AN ENGINEERING METHOD FOR PROCESS IDENTIFICATION

By

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I. Introduction

The plotting of control system frequency functions by measurements is often met with serious difficulties. On the one hand there isn't always an adequate sinusoid-signal generator available, on the other hand the system to be investigated cannot always be fed by a signal with frequencies varying over a wide range. In such cases the frequency response may be determined by different approximation methods. By one class of these approximation methods the system frequency response or transfer function is determined on the basis of unit-step response that is from the output appearing under the effect of the unit-step input [1, 2, 3].

Considering the serious calculation work required by these processes it seems to be advisable to approximate the given time responses of the process by assumed simpler time responses of models.

A few years ago an approximation by the time response of a single time constant model with dead time was used in spite of the fact that short period deviations were high (Fig. 1). Another possibility is the approximation of the process by a pure second order element, i.e. by a model with the transfer function

$$G(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{1}{1 + 2\zeta sT + s^2 T^2} \tag{1}$$

where $\omega_0 = 1/T$ is the natural frequency. A method for the determination of the time constants of overdamped ($\zeta > 1$) systems of second order was developed by OLDENBOURG and SARTORIUS [4] and an approximation for underdamped ($\zeta < 1$) systems was suggested by SMITH and MURRILL [5]. As most controlled processes are multi-time constant systems or contain also elements with dead time, the best simple approximation is offered by a model second order with dead time, i.e.

$$G(s) = \frac{e^{-sT_D}}{1 + 2\zeta sT + s^2 T^2} \tag{2}$$

A method for the determination of the parameters of such an approximative transfer function was developed by J. R. MEYER et al. [6].

The purpose of the present study is to give a precalculated set of curves in the form of nomograms, by extending the above mentioned methods. The parameters of the approximative transfer function (2) are easily determined by the presented graphs.

The application of the method will be demonstrated by numerical examples. The deviations between the frequency responses of the original processes and of the approximative models will be studied.

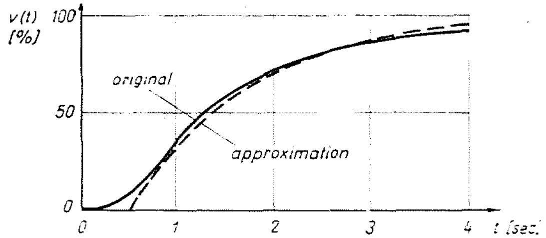

Fig. 1. First order approximation with dead time

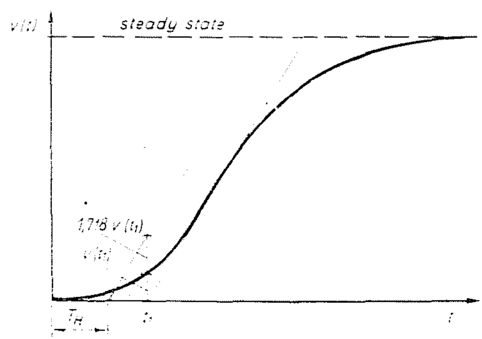

Fig. 2. Determination of the dead time $T_D = T_H$

II. The proposed method

The first step when approximating the measured unit-step response of the unknown system is to determine the dead time. The method was developed by Cox et al. [7]. The dead time may be determined on the basis of Fig. 2 as follows:

1. A tangential line is drawn through the inflection point of the unit-step response

2. At a certain time t_1 the time axis is intersected by this tangent. Let the value of the unit-step response at time t_1 be $v(t_1)$.

3. A straight line is drawn parallel with the tangent through the point $v_*(t_1) = 2.718 \ v(t_1)$.

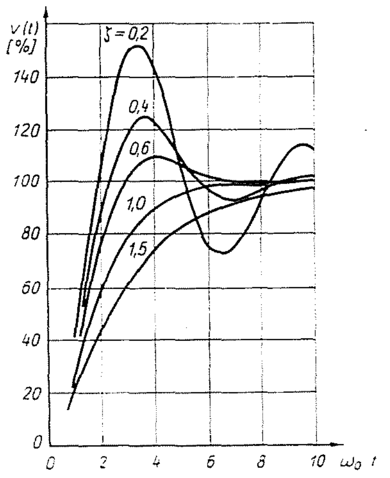

Fig. 3. Unit-step response of a system of second order at different values of the damping factor

