THREE-PHASE TRANSMISSION LINE SYSTEMS WITH TRANSPOSITION

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Introduction

Power transmission lines consist in practice mostly of once or twice three phase systems, in which the position of the individual wires is interchanged after a certain length. With the usual calculation method of such transmission lines the symmetrical components are employed [1, 2, 3]. In the present paper the well-known theory of coupled transmission lines [4, 5] is connected with the usual calculation method for three-phase lines built with transposition. Once and twice three-phase wire systems are examined whereby the influence of the ground wires is being disregarded. The calculation of the effect of the ground wires will be published in a subsequent paper.

Coupled transmission line system without transposition

In the followings the theory of such a transmission line system is summarized briefly which consists of n wires arranged above the earth parallel with each other and with the earth. The earth is supposed to be limited by a plane, homogeneous, and lossy. The electromagnetic fields of the wire currents are coupled with each other. For a coupled transmission line system of this kind the system of differential equations

$$\begin{aligned} \frac{\mathrm{d}i}{\mathrm{d}z} &= \mathbf{Y}_p\, u \\ \frac{\mathrm{d}u}{\mathrm{d}z} &= \mathbf{Z}_s\, i \end{aligned} \tag{1}$$

is valid, where z denotes the co-ordinate of the place in the direction of the transmission line, i the column vector formed of the currents of the wires, u that of the voltages between the wires and the earth surface, $\mathbf{Y}_p$ the parallel admittance matrix related to unit length, while $\mathbf{Z}_s$ the series impedance matrix related to unit length. These are square matrices of the nth order. Our calculations are performed on the basis of relationship

$$
\begin{aligned}
\mathbf{Y}_p &= j\omega\varepsilon\pi\mathbf{M}^{-1} \\
\mathbf{Z}_s &= \frac{j\omega\mu}{\pi}\mathbf{M} + \mathbf{Z}_b
\end{aligned}
\tag{2}
$$

Here ε and μ are the permittivity and permeability of the air, respectively, $\mathbf{M}$ a symmetrical square matrix depending on the geometrical data of the arrangement, in which the kth element in the jth row is found to be

$$m_{jk} = \ln \frac{\varrho_{jk}}{r_{jk}} \tag{3}$$

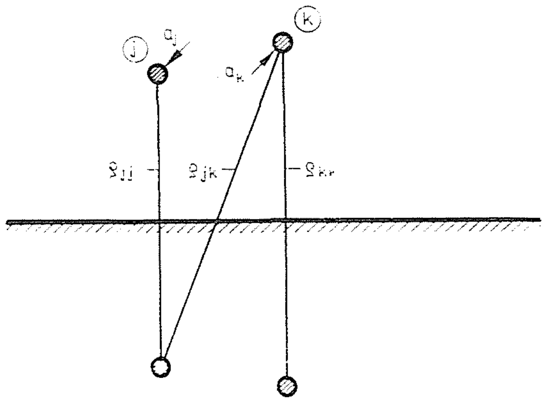

Fig. 1

r_{jk} is the distance of the jth wire from the kth wire, and ϱ_{jk} the distance of the mirror image of the jth wire from the kth wire (Fig. 1). The symmetrical square matrix $\mathbf{Z}_b$ is the sum of two matrices.

$$\mathbf{Z}_b = \mathbf{Z}_\nu + \mathbf{Z}_f \tag{4}$$

$\mathbf{Z}_\nu$ is a diagonal matrix, the elements in the main diagonal are the skin impedances of the individual wires.

$$\mathbf{Z}_\nu = \langle Z_{\nu 1}\, Z_{\nu 2} \dots Z_{\nu n} \rangle \tag{5}$$

$\mathbf{Z}_f$ is the earth impedance matrix, its elements can be determined by the help of the rows [5].

The solution of the system of differential equations (1) is found to be

$$
\begin{aligned}
u(z) &= e^{-\mathbf{\Gamma} z}\, U_0^{(+)} + e^{\mathbf{\Gamma} z}\, U_0^{(-)} \\
i(z) &= \mathbf{Y}_0 (e^{-\mathbf{\Gamma} z}\, U_0^{(+)} - e^{\mathbf{\Gamma} z}\, U_0^{(-)}) \,,
\end{aligned}
\tag{6}
$$

where $U_0^{(+)}$, and $U_0^{(-)}$ are the column vectors formed of the value assumed at the place $z = 0$ by voltages propagating in the directions $+z$ and $-z$, respectively, $\mathbf{\Gamma}$ is the propagation coefficient matrix the square of which is

$$\mathbf{\Gamma}^2 = \mathbf{Z}_s \mathbf{Y}_p , \tag{7}$$

and the expression of the wave admittance matrix is

$$\mathbf{Y}_0 = \mathbf{Z}_s^{-1} \mathbf{\Gamma} . \tag{8}$$

The matrix functions figuring in (6) can be expressed by the matrix Lagrange polynomials.

