloop circuit of negative feedback containing a linear and a non-linear element. The transfer function $G(s)$ of the linear

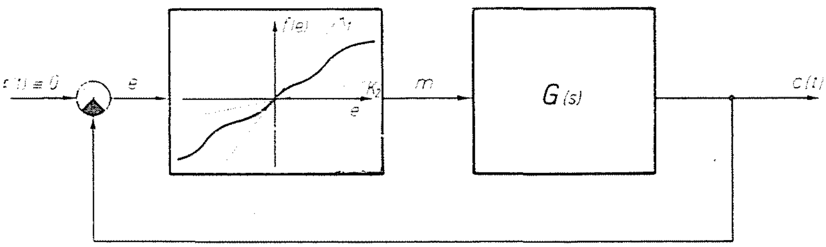

Fig. 1—1

element may only have a single pole in the origin at the most and its numerator must be of a higher order than its denominator. The non-linearity $f(e)$ is a sectionally continuous one-value function. AIZERMAN reduced this non-linearity to the sector limited by straight lines of slopes K_1 and K_2 (Fig. 1—1).

$$f(0) \equiv 0\,; K_1 \leqq \frac{f(e)}{e} \leqq K_2 . \tag{1—1}$$

As a result of his investigations he established the following: If non-linearity is substituted by a linear element of transfer factor K so that

$$K_1 \leqq K \leqq K_2 \tag{1—2}$$

and the linear system obtained in this way is stable, then this may be assumed to constitute a sufficient condition of the global asymptotic stability of the initial non-linear system limited to the sector $[K_1; K_2]$. This assumption of AIZERMAN is known as the "AIZERMAN *conjecture*" and the system shown in Fig. 1—1 is generally called the "AIZERMAN *system*."

Later research has proved that the AIZERMAN conjecture — even if it gives correct results in many cases — cannot be accepted generally. The "*in the small stability*" (in a *Ljapunovian sense*) cannot be extended to "*in the large stability*" of the system. In other words the *asymptotic stability* in the vicinity of the work-point does not mean the *global asymptotic stability* of the system.

As an example, if

$$G(s) = \frac{1}{(s+p_1)(s+p_2)\ldots(s+p_n)} \tag{1—3}$$

and p_i $(i = 1,2, \ldots, n)$ is a real negative pole, among which at most one is in the origin, then the AIZERMAN conjecture holds. It also holds for a third-order system of the form

$$G(s) = \frac{s+z_1}{(s+p_1)(s+p_2)(s+p_3)} \tag{1—4}$$

if the zero-point z_1 is on the left-hand side. With fourth-order systems of the form

$$G(s) = \frac{1}{(s+p_1)(s+p_2)(s+p_3)(s+p_4)} \tag{1—5}$$

where two of the four poles are left-side real ones and two complex ones, or if the four poles form a conjugate pair of radicals, the fulfilment of the conjecture depends on the position of the radicals as well.

In 1957 KALMAN [3] published a hypothesis stricter than the AIZERMAN conjecture. Also KALMAN examined the system of AIZERMAN and as a further constraint he restricted also the slope of the non-linearity between two limits:

$$k_1 \leqq \frac{d}{de} f(e) = f'(e) \leqq k_2. \tag{1—6}$$

In addition he assumed that

$$k_1 \leqq K_1 < K_2 \leqq k_2 \tag{1—7}$$

The conjecture of KALMAN is the following: If the above conditions are satisfied and the linearized system is stable in case of transfer factor K

$$k_1 \leqq K \leqq k_2 \tag{1—8}$$

then the initial non-linear system is a globally asymptotically stable system. From the above it follows that all the non-linear characteristics which satisfy the conditions of KALMAN satisfy also those of AIZERMAN. In other words, the set of the KALMAN non-linearities is a partial set of the AIZERMAN non-linearities. This is naturally not valid conversely.

Further research proved that the conjecture of KALMAN does not hold in all cases either.

In 1966 FITTS [4] showed by the results of his analogue computer investigations and in the same year WILLEMS [5] proved also analytically that the above conjectures have no general validity.

About 1950, LURIE [6] utilized the direct method of LJAPUNOV to further investigate into the problem. He studied the AIZERMAN system in the sector $(0; \infty)$, choosing the sum of a quadratic form and of the interval of a non-linearity as LJAPUNOV function. He determined the quadratic form from the parameters of the linear part. In this way he tried to find the conditions for the global asymptotic stability of the system.

