PROPAGATION AND REFLECTION OF ELECTROMAGNETIC WAVES IN THE PRESENCE OF SUBSTANCES WITH ARBITRARY COMPLEX PERMITTIVITY

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Introduction

Maxwell's equations describing the electromagnetic phenomena have linear character in linear media. Therefore we can write these equations containing the time Fourier transforms of the field strengths. Choosing the usual time dependence exp ($j\omega t$), the complex effective values satisfy the following equations in a homogeneous, sourceless region of the space:

$$\operatorname{curl} \overline{\mathbf{H}} = j\omega\bar{\varepsilon}\,\overline{\mathbf{E}} \tag{1}$$

$$\operatorname{curl} \overline{\mathbf{E}} = -j\omega\mu_0\,\overline{\mathbf{H}} \tag{2}$$

$$\operatorname{div} \overline{\mathbf{H}} = 0 \tag{3}$$

$$\operatorname{div} \overline{\mathbf{E}} = 0 \tag{4}$$

Describing a substance of new type the equations obtain new physical content by the physical constants contained in this set of equations [1]. Furthermore we suppose that the substance is not magnetoactive. So the permeability is that of the free space $\mu = \mu_0 = 4\,\pi \cdot 10^{-7}$ Vs/Am. At the same time the permittivity is a complex quantity, considering the conduction and polarization currents in isotropic substances:

$$\bar{\varepsilon} = \varepsilon_0\ (\varepsilon' - j\varepsilon'')\,. \tag{5}$$

Here $\varepsilon_0 = 8.854 \cdot 10^{-12}$ As/Vm is the permittivity of the free space.

The imaginary part of the complex relative permittivity $\varepsilon' - j\varepsilon''$ may be written as

$$\varepsilon'' = \frac{\sigma}{\omega\varepsilon_0} \tag{6}$$

which means that the function $\varepsilon''(\omega)$ is replaced by another one $\sigma(\omega)$, known as generalized conductivity [2].

ε' describing electromagnetic phenomena in ionized gases and plasmas may be smaller than 1 or even a negative number. In accordance with the linear theory the coherent induced emission may be described by $\varepsilon'' < 0$ and so $\sigma < 0$, i.e. by negative conductivity. Theoretically there is no objection for the permittivity to be any complex quantity. In the following we shall investigate the behaviour of the quantities characterizing the wave propagation in the case of arbitrary complex permittivity. A particular emphasis will be spent to cases that differ by the sign of the conductivity only.

Homogeneous plane wave

It follows from Eqs. (1)—(5) that both $\overline{\mathbf{E}}$ and $\overline{\mathbf{H}}$ satisfy the vectorial Helmholtz's equation:

$$\nabla^2 \overline{\mathbf{F}} + \bar{k}^2 \overline{\mathbf{F}} = 0 \tag{7}$$

where

$$\bar{k}^2 = \omega^2 \varepsilon_0 \mu_0 (\varepsilon' - j\varepsilon'') . \tag{8}$$

$\bar{k} = k' - jk''$ is the *wave number.*

The homogeneous plane wave solution of Eq. (7) propagating toward the direction of the positive x axis is:

$$\overline{\mathbf{E}} = \overline{\mathbf{E}}_0 \exp [j(\omega t - \bar{k}x)] = \overline{\mathbf{E}}_0 \exp (-k''x) \exp [j(\omega t - k'x)] \tag{9}$$

$$\overline{\mathbf{H}} = \frac{\bar{k}}{\omega\mu_0} (\mathbf{i} \times \overline{\mathbf{E}}) = \frac{1}{\overline{\mathbf{Z}}} (\mathbf{i} \times \overline{\mathbf{E}}) \cdot \tag{10}$$

So the propagation of the homogeneous plane wave is characterized either by the wave number

$$\bar{k} = k' - jk'' = k_0 \sqrt{\varepsilon' - j\varepsilon''} \tag{11}$$

or by the *wave impedance*

$$\overline{Z} = R + jX = \frac{Z_0}{\sqrt{\varepsilon' - j\varepsilon''}} . \tag{12}$$

Here

$$k_0 = \omega \sqrt{\varepsilon_0 \mu_0} = \frac{\omega}{c} = \frac{2\pi}{\lambda} \, [m^{-1}] \tag{13}$$

is the wave number of the free space and

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cong 120\pi\,[\Omega] \tag{14}$$

is the wave impedance of the free space.

