STABILITY TEST OF LINEAR CONTROL SYSTEMS WITH DEAD TIME COMPENSATED BY PID ACTION CONTROLLER

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(Received September 30, 1969)

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The stability of linear control systems with dead time is rather difficult to determine because of the existence of the exponential function appearing in the system characteristic equation. The use of a digital computer facilitates, however, to determine the variation of the stability region of a given control system with dead time for various parameter values. These critical loop gain values make it easier to design a given control system.

In a previous paper [1] it was investigated how to determine the stability region variation of linear control systems containing second order lag and dead time with a unit feedback compensated, generally speaking, by a PID action controller as shown in Fig. 1. Later graphs were presented giving the

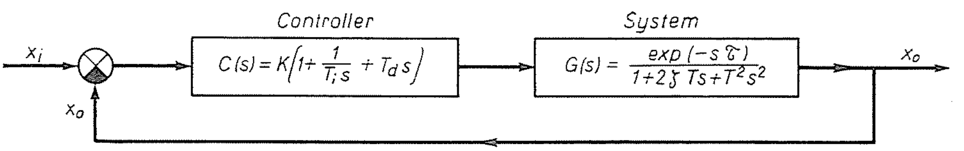


Fig. 1

critical loop gain K_{cr} versus dead time with system time constants as parameters for various damping factor ζ values. In these papers the above system was compensated by P, I [10] and PI [11] action controllers, respectively.

The present paper gives the values of K_{cr} versus dead time in case of a PID controller for different parameter values.

Proportional-plus-integral-plus-derivative control

The transcendental equation determining the critical angular frequency ω_{cr} belonging to K_{cr} with the assumption $T_i = 1$ from (11) of [11] is:

$$(T^2\omega_{cr}^3-\omega_{cr}+2\zeta T\omega_{cr}-2\zeta T\omega_{cr}^3T_d)\tan\omega_{cr}\tau = \\ = 2\zeta T\omega_{cr}^2-T^2\omega_{cr}^2+1+\omega_{cr}^4T_dT^2-\omega_{cr}^2T_d. \quad (1)$$

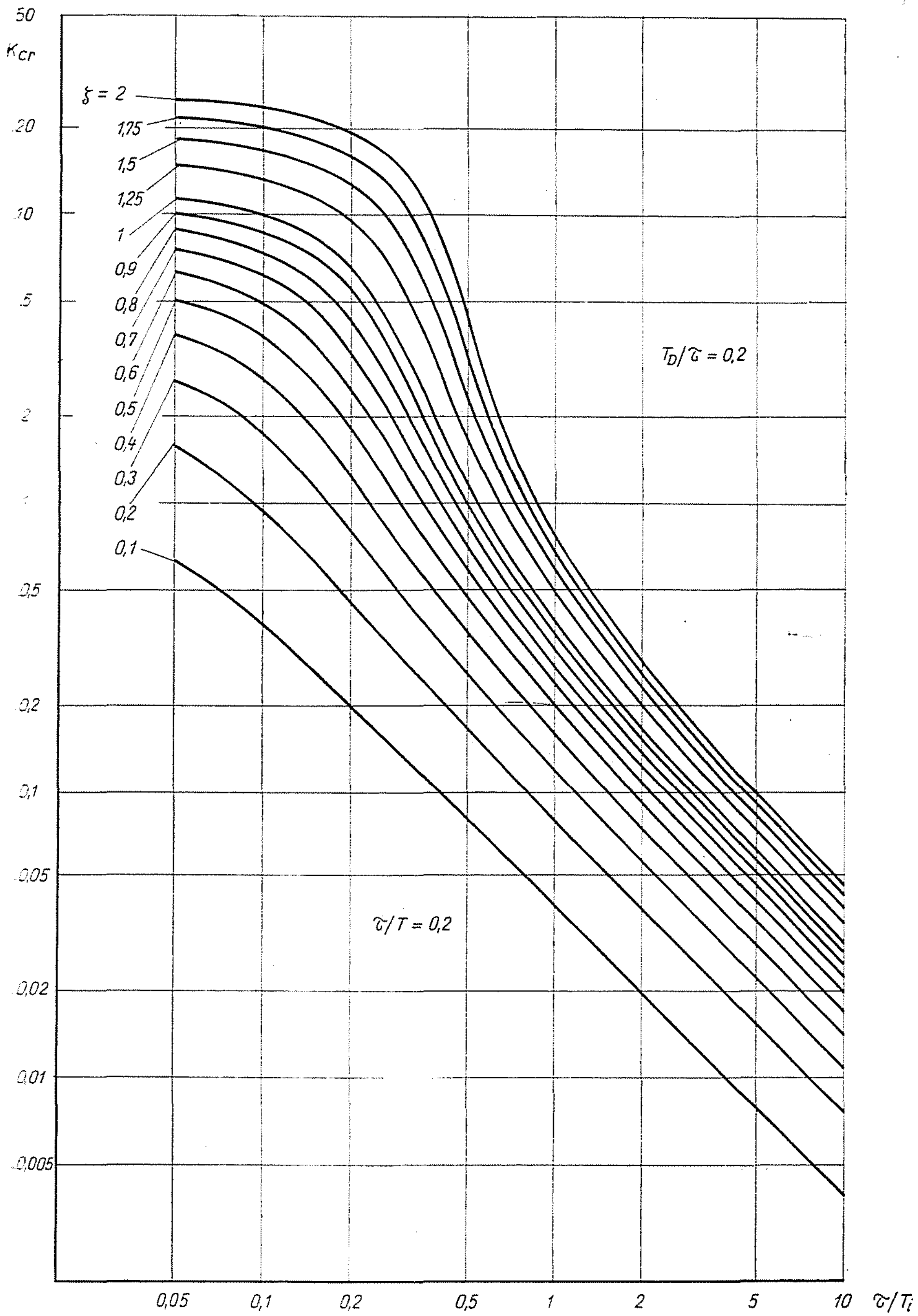

50
K_{cr}
20
10
5
2
0,5
0,2
0,1
0,05
0,02
0,01
0,005
ξ = 2
1,75
1,5
1,25
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
$T_D/\tau = 0,2$
$\tau/T = 0,2$
0,05
0,1
0,2
0,5
1
2
5
10
τ/T_i