In 1961 LA SALLE and LEFSCHETZ [7] continued experimenting with LJAPUNOV functions of the type: quadratic form + integral.

While efforts to solve the problems which arose up to that date were carried on in the USSR and in the USA, in Roumania the mathematician POPOV [8, 9] determined a totally new, comprehensive criterion. He determined the sufficient conditions of the absolute stability of the AIZERMAN system.

The AIZERMAN *system* is absolute stable in the sector $[K_1; K_2]$ if it is — with every sectionally continuous, one-value non-linear characteristic existing between both limiting straight lines — globally asymptotically stable in the Ljapunovian sense for any initial condition.

The system is stable in the *Ljapunovian sense* if a positive real quality $\delta(\varepsilon) > 0$ can be given, for which with the initial condition of $|c(0)| \leqq \delta$, the inequality $c(t) \leqq \varepsilon$ is satisfied. If δ is infinitely great and $\varepsilon \to 0$ we speak of a global asymptotic stability.

Instead of the sector $[K_1; K_2]$ generally the sector $[0; K]$ is taken for a basis, as $G(s)$ might have a pole in the origin. If $G(s)$ is limit-stable, the system may be examined in the sector $[0; K]$ as well. It is easily seen that the sectors $[0; K]$ and $[0; K]$ respectively may be reduced by transformation into the sector $[K_1. K_2]$.

The theorem of POPOV is as follows:

The sufficient condition of the absolute stability of the AIZERMAN system in the sector under investigation is:

$$\mathrm{Re}\,[1+j\omega q]\,G(j\omega)+\frac{1}{K}\geqq 0 \qquad (1-9)$$

where $G(j\omega)$ is the frequency function of the linear part in case of positive frequencies and q is a positive number, $q \geqq 0$.

We note that AIZERMAN and GANTMACHER proved [2] that the POPOV criterion holds also when $q < 0$.

2. The absolute stability of systems with limited slope non-linear characteristics

In 1966 DEWEY and JURY [10] studied the AIZERMAN system for limited slope non-linear characteristics:

$$\begin{aligned} -k_1 \leqq \frac{d}{de} f(e) \leqq k_2 \\ k_1 \geqq 0; \quad k_2 \geqq 0. \end{aligned} \tag{2—1}$$

In this case the sufficient condition of the absolute stability for the sectors $(0, K]$ and $[-k_1; k_2]$ is:

$$\operatorname{Re}[1+j\omega q]\,G(j\omega)+\frac{1}{K}+\mu\omega^2\{1+(k_2-k_1)\quad \operatorname{Re}G(j\omega)-k_1k_2|G(j\omega)|^2\}>0 \tag{2—2}$$

$$\overline{\lim_{|e|\to\infty}}\, q\left[\int_0^e f(x)\,dx-\frac{ef(e)}{2}\right]=+\infty \tag{2—3}$$

where q is an arbitrary real number, μ is a finite positive number ($\mu \geqq 0$) and $G(j\omega)$ is the frequency function of the linear part for positive frequencies. The criterion may be written in a simpler form, if the slope of the non-linearity is limited to the sector $[0; K]$, i.e. $-k_1 = 0$ and $k_2 = K$.

Then, instead of condition (2—2), the following inequality is obtained:

$$\operatorname{Re}\left[1+j\omega q\frac{1}{1+\mu\omega^2}\quad G(j\omega)\quad\right]+\frac{1}{K}>0. \tag{2—2)*}$$

It is easily admitted that if $\mu = 0$, the conditions (2—2) and (2—2)* agree with the inequality (1—9). DEWEY and JURY gave simpler inequalities than the general conditions (2—2) also for other special cases, not to be discussed here.

We chose the above inequality (2—2)* from among the special cases, because on the one hand the two sectors limiting the non-linearity satisfy the conditions of KALMAN's hypothesis, on the other hand the geometrical demonstration of inequality (2—2)* is similar to that of the original inequality (1—9) of the POPOV criterion.