Let us define the complex *refractive index* as

$$\bar{n} = n' - jn'' = \sqrt{\varepsilon' - j\varepsilon''}\,. \tag{15}$$

Thus

$$\bar{k} = k_0 \bar{n} \text{ and } \bar{Z} = \frac{Z_0}{\bar{n}}\,. \tag{16)—(17}$$

Consequently, the complex refractive index alone is sufficient to characterize the propagation. But it is the permittivity that is directly determined by microphysical phenomena. Fortunately Eqs. (10)—(11) and (14)—(16) show that very simple conformal mappings establish the connection between $\bar{\varepsilon}/\varepsilon_0$, $\bar{k}$, $\bar{Z}$, and $\bar{n}$. The properties of these mappings are well known and their diagrams are available in textbooks (see e.g. [3]). Let us use them and examine the behaviour of the quantities mentioned above.

After squaring both sides of Eq. (14) and separating the real and imaginary parts we obtain

$$n'^2 - n''^2 = \varepsilon' \tag{18a}$$

$$2\,n'n'' = \varepsilon''. \tag{18b}$$

From here it follows that

$$n' = \sqrt{\frac{\sqrt{\varepsilon'^2 + \varepsilon''^2} + \varepsilon'}{2}} \tag{19a}$$

$$n'' = \pm\sqrt{\frac{\sqrt{\varepsilon'^2 + \varepsilon''^2} - \varepsilon'}{2}} \tag{19b}$$

Fig. 1 shows the curves $\varepsilon' =$ const. and $\varepsilon'' =$ const. of the mapping. The direction of the n'' axis was chosen so that the arcus of $n'—jn''$ should be the proper one.

How may we chose the signs of n' and n''? The phase velocity of the wave travelling to the positive x direction is, from Eq. (8):

$$v = \frac{\omega}{k'} = \frac{\omega}{n'\,k_0} = \frac{c}{n'}\,. \tag{20}$$

This velocity is non-negative, therefore

$$n' \geq 0. \tag{21}$$

From Eq. (17b) it follows directly that

$$\operatorname{sign} n'' = \operatorname{sign} \varepsilon'' . \tag{22}$$

The important consequence of Eq. (22) is the fact that the sign of the attenuation factor depends only on the sign of the conductivity independently of the ε' value.

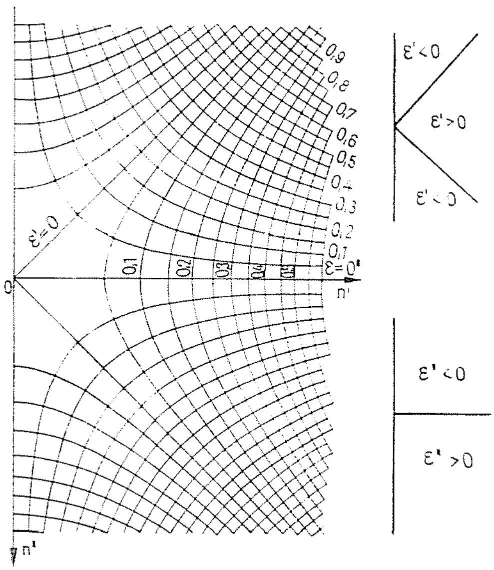

Fig. 1

Let us see the role of ε' in the value of the refractive index and the wave number. For $\varepsilon' = 0$ we get $n' = |n''|$. It is approximately valid for real (non-ideal) metals.

The reversal of ε' means the mutual change of the absolute values of n' and n''. (In Fig. 1 it means a reflection on one of the lines $\varepsilon' = 0$.) So for $\varepsilon' > 0$ and $\varepsilon'' = 0$, we get $n'' = 0$ and in the theoretically imaginable case where $\varepsilon' < 0$ and $\varepsilon'' = 0$, we obtain $n' = 0$. It means on the basis of Eq. (20) that the phase velocity is infinitely large. A collective oscillation arises in the medium with exponentially varying amplitude. It is the first approximation of a type of collective plasma oscillations.

The inverse of the mapping discussed above is $\varepsilon' - j\varepsilon'' = (n' - jn'')^2$. The curves $n' = \text{const.}$ and $n'' = \text{const.}$ are represented in Fig. 2. The reversal of n'' results in the reversal of ε''. It is clear from the diagram that $\varepsilon' < 1$ is the condition of the appearance of a phase velocity greater than that of light.

Let us see now the mapping given by Eq. (11):

$$\frac{\overline{Z}}{Z_0} = \frac{R}{Z_0} + j\,\frac{X}{Z_0} = \frac{1}{\sqrt{\varepsilon' - j\varepsilon''}} \; . \tag{23}$$

The diagram is given in Fig. 3.

