INFLUENCE OF THE GROUND WIRE ON THE ELECTROMAGNETIC WAVES PROPAGATING IN TRANSMISSION LINES

By

I. VÁGÓ

Department of Theoretical Electricity, Technical University, Budapest

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Introduction

The wave theory of coupled transmission lines is known from the technical literature [1, 5]. According to this, as many modes with different propagation coefficients arise in general in the transmission line, as the number of over-ground wires. In three-phase transmission lines built with transposition the symmetrical components correspond to the modes [2, 3, 4]. Among the wires the ground wire has a special rule, it is earthed at each pole. In the present paper it is examined how the influence of the ground wire on the electromagnetic waves propagating in the transmission line can be taken into consideration.

Theory of the coupled transmission line

Our considerations are based on the theory of the coupled transmission line. (For a concise summary see paper [4].) According to this, for a system consisting of n wires arranged above the earth, parallel with the earth and with each other, the system of differential equations

$$
\begin{aligned}
-\frac{\partial \boldsymbol{i}}{\partial z} &= \mathbf{Y}_p \boldsymbol{u} \\
-\frac{\partial \boldsymbol{u}}{\partial z} &= \mathbf{Z}_s \boldsymbol{i}
\end{aligned}
\tag{1}
$$

is valid, where z is the spot co-ordinate in the direction of the transmission line, $\boldsymbol{i}$ the column vector formed of the currents in the wires, $\boldsymbol{u}$ that of the voltages between the wires and the earth surface, $\mathbf{Y}_p$ the parallel admittance matrix related to unit length, and $\mathbf{Z}_s$ the series impedance matrix related to unit length. $\mathbf{Y}_p$ and $\mathbf{Z}_s$ are square matrices of the nth order. (The method of their determination is to be found in [1] and [4].)

The solution of the system of differential equations (1) is given by

$$\begin{aligned} u(z) &= e^{-\mathbf{\Gamma} z}\, U_0^{(+)} + e^{\mathbf{\Gamma} z}\, U_0^{(-)} \\ i(z) &= \mathbf{Y}_0 \left[e^{-\mathbf{\Gamma} z}\, U_0^{(+)} - e^{\mathbf{\Gamma} z}\, U_0^{(-)} \right] \end{aligned} \tag{2}$$

where $U_0^{(+)}$ and $U_0^{(-)}$ designate the colum vectors formed of the values assumed at the place $z = 0$ by the voltages passing in directions $+z$ and $-z$, respectively, $\mathbf{\Gamma}$ is the propagation coefficient matrix the square of which is

$$\mathbf{\Gamma}^2 = \mathbf{Z}_s \mathbf{Y}_p \tag{3}$$

and the expression of the wave admittance matrix is given as

$$\mathbf{Y}_0 = \mathbf{Z}_s^{-1} \mathbf{\Gamma}. \tag{4}$$

Matrix functions figuring in (2) can be expressed by the matrix Lagrange polynomials. The Lagrange polynomials of the nth order square matrix $\mathbf{X}$ can be written as follows.

$$\mathbf{L}_k(\mathbf{X}) = \prod_{\substack{j=1 \\ j \neq k}}^{n} \frac{\mathbf{X} - \lambda_k \mathbf{E}}{\lambda_j - \lambda_k}, \tag{5}$$

where $\mathbf{E}$ is the unit matrix of the nth order and λ_k denotes $(k = 1, 2, \ldots n)$ the eigenvalues of matrix $\mathbf{X}$ which can be determined of the equation

$$\det |\mathbf{X} - \lambda \mathbf{E}| = 0. \tag{6}$$

The matrix functions of $\mathbf{X}$ can be expressed by the Lagrange polynomials as follows.

$$\mathrm{f}(\mathbf{X}) = \sum_{k=1}^{n} f(\lambda_k)\, \mathbf{L}_k(\mathbf{X}). \tag{7}$$

Accordingly the relationships given under (2) can be written also in the following form.