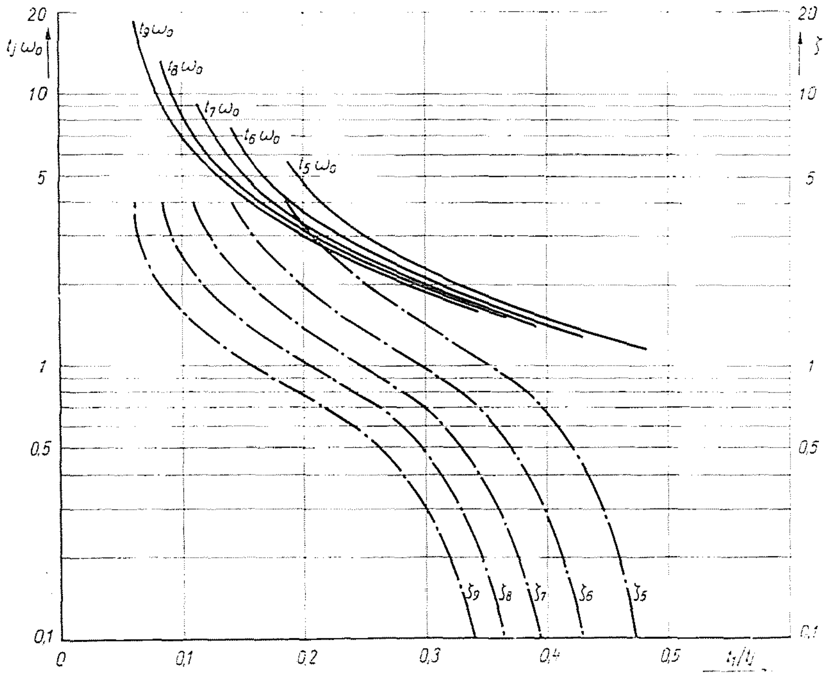

Fig. 4

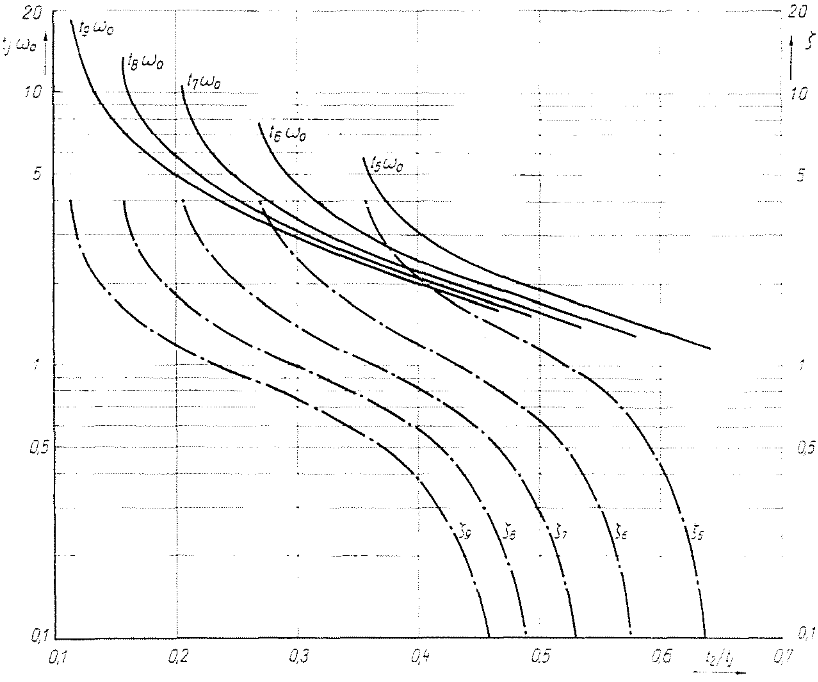

Fig. 5

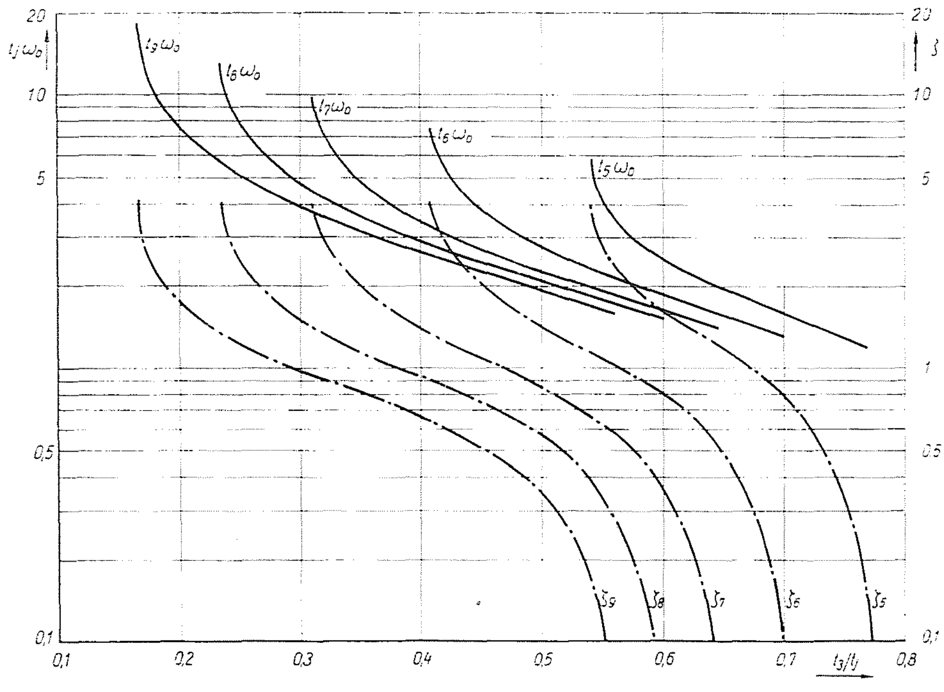

Fig. 6

Figs. 4, 5, 6. Damping factor and natural frequency vs. t_i/t_j

4. The dead time T_D is cut out on the time axis by this parallel straight line.

The second step is to determine the parameters of the transfer function of second order expressed by (1). The approximative system given in this way contains both the underdamped $\zeta < 1$ and the overdamped $\zeta > 1$ cases, so no special provision is required for this distinction in the following treatment. Fig. 3 shows the unit-step responses plotted for different values of ζ.

By the transient responses the values $t_i\,\omega_0$ and $t_j\omega_0$ may be determined. These values belong to the $10i$, and $10j$ percentage values of the steady state transient response value. In our study the values $i = 1,2,3$ and $j = 5,6,7,8,9$ were chosen in correspondence with the values of 10%, 20%, 30% and 50%, 60%, 70%, 80% 90%, respectively.

A ratio $t_i\omega_0/t_j\omega_0 = t_i/t_j$ depends only on ζ, but is independent of ω_0. By plotting the values $t_j\omega_0$ and ζ vs. the quotient t_i/t_j in logarithmic scale we obtain the set of curves shown in Figs 4, 5, 6. The parameters of the model approximating the unknown system are determined on the basis of the curves as follows:

The pair of values t_i and t_j are determined on the basis of the unit-step response to be approximated, then the quotient t_i/t_j is evaluated. On the basis of the ratio t_i/t_j the damping factor of the approximating model may be determined directly by Figs 4, 5, 6 and its natural frequency from the value $t_j\omega_0$. To every quotient t_i/t_j the coherent ζ and ω_0 values referring to an approximative model of second order are determined. Taking into account all the possibilities in this way, $3 \times 5 = 15$ approximations may be obtained altogether by Figs 4, 5, 6. From these approximations the most appropriate one must be chosen.

III. Illustrative examples

Example 1.

Let us assume that the transfer function of the process to be identified has three different time constants:

$$G(s) = \frac{1}{(1 + sT_a)(1 + sT_b)(1 + sT_c)}.$$

Let us assume that the time constants $T_a = 0.5$ sec, $T_b = 1$ sec and $T_c = 2$ sec are known. The dead time of the approximating model is determined on the basis of the unit-step response by the method described in Ch. II as

