AN AUTOMATIC PROCESS IDENTIFICATION METHOD

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Introduction

An ever more frequent demand arising in designing control systems is to ensure optimum operation in a definite sense in spite of both external disturbances acting on the system and possible variations in the environment or within the system.

This task may be solved on the basis of the following principle: The structure and the parameters of the variable part of the system are identified and the obtained values utilized for adjusting the control parameters in a way that the figure of merit characterizing the behaviour of the system (e.g. the expected value of the quadratic error) reaches an extreme value. Our purpose is best served by identifying without any external disturbance of the system, by utilizing the actually operating (mostly stochastic) signals. A certain a priori knowledge on the variable part of the system is always available, it is possible to assume a structure and so the identification may be reduced to the determination of the parameters.

The stochastic approximation method

Be the input and output signals of the plant to be identified x and y, respectively. The signals x and y are related by an operator W, the structure of which is assumed to be known (Fig. 1.). In the course of the identification we try to determine the unknown parameters of W.

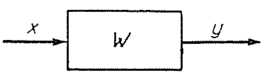

Fig. 1. Block-representation of the process

The task may be approximated in the following way: A model is established and its c_i parameters kept varying until the functional $I(\mathbf{c})$, the expected value of the quadratic deviation Q, characterizing the deviation between y and $\hat{y}$, the output signals of the system and of the model, respectively, reaches a minimum value.

$$I(\mathbf{c}) = M[(y - \hat{y})^2] = M(Q(x,\mathbf{c})] = \min \tag{1}$$

i.e.

$$\nabla I(\mathbf{c}) = -2M\left[(y - \hat{y})\frac{\partial \hat{y}}{\partial \mathbf{c}}\right] = M[\nabla_c Q[x, \mathbf{c})] = 0. \tag{2}$$

According to TSYPKIN let us conceive the output $\hat{y}$ of the model in the following form [1, 3]:

$$\hat{y} = \sum_{i=1}^{m} c_i \Theta_i(x, \hat{y}) \tag{3}$$

with Θ_i being the transfer operator.

In this case expression (2) is reduced to

$$\nabla I(\mathbf{c}) = -2M[(y - \hat{y})\boldsymbol{\Theta}(x,\hat{y})] = 0. \tag{4}$$

Several iterative approximation procedures are known for determining the vector $\mathbf{c}^*$ best approximating the actual vector of parameters. Very often the gradient method is used according to which

$$\mathbf{c}[n] = \mathbf{c}[n-1] - \gamma[n] \nabla I(\mathbf{c}[n-1]) \tag{5}$$

where $\gamma[n]$ denotes the step length. The convergence of the process depends on the choice of $\gamma[n]$.

The gradient method may be applied easily only in cases where the probability density function is known in advance, i.e. the expected value can be evaluated. If there is no a priori knowledge concerning the probability density function, the stochastic approximation method may be applied (among others) [2], according to which the following algorithm may be utilized for the determination of $\mathbf{c}^*$:

$$\mathbf{c}[n] = \mathbf{c}[n-1] - \gamma[n] \nabla_c Q(x[n-1], \mathbf{c}[n-1]) \tag{6}$$

or in difference form:

$$\Delta\mathbf{c}[n] = -\gamma[n] \nabla_c Q(x[n-1], \mathbf{c}[n-1]).$$

The convergence of the process depends again on the choice of γ.

The discrete algorithm may be converted into a continuous algorithm, which is more suitable for analog modelling:

$$\frac{d\mathbf{c}(t)}{dt} = -\gamma(t)\nabla_c Q(x(t), \mathbf{c}(t)) = \gamma(t)(y - \hat{y})\Theta(x, \hat{y}). \tag{8}$$

Applications

In the following a few simple examples of identification with the help of the stochastic approximation are described.