$$\begin{aligned} u(z) &= \sum_{k=1}^{n} \mathbf{L}_k(\mathbf{\Gamma}^2) \left[U_0^{(+)} e^{-\gamma_k z} + U_0^{(-)} e^{\gamma_k z} \right] \\ i(z) &= \mathbf{Y}_0 \sum_{k=1}^{n} \mathbf{L}_k(\mathbf{\Gamma}^2) \left[U_0^{(+)} e^{-\gamma_k z} - U_0^{(-)} e^{\gamma_k z} \right] \end{aligned} \tag{8}$$

γ_k $(k = 1, 2, \ldots, n)$ denotes that square root of the characteristic values of $\mathbf{\Gamma}^2$ which falls into the first quarter of the plane of complex numbers. These are the propagation coefficients pertaining to the individual modes. (For the physical interpretation of relationships (5) see [4].)

Consideration of the influence of the ground wire

The ground wires are short circuited at the individual transmission line poles with the earth. Consequently their potential is identical at these places with the potential of the earth surface. In the followings the ground wires are taken into consideration in such a way that their potential is assumed to be identical in each cross section with that of the earth surface. This is an approximation which is honoured exactly only at the place of the transmission line poles.

At the place of the ground wires the current of the phase wires produces a certain potential. The aforesaid condition on the potential of the ground wires necessitates that the current in the ground wires should be so high that the resultant potential at the place of the ground wires is zero.

Let us number the wires in such a way that the ground wires receive the first order numbers, and the phase wires the further ones. Let us designate by $\boldsymbol{u}_f$ the column vector formed of the potential of the phase wires at the examined place z, by $\boldsymbol{i}_f$ the column vector formed of the current of the phase wires, by $\boldsymbol{i}_0$ the column vector formed of the current of the ground wires. Then the voltage and current column vectors of the system can be written in the following form.

$$\boldsymbol{u}(z) = \begin{pmatrix} \boldsymbol{O} \\ \boldsymbol{u}_f \end{pmatrix} \quad \text{and} \quad \boldsymbol{i}(z) = \begin{pmatrix} \boldsymbol{i}_0 \\ \boldsymbol{i}_f \end{pmatrix}, \tag{9}$$

where $\boldsymbol{O}$ is a column vector of similar order as $\boldsymbol{i}_0$.

Matrices $\mathbf{Z}_s$ and $\mathbf{Y}_p$ are partitioned in such a way that the separating lines are drawn behind the first rows and columns representing the ground wires. Thus e.g. the series impedance matrix of the system consisting of two ground and three phase wires can be written as follows.

$$\mathbf{Z}_{s0} = \left[\begin{array}{cc|ccc} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} \\ \hline Z_{31} & Z_{32} & Z_{33} & Z_{34} & Z_{35} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} & Z_{45} \\ Z_{51} & Z_{52} & Z_{53} & Z_{54} & Z_{55} \end{array}\right] \tag{10}$$

In the sense of the foregoing the partitioned series impedance matrix of any transmission line system having ground wires is given by

$$\mathbf{Z}_{s0} = \begin{bmatrix} \mathbf{Z}_n & \mathbf{Z}_{ns} \\ \mathbf{Z}_{sn} & \mathbf{Z}_s \end{bmatrix}. \tag{11}$$

In (10) $\mathbf{Z}_n$ and $\mathbf{Z}_s$ are square matrices of the second and third order, respectively.

The parallel admittance matrix of the transmission line system can be written in the form of a hypermatrix similar to $\mathbf{Z}_{s0}$.

$$\mathbf{Y}_{p0} = \begin{bmatrix} \mathbf{Y}_n & \mathbf{Y}_{np} \\ \mathbf{Y}_{pn} & \mathbf{Y}_p \end{bmatrix}. \tag{12}$$

Let us substitute (9), (11) and (12) in Eqs (1). Thus we find that

$$-\frac{\partial}{\partial z}\begin{bmatrix} \mathit{0} \\ \boldsymbol{u}_f \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_n & \mathbf{Z}_{ns} \\ \mathbf{Z}_{sn} & \mathbf{Z}_s \end{bmatrix}\begin{bmatrix} \boldsymbol{i}_0 \\ \boldsymbol{i}_f \end{bmatrix} \tag{13}$$

$$-\frac{\partial}{\partial z}\begin{bmatrix} \boldsymbol{i}_0 \\ \boldsymbol{i}_f \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_n & \mathbf{Y}_{np} \\ \mathbf{Y}_{pn} & \mathbf{Y}_p \end{bmatrix}\begin{bmatrix} \mathit{0} \\ \boldsymbol{u}_f \end{bmatrix}, \tag{14}$$

where in consequence of reciprocity $\mathbf{Z}_{ns}$ is the transposed of $\mathbf{Z}_{sn}$, and $\mathbf{Y}_{np}$ that of $\mathbf{Y}_{pn}$. Of these we can write the following equations.