$$\mathrm{f}(\mathbf{X}) = \sum_{k=1}^{n} f(\lambda_k) \mathbf{L}_k \tag{9}$$

λ_k ($k = 1, 2, \ldots, n$) denotes the characteristic values of $\mathbf{X}$, these can be determined from the equation

$$\det |\mathbf{X} - \lambda \mathbf{E}| = 0 , \tag{10}$$

where E is the unit matrix of the nth order. The definition of the matrix Lagrange polynomials is given by

$$\mathbf{L}_k(\mathbf{X}) = \prod_{\substack{j=1 \\ j \neq k}}^{n} \frac{\mathbf{X} - \lambda_k \mathbf{E}}{\lambda_j - \lambda_k} . \tag{11}$$

Accordingly the relationships (6) can be written also in the following form.

$$\begin{aligned} u(z) &= \sum_{k=1}^{n} \mathbf{L}_k(\mathbf{\Gamma}^2) \left[U_0^{(+)} e^{-\gamma_k z} + U_0^{(-)} e^{\gamma_k z} \right] \\ i(z) &= \mathbf{Y}_0 \sum_{k=1}^{n} \mathbf{L}_k(\mathbf{\Gamma}^2) \left[U_0^{(+)} e^{-\gamma_k z} - U_0^{(-)} e^{\gamma_k z} \right] \end{aligned} \tag{12}$$

γ_k is that square root from the eigenvalues of $\mathbf{\Gamma}^2$ which lies in the first quarter of the plane of complex numbers.

On the basis of Eqs. (12), phenomena in the transmission line system can be interpreted as follows. The solutions both for the voltage and the current consist of two parts: One consists of waves propagating in the direction $+z$, while the other of those in direction $-z$, which are in general attenuated. The members of the sum correspond to one mode each. A propagation coefficient (γ_k) belongs to the individual modes. The number of modes cannot be higher than the number of wires. If the characteristic equation of $\mathbf{\Gamma}^2$ has identical roots too then the number of modes is lower than the number of wires.

The values $U_0^{(-)}$ and $U_0^{(-)}$ can be determined from the conditions arising at the termination of the transmission line system.

Once three phase transmission line with transposition

The compensation

A certain compensation takes place in consequence of the phase change. This can be taken into consideration in matrices $\mathbf{Y}_p$ and $\mathbf{Z}_s$ as follows. The elements in the main diagonal of these matrices originate from the own characteristics of the single lines, while the elements outside the main diagonal from the mutual correlation between the lines. Accordingly the compensation is taken into consideration in such a way that the elements of the main diagonal are substituted by the average value of the elements in the main diagonal, while the other elements substituted by the average of elements outside the main diagonal. This means that matrices $\mathbf{Y}_p$ and $\mathbf{Z}_s$ are transformed so as to have the structure

$$\mathbf{X} = \begin{bmatrix} \alpha & \beta & \beta \\ \beta & \alpha & \beta \\ \beta & \beta & \alpha \end{bmatrix}. \tag{13}$$

The matrix given under (13) will be named the type f. Some characteristics of matrices of type f are discussed in the Appendix.

Consider now the conditions in the arrangement consisting of three wires of radius a_0, of circular cross section, parallel with the earth surface and with each other. The elements of matrix $\mathbf{M}$ characterizing the geometrical conditions of the arrangement can be calculated on the basis of (3). Matrix $\mathbf{Z}_s$ considering the compensation as well can be obtained by forming matrices $\mathbf{M}$ and $\mathbf{Z}_b$ in the form corresponding to (13), and from these matrix $\mathbf{Z}_s$ is calculated on the basis of (2). By averaging the reciprocal of matrix $\mathbf{M}$, similarly a matrix of the form (13) can be obtained and of this, on the basis of (2), matrix $\mathbf{Y}_p$ in which compensation is taken into consideration, can be determined.