1. *Identification of a lag element with one time constant*

The transfer function W i.e. the quotient of the Laplace transforms of the output and input signals:

$$W(s) = \frac{Y(s)}{X(s)} = \frac{A}{1 + sT} . \tag{9}$$

A and T are the gain and the time constant, respectively, to be identified. Turning to the time domain we have the following differential equation:

$$T \frac{dy}{dt} + y = Ax .$$

Division by T, arranging and integration result in the relationship:

$$y = \frac{A}{T} \int_0^t x dt - \frac{1}{T} \int_0^t y dt .$$

The structure of the model is similar, therefore the equation for $\hat{y}$ in a form corresponding to (3) is:

$$\hat{y} = c_1 \int_0^t x dt + c_2 \int_0^t \hat{y} dt$$

where (10)

$$c_1^* = \frac{A}{T} \quad \text{and} \quad c_2^* = -\frac{1}{T} .$$

On the basis of (8) c_1 and c_2 must be varied according to the following relationships:

$$\frac{dc_1}{dt} = \gamma(y - \hat{y}) \int_0^t x dt$$

$$\frac{dc_2}{dt} = \gamma(y - \hat{y}) \int_0^t \hat{y} dt. \tag{11}$$

The scheme is shown in Fig. 2.

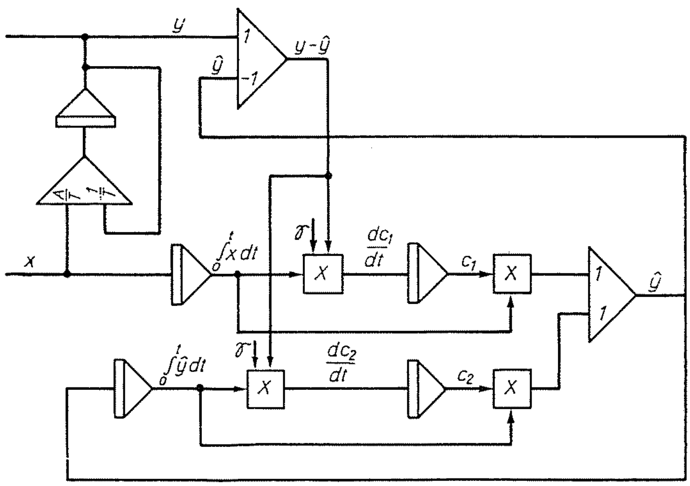

Fig. 2. Identification scheme of a lag element with one time constant

The process was checked for deterministic periodic signals on a type MEDA 80 analog computer, and with the help of a BOCS program developed for a type MINSK 22 digital computer. (Excitation with stochastic signals is to be the subject of a further study.)

For the sake of simple realizability constant γ-values were assumed. By varying the frequency of the sinusoidal input signal, we tried to find the γ-range where the process proved to be convergent. (Naturally the γ-values associated with the individual parameters may also differ from each other.)

According to the measurements the convergence was ensured by γ-values of a lower (2—5) or higher (20—1000) order of magnitude at frequencies lower or higher than the corner frequency respectively (Fig. 3). With an appropriate γ-value the process proved to be convergent independently of the phase angle of signal x, which influenced only the initial dynamics. As the model structure — because of the multipliers — is non-linear, the dynamics depend on the amplitude of signal x too.

On starting from arbitrary initial values of c_1 and c_2, the parameters reach the actual values (Fig. 4). If the system parameters are varied after self-adjustment, the variation is followed up by the parameters of the model (Figs 5, 6).