$$\mathit{0} = \mathbf{Z}_n \boldsymbol{i}_0 + \mathbf{Z}_{ns} \boldsymbol{i}_f \tag{15}$$

$$-\frac{\partial}{\partial z} \boldsymbol{u}_f = \mathbf{Z}_{sn} \boldsymbol{i}_0 + \mathbf{Z}_s \boldsymbol{i}_f \tag{16}$$

$$-\frac{\partial}{\partial z} \boldsymbol{i}_0 = \mathbf{Y}_{np} \boldsymbol{u}_f \tag{17}$$

$$-\frac{\partial}{\partial z} \boldsymbol{i}_f = \mathbf{Y}_p \boldsymbol{u}_f. \tag{18}$$

From (15) we obtain that

$$\boldsymbol{i}_0 = -\mathbf{Z}_n^{-1} \mathbf{Z}_{ns} \boldsymbol{i}_f. \tag{19}$$

Upon substituting this into (16) we find that

$$-\frac{\partial}{\partial z} \boldsymbol{u}_f = (\mathbf{Z}_s - \mathbf{Z}_{sn} \mathbf{Z}_n^{-1} \mathbf{Z}_{ns}) \boldsymbol{i}_f \tag{20}$$

(18) and (20) represent a system of differential equations, similar to (1), in which only the currents and the voltages of the phase wires are figuring, further

$$\mathbf{Y}'_p = \mathbf{Y}_p \tag{21}$$

corresponds to the parallel admittance matrix and

$$\mathbf{Z}'_s = \mathbf{Z}_s - \mathbf{Z}_{sn}\,\mathbf{Z}_n^{-1}\,\mathbf{Z}_{ns} - \mathbf{Z}_s = \Delta\mathbf{Z}_s\,. \tag{22}$$

The meaning of these results is that at the approximate calculation of the transmission line system having ground wires these are not to be taken into consideration at determining the parallel admittance matrix, while in the series impedance matrix the correction member

$$-\,\Delta\mathbf{Z}_s = -\mathbf{Z}_{sn}\,\mathbf{Z}_n^{-1}\,\mathbf{Z}_{ns} \tag{23}$$

is added to the series impedance matrix $\mathbf{Z}_s$ of the system without ground wires. This can be interpreted in that manner that the current in the wires is not influenced by the potential of the ground wire, since it is zero, the potential of the wires, however, is influenced by the current of the ground wires.

It follows from the aforesaid that the number of modes arising in the system is not influenced by the ground wires, $\mathbf{\Gamma}^2$ and $\mathbf{Y}_0$ however are modified with respect to the system without ground wires, in consequence of the correction member added to $\mathbf{Z}_s$.

It can be seen from the foregoing that the calculation of transmission line systems containing ground wires too can be reduced to that of the system without ground wires.

It should be noted that Eq. (17) has not been discussed. This equation is in general in contradiction with Eqs (15), (16) and (18). These are namely satisfied jointly if

$$\mathbf{Z}_{ns}\,\mathbf{Y}_p + \mathbf{Z}_n\,\mathbf{Y}_{np} = \mathbf{0}, \tag{24}$$

what is generally not valid. This contradiction originates from the fact that we calculated with an approximation. We have namely assumed that the ground wire is at zero potential and this makes the problem redundant. In fact the potential of the ground wires is zero only at the earthing places, i.e. at the transmission line poles. Since however the distance between two poles is very small in comparison with the wave length, the potential of the ground wire can actually be regarded as zero.