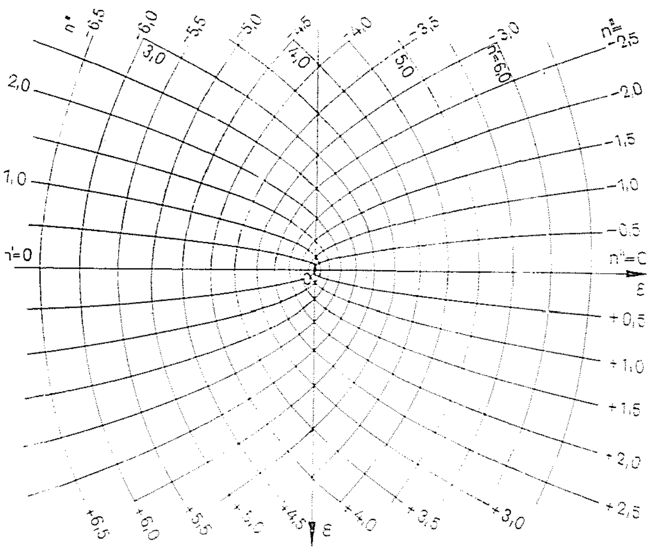

Fig. 2

Considering the allowable region of $\sqrt{\varepsilon' - j\varepsilon''} = \overline{n}$ in the denominator we can state that

$$\frac{R}{Z_0} = \frac{n'}{n'^2 + n''^2} \geq 0 \tag{24}$$

and

$$\frac{X}{Z_0} = \frac{n''}{n'^2 + n''^2} \; .$$

Therefore

$$\operatorname{sign} X = \operatorname{sign} \varepsilon'' \, . \tag{25}$$

The inverse of this mapping is shown in Fig. 4.

Finally we show the simple mapping

$$\frac{R}{Z_0} + j\,\frac{X}{Z_0} = \frac{1}{n' - jn''} \tag{26}$$

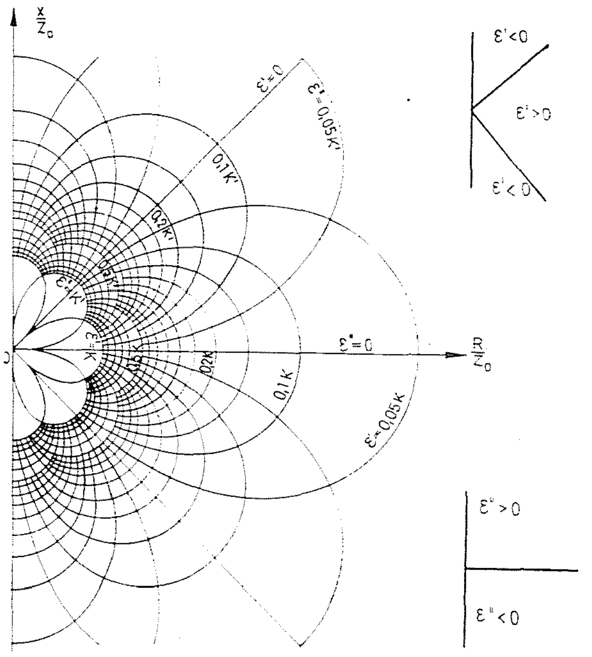
x/Z0
ε'=0
ε''=0,05K'
0,1K'
0,2K'
ε''=K'
ε'=K
0,2K
0,1K
ε''=0
R/Z0
ε'=0,05K
ε'<0
ε'>0
ε'<0
ε''>0
ε''<0

Fig. 3

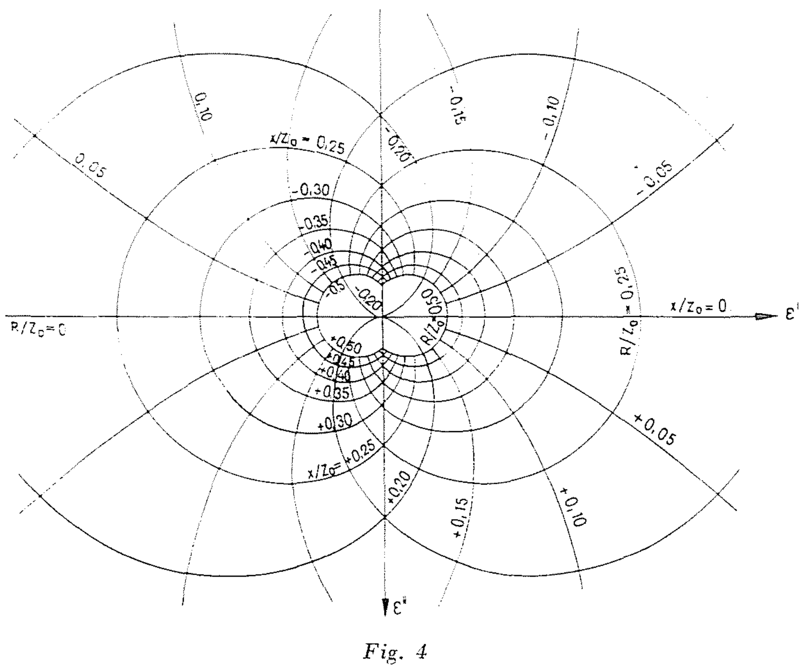
0,10
0,05
x/Z0 = 0,25
-0,20
-0,15
-0,10
-0,05
-0,30
-0,35
-0,40
-0,45
-0,50
-0,20
R/Z0 = 0,50
R/Z0 = 0
R/Z0 = 0,25
x/Z0 = 0
ε'
+0,50
+0,45
+0,40
+0,35
+0,30
x/Z0 = +0,25
+0,20
+0,15
+0,10
+0,05
ε''