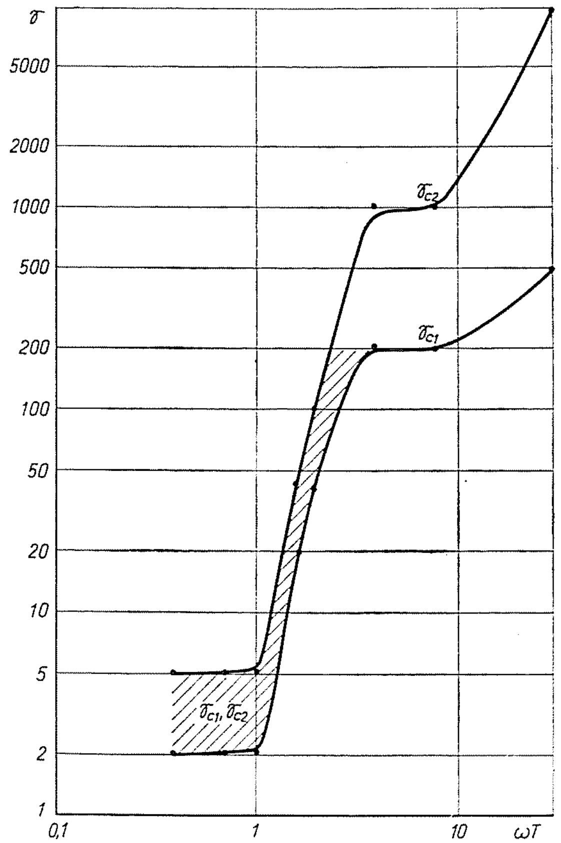

Fig. 3. γ convergency-constant as a function of frequency

The identification is also correct when the system is excited by triangle or square waves; in these cases the corresponding accuracy is ensured by γ-values somewhat higher than those given in Fig. 3, because of the harmonics contained by the signal.

The described process may also be applied when the output of the element to be identified is mixed with a zero-centered noise (Fig. 7). In this case the parameters oscillate around the effective values. The nearer the noise frequency to that of the signal, the greater is the amplitude of the oscillations. In our measurements a sinusoidal noise signal was chosen (Figs 8, 9).

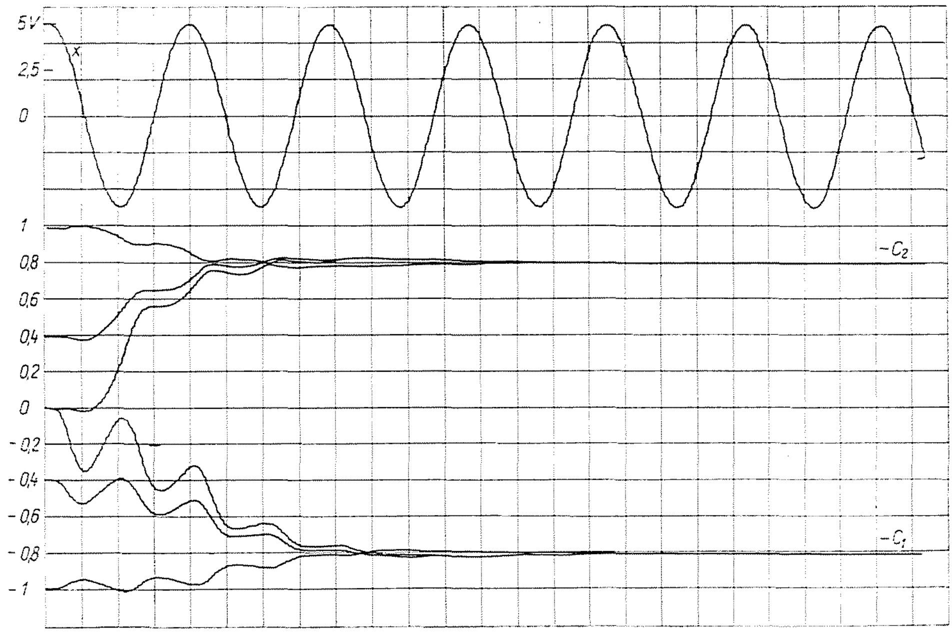

Fig. 4. Starting from arbitrary values c_1 and c_2 reach their actual values. $A = 1$; $T = 1.25$s; $f_x = 0.128$ c/s; $\gamma = 5$; $c_1^* = 0.8$; $c_2^* = -0.8$