For a more exact description of conditions in the system we should take into consideration that the number of wires is increased by the number of ground wires and thus also the number of modes is generally increasing. If the potential of the ground wires at a place with co-ordinate z is zero, then by force of (8) the potential of the ground wires at a distance of Δz from the former place is in general different from zero. Accordingly such a reflection comes into existence at the further earthing places that the potential of the ground wires will be zero there. This means that at more exact calculations sections between two neighbouring transmission line poles should be handled separately.

Three-phase transmission line with ground wires built with transposition

By employing the theory of the coupled transmission line, conditions of transmission lines built with transposition can be discussed [4]. In the followings the influence of the ground wires in a transmission line with transposition will be examined by employing the same theory.

In the case of transposition the individual blocks of hypermatrices (11) and (12) have a certain symmetry. In the case of an once three-phase system $\mathbf{Z}_s$, $\mathbf{Y}_p$, $\mathbf{\Gamma}^2$, $\mathbf{\Gamma}$ and $\mathbf{Y}_0$ are of the form

$$\mathbf{X}_1 = \begin{bmatrix} \alpha & \beta & \beta \\ \beta & \alpha & \beta \\ \beta & \beta & \alpha \end{bmatrix} \tag{25}$$

while in the case of twice three-phase matrices the build-up of the above mentioned matrices is the following.

$$\mathbf{X}_2 = \begin{bmatrix} \alpha & \beta & \beta & \gamma & \gamma & \gamma \\ \beta & \alpha & \beta & \gamma & \gamma & \gamma \\ \beta & \beta & \alpha & \gamma & \gamma & \gamma \\ \delta & \delta & \delta & \varepsilon & \zeta & \zeta \\ \delta & \delta & \delta & \zeta & \varepsilon & \zeta \\ \delta & \delta & \delta & \zeta & \zeta & \varepsilon \end{bmatrix} . \tag{26}$$

An eigenvector and eigenvalue system of matrices $\mathbf{X}_1$ and $\mathbf{X}_2$ is described in the Appendix.

The columns of matrices $\mathbf{Z}_{ns}$ and $\mathbf{Y}_{np}$ are equal with each other, similarly the rows in matrices $\mathbf{Z}_{sn}$ and $\mathbf{Y}_{pn}$ are equal. In consequence of reciprocity $\mathbf{Z}_{ns}$ is the transpose of $\mathbf{Z}_{sn}$, and $\mathbf{Y}_{np}$ that of $\mathbf{Y}_{pn}$, i.e.

$$\mathbf{Z}_{ns} = [Z_k \ Z_k \ \cdot \ \cdot \ \cdot \ Z_k] = \mathbf{Z}_{sn} \tag{27}$$

$$\mathbf{Y}_{np} = [Y_k \ Y_k \ \cdot \ \cdot \ \cdot \ Y_k] = \mathbf{Y}_{np} \tag{28}$$

(The asterisk * designates the transpose.) These matrices take the interaction of ground wires and phase wires into consideration. The elements of $\boldsymbol{Z}_k$ and $\boldsymbol{Y}_k$ are supplied by the arithmetic means of the corresponding rows in matrices $\boldsymbol{Z}_{ns}$ and $\boldsymbol{Y}_{np}$, respectively, calculated for the case without transposition.

It should be noted that in the case of a single ground wire $\boldsymbol{Z}_k$ and $\boldsymbol{Y}_k$ consist of a single element.

In the case of transposition the correction member of the series impedance as written in (23) can be calculated relatively simply. In this case namely $\Delta\mathbf{Z}_s$ is a square matrix, all the elements of which are equal to each other and the order of which is identical with the number of phase wires.

This can be understood as follows. Let $\mathbf{Y}_1^*, \mathbf{Y}_2^*, \ldots, \mathbf{Y}_i^*$ denote the individual rows of $\mathbf{Z}_n^{-1}$, where i is the number of ground wires.

$$\mathbf{Z}_n^{-1} = \begin{bmatrix} \mathbf{Y}_1^* \\ \mathbf{Y}_2^* \\ \vdots \\ \mathbf{Y}_i^* \end{bmatrix}. \tag{29}$$