$$T_D = 0.32 \text{ sec.}$$

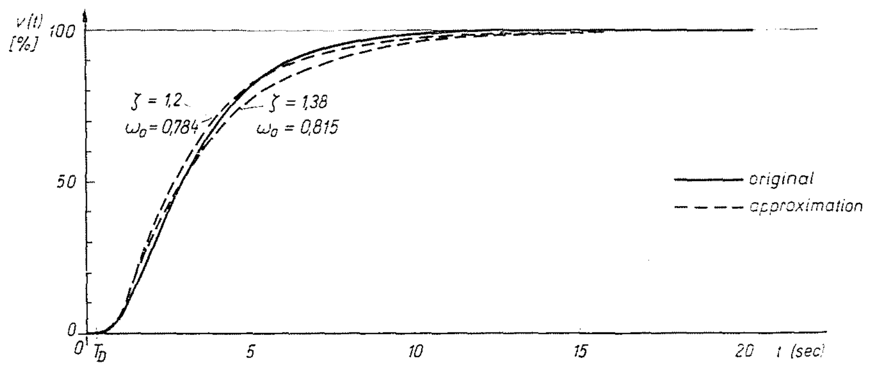

Fig. 7. Approximation of a system or process of third order. $T_a = 0.5s$; $T_b = 1s$; $T_c = 2s$. Unit-step response of the original process and of the approximative model obtained on the basis of the t_1/t_j quotients

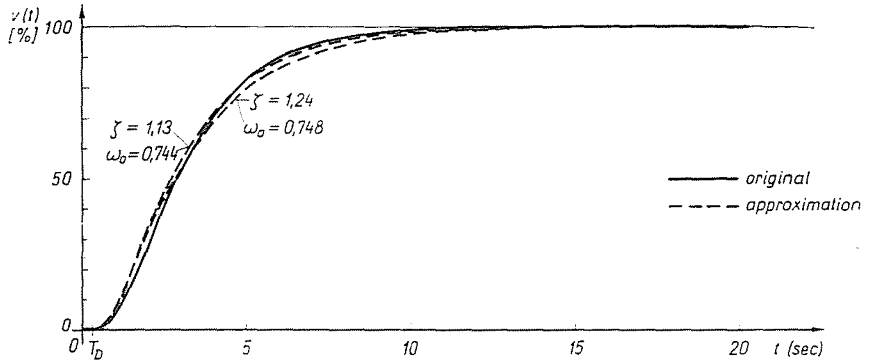

Fig. 8. Approximation of a system or process of third order. $T_a = 0.5s$; $T_b = 1s$; $T_c = 2s$. Unit-step response of the original process and of the approximative model obtained on the basis of the t_2/t_j quotients

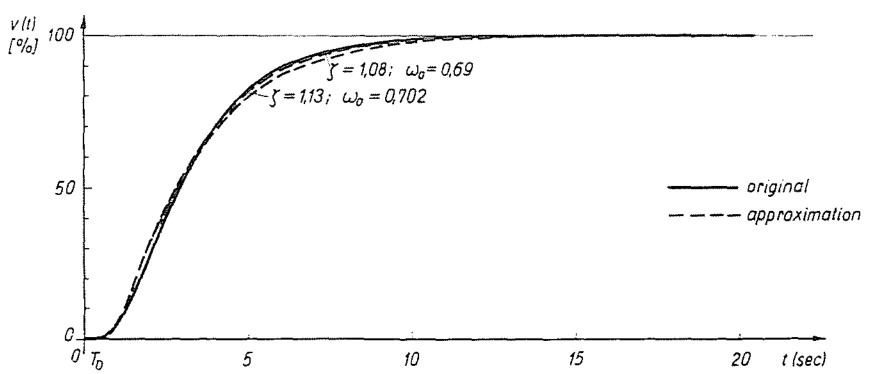

Fig. 9. Approximation of a system or process of third order. $T_a = 0.5s$; $T_b = 1s$; $T_c = 2s$. Unit-step response of the original process and of the approximative model obtained on the basis of the t_3/t_j quotients

Now the values t_i and t_j are determined also from the transient response taking into consideration that the approximation of second order starts from the instance $T_D = 0.32$ sec, i.e.

$$t_i = T_i - T_D$$

$$t_j = T_j - T_D$$

where T_i and T_j are two time moments read of the unit-step response to be approximated, reaching $10i$ and $10j$ percent, resp., of the steady state value. After evaluating the quotient t_i/t_j the values ω_0 and ζ were determined by Figs