Matrix $\mathbf{\Gamma}^2$ characterizing the transmission line system is found to be on the basis of (7) and (2)

$$\mathbf{\Gamma}^2 = j\omega\varepsilon\pi\mathbf{Z}_b\mathbf{M}^{-1} + k_0^2\mathbf{E} = \begin{bmatrix} \Gamma_s^2 & \Gamma_k^2 & \Gamma_k^2 \\ \Gamma_k^2 & \Gamma_s^2 & \Gamma_k^2 \\ \Gamma_k^2 & \Gamma_k^2 & \Gamma_s^2 \end{bmatrix}, \tag{14}$$

where

$$k_0^2 = j\omega\varepsilon j\omega\mu\,. \tag{15}$$

On the basis of the characteristic of type f matrices named under 1 in the Appendix, $\mathbf{\Gamma}^2$ as a product and sum of type f matrices is similarly of type f. Similarly, by force of what is described under 1, 2 and 5 in the Appendix, $\mathbf{\Gamma}$ and $\mathbf{Y}_0 = \mathbf{Z}_s^{-1}\mathbf{\Gamma}$ is also of type f, since $\mathbf{\Gamma}$ is the matrix function of $\mathbf{\Gamma}^2$.

It can be concluded from the foregoing that matrices $\mathbf{\Gamma}^2$, $\mathbf{\Gamma}$ and $\mathbf{Y}_0$ characterizing the once three-phase transmission line built with transposition, are of the type f.

Decomposition of currents and voltages according to the eigenvectors of the characteristic matrices

The characteristic values of matrix $\mathbf{\Gamma}^2$ given under (14) can be written on the basis of (10) and correlation (A1) in the Appendix.

$$\begin{aligned} \gamma_0^2 &= \Gamma_s^2 + 2\Gamma_k^2 \\ \gamma_{12}^2 &= \Gamma_s^2 - \Gamma_k^2, \end{aligned} \tag{16}$$

where γ_{12}^2 is a double characteristic value.

Let us examine the wave bundle passing in the direction $+z$. Let $U_1^{(+)}$, $U_2^{(+)}$, $U_3^{(+)}$ denote the voltages between the individual wires and the earth at the place $z = 0$, and form of these the column vector

$$U_0^{(+)} = \begin{bmatrix} U_1^{(+)} \\ U_2^{(+)} \\ U_3^{(+)} \end{bmatrix}. \tag{17}$$

The dependence of voltages passing in the direction $+z$ on the place can be written by force of (12) in the following form.

$$\begin{aligned} u^{(+)}(z) &= e^{-\gamma_0 z}\mathbf{L}_0 U_0^{(+)} + e^{-\gamma_{12} z}\mathbf{L}_{12} U_0^{(+)} = \\ &= \frac{e^{-\gamma_0 z}}{3} \cdot (U_1^{(+)} + U_2^{(+)} + U_3^{(+)}) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \\ &+ \frac{e^{-\gamma_{12} z}}{3} \begin{bmatrix} 2U_1^{(+)} - U_2^{(+)} - U_3^{(+)} \\ -U_1^{(+)} + 2U_2^{(+)} - U_3^{(+)} \\ -U_1^{(+)} - U_2^{(+)} + 2U_3^{(+)} \end{bmatrix}. \end{aligned} \tag{18}$$

In this equation the first term at the right side describes the spot dependence of the zero phase-sequence component of the voltage passing in direction $+z$, while the second term that of the sum of the positive and negative phase-sequence component. For our following calculations it is advisable to separate the positive and negative phase-sequence components in the second term. To this end let us write the voltage figuring in (18), by the help of eigenvectors S_0, S_1, S_2 as defined by correlations (A6) in the Appendix, i.e. by the unit vectors of the symmetrical components. So we can write that [6]

$$
\begin{aligned}
u^{(+)}(z) &= e^{-\gamma_0 z}\, \boldsymbol{S}_0 \,(\tilde{\boldsymbol{S}}_0^* \boldsymbol{U}_0^{(+)}) + e^{-\gamma_{12} z}\, \boldsymbol{S}_1 (\tilde{\boldsymbol{S}}_1^* \boldsymbol{U}_0^{(+)}) + e^{-\gamma_{12} z}\, \boldsymbol{S}_2 (\tilde{\boldsymbol{S}}_2^* \boldsymbol{U}_0^{(+)}) = \\
&= \frac{e^{-\gamma_0 z}}{3} \,(U_1^{(+)} + U_2^{(+)} + U_3^{(+)}) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \\
&+ \frac{e^{-\gamma_{12} z}}{3} \,(U_1^{(+)} + a U_2^{(+)} + a^2 U_3^{(+)}) \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} + \\
&+ \frac{e^{-\gamma_{12} z}}{3} \,(U_1^{(+)} + a^2 U_2^{(+)} + a U_3^{(+)}) \begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix} .
\end{aligned} \tag{19}
$$