Fig. 2

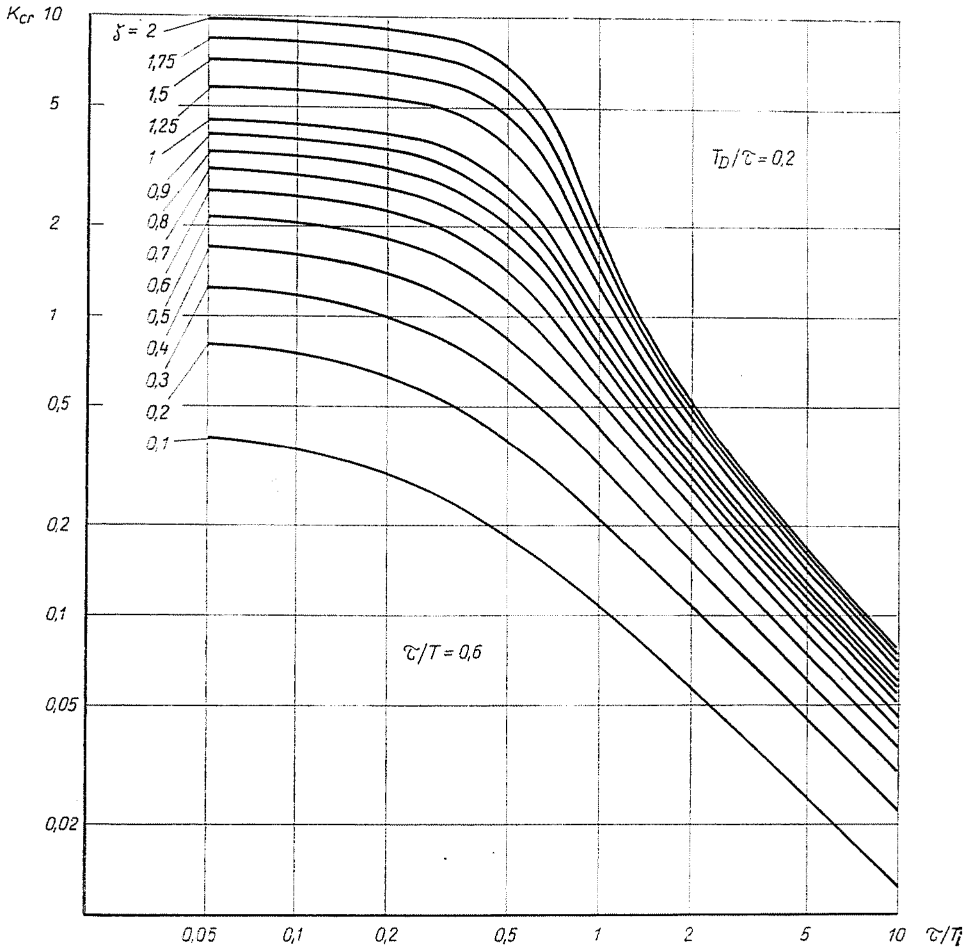


Fig. 3

The critical loop gain K_{cr} with arbitrarily accurate approximation values of ω_{cr} may be determined by the following simple algebraic equation:

$$K_{cr} = \frac{\omega_{cr}\sqrt{(1-\omega_{cr}^2 T^2)^2+(2\zeta T\omega_{cr})^2}}{\sqrt{(1-T_d\,\omega_{cr}^2)^2+\omega_{cr}^2}} \,. \tag{2}$$

Figs 2 through 16 show the variation of the stability region as function of the dead time if $0.05 \leq \tau/T_i \leq 10$ for the following time constant ratios and parameter values:

$T_d/\tau = 0.2,\ 0.6,\ 1,$

$\tau/T = 0.2,\ 0.6,\ 1,\ 4,\ 10,$

$\zeta = 0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9,\ 1,\ 1.25,\ 1.5,\ 1.75,\ 2.$