On the basis of (23) and (27)

$$-\Delta \mathbf{Z}_s = -\begin{bmatrix} \mathbf{Z}_k^* \\ \mathbf{Z}_k^* \\ \vdots \\ \mathbf{Z}_k^* \end{bmatrix} \begin{bmatrix} \mathbf{Y}_1^* \mathbf{Z}_k & \ldots & \mathbf{Y}_1^* \mathbf{Z}_k \\ \mathbf{Y}_2^* \mathbf{Z}_k & \ldots & \mathbf{Y}_2^* \mathbf{Z}_k \\ \vdots & & \vdots \\ \mathbf{Y}_i^* \mathbf{Z}_k & \ldots & \mathbf{Y}_1^* \mathbf{Z}_k \end{bmatrix}. \tag{30}$$

Thus the elements of $\Delta \mathbf{Z}_s$ are equal to each other and the value of such an element is

$$\Delta Z_s = \mathbf{Z}_k^* \begin{bmatrix} \mathbf{Y}_1^* \mathbf{Z}_k \\ \mathbf{Y}_2^* \mathbf{Z}_k \\ \vdots \\ \mathbf{Y}_i^* \mathbf{Z}_k \end{bmatrix}. \tag{31}$$

The expression of the propagation coefficient matrix on the basis of (3), (21) and (22) is found to be

$$\mathbf{\Gamma}'^2 = \mathbf{Z}_s' \mathbf{Y}_p = (\mathbf{Z}_s - \Delta \mathbf{Z}_s) \mathbf{Y}_p = \mathbf{Z}_s \mathbf{Y}_p - \Delta \mathbf{Z}_s \mathbf{Y}_p = \mathbf{\Gamma}^2 - \Delta \mathbf{\Gamma}^2, \tag{32}$$

where

$$\mathbf{\Gamma}^2 = \mathbf{Z}_s \mathbf{Y}_p \quad \text{and} \quad \Delta \mathbf{\Gamma}^2 = \Delta \mathbf{Z}_s \mathbf{Y}_p. \tag{33}$$

$\mathbf{\Gamma}$ is the propagation coefficient matrix of the system without ground wire. It was seen that all the elements of $\Delta \mathbf{Z}_s$ were equal to each other. In the case of an once three-phase system $\mathbf{Y}_p$ has the form (25), in that of a twice three-phase system the form (26), and is symmetrical. It follows of this that the sum of the elements in a column of $\mathbf{Y}_p$ is the same for all columns. An element of $\Delta \mathbf{\Gamma}^2$ as written in (33) is equal to the product of ΔZ_s and of the sum of the elements in a column of $\mathbf{Y}_p$. Thus the elements of $\Delta \mathbf{\Gamma}^2$ are equal to each other. Let us designate these by $\Delta \Gamma^2$.

In the followings we shall first examine the once three-phase system. Then $\mathbf{\Gamma}^2$ is an $\mathbf{X}_1$-type matrix of the form (25). Thus

$$\mathbf{\Gamma}'^2 = \mathbf{\Gamma}^2 - \Delta \mathbf{\Gamma}^2 = \begin{bmatrix} \Gamma_s^2 - \Delta \Gamma^2 & \Gamma_k^2 - \Delta \Gamma^2 & \Gamma_k^2 - \Delta \Gamma^2 \\ \Gamma_k^2 - \Delta \Gamma^2 & \Gamma_s^2 - \Delta \Gamma^2 & \Gamma_k^2 - \Delta \Gamma^2 \\ \Gamma_k^2 - \Delta \Gamma^2 & \Gamma_k^2 - \Delta \Gamma^2 & \Gamma_s^2 - \Delta \Gamma^2 \end{bmatrix} \tag{34}$$

are also of type $\mathbf{X}_1$. The eigenvalues on the basis of relationship (A3) in the Appendix are found to be

$$\gamma_0'^2 = \Gamma_s^2 - \Delta\Gamma^2 + 2\,(\Gamma_k^2 - \Delta\Gamma^2) = \Gamma_s^2 + 2\Gamma_k^2 - 3\,\Delta\Gamma^2 \tag{35}$$

$$\gamma_{12}'^2 = \Gamma_s^2 - \Delta\Gamma^2 - (\Gamma_k^2 - \Delta\Gamma^2) = \Gamma_s^2 - \Gamma_k^2\,. \tag{36}$$

Upon comparing these results with the eigenvalues of matrix $\mathbf{\Gamma}^2$ valid for the system without ground wire (see [4]) it is seen that the ground wire modifies only the propagation coefficient belonging to the zero phase-sequence component, those belonging to the positive and negative phase-sequence components are not influenced by it.