This equation can be written by the Lagrange polynomials as well. To this end matrix $\mathbf{L}_{12}$ is to be written in accordance with Eqs. (A4) and (A5) in the Appendix as a sum of two matrices, of $\mathbf{L}_1$ and $\mathbf{L}_2$.

$$
u^{(+)}(z) = e^{-\gamma_0 z}\, \mathbf{L}_0\, \boldsymbol{U}_0^{(+)} + e^{-\gamma_{12} z}\, \mathbf{L}_1\, \boldsymbol{U}_0^{(+)} + e^{-\gamma_{12} z}\, \mathbf{L}_2\, \boldsymbol{U}_0^{(+)} . \tag{20}
$$

Current can also be decomposed to symmetrical components in a similar way. To this the zero, the positive, and the negative phase-sequence wave admittance should also be taken into consideration. The characteristic values of wave admittance matrix $\mathbf{Y}_0$ can be written on the basis of (A1).

$$
Y_0^{(0)} = Y_{0s} + 2\, Y_{0k} \tag{21}
$$

gives the zero phase-sequence and

$$
Y_0^{(12)} = Y_{0s} - Y_{0k} \tag{22}
$$

the positive and negative phase-sequence wave admittances, respectively Thus the spot dependence of the current passing in direction $+z$ can be described by the relationship

$$
i(z) = \frac{e^{-\gamma_0 z}}{3 Y_0^{(0)}} \,(U_1^{(+)} + U_2^{(+)} + U_3^{(+)}) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} +
$$

$$+\frac{e^{-\gamma_{12}z}}{3Y_0^{(12)}}\left(U_1^{(+)}+aU_2^{(+)}+a^2U_3^{(+)}\right)\begin{bmatrix}1\\a^2\\a\end{bmatrix}+$$

$$+\frac{e^{-\gamma_{12}z}}{3Y_0^{(12)}}\left(U_1^{(-)}+a^2U_2^{(-)}+aU_3^{(-)}\right)\begin{bmatrix}1\\a\\a^2\end{bmatrix}. \tag{23}$$

It has been taken into consideration that various phase-sequence voltage components are generating only currents of the same phase-sequence.

Twice three-phase transmission line with transposition

Matrices characterizing the transmission line section

In the case of twice three-phase transmission lines transposition is usually employed within the individual three-phase systems. Accordingly equalization that takes place can be taken into consideration in such a way that in place of matrices $\mathbf{M}$, $\mathbf{Z}_b$, $\mathbf{Z}_s$, $\mathbf{Y}_p$ which are valid for the system built without transposition, the matrix of the form

$$\mathbf{X}=\begin{bmatrix}\alpha & \beta & \beta & \gamma & \gamma & \gamma\\ \beta & \alpha & \beta & \gamma & \gamma & \gamma\\ \beta & \beta & \alpha & \gamma & \gamma & \gamma\\ \delta & \delta & \delta & \varepsilon & \zeta & \zeta\\ \delta & \delta & \delta & \zeta & \varepsilon & \zeta\\ \delta & \delta & \delta & \zeta & \zeta & \varepsilon\end{bmatrix} \tag{24}$$

is taken where $\alpha, \beta, \ldots, \zeta$ are the arithmetic means of the corresponding elements in the original matrix. Matrices having the structure (24) will be called the type g. In the Appendix some characteristics of type g matrices are enumerated.

The square matrix of the sixth order given under (24) can be partitioned in the following way.

$$\mathbf{X}=\begin{bmatrix}\mathbf{X}_{11} & \mathbf{X}_{12}\\ \mathbf{X}_{21} & \mathbf{X}_{22}\end{bmatrix} \tag{25}$$

where $\mathbf{X}_{11}$, $\mathbf{X}_{12}$, $\mathbf{X}_{21}$, $\mathbf{X}_{22}$ are square matrices of the third order. Upon comparing with (24) it can be seen that $\mathbf{X}_{11}$ and $\mathbf{X}_{22}$ are of type f, further that all the elements of $\mathbf{X}_{12}$ and $\mathbf{X}_{21}$ are equal. Such matrices which are built up of identical elements will be called type h.

In the examined system (Fig. 2) the radius of one of the three-phase systems is a_1, that of the other is a_2. Matrices of the arrangement are written in such a way that the first three elements in the columns and rows correspond to the individual wires of one of the three-phase systems, while elements 4, 5 and 6 to those of the other system.