Fig. 4

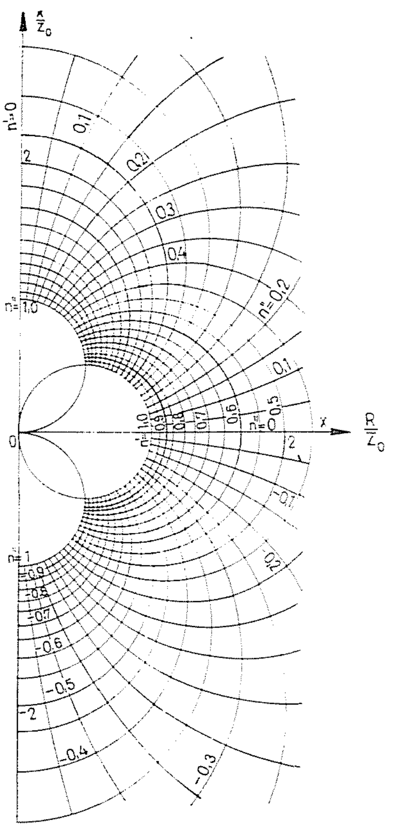

Fig. 5

in Fig. 5. Naturally, this set of diagrams may be applied for the inverse mapping $\bar{n} = \dfrac{Z_0}{\bar{Z}}$ too.

Let us summarize the results obtained for the quantities characterizing the propagation of the homogeneous plane wave:

1. The real part of the refractive index and that of the wave impedance cannot be negative. This property is independent of the value of the permittivity.

2. The permittivity, the refractive index and the wave impedance change to their conjugate at the same time.

Inhomogeneous plane wave

Till now the wave vector characterizing the propagation of the plane wave has been the product of the wave number and the unit vector indicating the direction of the propagation. The vectors of real and imaginary complex values point to the same direction. It is a very strong restriction. Let us suppose that the wave vector has the form

$$\bar{\mathbf{K}} = \mathbf{K}' - j\mathbf{K}'' \tag{27}$$

where

$$\frac{\mathbf{K}'\,\mathbf{K}''}{K'\,K''} = \cos\psi \tag{28}$$

i.e. the pure real vector and the pure imaginary vector include an angle ψ. If this vector satisfies the condition

$$\bar{\mathbf{K}}^2 = \bar{k}^2 \tag{29}$$

the solution satisfies Eq. (7) in the following form:

$$\bar{\mathbf{E}} = \bar{\mathbf{E}}_0 \exp\,[j(\omega t - \bar{\mathbf{K}}\mathbf{r})] = \bar{\mathbf{E}}_0 \exp\,(-\mathbf{K}''\,\mathbf{r}) \exp\,[j(\omega t - \mathbf{K}'\mathbf{r})] \tag{30}$$

The equi-amplitude planes ($\mathbf{K}''\mathbf{r} =$ const.) and the equiphase planes ($\mathbf{K}'\mathbf{r} =$ $=$ const.) do not coincide. The plane wave is inhomogeneous. This plane wave solution of Eq. (7) is the most general one.

Using Eqs. (28) and (29)

$$K'^2 - K''^2 = k_0^2\,(n'^2 - n''^2) = k_0^2\varepsilon' \tag{31}$$

$$2K'K''\cos\psi = 2k_0^2\,n'n'' = k_0^2\varepsilon'' \,. \tag{32}$$

Let $\mathbf{K}'$ point to the direction of the propagation and let us stipulate that $0 \leq \psi \leq \pi/2$. On these conditions we get

$$K' \geq 0 \tag{33}$$

and

$$\operatorname{sign} K'' = \operatorname{sign} n'' = \operatorname{sign} \varepsilon'' \,. \tag{34}$$

The behaviour of the wave vector of the inhomogeneous plane wave is perfectly analogous to that of its homogeneous counterpart.