3. Conditions for the fulfilment of the conjecture of Aizerman and Kalman

From the frequency function $G(j\omega)$ of the linear part a modified frequency function $G^{M}(j\omega)$ may be constructed on the basis of the following relationships:

$$\begin{aligned} \operatorname{Re} G^{M}(j\omega) &= \operatorname{Re} G(j\omega) \\ \operatorname{Im} G^{M}(j\omega) &= \frac{\omega}{1+\mu\omega^2} \operatorname{Im} G(j\omega). \end{aligned} \tag{3—1}$$

This modified frequency function $G^{M}(j\omega)$ may be utilized for the geometrical illustration of the inequalities (1—9) and (2—2)*. For the geometrical illustration of the Popov criterion:

$$\begin{aligned} \operatorname{Re} G^{M}(j\omega) &= \operatorname{Re} G(j\omega) = x \\ \operatorname{Im} G^{M}(j\omega)|_{\mu=0} &= \omega \operatorname{Im} G(j\omega) = y. \end{aligned} \tag{3—2}$$

With relationships (3—2), the inequality (1—9) may be written as

$$x - qy + \frac{1}{K} > 0. \tag{3—3}$$

The equation of the straight line of slope $1/q$ intersecting the real axis at point $-1/K$ is:

$$x - qy + \frac{1}{K} = 0. \tag{3—4}$$

This is called the Popov *straight line.*

The Popov straight line divides the complex plane into two parts. Inequality (3—3) represents the half-plane to the right of the straight line. So the Popov criterion demands all of the modified frequency characteristic curve $G^{M}(j\omega)$ to be positioned to the right of the Popov straight line, for the frequency varying from 0 to ∞. Also the inequality (2—2)* of Dewey and Jury may be illustrated in a similar way.

$$\begin{aligned} \operatorname{Re} G^{M}(j\omega) &= \operatorname{Re} G(j\omega) = x \\ \operatorname{Im} G^{M}(j\omega) &= \frac{\omega}{1+\mu\omega^2} \operatorname{Im} G(j\omega) = y^* \end{aligned} \tag{3—5}$$

and

$$x - qy^* + \frac{1}{K} > 0. \tag{3—6}$$

Our further discussion extends only to systems whose frequency characteristic curve $G^M(j\omega)$ intersects the negative real axis of the complex numerical plane in a single point, while it approaches the origin

$$\lim_{\omega \to \infty} |G^M(j\omega)| = 0 .$$

Equations (3—2) and (3—5) show the negative real axis to be intersected at the same point by both the modified frequency function and the frequency function $G(j\omega)$. In the following let us summarize two definitions: A Hurwitz *sector* is called the sector $(0; K_H]$ in which the Aizerman *system* is stable, if the non-linearity is substituted by a straight line of a slope varying between 0 and K_H and which passes through the origin. A Popov *sector* is called the sector $(0; K_p]$, for which the sufficient condition of the absolute stability according to the Popov *criterion* is satisfied. If the absolute stability of the sector is examined by the Dewey and Jury criterion (2—2)*, the sector obtained in this way will be denoted by $(0; K_p^*]$. The Aizerman *conjecture* is satisfied if the Hurwitz-*sector* is coincident with the Popov-*sector*.

$$K_p = K_H. \tag{3—7}$$

The value of K_H may be determined from the point of intersection of the modified frequency characteristic curve $G^M(j\omega)$ with the negative real axis.

$$-1/K_H = X^0.$$

The value of K_H is determined from the modified frequency characteristic curve $G^M(j\omega)|_{\mu=0}$. The Popov *straight line* intersects the negative real axis at point $-1/K_p$ (Fig. 4—1).

The geometrical conditions for the fulfilment of the Aizerman *conjecture* are:

Necessary condition: The negative real axis is intersected at the same point both by the frequency function $G(j\omega)$ of the linear section and by the modified frequency characteristic curve $G^M(j\omega)|_{\mu=0}$.

Sufficient condition: The tangent to the intersection point of the modified frequency characteristic curve $G^M(j\omega)$ with the negative real axis is entirely to the left of the characteristic curve, without intersecting it. In other words the tangent is a Popov *straight line.*

The geometrical conditions for the fulfilment of the Kalman are:

Necessary condition: The negative real axis is intersected at the same point both by the frequency function $G(j\omega)$ of the linear section and by the modified frequency function $G^M(j\omega)|_{\mu>0}$.

Sufficient condition: The tangent to the intersection point of anyone of the modified frequency characteristics $G^M(j\omega)|_{\mu>0}$ with the real axis must be entirely to the left of the characteristic curve without intersecting it.

After all this the question arises: How does the sufficient condition for the fulfilment of the KALMAN *conjecture* depend on the value of μ? This question is discussed in the following chapter in case of fourth-order systems.