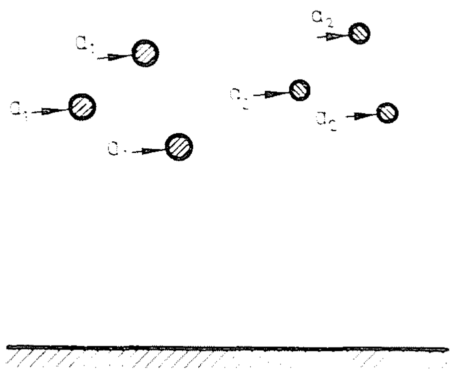

Fig. 2

Matrix $\mathbf{M}'$ characterizing the geometry of the system can be written on the basis of (3), and from this the reciprocal value $\mathbf{M}'^{-1}$ can be formed. Matrices $\mathbf{M}_0$, $\mathbf{M}^{-1}$, and $\mathbf{Z}_b$ of type g are formed of matrices $\mathbf{M}', \mathbf{M}'^{-1}_s, \mathbf{Z}'_b$ by averaging the corresponding elements. Matrix $\mathbf{\Gamma}^2$ characterizing the system can be calculated in the knowledge of $\mathbf{M}^{-1}$ and $\mathbf{Z}_b$ on the basis of (7) and (2). According to characteristic 1 of type g matrices enumerated in the Appendix matrix $\mathbf{\Gamma}^2$ is also of type g. Similarly of type g is $\mathbf{\Gamma}$ which can be determined of $\mathbf{\Gamma}^2$, further $\mathbf{Z}_s$ which can be calculated on the basis of (2), and the wave impedance matrix $\mathbf{Y}_0$ given under (8).

Decomposition of voltages and currents

Let us examine the voltages passing the system in direction $+z$. Decompose these according to the eigenvectors of matrix $\mathbf{\Gamma}$. In this case

$$\begin{aligned} u^{(+)}(z) = {} & e^{-\gamma_{01} z}\, \mathbf{L}_{01}\, \boldsymbol{U}_0^{(+)} + e^{-\gamma_{02} z}\, \mathbf{L}_{02}\, \boldsymbol{U}_0^{(+)} + \\ & + e^{-\gamma_{11} z}\, \mathbf{L}_{11}\, \boldsymbol{U}_0^{(+)} + e^{-\gamma_{12} z}\, \mathbf{L}_{12}\, \boldsymbol{U}_0^{(+)} + \\ & + e^{-\gamma_{21} z}\, \mathbf{L}_{21}\, \boldsymbol{U}_0^{(+)} + e^{-\gamma_{22} z}\, \mathbf{L}_{22}\, \boldsymbol{U}_0^{(+)} . \end{aligned} \tag{26}$$

Here $\gamma_{01}, \gamma_{02}, \gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}$ are the characteristic values of matrix $\mathbf{\Gamma}$ taken on the basis of relationship (A9) in the Appendix, further

$$\mathbf{L}_{01} = \mathcal{A}_{01}\,\widetilde{\mathcal{A}}_{01}^{*} = \begin{bmatrix} |A_{11}|^2\,\mathbf{L}_0 & A_{11}\,A_{12}\,\mathbf{L}_0 \\ A_{12}\,\tilde{A}_{11}\,\mathbf{L}_0 & |A_{12}|^2\,\mathbf{L}_0 \end{bmatrix} \tag{27}$$

$$\mathbf{L}_{02} = \mathcal{A}_{02}\,\widetilde{\mathcal{A}}_{02} = \begin{bmatrix} |A_{21}|^2\,\mathbf{L}_0 & A_{21}\,\tilde{A}_{22}\,\mathbf{L}_0 \\ A_{22}\,\tilde{A}_{21}\,\mathbf{L}_0 & |A_{22}|^2\,\mathbf{L}_0 \end{bmatrix} \tag{28}$$

$$\mathbf{L}_{11} = \mathcal{A}_{11}\,\widetilde{\mathcal{A}}_{11}^{*} = \begin{bmatrix} \mathbf{L}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{29}$$

$$\mathbf{L}_{12} = \mathcal{A}_{12}\,\widetilde{\mathcal{A}}_{12}^{*} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_1 \end{bmatrix} \tag{30}$$

$$\mathbf{L}_{21} = \mathcal{A}_{21}\,\widetilde{\mathcal{A}}_{21}^{*} = \begin{bmatrix} \mathbf{L}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{31}$$

$$\mathbf{L}_{22} = \mathcal{A}_{22}\,\widetilde{\mathcal{A}}_{22}^{*} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_2 \end{bmatrix} \tag{32}$$

$\mathbf{L}_0$, $\mathbf{L}_1$ and $\mathbf{L}_2$ are the Lagrange polynomials pertaining to the unit vectors $\boldsymbol{S}_0$, $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ of the three-phase symmetrical components, respectively.