Reflection and refraction of homogeneous plane waves on a flat surface

As we have seen from the aspect of wave propagation, any linear, homogeneous and isotropic medium may be characterized by the refractive index. Its imaginary part reverses the sign together with the conductivity. Further we shall investigate how this reversal affects the reflection and the refraction on a flat surface. Let a homogeneous plane wave travel from vacuum in the way indicated in Fig. 6.

$$\overline{\mathbf{E}}_0 = \overline{\mathbf{E}}_{00} \exp [j(\omega_0 t - \overline{\mathbf{K}}_0 \mathbf{r})] \cdot \tag{35}$$

In our case $K_0' = k_0$, $K_0'' = 0$.

Two other plane waves (and generally inhomogeneous ones) are necessary to satisfy the boundary conditions. If their propagation vectors point away

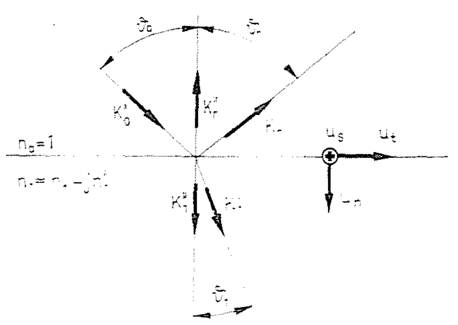

Fig. 6

from the boundary surface we can chose only one set of waves: one in both regions. The waves travelling away from the surface must not be on the same side of the boundary surface [4]. Let these two waves be the ones represented in Fig. 6.

$$\overline{\mathbf{E}}_r = \overline{\mathbf{E}}_{0r} \exp [j(\omega_r t - \overline{\mathbf{K}}_r \mathbf{r})] \tag{36}$$

$$\overline{\mathbf{E}}_1 = \overline{\mathbf{E}}_{01} \exp [j(\omega_1 t - \overline{\mathbf{K}}_1 \mathbf{r})] \, . \tag{37}$$

To satisfy the boundary conditions [1] let us assume that

$$\overline{E}_1 = \bar{t}\overline{E}_0 \quad \text{and} \quad \overline{E}_r = \bar{r}\overline{E}_0 \qquad (38)\ (39)$$

at any time and for all the points of the surface. Here $\bar{t}$ is the complex transmission coefficient and $\bar{r}$ is the complex reflection coefficient.

The necessary conditions of the fulfilment of Eqs. (38—39) are the following [5]:

$$\omega_0 = \omega_r = \omega_1 \tag{40}$$

$$\mathbf{K}_0' \mathbf{u} = \mathbf{K}_r' \, \mathbf{u} = \mathbf{K}_1' \, \mathbf{u} \tag{41a}$$

$$\mathbf{K}_0'' \, \mathbf{u} = \mathbf{K}_r'' \, \mathbf{u} = \mathbf{K}_1'' \, \mathbf{u} \tag{41b}$$

where $\mathbf{u}$ is an arbitrary vector parallel to the surface.

Since the complex wave vectors are coplanar, on the basis of Eq. (41), the reflection law is valid

$$\vartheta_r = \vartheta_0 \tag{42}$$

and Snell's law remains valid as well:

$$\frac{n_0}{\overline{n}_1} = \frac{\sin \overline{\Theta}_1}{\sin \vartheta_0} \tag{43}$$

or in an equivalent form

$$\overline{n}_1 \sin \overline{\Theta}_1 = n_0 \sin \vartheta_0 \,. \tag{44}$$

In our case $n_0 = 1$ and ϑ_0 is real. Nevertheless $\overline{n}_1$ is a complex number of non-negative real part, so $\overline{\Theta}_1$ is complex too. From Eqs. (43)—(44) we can see that $\overline{n}_1$, $\sin \overline{\Theta}_1$ and therefore $\overline{\Theta}_1$ too change to their conjugate together, but the product $\overline{n}_1 \sin \overline{\Theta}_1$ remains unchanged by the same ϑ_0.