4. The investigation of fourth order systems

Investigations referred to fourth-order systems with two negative real poles and two complex conjugate poles. From among the real poles only one may be in the origin. The transfer function of the linear part is:

$$G(s) = \frac{1}{(s+h)\,(s+p)\,(s+\alpha+j\sigma)\,(s+\alpha-j\sigma)}\,. \tag{4—1}$$

Fig. 4—1 shows the modified characteristic curve $G^{M}(j\omega)|_{\mu=0}$ and the POPOV *straight line* for $h = 0$, $p = 1$, $\alpha = 0.4$ and $\sigma = 10$.

The HURWITZ-*sector* is: (0; 2500]

The POPOV-*sector* is: (0; 179.5] Let us follow now the variation of K_p^* versus μ. The characteristic curves $G^{M}(j\omega)|_{\mu>0}$ were determined for $\mu = 0.2$;

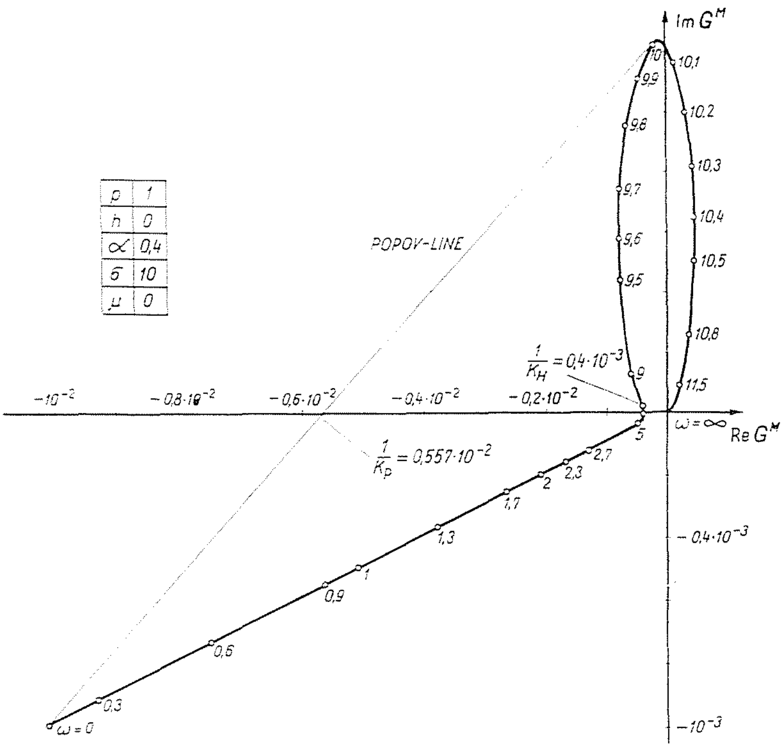

Fig. 4—1

0.4; 0.6; 0.8; 1; 2; 5; 10; 15; 20; 50; 100, then form them the values of K_p^* different μ values were established.

The function $K_p^*(\mu)$ is shown in Fig. 4—2. As a result it may be established that with increasing μ the value of K_p^* tends to the limiting value 1050. Examination of several similar cases showed that with positive q values the value of K_p^* approaches the value to be assumed for $\mu = \infty$, the maximum K_p^* value and this for $\mu = 50$ already at the required accuracy:

$$K_p^*(\mu)|_{\max} = K_p^*(\infty) \approx K_p^*(50) \,. \qquad (4\text{—}2)$$

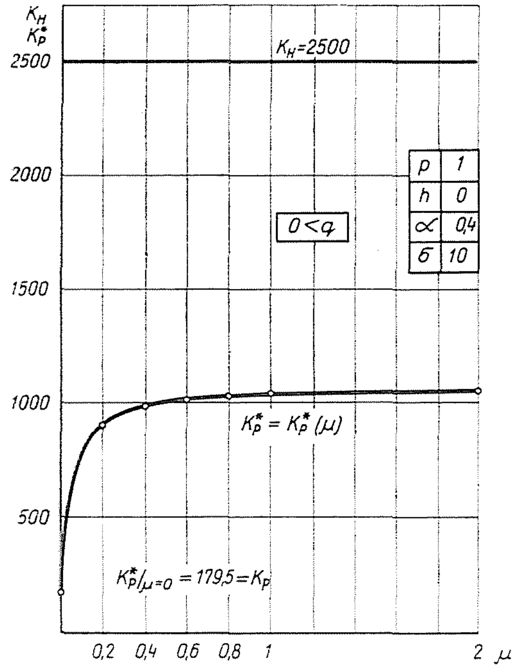

Fig. 4—2

Interesting results were obtained in case $q < 0$. Contrarily to the previous results, K_p^* decreased with increasing μ. Fig. 4—3 shows the modified frequency functions for $p = 1$; $h = 1$; $\alpha = 0.4$ and $\sigma = 4$ ($\mu = 0$; 0.2; 0.4; 0.6; 0.8; 1; 2). Fig. 4—4 shows the functional relationship $K_p^*(\mu)$. For this system the condition of the AIZERMAN *conjecture* is satisfied, as $K_p^*|_{\mu=0} = K_p = K_H$. So the sector $[0; K_H]$ is a POPOV-*sector*. In other words, a non-linearity in the sector $[0; K_H]$ is absolutely stable. If its slope is limited by the sector $[0; K_p^*]$, the absolute stability range is reduced to the sector $[0; K_p^*]$ where $K_p^* < K_H$.

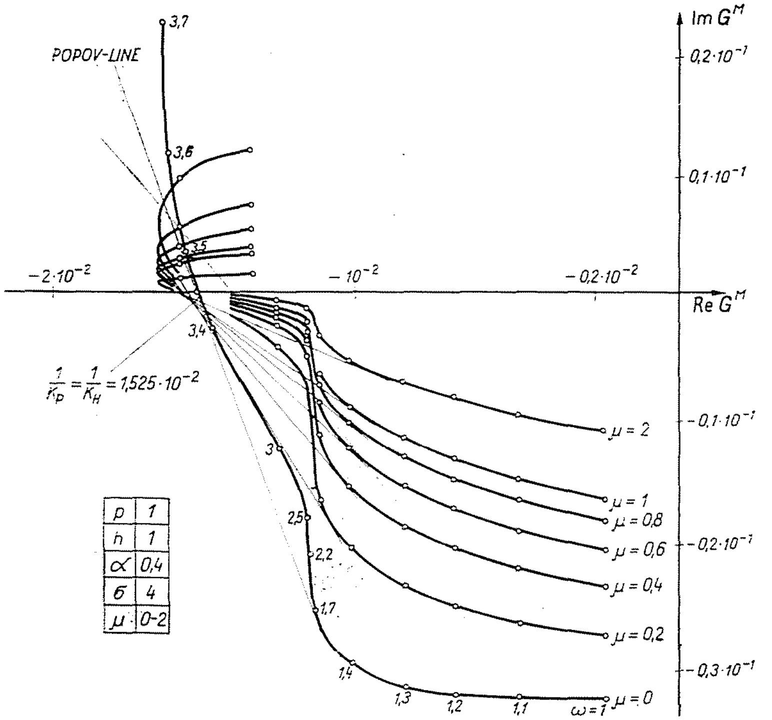
Im G^M
$0{,}2\cdot10^{-1}$
$0{,}1\cdot10^{-1}$
POPOV-LINE
3,7
3,6
3,5
3,4
$-2\cdot10^{-2}$
-10^{-2}
$-0{,}2\cdot10^{-2}$
Re G^M
$\frac{1}{K_P}=\frac{1}{K_H}=1{,}525\cdot10^{-2}$
$-0{,}1\cdot10^{-1}$
$-0{,}2\cdot10^{-1}$
$-0{,}3\cdot10^{-1}$
μ = 2
μ = 1
μ = 0,8
μ = 0,6
μ = 0,4
μ = 0,2
μ = 0
3
2,5
2,2
1,7
1,4
1,3
1,2
1,1
ω = 1
p 1
h 1
α 0,4
σ 4
μ 0-2