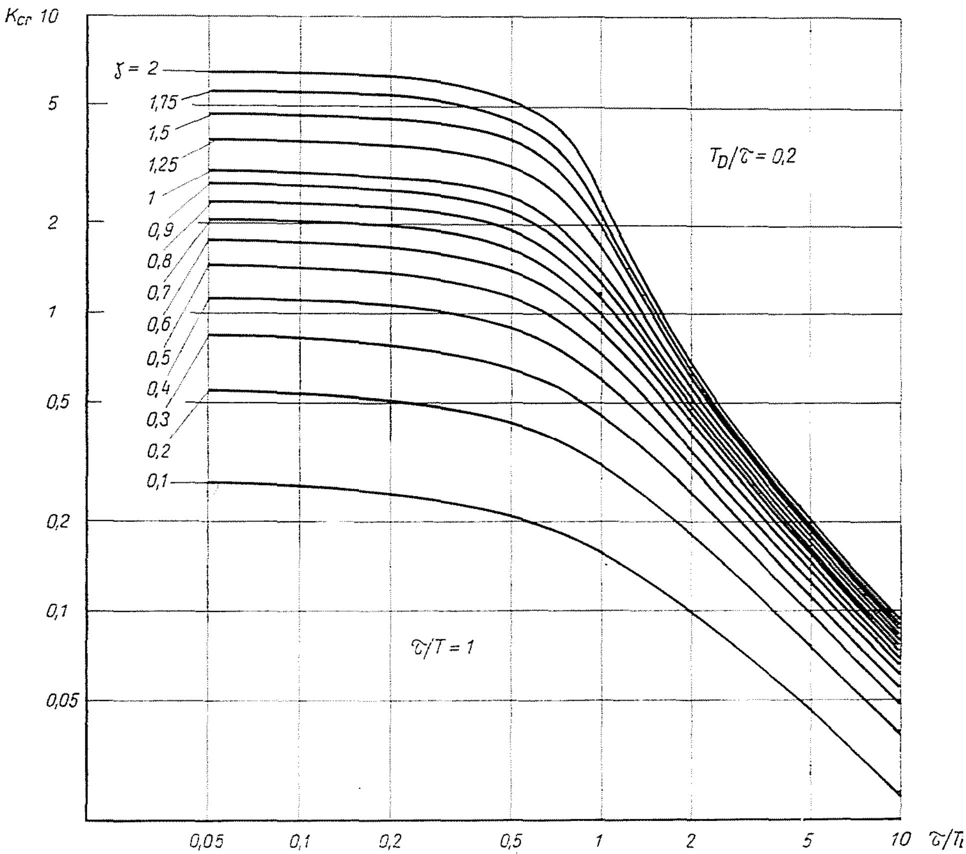


Fig. 4

The figures are traced in log-log scale for the sake of clearness. In some figures [11, 16.] K_{cr} curves belonging to some ζ parameters are omitted to make the diagrams clearer.

The figures demonstrate that in case of low T_d/τ values the behaviour of the control system shown in Fig. 1 may be substituted by a control system compensated by a PI action controller.

The correctness of the latter statement is obvious. By the help of (12) from [1] the critical loop gain using the over-all transfer function of the open loop system is:

$$K_{cr} = -\frac{T_i s}{1+T_i s+T_d T_i s} \cdot \frac{1+2\zeta Ts+T^2 s^2 2\zeta s}{e^{-s\tau}} .$$

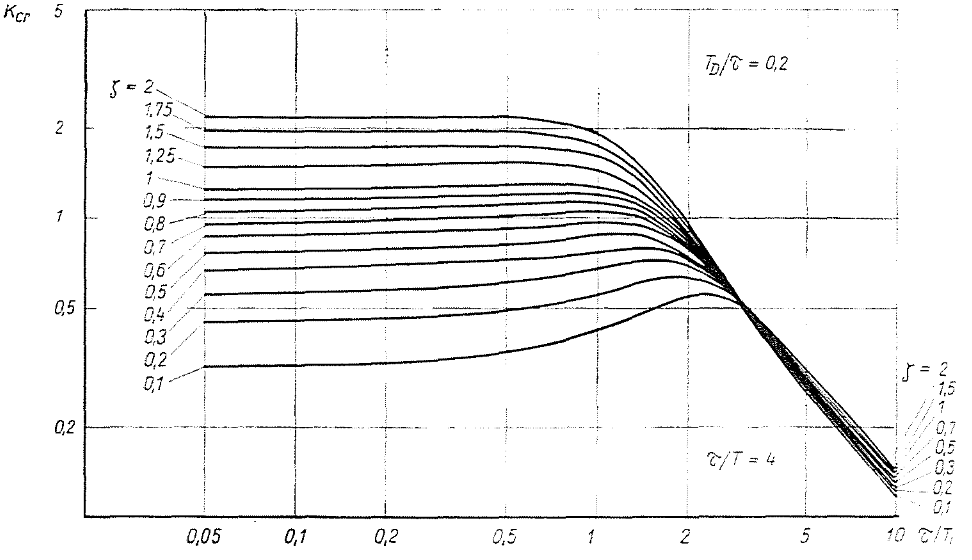

K_{cr}
5
2
1
0,5
0,2
ζ = 2
1,75
1,5
1,25
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
T_D/τ = 0,2
τ/T = 4
ζ = 2
1,5
1
0,7
0,5
0,3
0,2
0,1
0,05
0,1
0,2
0,5
1
2
5
10
τ/T_i