An inhomogeneous plane wave may be connected with the complex refractive angle. Let us find the connection between $\overline{\mathbf{K}}_1$ and $\overline{\Theta}_1$. Let us choose the coordinate system as characterized by the unit vectors $\mathbf{u}_t$, $\mathbf{u}_n$ and $\mathbf{u}_s$ in Fig. 6. It follows from Eq. (41b) that

$$\mathbf{K}_r'' \mathbf{u}_t = \mathbf{K}_1'' \mathbf{u}_t \tag{45}$$

because $K_0'' = 0$. It can be proved [5] that for the reflected wave in free space, $K_r'' = 0$. In this case the vector $\mathbf{K}_1''$ is perpendicular to the boundary surface, i.e. the equi-amplitude planes of the refracted waves are parallel to the boundary surface.

It follows from Eq. (41a), that

$$K_0' \sin \vartheta_0 = K_r' \sin \vartheta_r = K_1' \sin \vartheta_1 \,. \tag{46}$$

Using Eq. (42) we directly get the equality of K_0' and K_r'. For $\overline{\mathbf{K}}_1$ is valid, that

$$\overline{\mathbf{K}}_1 = (K_1' \sin \vartheta_1)\mathbf{u}_t + (K_1' \cos \vartheta_1 - jK_1'')\mathbf{u}_n \,. \tag{47}$$

On the other hand we can write the following formula in analogy with the homogeneous plane wave

$$\overline{\mathbf{K}}_1 = k_0\, \overline{n}_1[(\sin \overline{\Theta}_1)\mathbf{u}_t + (\cos \overline{\Theta}_1)\mathbf{u}_n] \,. \tag{48}$$

Comparing Eqs. (47) and (48) we get

$$k_0 \bar{n}_1 \sin \bar{\Theta}_1 = K_1' \sin \vartheta_1 \tag{49}$$

$$k_0 \bar{n}_1 \cos \bar{\Theta}_1 = K_1' \cos \vartheta_1 - jK_1'' \tag{50}$$

and it is true for complex angles too that

$$\sin^2 \bar{\Theta}_1 + \cos^2 \bar{\Theta}_1 = 1 . \tag{51}$$

From Eqs. (44) and (49)—(51) we can obtain with some algebra

$$K_1' \cos \vartheta_1 - jK_1'' = k_0 \sqrt{n_1'^2 - n_1''^2 - \sin^2 \vartheta_0 - 2n'n''} . \tag{52}$$

From this and Eq. (30) we can get the values of K_1', K_1'' and $\cos \vartheta_1$ as the functions of n_1', n_1'' and ϑ_0, respectively. But these lengthy formulae are not necessary to realize the truth of the following statement:

The complex permittivity and the wave vector of the inhomogeneous plane wave produced by the refraction of a homogeneous one change to their conjugate together. In the case of this reversal the refractive angle characterizing the equiphase planes remains unchanged.

We have seen that $\bar{n}_1$ and, on the basis of Eq. (43), $\bar{\Theta}_1$ too change to their conjugate but Eq. (44) shows that $\bar{n}_1 \sin \bar{\Theta}_1$ is unchanged. It follows from Eq. (50) that $K_1' \cos \vartheta_1$ remains unchanged too but K_1'' reverses the sign. So the statement is theoretically proved — in a total accordance with Eqs. (33)—(34).

Naturally Eqs. (31) and (32) are valid also for the refracted wave. From them

$$I_1 = \left(\frac{K_1'}{k_0}\right)^2 - \left(\frac{K_1''}{k_0}\right)^2 = n_1'^2 - n_1''^2 = \varepsilon_1' \tag{53}$$

$$I_2 = \frac{K_1'}{k_0} \cdot \frac{K_1''}{k_0} \cos \vartheta_1 = n_1' n_1'' = \frac{\varepsilon_1''}{2} . \tag{54}$$

These are the so-called Ketteler's equations. The deduction given here is far simpler than the usual one [6, 7]. The optical invariants I_1 and I_2 are in direct connection with the complex permittivity. This recognition is absent from the optical literature of Ketteler's equations.

For the refraction angle ϑ_1 of the equiphase planes we get

$$\frac{\sin \vartheta_0}{\sin \vartheta_1} = \frac{K_1'}{k_0} = n_1(\vartheta_0) \tag{55}$$

where we used Eqs. (44), (49) and $n_0 = 1$. The refractive index thus defined depending on ϑ_0, satisfies a law which is very similar to Snell's law.

Reflection and transmission coefficients

Till now we have investigated the change of the amplitude and phase of the reflected and refracted wave independently from each other. Let us see now the relations between the amplitudes and phases of the three partial waves mentioned above. We want particularly to determine the complex quantities $\bar{t}$ and $\bar{r}$ in Eqs. (38)—(39). These are the so-called Fresnel coefficients.