Fig. 4—3

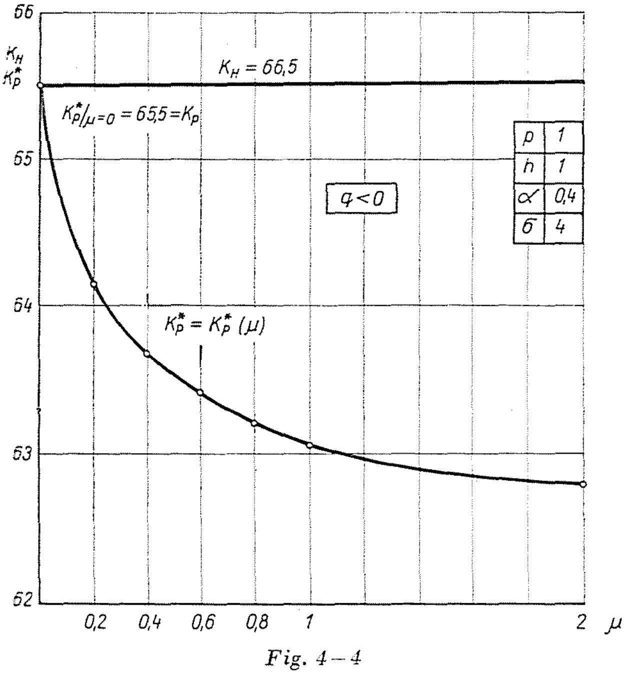
66
K_H
K_P^*
$K_H = 66{,}5$
$K_P^*/_{\mu=0} = 65{,}5 = K_P$
65
q<0
p 1
h 1
α 0,4
σ 4
64
$K_P^* = K_P^*(\mu)$
63
62
0,2
0,4
0,6
0,8
1
2
μ

Fig. 4—4

Also SCHMIDT and PREUSCHE [11] studied the system (4—1).

They determined the geometrical position of the pair of complex conjugate poles, satisfying the AIZERMAN *conjecture* for $p = 1$; $h = 0, 1, 2, 3, 4$. Such limiting curves are shown in Fig. 4—5 for $p = 4$.

The AIZERMAN *conjecture* is satisfied if the complex pole is below the limiting curve. The curves were determined by the method described in the Appendix. The q values belonging to each point were also evaluated. The curve points $q = 0$ are connected by dash-and-dot lines.

The case of $q < 0$ is drawn in a dashed line, that of $0 < q$ in a continuous line.

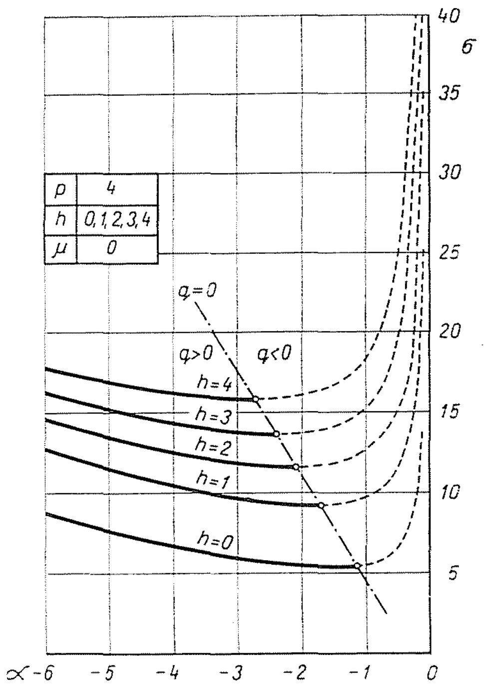

Fig. 4—5

Let us investigate now the case where the system (4—1) satisfies the conditions of the KALMAN *conjecture*. Fig. 4—6 shows the curve limiting the fulfilment of the conjecture for $p = 1$; $h = 4$. For comparison also the limiting curve for the fulfilment of the AIZERMAN *conjecture* is shown indicating also q values for some of the calculated points. For $0 < q$, the limiting curve of the KALMAN *conjecture* is above that of the AIZERMAN *conjecture*. The imaginary part of the complex radicals may be greater with the fulfilment of the KALMAN *conjecture* than when the AIZERMAN *conjecture* is satisfied.

On the other hand, for $q < 0$, the limiting curve of the KALMAN *conjecture* gives a smaller imaginary part than that of the AIZERMAN *conjecture*. Analyzing this fact and the behaviour shown in Fig. 4—3, it can be stated that the theorem of POPOV and the DEWEY—JURY relations are not congruent. The results obtained with the restriction of the slope of the non-linearity are only acceptable for $0 < q$, but for $q < 0$ they lead to contradictions.

As an illustration Fig. 4—6 shows the modified frequency function $G^{M}(j\omega)$ and the tangent to its point of intersection with the negative real axis at six calculated points.