In the case of a twice three-phase system the build-up of matrix $\mathbf{\Gamma}^2$ is identical with that of matrix $\mathbf{X}_2$ given under (26). Then

$$\mathbf{\Gamma}'^2 = \mathbf{\Gamma}^2 - \Delta\mathbf{\Gamma}^2 = \begin{bmatrix} \alpha-\Delta\Gamma^2 & \beta-\Delta\Gamma^2 & \beta-\Delta\Gamma^2 & \gamma-\Delta\Gamma^2 & \gamma-\Delta\Gamma^2 & \gamma-\Delta\Gamma^2 \\ \beta-\Delta\Gamma^2 & \alpha-\Delta\Gamma^2 & \beta-\Delta\Gamma^2 & \gamma-\Delta\Gamma^2 & \gamma-\Delta\Gamma^2 & \gamma-\Delta\Gamma^2 \\ \beta-\Delta\Gamma^2 & \beta-\Delta\Gamma^2 & \alpha-\Delta\Gamma^2 & \gamma-\Delta\Gamma^2 & \gamma-\Delta\Gamma^2 & \gamma-\Delta\Gamma^2 \\ \delta-\Delta\Gamma^2 & \delta-\Delta\Gamma^2 & \delta-\Delta\Gamma^2 & \varepsilon-\Delta\Gamma^2 & \zeta-\Delta\Gamma^2 & \zeta-\Delta\Gamma^2 \\ \delta-\Delta\Gamma^2 & \delta-\Delta\Gamma^2 & \delta-\Delta\Gamma^2 & \zeta-\Delta\Gamma^2 & \varepsilon-\Delta\Gamma^2 & \zeta-\Delta\Gamma^2 \\ \delta-\Delta\Gamma^2 & \delta-\Delta\Gamma^2 & \delta-\Delta\Gamma^2 & \zeta-\Delta\Gamma^2 & \zeta-\Delta\Gamma^2 & \varepsilon-\Delta\Gamma^2 \end{bmatrix} \tag{37}$$

is similarly of type $\mathbf{X}_2$ what means that the ground wires are not influencing the system of eigenvectors. The eigenvalues belonging to the two zero phase-sequence components are, according to (A7) in the Appendix,

$$\varphi_0 = \frac{\lambda_{11}^0 + \lambda_{22}^0}{2} \pm \frac{1}{2} \sqrt{(\lambda_{11}^0 - \pi_{22}^0)^2 + 4\,\lambda_{12}^0\,\lambda_{21}^0}, \tag{38}$$

where

$$\begin{aligned} \lambda_{11}^0 &= \alpha + 2\,\beta - 3\Delta\Gamma^2 \\ \lambda_{22}^0 &= \varepsilon + 2\zeta - 3\Delta\Gamma^2 \\ \lambda_{12}^0 &= 3\gamma - 3\Delta\Gamma^2 \\ \lambda_{21}^0 &= 3\delta - 3\Delta\Gamma^2. \end{aligned} \tag{39}$$

The eigenvalue belonging to A_{11} and A_{12} (see Appendix) is

$$\varphi_{11} = \varphi_{21} = \alpha - \beta \tag{40}$$

and the eigenvalue belonging to A_{12} and A_{22} is

$$\varphi_{12} = \varphi_{22} = \varepsilon - \zeta. \tag{41}$$

This means that the eigenvalue belonging to the positive and negative phase-sequence component is not influenced by the ground wire (cf. [4]).

We can accordingly establish that the ground wire is influencing only the propagation coefficient of the zero phase-sequence component in the case of both once and twice three-phase transmission lines built with transposition. The values of wave admittances of various phase-sequence however are not independent of the presence of ground wires.

In the foregoing the calculation of three-phase transmission lines built with ground wire was reduced to the theory of systems without ground wires.