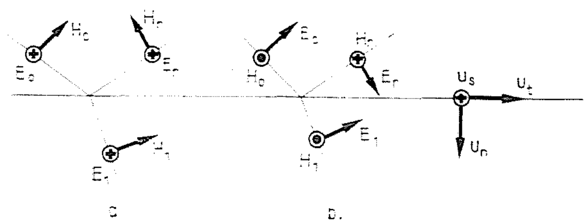

Fig. 7

For perpendicular polarization (Fig. 7*a*) we get [5]

$$\bar{r}_\perp = \frac{1-\bar{\varkappa}_\perp}{1+\bar{\varkappa}_\perp}; \quad \bar{t}_\perp = \frac{2}{1+\bar{\varkappa}_\perp}; \quad \text{where} \quad \bar{\varkappa}_\perp = \frac{\bar{n}_1 \cos \bar{\Theta}_1}{n_0 \cos \vartheta_0} \quad (56\,\text{a,b,c}).$$

For parallel polarization (Fig. 7*b*) similarly

$$\bar{r}_{\shortparallel} = \frac{1-\bar{\varkappa}_{\shortparallel}}{1+\bar{\varkappa}_{\shortparallel}}; \quad \bar{t}_{\shortparallel} = \frac{2}{1+\bar{\varkappa}_{\shortparallel}} \, \frac{\cos \vartheta_0}{\cos \bar{\Theta}_1}; \quad \text{where} \quad \bar{\varkappa}_{\shortparallel} = \frac{\bar{n}_1 \cos \vartheta_0}{n_0 \cos \bar{\Theta}_1}. \quad (57\text{a,b,c}$$

Let the conductivity and together with it n_1'' reverse the sign. Then $\bar{\Theta}_1$ and therefore $\bar{\varkappa}$ change to their conjugate. On the basis of Eqs. (56)—(57) all the Fresnel coefficients change to their conjugate too.

This is very advantageous from a practical standpoint. All the tables and diagrams calculated for positive conductivity (see e.g. [8]) lend themselves to determine the Fresnel coefficients. The absolute value is not changed, only the phase angle reverses the sign.

At the same time this result seems to be incorrect. The power density carried by the reflected wave is proportional with the square of the absolute value of the reflection coefficient. But it has proved independent from the sign of the conductivity. The physically unrealistic result is that the power reflection is the same for the active and passive half-spaces.

The problem is in close connection with the physical realizability of the active half-space. Consequently let us find a configuration where the limit of the geometry is the half-space and its reflection coefficient tends to the Fresnel reflection coefficient in the case of positive conductivity. Let us examine the limit of the reflection coefficient measurable on the surface in the case of negative conductivity and consider it as the real reflection coefficient of the half-space with negative conductivity.

The proper model is the plan-parallel layer. The total change of the complex amplitude of the wave travelling across the layer of thickness l is

$$\bar{a} = \exp(-j\bar{\mathbf{K}}l\mathbf{u}_n) \tag{58}$$

where $\mathbf{u}_n$ is the unit vector perpendicular to the flat surface.

Considering Eqs. (48) and (50) we get

$$\bar{a} = \exp(-jk_0\,\bar{n}l\cos\bar{\Theta}) = \exp(-K''l)\exp(-jK'\,l\cos\vartheta)\,. \tag{59}$$

With the help of this quantity the proportion of the amplitudes of the reflected and the incident waves is [6, 9]:

$$\bar{R} = \frac{\bar{r}_{12} + \bar{a}^2\bar{r}_{23}}{1 + \bar{a}^2\bar{r}_{12}\bar{r}_{23}} \tag{60}$$

and this proportion for the transmitted and the incident waves is:

$$\bar{T} = \frac{\bar{a}(1+\bar{r}_{12})(1+\bar{r}_{23})}{1+\bar{a}^2\bar{r}_{12}\bar{r}_{23}} \tag{61}$$

where $\bar{r}_{12}$ and $\bar{r}_{23}$ are the Fresnel reflection coefficients in the front and the back sides of the layer.

The examination of Eq. (60) gives an interesting result in the case of $\bar{r}_{23} = -1$. It is the case of the reflection from an ideal metal plane. For positive conductivity

$$\bar{R}^{+} = \frac{\bar{r}_{12}^{+} - (\bar{a}^{+})^2}{1 - \bar{r}_{12}^{+}(\bar{a}^{+})^2}\,. \tag{62}$$

Let us reverse the sign of the conductivity. As it follows from Eq. (59) the phase of $\bar{a}$ remains unchanged, but its absolute value changes to its reciprocal. At the same time $\bar{r}_{12}$ changes to its conjugate. We obtain that

$$\bar{R}^{-} = \frac{\bar{r}_{12}^{-} - (\bar{a}^{-})^2}{1 - \bar{r}_{12}^{-}(\bar{a}^{-})^2} = \frac{(\bar{r}_{12}^{+})^* - \left[\dfrac{1}{(\bar{a}^{+})^*}\right]^2}{1 - (\bar{r}_{12}^{+})^*\left[\dfrac{1}{(\bar{a}^{+})^*}\right]^2} = \frac{1}{(\bar{R}^{+})^*} \tag{63}$$

where the asterisk denotes the complex conjugate.