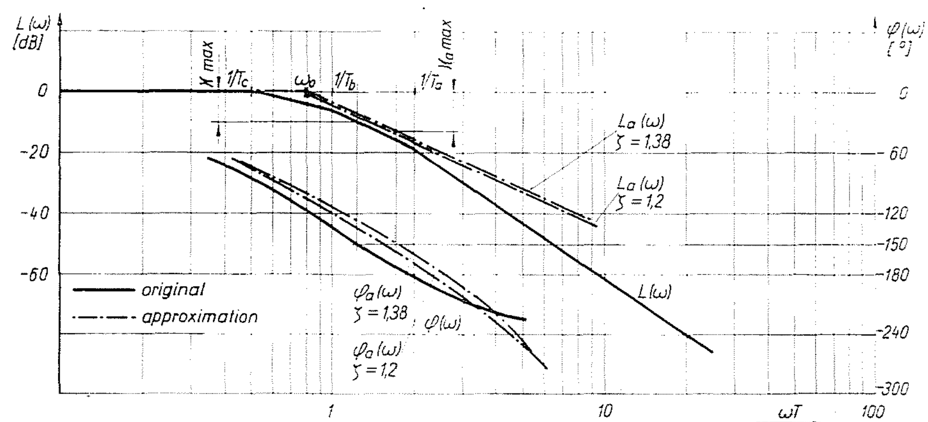

Fig. 10. Approximation of a system or process of third order. $T_a = 0.5s$; $T_b = 1s$; $T_c = 2s$. BODE-diagrams of the original process and of the approximative model obtained on the basis of the t_1/t_j quotients

4, 5, 6. The values ω_0 and ζ are summarized in Table 1. Figs 7, 8, 9 show some of the original and the approximative curves denoted by asterisks in the table for demonstration. It is seen that the best approximation is given by the models, whose parameters are obtained from the set of curves t_3/t_j (Fig. 6).

This statement is also born out by the results obtained from the BODE-diagrams (Figs 10, 11, 12). Here the amplitude $L(\omega) = 20 \log | G(j\omega) |$ and the phase diagram $\varphi(\omega) = \tan^{-1}[ImG(j\,\omega)/ReG(j\,\omega)]$ are shown in the usual single logarithmical plots (BODE-diagrams). On the basis of these graphs the gain margin belonging to the phase margin of $\varphi_t = 30°$ were also determined. It is seen by these data that the gain margin of the approximative models surpass by about 10% those of the original system.

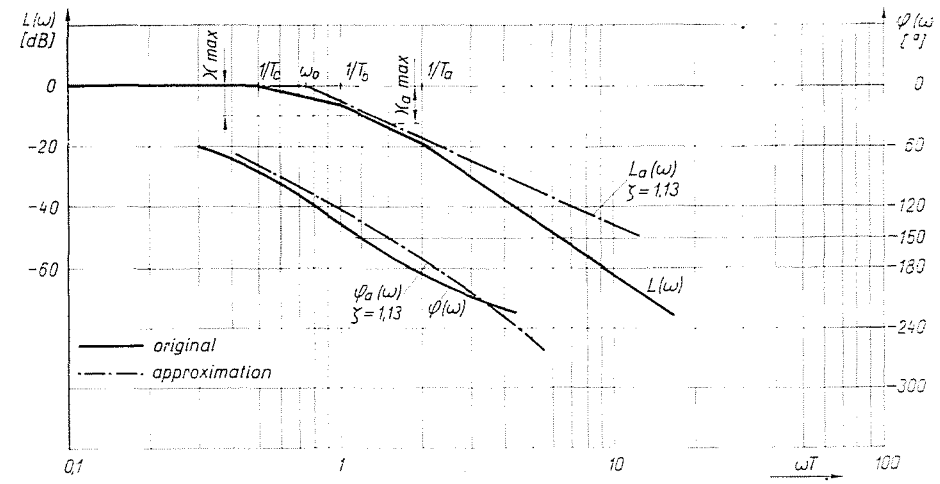

Fig. 11. Approximation of a system or process of third order. $T_a = 0.5s$; $T_b = 1s$; $T_c = 2s$. BODE-diagrams of the original process and of the approximative model obtained on the basis of the t_2/t_j quotients