Fig. 5

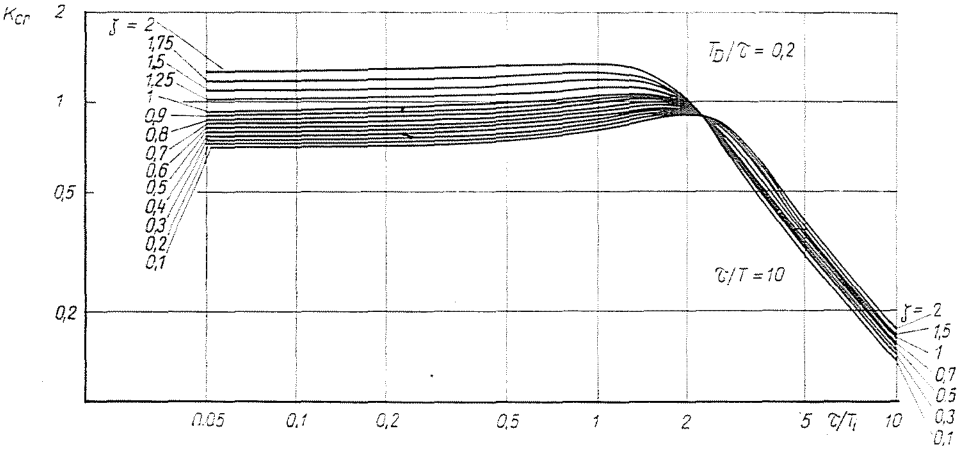
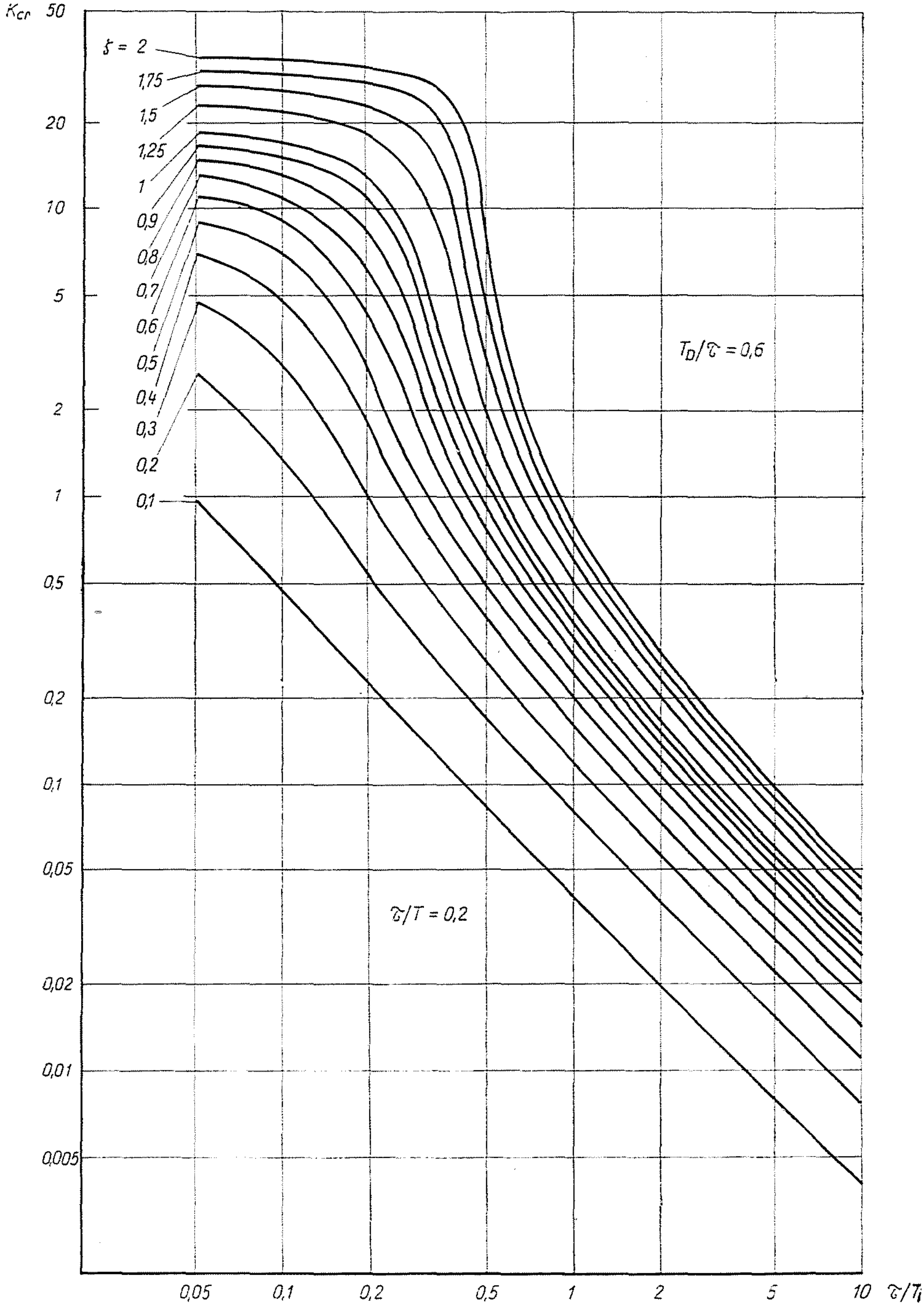

K_{cr}
2
1
0,5
0,2
ζ = 2
1,75
1,5
1,25
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
T_D/τ = 0,2
τ/T = 10
ζ = 2
1,5
1
0,7
0,5
0,3
0,1
0,05
0,1
0,2
0,5
1
2
5
τ/T_i
10

Fig. 6



K_{cr}
50
20
10
5
2
1
0,5
0,2
0,1
0,05
0,02
0,01
0,005
ζ = 2
1,75
1,5
1,25
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
$T_D/\tau = 0,6$
$\tau/T = 0,2$
0,05
0,1
0,2
0,5
1
2
5
10
τ/T_i