Eq. (63) means that the absolute value of the reflectivity changes to its reciprocal and its phase remains unchanged in the case of a plan-parallel layer that reflects totally on the back side, if the conductivity reverses the sign. This configuration exists in a group of lasers of the Fabry—Perot type. If we want to examine their mechanism we can use the formulae or diagrams for positive conductivity (see e.g. [10]).

To realize our original intention let l tend to the infinity. In the case of positive conductivity $K'' > 0$ and consequently $|\bar{a}| \to 0$. It follows from Eqs. (60)—(61) that

$$\bar{R}^{+} \to \bar{r}_{12}^{+}; \quad \bar{T}^{+} \to 0 . \tag{64a, b}$$

as it has been expected. At the same time, for negative conductivity $K'' < 0$, i.e. $|\bar{a}| \to \infty$. In this case

$$\bar{R}^{-} \to \frac{1}{\bar{r}_{12}^{-}}; \quad \bar{T}^{-} \to 0 . \tag{65a,b}$$

The above limits of $\bar{R}^{+}$ and $\bar{R}^{-}$ are the measurable real reflection coefficients. For positive conductivity this limit coincides with the Fresnel coefficient

$$\bar{R}_{\infty}^{+} = \bar{r}^{+} \tag{66}$$

for negative conductivity it is the reciprocal of the same coefficient. But we have seen that

$$\bar{r}^{-} = (\bar{r}^{+})^{*} \tag{67}$$

and so

$$\bar{R}_{\infty}^{-} = \frac{1}{(\bar{r}^{+})^{*}} = \frac{1}{(\bar{R}_{\infty}^{+})^{*}} . \tag{68}$$

We can state the following:

If the complex permittivity changes to its conjugate the absolute value of the reflection coefficient changes to its reciprocal and its phase angle remains unchanged. Consequently the absolute value of the reflection coefficient of the half-space of negative conductivity cannot be smaller than 1. This fact coincides with the picture created about the active materials.

Now it is also obvious why the Fresnel reflection coefficients fail to describe correctly the power reflection. In the course of its deduction the backward travelling wave was neglected. It is permissible in the case of positive conductivity but it is a very bad approximation for negative conductivity.

Here the backward travelling wave is dominant in the medium. It can easily be proved for perpendicular incidence that the surface impedance is $-\bar{Z}^-$, i.e. the wave impedance of the backward travelling wave. But according to Eq. (24) $Re(-\bar{Z}^-) \leq 0$ and this is just what we can accept for the case of negative conedutivity.

The statement of Eq. (65b) is very surprising. The fact that the amplitude of a wave transmitted by an active medium of very large extension tends to zero emphasizes the strong interference character of the phenomenon.

Conclusions

1. The basic parameter of the electromagnetic wave phenomena taking place in arbitrary linear substances is the properly chosen refractive index.
2. In the case of a single frequency sinusoidal wave the real part of the refractive index is non-negative and the sign of its imaginary part coincides with that of the conductivity. The same is true for the wave impedance as well as the wave vector.
3. The permittivity, the refractive index, the wave vector, the wave impedance and the Fresnel coefficients change to their conjugate together.
4. The absolute value of the surface reflection coefficient of a medium of great extension with negative conductivity is the reciprocal of that in the case of positive conductivity. The phases are equal in the two cases.
5. The amplitude of a wave transmitted by a medium of very large extension tends to zero independently of the sign of the conductivity.

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Summary

The propagation of plane waves of sinusoidal time dependence in linear isotropic media of arbitrary complex permittivity is investigated. Relations are stated between the real and imaginary parts of the complex permittivity, the refractive index, the wave vector and the wave impedance, emphasizing the effect of the reversal of the conductivity. Use is made of these data in the examination of the reflection on the plane surface of a medium with arbitrary complex refractive index. The behaviour of the Fresnel coefficients in the case of negative conductivity is examined demonstrating that the reflection coefficient on the surface of media of large extension with negative conductivity does not coincide with the Fresnel coefficient; finally, a connection is established between both quantities.

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