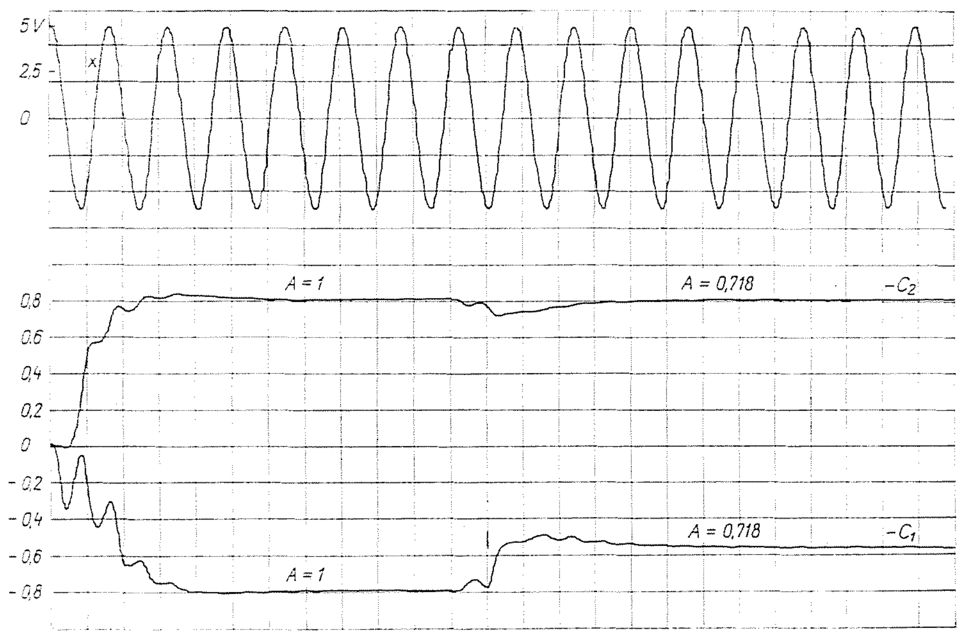

Fig. 5. The parameters follow the changes in the gain factor. $A = 1 \rightarrow 0.718$; $T = 1.25$s; $f_x = 0.128$ c/s; $\gamma = 5$; $c_1^* = 0.8 \rightarrow 0.574$; $c_2^* = -0.8$