The eigenvector attached to γ_{11} is

$$\mathcal{A}_{11} = \begin{bmatrix} \boldsymbol{S}_1 \\ \mathbf{0} \end{bmatrix} \tag{33}$$

This means that in (26) the third term at the right side describes a voltage which corresponds to the positive phase-sequence component of the first three-phase system and in the second three-phase system the voltages belonging to this are equal to zero. Similarly the fourth term describes that the voltage belonging to the eigenvector $\mathcal{A}_{12}$ is the positive phase-sequence component of the second three-phase system. The fifth term is the negative phase-sequence component of the first system, the sixth term that of the second system. It follows from all this that the positive and negative phase-sequence symmetrical components of the two three-phase systems can be calculated independently of each other.

The first two terms at the right side of equation (26) represent the zero phase-sequence components of the voltage. To this, two modes are seen to pertain.

Symmetrical arrangement

Let us examine the practically often occurring case when the two three-phase systems are symmetrical with respect to each other (Fig. 3). In this case

$$\mathbf{\Gamma}^2 = \begin{bmatrix} \mathbf{\Gamma}_1^2 & \mathbf{\Gamma}_2^2 \\ \mathbf{\Gamma}_2^2 & \mathbf{\Gamma}_1^2 \end{bmatrix} \tag{34}$$

where $\mathbf{\Gamma}_1^2$ and $\mathbf{\Gamma}_2^2$ are symmetrical matrices of the third order. We were able to see that the two systems are not acting on each other in the aspect of the positive and negative phase-sequence symmetrical components of the voltages, accordingly these are not influenced by the conditions of symmetry, in the case of a matrix of form (34) however $\lambda_{11}^0 = \lambda_{22}^0$ and $\lambda_{12}^0 = \lambda_{21}^0$ and thus on the basis of (A10)

$$\frac{A_{11}}{A_{12}} = 1 \quad \text{and} \quad \frac{A_{21}}{A_{22}} = -1 . \tag{35}$$

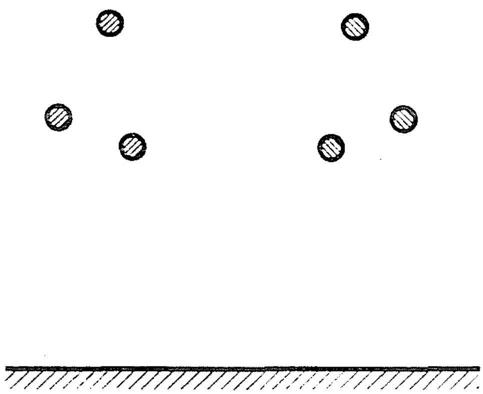

Fig. 3

In addition, on the basis of (A9)

$$\begin{aligned} \varphi_{01} &= \lambda_{11}^0 + \lambda_{12}^0 \\ \varphi_{02} &= \lambda_{11}^0 - \lambda_{12}^0 . \end{aligned} \tag{36}$$

Also in this case two modes belong to the zero order component. The eigenvector of these is

$$A_{01} = \begin{bmatrix} \mathbf{S}_0 \\ \mathbf{S}_0 \end{bmatrix} \quad \text{and} \quad A_{02} = \begin{bmatrix} \mathbf{S}_0 \\ \mathbf{S}_0 \end{bmatrix} . \tag{37}$$

The two modes are propagating with different propagation coefficients (γ_{01} and γ_{02}). The appertaining Lagrange polynomials can be obtained from (27) and (28) upon considering (35).

$$\mathbf{L}_{01} = \begin{bmatrix} \mathbf{L}_0 & \mathbf{L}_0 \\ \mathbf{L}_0 & \mathbf{L}_0 \end{bmatrix} ; \quad \mathbf{L}_{02} = \begin{bmatrix} \mathbf{L}_0 & -\mathbf{L}_0 \\ -\mathbf{L}_0 & \mathbf{L}_0 \end{bmatrix} . \tag{38}$$

If at $z = 0$ the two systems are connected in parallel then

$$U_0^{(+)} = \begin{bmatrix} U^{(+)} \\ U^{(+)} \end{bmatrix}, \tag{39}$$

where

$$U^{+} = \begin{bmatrix} U_1^{(+)} \\ U_2^{(+)} \\ U_3^{(+)} \end{bmatrix}. \tag{40}$$