Fig. 7

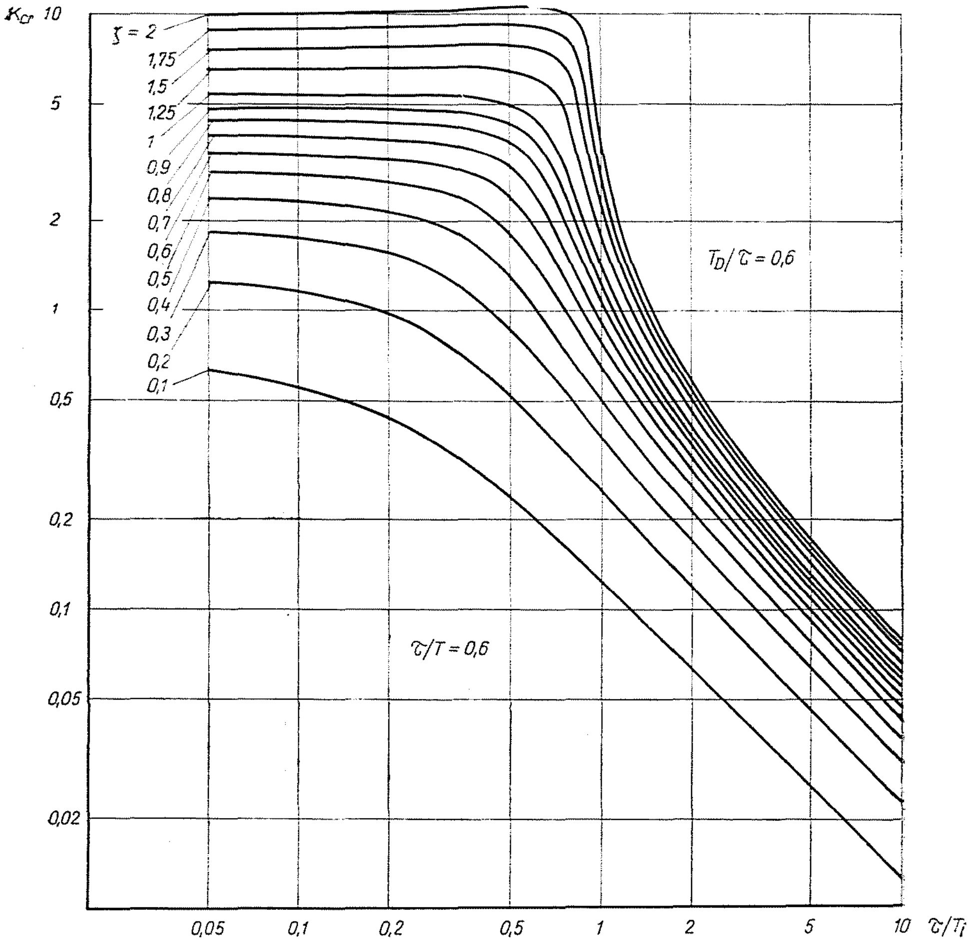


Fig. 8

After some arrangements:

$$K_{cr} = -\frac{(T_i/\tau)(s\tau)}{1+(T_i/\tau)(s\tau)+(T_d/\tau)(T_i/\tau)(s\tau)^2} \cdot \frac{1+2\zeta(T/\tau)(s\tau)+(T/\tau)^2(s\tau)^2}{e^{-st}} . \tag{2}$$

On the basis of equation (3) it can be stated that with decreasing T_d/τ the critical loop gain more and more approaches the values of K_{cr} belonging to a control system with second order lag and dead time compensated by a PI controller. Relation (3) indicates at the same time that with increasing dead time the latter behaviour more and more applies.

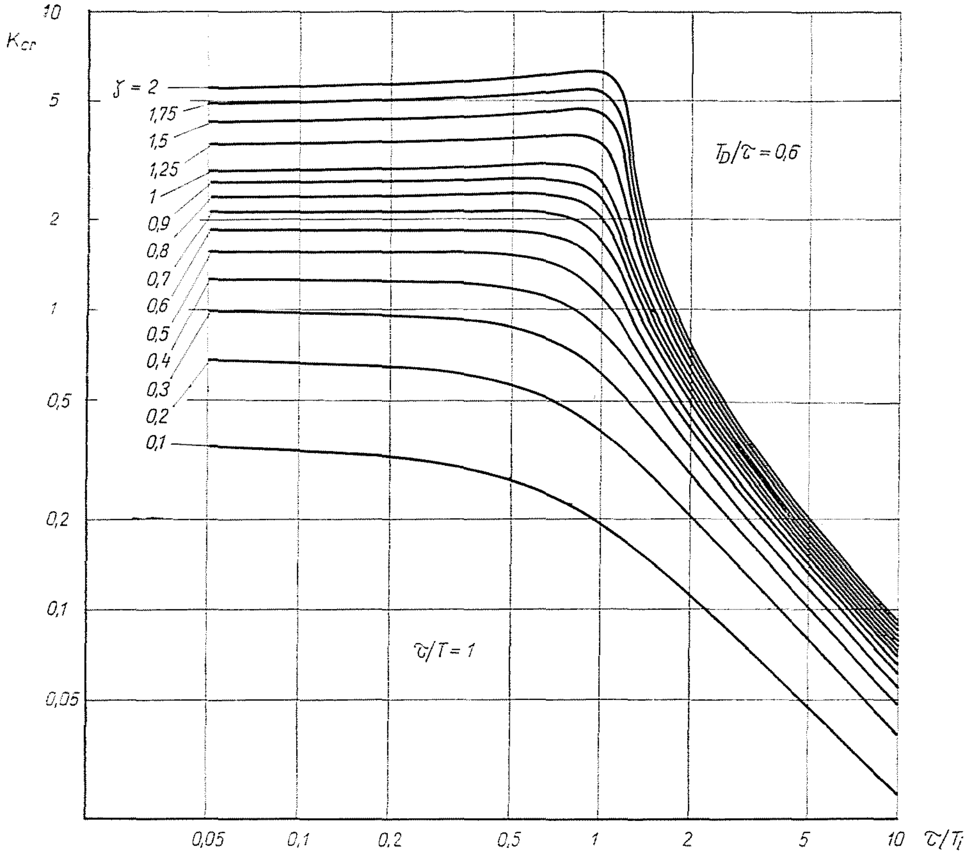


Fig. 9

For instance, let be: $T_d/\tau = 0.2$, $\tau/T = 1$, $\zeta = 0.5$, $\tau = 10$. On the basis of Fig. 4 of [11] the critical loop gain is: $K_{crPI} = 0.06$. From Fig. 4 of present paper $K_{crPID} = 0.061$. The difference between $K_{crPI}|_{\tau=10}$ and $K_{crPID}|_{\tau=10}$ is seen to be rather small.

On the other hand let be: $T_d/\tau = 0.2$, $\tau/T = 1$, $\zeta = 0.5$, $\tau = 1$. The corresponding critical loop gain values are: $K_{crPI}|_{\tau=1} = 0.628$ and $K_{crPID}|_{\tau=1} = 0.73$, respectively. The difference between the last two K_{cr} is more significant.