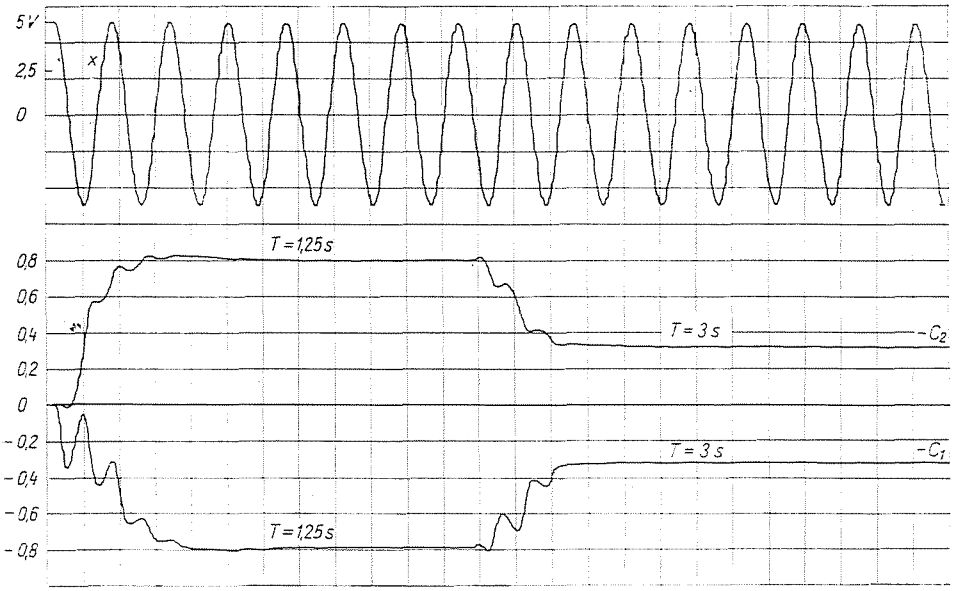

Fig. 6. The parameters follow the changes in the time constant. $A = 1$; $T = 1.25 \rightarrow 3$s; $f_x = 0.128$ c/s; $\gamma = 5$; $c_1^* = 0.8 \rightarrow 0.333$; $c_2^* = -0.8 \rightarrow -0.333$

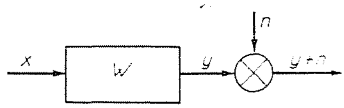

Fig. 7. Block representation of a noisy process

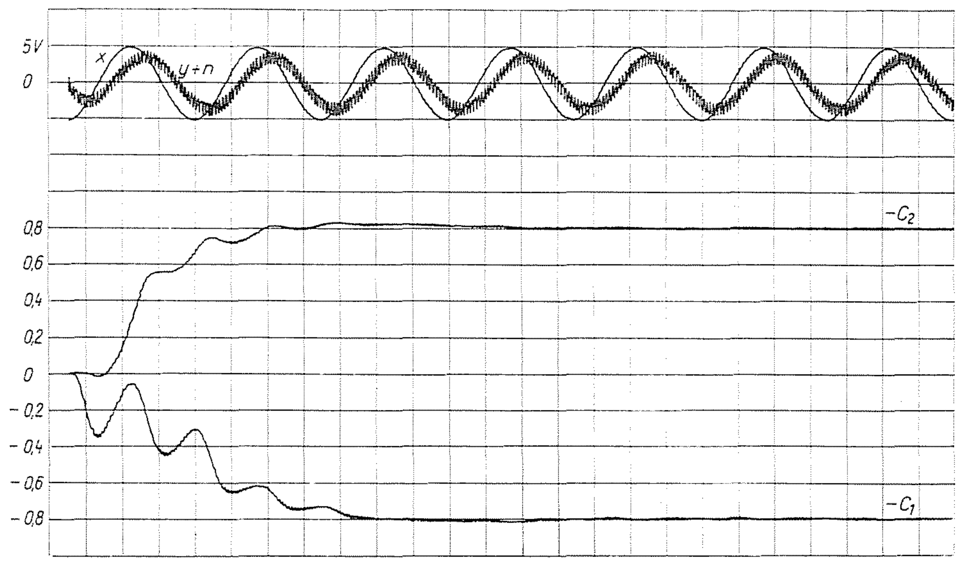

Fig. 8. The dynamics of the identification process with sinusoidal input and noise. $x_{max} = 5$V; $f_x = 0.128$ c/s; $n_{max} = 1$V; $f_n = 5$ c/s; $A = 1$; $T = 1.25$ s; $\gamma = 5$; $c_1^* = 0.8$; $c_2^* = -0.8$

2. *Identification of a rate element with one time constant*

The transfer function of the element is:

$$W(s) = \frac{Y(s)}{X(s)} = \frac{sT_D}{1+\ sT}\,. \tag{12}$$

In the time domain

$$y = \frac{T_D}{T}\,x - \frac{1}{T}\int_0^t y\,dt\,.$$

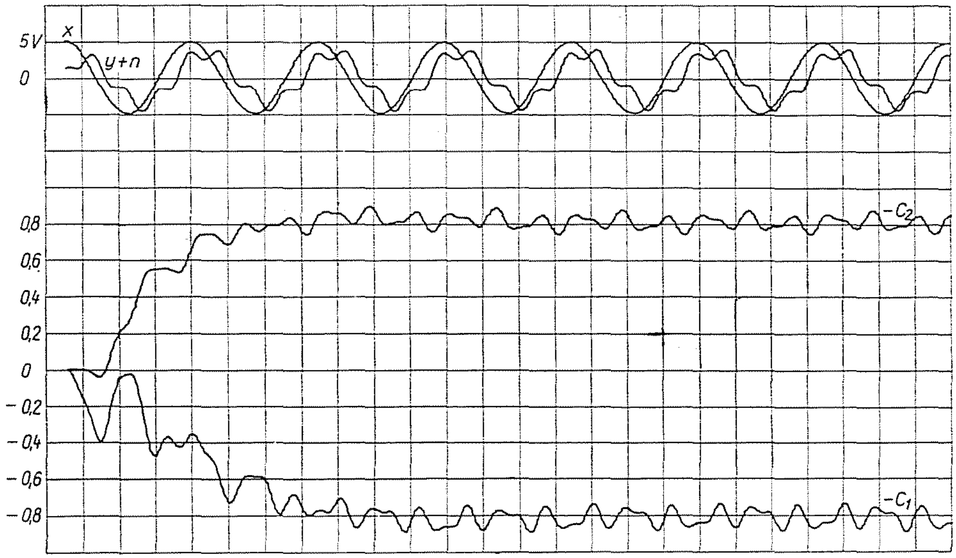

Fig. 9. The dynamics of the identification process with sinusoidal input and noise. $x_{max} = 5$V; $_x = 0.128$ c/s; $n_{max} = 1$V; $f_n = 0.5$ c/s; $A = 1$; $T = 1.25$s; $\gamma = 5$; $c_1^* = 0.8$; $c_2^* = -0.8$

As the structure of the model is similar, the relation for the model output signal is

$$\hat{y} = c_1\,x + c_2 \int_0^t \hat{y}\ \ dt \tag{13}$$

where

$$c_1^* = \frac{T_D}{T} \quad \text{and} \quad c_2^* = -\frac{1}{T}\,.$$

According to (8)

$$\begin{aligned} \frac{dc_1}{dt} &= \gamma(y-\hat{y})\,x \\ \frac{dc_2}{dt} &= (y-\hat{y}) \int_0^t \hat{y}\ \ dt\,. \end{aligned} \tag{14}$$