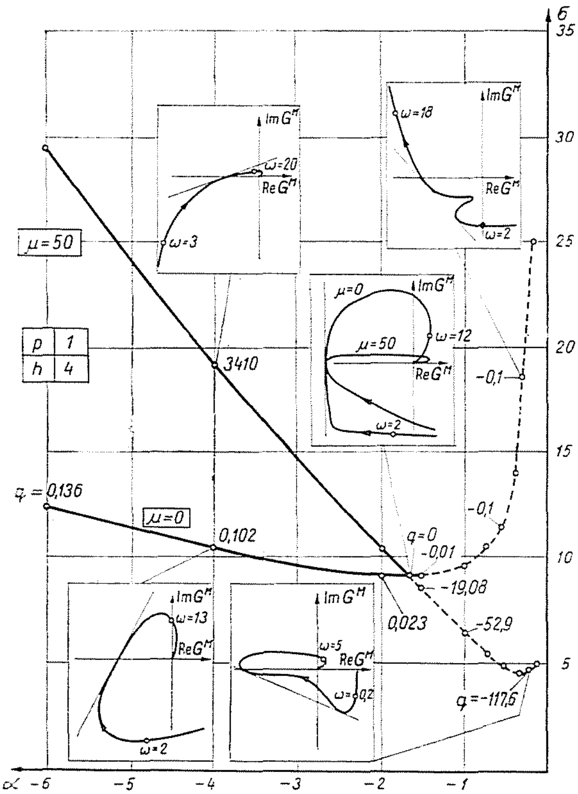

Fig. 4—6

5. Appendix

The evaluation of the limiting curves of the AIZERMAN and KALMAN *conjecture* is demonstrated in Fig. 5—1. The modified frequency function is:

$$G^{M}(j\omega) = \operatorname{Re} G(j\omega) + j \frac{\omega}{1+\mu\omega^2} \operatorname{Im} G(j\omega) = x + jy\,. \tag{5—1}$$

The negative real axis is intersected by this curve at point x_0 of the frequency:

$$\omega_0 = \sqrt{\frac{(h+p)(\sigma^2+\alpha^2)+2\alpha ph}{h+p+2\alpha}} \tag{5—2}$$

hence:

$$x_0 = -\frac{1}{K_H} = \frac{1}{\omega_0^4 - \omega_0^2[ph+\sigma^2+\alpha^2+2\alpha(p+h)]+ph(\sigma^2+\alpha^2)} \tag{5—3}$$

The slope of the tangent at the intersection point is:

$$q^* = \frac{dy}{dx} = \frac{1}{q}\,. \tag{5—4}$$

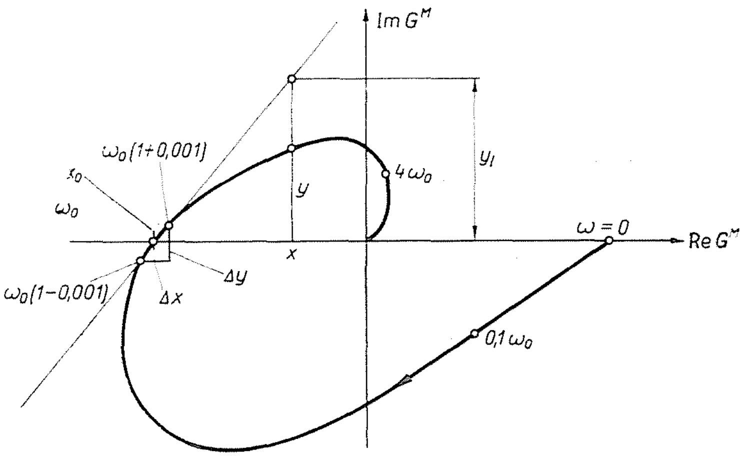

Fig. 5—1

Replacing the tangent by the secant, $q^* = \Delta y/\Delta x$, we have:

$$\Delta y = \operatorname{Im} G^M(j\,1{,}001\,\omega_0) - \operatorname{Im} G^M(j\,0{,}999\,\omega_0) \tag{5—5}$$

$$\Delta x = \operatorname{Re} G^M(j\,1{,}001\,\omega_0) - \operatorname{Re} G^M(j\,0{,}999\,\omega_0). \tag{5—6}$$