From the transfer function (3) of the open loop system for very high T_d/τ values, i.e. for $T_d/\tau \to \infty$, the critical loop gain is approximatively:

$$K_{cr} \approx -\frac{1}{T_d s} \frac{1+2\zeta(T/\tau)(s\tau)+(T/\tau)^2(s\tau)^2}{e^{-s\tau}} .$$

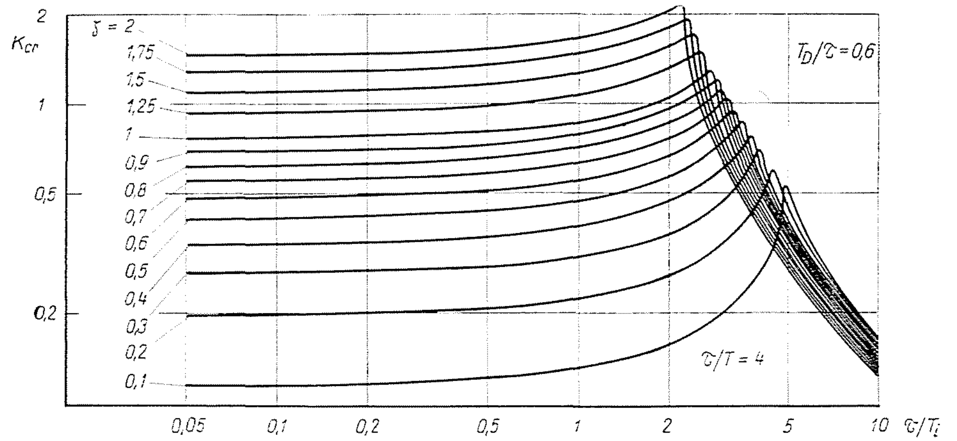


Fig. 10

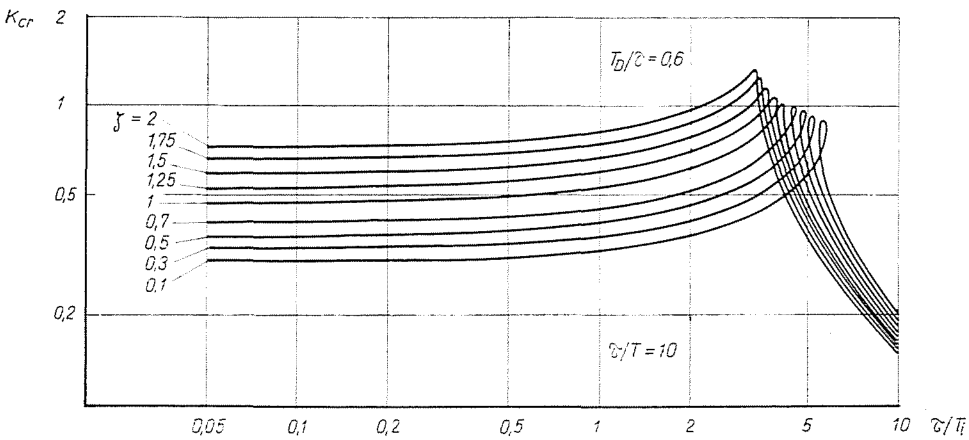


Fig. 11

Consequently, with increasing T_d/τ the control system may more and more be regarded a system with second order lag and dead time compensated by a PD type controller. For very high values of T_d/τ in comparison with the T/τ ratio the system may be substituted by a system with dead time compensated by a PD action controller.