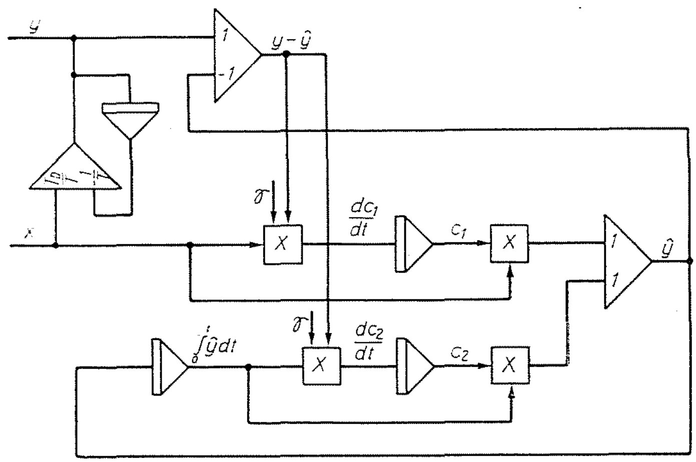

Fig. 10. Identification scheme of a rate element with one time constant

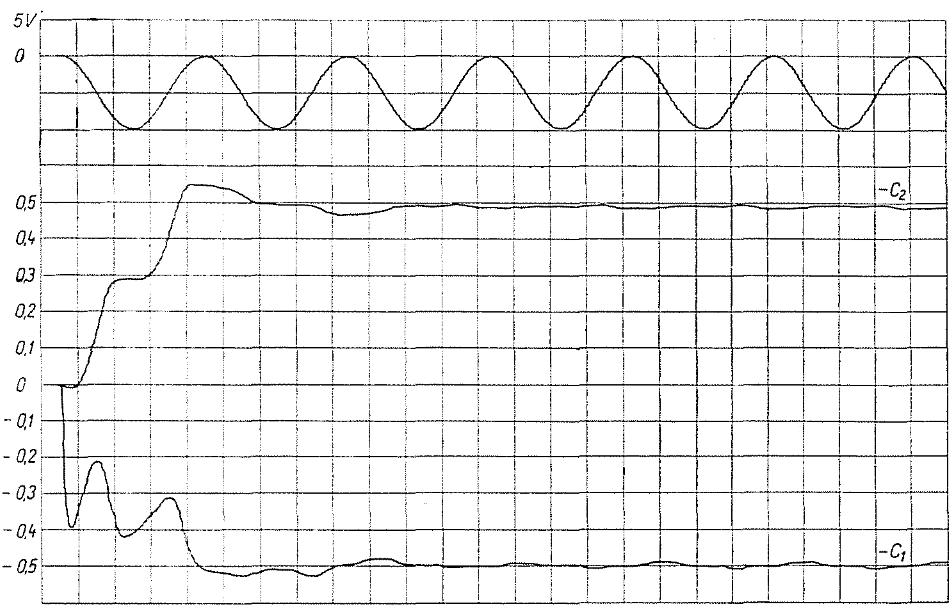

Fig. 11. The dynamics of the identification process of a rate element with one time constant. $T_D = 1$s; $T = 2$s; $f_x = 0.128$ c/s; $\gamma = 20$; $c_1^* = 0.5$; $c_2^* = -0.5$

The scheme is shown in Fig. 10. Fig. 11 gives the dynamics of the parameter setting in process.

3. *Identification of a lag element with two time constants*

Let us discuss now the process when proportional elements with one time constant each are connected in series and besides input signal x and output signal y also the intermediate signal u can be measured (Fig. 12). This case applies e.g. to series connected d.c. generators (generator and exciter).

The transfer function of the element is:

$$W(s) = \frac{Y(s)}{X(s)} = \frac{U(s)}{X(s)} \frac{Y(s)}{U(s)} = \frac{A}{(1+sT_1)(1+sT_2)} . \tag{15}$$

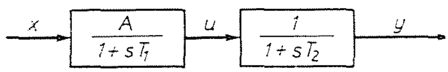

Fig. 12. Block diagram of a lag element with two time constants. The u intermediate signal is also measurable

The structure of the model is conceived similarly. The relations between the individual signals of the model in the time domain are:

$$\hat{u} = c_1 \int_0^t x dt + c_2 \int_0^t \hat{u} dt \tag{16}$$

where

$$c_1^* = \frac{A}{T_1} \quad \text{and} \quad c_2^* = -\frac{1}{T_1}$$

and

$$\hat{u} = c_3 \int_0^t (\hat{u} - \hat{y})\, dt \quad \text{where} \quad c_3^* = \frac{1}{T_2} . \tag{17}$$

The relations giving the variation rule of the parameters on the basis of (8) are:

$$\begin{aligned} \frac{dc_1}{dt} &= \gamma(u - \hat{u}) \int_0^t x dt \\ \frac{dc_2}{dt} &= \gamma(u - \hat{u}) \int_0^t \hat{u} dt \\ \frac{dc_3}{dt} &= \gamma(y - \hat{y}) \int_0^t (\hat{u} - \hat{y})\, dt . \end{aligned} \tag{18}$$