In this case, from Eq. (26), the two members corresponding to the zero phase-sequence component of voltage $u(z)$ are found to be

$$e^{-\gamma_{01} z}\, \mathbf{L}_{01}\, U_0^{(+)} = e^{-\gamma_{01} z} \begin{bmatrix} \mathbf{L}_0 & \mathbf{L}_0 \\ \mathbf{L}_0 & \mathbf{L}_0 \end{bmatrix} \begin{bmatrix} U^{(+)} \\ U^{(+)} \end{bmatrix} = e^{-\gamma_{01} z} \begin{bmatrix} 2\,\mathbf{L}_0\, U^{(+)} \\ 2\,\mathbf{L}_0\, U^{(+)} \end{bmatrix} =$$

$$= 2\, e^{-\gamma_{01} z} \left(U_1^{(+)} + U_2^{(+)} + U_3^{(+)}\right) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{41}$$

and

$$e^{-\gamma_{02} z}\, \mathbf{L}_{02}\, U_0^{(+)} = e^{-\gamma_{02} z} \begin{bmatrix} \mathbf{L}_0 & -\mathbf{L}_0 \\ \mathbf{L}_0 & \mathbf{L}_0 \end{bmatrix} \begin{bmatrix} \mathbf{U}^{(+)} \\ U^{(+)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{42}$$

i.e. in the case of a symmetrical build-up the zero phase-sequence voltages corresponding to one of the modes are all equal to zero.

On the basis of the foregoing it can be stated that in the case of a twice three-phase system built with transposition the positive and negative phase-sequence components of the two three-phase systems are independent of each other, and two modes belong to the zero phase-sequence component. If the two three-phase systems are of symmetrical arrangement with respect to each other and are connected at least at one of the terminals in parallel, then only one of the two modes of the zero phase-sequence components comes into existence. In this case the corresponding voltages and currents in the two systems show a complete symmetry.

Network consisting of three-phaase transmission lines

The topological theory of transmission line networks is well known [7]. By the help of this theory, in the knowledge of the length, wave impedance, and propagation coefficient of the transmission lines, further of the impedance and source voltage of impedances and generators connected to the connection places, vertices, the voltages arising at the vertices can be determined by a single matrix equation. Beyond this currents to be measured at the ends of the transmission line can also be calculated.

The impedances of network parts being at the vertices can be decomposed to components corresponding to the symmetrical components [3]. The voltage of the generators can also be decomposed to symmetrical components. Such a decomposition was seen to be possible both in the case of once three phase and of symmetrical twice three phase systems. Accordingly for the calculation of three-phase transmission line networks an one-phase connection can be given which is valid for the individual symmetrical components. Here the network parts connected to the vertices should be taken into consideration with their impedances and voltages of the respective phase-sequence. Twice three-phase sections can be calculated as two systems connected in parallel. At the calculation of positive and negative phase-sequence components. however. we should take into consideration that propagation coefficients and wave admittances belonging to the zero phase-sequence components are influenced by the coupling of the two three-phase systems.

Appendix

In the following some characteristics of matrices of type f having the orm (13). and of type g having the form (24) are summarized.

a) *Type f matrices*

1. The sum, difference, and product of two matrices of type f is similarly of type f.

2. If $\mathbf{X}$ is a matrix of type f then its reciprocal $\mathbf{X}^{-1}$ is similarly of type f.

3. The eigenvalues of type f matrix (13) are

$$\begin{aligned} \lambda_0 &= \alpha + 2\beta \\ \lambda_{12} &= \alpha - \beta \end{aligned} \tag{A1}$$

where λ_{12} is a double eigenvalue. The corresponding Lagrange polynomials are

$$\mathbf{L}_0 = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{A2}$$

and

$$\mathbf{L}_{12} = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \tag{A3}$$

$\mathbf{L}_{12}$ can be written as the sum of

$$\mathbf{L}_{1} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{bmatrix} \tag{A4}$$

and of

$$\mathbf{L}_{2} = \frac{1}{3}\begin{bmatrix} 1 & a^2 & a \\ a & 1 & a^2 \\ a^2 & a & 1 \end{bmatrix}, \tag{A5}$$

where

$$a = e^{j\frac{2}{3}\pi}$$

denotes the eigenvectors of $\mathbf{X}$ belonging to the Lagrange polynomials $\mathbf{L}_0$, $\mathbf{L}_1$ and $\mathbf{L}_2$, respectively.

$$\mathbf{S}_0 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; \quad \mathbf{S}_1 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}; \quad \mathbf{S}_2 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix}. \tag{A6}$$

The decomposition of type f matrices with respect to these eigenvectors corresponds to the decomposition to symmetrical components employed at the calculation of three-phase networks.