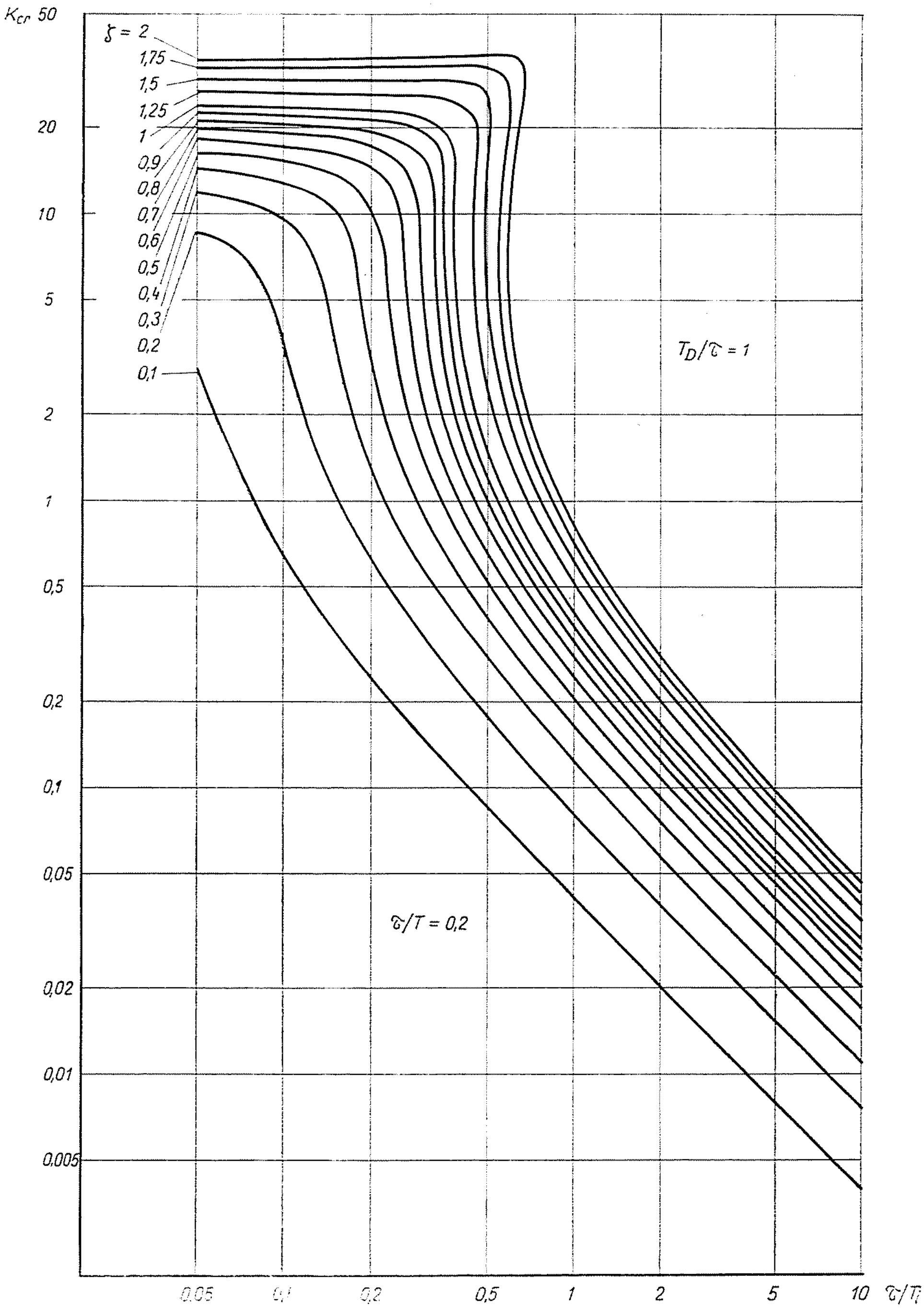

K_{cr} 50
20
10
5
2
1
0,5
0,2
0,1
0,05
0,02
0,01
0,005
ζ = 2
1,75
1,5
1,25
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
$T_D/τ = 1$
$τ/T = 0,2$
0,05
0,1
0,2
0,5
1
2
5
10
$τ/T_i$

Fig. 12

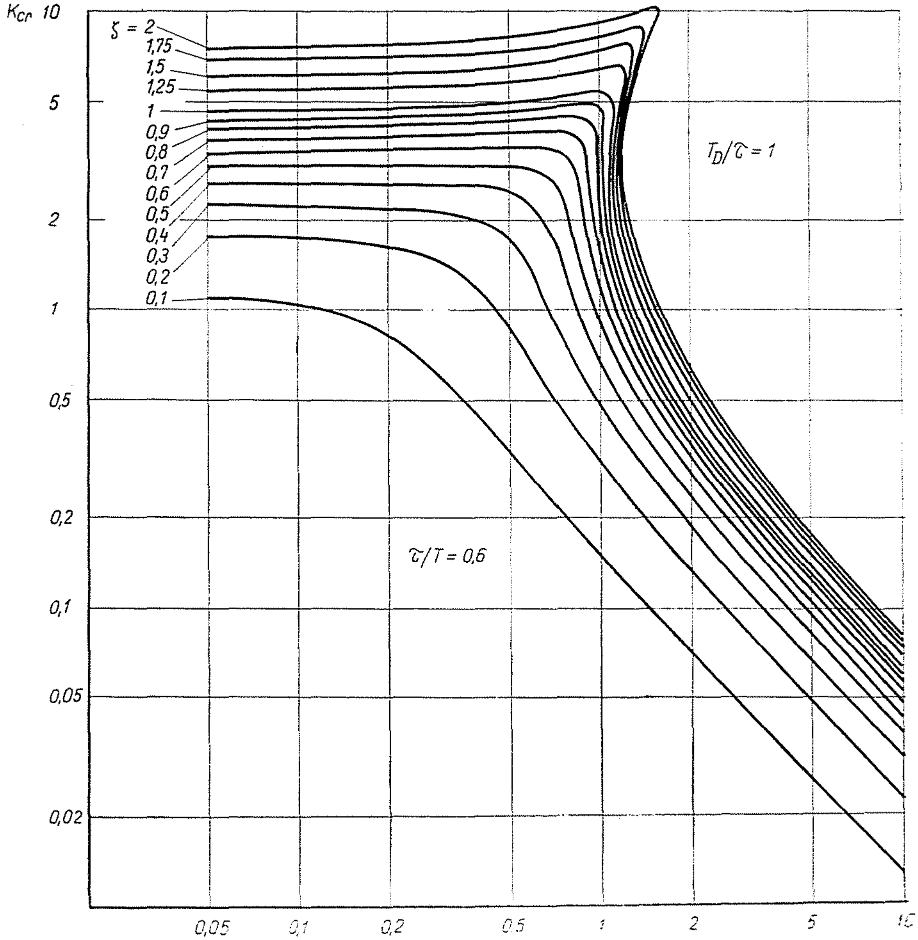
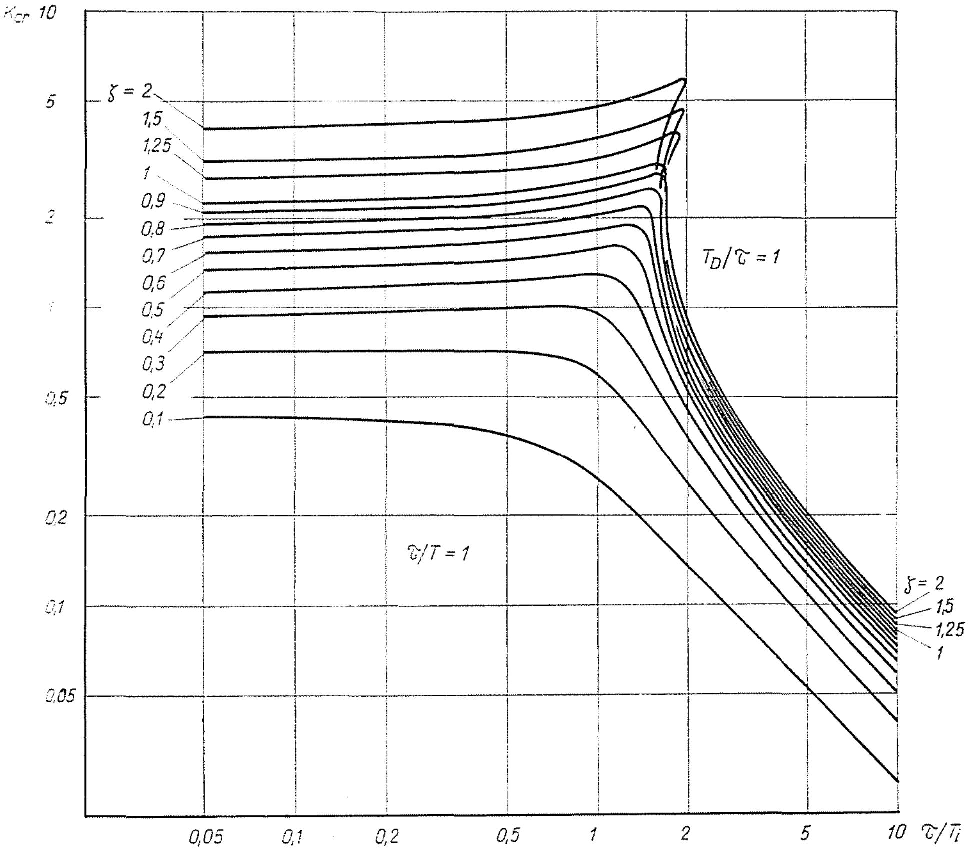
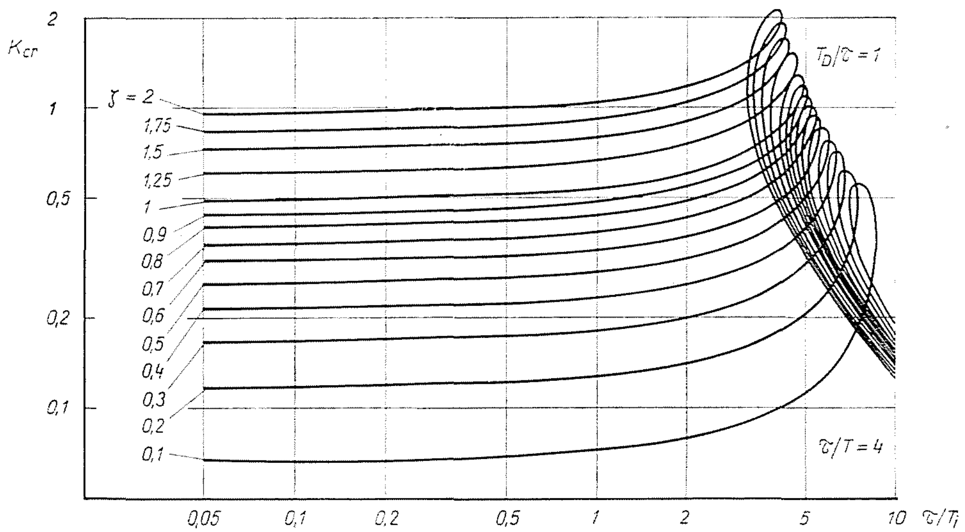