Appendix

A possible system of eigenvectors for matrix $\mathbf{X}_1$ given under (25) is the following.

$$\mathbf{S}_0 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; \quad \mathbf{S}_1 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}; \quad \mathbf{S}_2 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix}, \tag{A1}$$

where

$$a = e^{j\frac{3}{2}\pi}. \tag{A2}$$

The eigenvalues appertaining to this eigenvector are

$$\lambda_0 = \alpha + 2\beta; \quad \lambda_1 = \alpha - \beta; \quad \lambda_2 = \alpha - \beta. \tag{A3}$$

For writing one of the eigenvector systems of matrix $\mathbf{X}_2$ given under (26) let us partition $\mathbf{X}_2$ as follows.

$$\mathbf{X}_2 = \begin{bmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} \\ \mathbf{X}_{21} & \mathbf{X}_{22} \end{bmatrix}. \tag{A4}$$

where the matrix blocks are square matrices of the third order. Among these $\mathbf{X}_{11}$ and $\mathbf{X}_{22}$ are of identical build-up with $\mathbf{X}_1$ given under (25). Thus the eigenvectors of these are as given under (A1) and the eigenvalues can be determined on the basis of (A3). Be the signs of these eigenvalues $\lambda_{11}^0, \lambda_{11}^{(1)}, \lambda_{11}^{(2)}$ and $\lambda_{22}^0, \lambda_{22}^{(1)}, \lambda_{22}^{(2)}$, respectively. $\mathbf{X}_{12}$ and $\mathbf{X}_{21}$ are matrices each element of which is equal to that in the other. This can be regarded as a special case of matrix $\mathbf{X}_1$ given under (25). Among the eigenvalues of these only the one appertaining to eigenvector S_0 is different from zero. Let us designate this with λ_{12}^0 and λ_{21}^0, respectively.

One of the eigenvector systems of matrix $\mathbf{X}_2$ given under (26) is given by

$$\Lambda_0 = \begin{bmatrix} A_1 \mathbf{S}_0 \\ A_2 \mathbf{S}_0 \end{bmatrix}; \quad \Lambda_{11} = \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{O} \end{bmatrix}; \quad \Lambda_{12} = \begin{bmatrix} \mathbf{O} \\ \mathbf{S}_1 \end{bmatrix}$$

$$\Lambda_{21} = \begin{bmatrix} \mathbf{S}_2 \\ \mathbf{O} \end{bmatrix}; \quad \Lambda_{22} = \begin{bmatrix} \mathbf{O} \\ \mathbf{S}_2 \end{bmatrix}; \tag{A5}$$

where

$$\frac{A_1}{A_2} = \frac{\lambda_{11}^0 - \lambda_{22}^0 \pm \sqrt{(\lambda_{11}^0 - \lambda_{22}^0)^2 + 4\,\lambda_{12}^0\,\lambda_{11}^0}}{2\,\lambda_{21}^0}\,. \tag{A6}$$

Since A_1/A_2 may assume two values, Λ_0 designates two eigenvectors.

The eigenvalues appertaining to the eigenvectors (A5) are

$$\begin{aligned} \gamma_0 &= \frac{\lambda_{11}^0 + \lambda_{22}^0}{2} \pm \frac{1}{2}\sqrt{(\lambda_{11}^0 - \lambda_{22}^0)^2 + 4\lambda_{12}^0\,\lambda_{21}^0} \\ \gamma_{11} &= \lambda_{11}^{(1)} \\ \gamma_{12} &= \lambda_{22}^{(1)} \\ \gamma_{21} &= \lambda_{11}^{(2)} \\ \gamma_{22} &= \lambda_{22}^{(2)} \end{aligned} \tag{A7}$$

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Summary

The wave theory of coupled transmission lines is known from the technical literature, as well as the application of this theory for the examination of once and twice three-phase systems built with transposition. The present paper gives a method for the consideration of the influence of ground wires, similarly on the basis of the wave theory. Ground wires are influencing the propagation coefficients of waves arising in the transmission line system and the wave impedances, but not the number of arising modes. In three-phase lines built with transposition the ground wire affects only the condition of the zero phase-sequence symmetrical component.

Dr. István Vágó; Budapest XI., Egry József u. 18—20. Hungary