Each point of the limiting curve was determined with p, h, α and μ values assumed constant. The σ value was varied (increased) from zero with one that internals. So the condition for evaluating $\sigma_{\lim}$ is: for $0 < q$:

$$y_l - y = q^*(x - x_0) - y \leqq 0 \tag{5—7}$$

for $q < 0$:

$$y_l - y = q^*(x - x_0) - y \geqq 0 \tag{5—8}$$

where the x and y values are to be determined in the frequency range $0.1\ \omega_0 < \omega < 4\ \omega_0$. Both conditions may also be written jointly:

$$q^*(y_l - y) \leqq 0 \qquad (5-9)$$

For $\Delta x = 0$, i.e. $g^* = \infty$, the condition (5—9) may be replaced by the inequality:

$$(x - x_0) \leqq 0. \qquad (5-10)$$

For the sake of completeness let us present the program used for the evaluation of σ_{lim}.

```
BEGIN REAL  B, H, P, M, A, OO, Xo, S, FELT, O, X, Y, Q, X1, X2,
             Y1, Y2, DX, OM, DY, V1, V2;
INTEGER  I, HN, PN, MN, AN, J, K, L, II, NQ;
ARRAY  HH, PP, MM, AA [1 : 20];
PROCEDURE  XY (O, S, H, P, M, A, X, Y),
VALUE  O, S, H, P, M, A;
REAL  O, S, H, P, M, A, X, Y;
BEGIN REAL  S1, S2, S3, S4;
S1: ≡ O**4—O**2*(P*H + S**2 + A**2 + 2*A*(P+H)) +
     + P*H*(S**2 +A**2);
S2: = —O**2*(H + P + 2*A) + (H+P) * (S**2+A**2) +
     2*A*P*H;
S3: = S2/S1,
S4: = —O**2/(1+M*O**2);
X: = 1/(S1+O**2*S2*S3);
Y: = X*S3*S4;
END XY;
PROCEDURE  XNUL (O1, S, H, P, A, Xo);
VALUE  O1, S, P, H, A;
REAL  O1, S, H, P, A, Xo;
BEGIN  Xo: = 1/(O1**4—O1**2* (P*H+S**2+
        A**2 + 2*A* (P+H)) +P*H* (S**2 + A**2));
END XNUL;
PROCEDURE  ONUL (S, H, P, A, OO);
VALUE  S, H, P, A;
REAL  OO, S, H, P, A;
BEGIN  OO: = SQRT (((H+P) * (S**+A**2)+
        2*A*P*H)/(H+P+2*A));
END  ONUL;
BEGIN  INPUT (HN, PN, MN, AN);
FOR  I: = 1 STEP 1 UNTIL HN DO
```

```
INPUT (HH[I]);
FOR I: = 1 STEP 1 UNTIL PN DO
INPUT (PP[I]);
FOR I: = 1 STEP 1 UNTIL MN DO
INPUT (MM[I]);
FOR I: = 1 STEP 1 UNTIL AN DO
INPUT (AA[I]);
FOR I: = STEP 1 UNTIL HN DO
FOR J: = 1 STEP 1 UNTIL PN DO
FOR K: = 1 STEP 1 UNTIL MN DO
FOR L: = 1 STEP 1 UNTIL AN DO
BEGIN
H: = HH[I];
P: = PP[J];
M: = MM[K];
A: = AA[L];
FOR B: = o, B+1 WHILE FELT > o AND S < 45,
      S—1, B+o.1 WHILE FELT > o AND S < 45
      DO
BEGIN
S: = B
ONUL (S, H, P, A, OO);
XNUL (OO, S, H, P, A, Xo);
V1: = OO+OO* 10^-3;
V2: = OO—OO*10^-3;
XY (U1, S, H, P, M, A, X1, Y1);
XY (V2, S, H, P, M, A, X2, Y2);
DX: = (X1—X2) * 10^6;
DY: = (Y1—Y2) * 10^6;
IF DX = o THEN BEGIN NQ: = 1 GOTO
E1 END ELSE IF (LN (ABS (DY)) — LN(ABS (DX)) >
                    42 THEN BEGIN
Q: = DY/DX;
NQ: = o;
E1; FOR OM: = 0.07 OM+0.05 WHILE OM < 10
         DO
BEGIN
O: = OO**OM;
XY (O, S, H, P, M, A, X, Y);
IF NQ ≠ 1 THEN FELT: = SIGN (Q) * (Q* (X—Xo) —
                    Y) ELSE
FELT: = X—Xo;
```

```
IF FELT ≦ o THEN GOTO E2;
END;
E2: END
Q: = 1/Q;
OUTPUT (S/4, H, P, M, A, Q);
LINE;
END;
END;
END;
```

Summary

A comprehensive picture is given of the conception and criteria of absolute stability. Results by POPOV and DEWEY—JURY are applied to demonstrate the conditions of the fulfilment of the KALMAN hypothesis besides those of the AIZERMAN conjecture. The study of fourth order systems is involved to demonstrate the contradiction arising from the application of the known criteria.

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