K_{cr} 10
5
2
1
0,5
0,2
0,1
0,05
0,02
ζ = 2
1,75
1,5
1,25
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
$T_D/\tau = 1$
$\tau/T = 0,6$
0,05
0,1
0,2
0,5
1
2
5
10

Fig. 13



K_{cr}
10
5
2
1
0,5
0,2
0,1
0,05
ζ = 2
1,5
1,25
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
$T_D/\tau = 1$
$\tau/T = 1$
ζ = 2
1,5
1,25
1
0,05
0,1
0,2
0,5
1
2
5
10
τ/T_i

Fig. 14



2
K_{cr}
1
0,5
0,2
0,1
ζ = 2
1,75
1,5
1,25
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
$T_D/\tau = 1$
$\tau/T = 4$
0,05
0,1
0,2
0,5
1
2
5
10
τ/T_i

Fig. 15

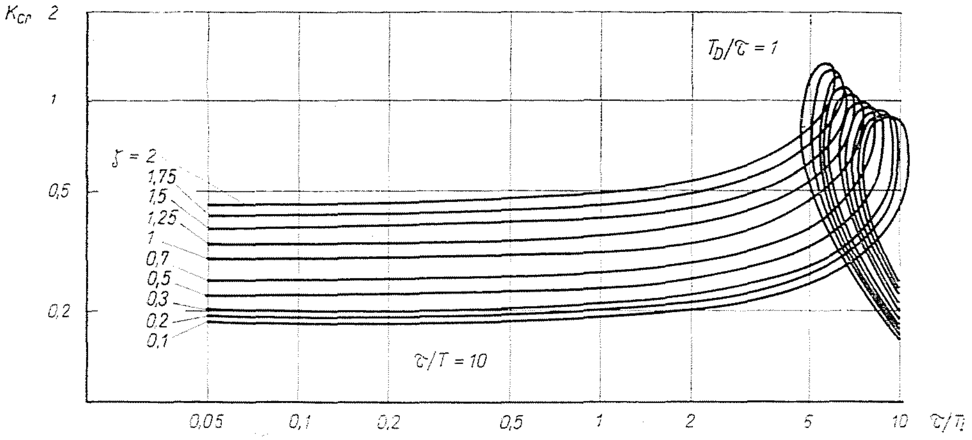


Fig. 16

Conclusion

Knowing the critical loop gain belonging to a given linear control system with second order lag and dead time compensated by P, I, PI and PID controller, respectively, one can easily determine what kind of controller proves to be convenient.

In [11], for low dead time values a proportional compensation was found to be preferable. For high dead time values the integral action control proves to be best. For medium dead times the most advantageous type of compensation will be determined by the parameters of the control system.

If the transfer function of the second order lag has the form:

$$Y(s) = \frac{1}{(1+sT_1)(1+sT_2)},$$

where T_1 and T_2 are the time constants of the second order lag, Figs. 8 on page 10—12 of [9] shows the preferable kind of compensation. Based on approximate measurements made on an analogue computer, the figure gives the advantageous compensations to be used with a second order lag and dead time system, as function of the dead time and the time constants.

Summary

The present paper gives the stability region variation of the linear continuous control system with second order lag and dead time compensated by proportional-plus-integral-plus--derivative PID action controller for different parameter values. The critical loop gain K_{cr}

values were evaluated by a digital computer. The K_{cr} diagrams for $0.05 \leq \tau/T_i \leq 10$ are plotted in log-log scale for the sake of clearness with the system time constants and the damping factor ζ as parameters.

Previous papers investigated the variation of the critical loop gain in cases of P, I [10] and PI [11] compensation.

The $K_{cr} = K_{cr}(\tau)$ diagrams of [10, 11] and the present paper help to choose the most advantageous action controller for different parameter values.

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