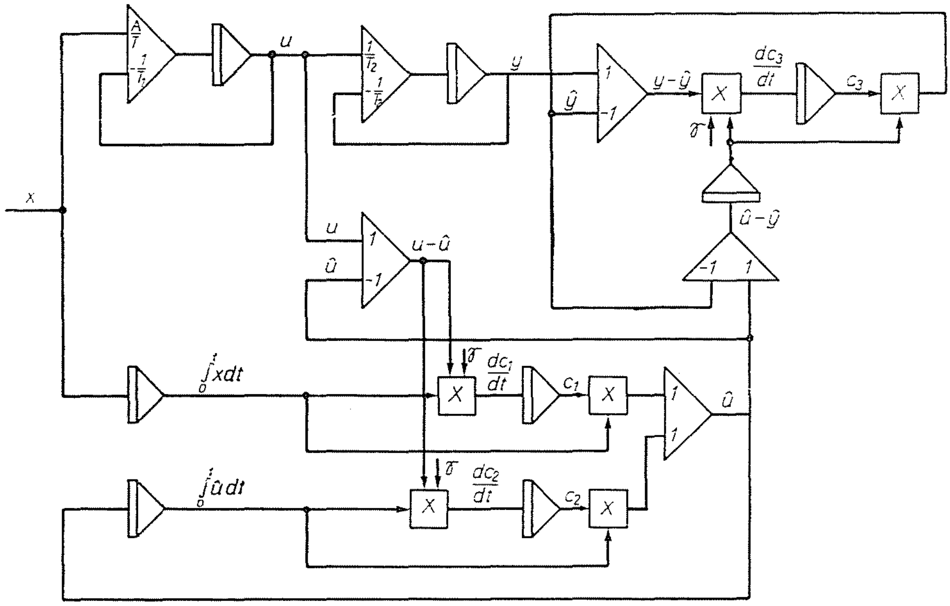

Fig. 13. **Identification scheme of a lag element with two time constants**

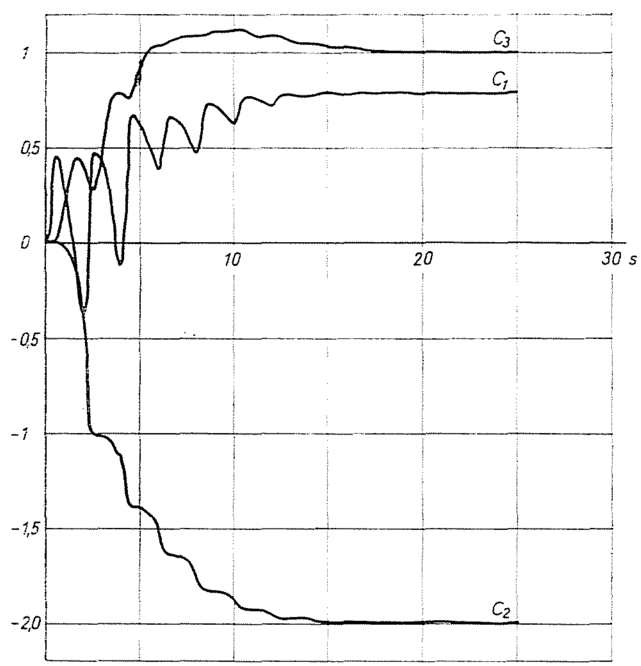

Fig. 14. The dynamics of the identification process of a lag element with two time constants. $A = 0.4$; $T_1 = 0.5$s; $T_2 = 1$s; $f_x = 0.25$ c/s; $\gamma = 50$; $c_1^* = 0.8$; $c_2^* = -2$; $c_3^* = 1$

Fig. 13 shows the identification scheme corresponding to equations (16) to (18) and Fig. 14 gives the variation of the parameters versus time. Adjustment is reached practically within three periods of the input signal.

In the following our intention is to extend the process and to determine its range of convergence for further elements occurring frequently in control engineering and to apply it also to stochastic signals.

Summary

Application of the stochastic approximation principle to the identification of some control elements is described The convergence of the identification process and the self-adjustment of the parameters are studied. The process is discussed in detail in the cases of lag elements with one and two time constants, and rate elements with one time constant. In the cases discussed the stability of the parameters' self-adjustment was ensured by the utilization of appropriate constant γ-values.

*

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