4. The Lagrange polynomial of a matrix of type f is similarly of type f.

5. A function of a type f matrix which is defined by the relationship

$$\mathbf{f}(\mathbf{X}) = \sum_{k=1}^{3} f(\lambda_k)\, \mathbf{L}_k(\mathbf{X}) \tag{A7}$$

is similarly of type f.

b) *Characteristics of type g matrices written under (24)*

1. The sum, difference, and product of type g matrices is similarly a matrix of type g.

2. The reciprocal of a type g matrix is similarly of type g.

3. For the determination of eigenvectors and characteristic values the partitioned form written in (25) for type g matrix written under (24) is used. The eigenvectors are

$$\Lambda_0 = \begin{bmatrix} A_1 \mathbf{S}_0 \\ A_2 \mathbf{S}_1 \end{bmatrix}; \ \Lambda_{11} = \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{0} \end{bmatrix}; \ \Lambda_{12} = \begin{bmatrix} \mathbf{0} \\ \mathbf{S}_1 \end{bmatrix}$$
$$\Lambda_{21} = \begin{bmatrix} \mathbf{S}_2 \\ \mathbf{0} \end{bmatrix}; \ \Lambda_{22} = \begin{bmatrix} \mathbf{0} \\ \mathbf{S}_2 \end{bmatrix}. \tag{A8}$$

The type g matrix can be partitioned according to (25). The eigenvectors of matrices $\mathbf{X}_{11}, \mathbf{X}_{12}, \mathbf{X}_{21}, \mathbf{X}_{22}$ involved are $\boldsymbol{S}_0, \boldsymbol{S}_1, \boldsymbol{S}_2$. The eigenvalues of the type g matrix can be expressed by the eigenvalues of the third order type f matrices $\mathbf{X}_{11}, \mathbf{X}_{12}, \mathbf{X}_{21}, \mathbf{X}_{22}$. Let the eigenvalues of these pertaining to $\boldsymbol{S}_0$ be denoted by $\lambda_{11}^0, \lambda_{12}^0, \lambda_{21}^0, \lambda_{22}^0$, those pertaining to $\boldsymbol{S}_1$ by $\lambda_{11}^{(+)}, \lambda_{12}^{(+)}, \lambda_{21}^{(+)}, \lambda_{22}^{(+)}$, and those pertaining to $\boldsymbol{S}_2$ by $\lambda_{11}^{(-)}, \lambda_{12}^{(-)}, \lambda_{21}^{(-)}, \lambda_{22}^{(-)}$. (It is easy to see that $\lambda_{12}^{(+)} = \lambda_{21}^{(+)} = 0$ and $\lambda_{12}^{(-)} = \lambda_{21}^{(-)} = 0$.) The six characteristic values of the matrix given under (24) are the following.

$$\begin{aligned} \varphi_0 &= \frac{\lambda_{11}^0 + \lambda_{22}^0}{2} \pm \frac{1}{2}\sqrt{(\lambda_{11}^0 - \lambda_{22}^0)^2 + 4\lambda_{12}^0 \lambda_{21}^0} \\ \varphi_{11} &= \lambda_{11}^{(+)} \\ \varphi_{12} &= \lambda_{22}^{(+)} \\ \varphi_{21} &= \lambda_{11}^{(-)} \\ \varphi_{22} &= \lambda_{22}^{(-)}. \end{aligned} \tag{A9}$$

Two eigenvalues appertain to the eigenvector Λ_0. For the ratio of A_1 to A_2 figuring in Λ_0 the relationship

$$\frac{A_1}{A_2} = \frac{\lambda_{11}^0 - \lambda_{22}^0 \sqrt{(\lambda_{11}^0 - \lambda_{22}^0)^2 + 4\lambda_{12}^0 \lambda_{21}^0}}{2\lambda_{21}^0} \tag{A10}$$

is valid.

4. A function of a type g matrix which is defined by the relationship

$$\mathrm{f}(\mathbf{X}) = \sum_{j=1}^{6} f(\varphi_j)\, \mathbf{L}_j(\mathbf{X}) \tag{A11}$$

is similarly of type g.

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Summary

Power transmission lines consist in practice mostly of once or twice three-phase systems in which the position of the individual wires is interchanged after a certain length. The topological theory of coupled transmission lines and of transmission networks is well known from the literature. In the present paper these two theories are employed for the above mentioned transmission networks.

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