Table 1

i	$t_i[s]$	j	$t_j[s]$	t_i/t_j	$t_j\omega_0$	$\omega_0[1/s]$	ζ
1	0.83	5	2.7	0.3075	2.2	0.815	1.38 *
1	0.83	6	3.28	0.253	2.65	0.808	1.32
1	0.83	7	3.98	0.2085	3.15	0.792	1.3
1	0.83	8	4.88	0.1695	3.85	0.787	1.24
1	0.83	9	6.38	0.13	5	0.784	1.2 *
2	1.31	5	2.7	0.485	2.03	0.748	1.24 *
2	1.31	6	3.28	0.3995	2.43	0.741	1.2
2	1.31	7	3.98	0.3293	2.95	0.74	1.18
2	1.31	8	4.88	0.268	3.6	0.737	1.15
2	1.31	9	6.38	0.205	4.75	0.744	1.13 *
3	1.78	5	2.7	0.659	1.9	0.704	1.1 *
3	1.78	6	3.28	0.543	2.3	0.702	1.13
3	1,78	7	3.98	0.447	2.78	0.698	1.12
3	1.78	8	4.88	0.364	3.4	0.696	1.11
3	1.78	9	6.38	0.2785	4.4	0.69	1.08 *

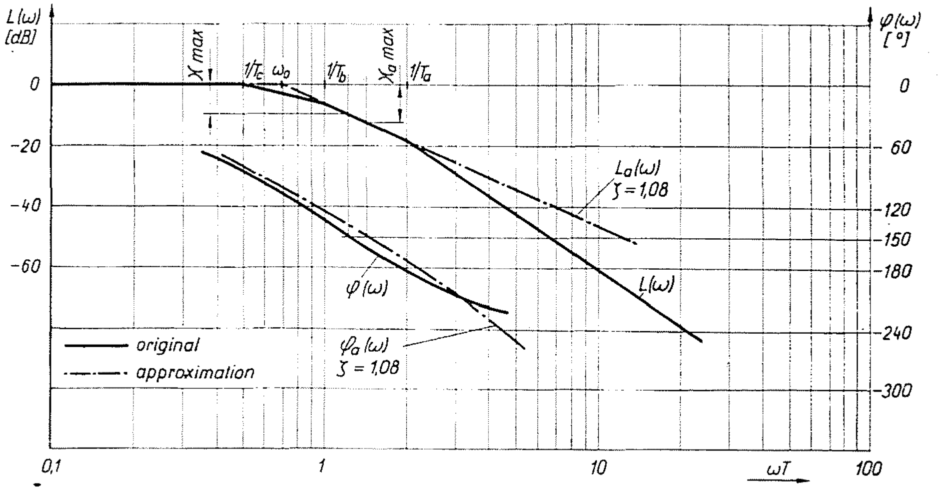

Fig. 12. Approximation of a system or process of third order. $T_a = 0.5s$; $T_b = 1s$; $T_c = 2s$. BODE-diagrams of the original process and of the approximative model obtained on the basis of the t_3/t_j quotients

Example 2

Whereas in the previous example the time constants of the system were close to each other, in this example they are distant from each other. Be

$$T_a = 0.1 \text{ sec}, \quad T_b = 1 \text{ sec}, \quad T_c = 10 \text{ sec}.$$

Table 2

i	$t_i[s]$	j	$t_j[s]$	t_i/t_j	$t_j\omega_0$	$\omega_0[1/s]$	ζ
1	1.8	5	7.8	0.2308	3.46	0.443	2.42 *
1	1.8	6	10	0.18	4.35	0.435	2.35
1	1.8	7	13	0.1385	5.8	0.446	2.35
1	1.8	8	17	0.1058	7.05	0.415	2.3
1	1.8	9	24.1	0.0747	10.6	0.439	2.25 *
2	3.1	5	7.8	0.3973	3.1	0.3975	2.21 *
2	3.1	6	10	0.31	4.05	0.405	2.2
2	3.1	7	13	0.2386	5.5	0.423	2.25
2	3.1	8	17	0.1825	6.8	0.4	2.16
2	3.1	9	24.1	0.1285	10.2	0.423	2.2 *
3	4.4	5	7.8	0.567	3.2	0.41	2.2
3	4.4	6	10	0.44	4.2	0.42	2.25
3	4.4	7	13	0.3382	5.6	0.431	2.3
3	4.4	8	17	0.2588	7	0.412	2.2
3	4.4	9	24.1	0.1825	10	0.415	2.25 *

The dead time is determined from the unit-step response as $T_D = 0.2$ sec. Table 2 shows the values ω_0 and ζ obtained from Figs 4, 5, 6 on the basis of the t_i/t_j quotients.

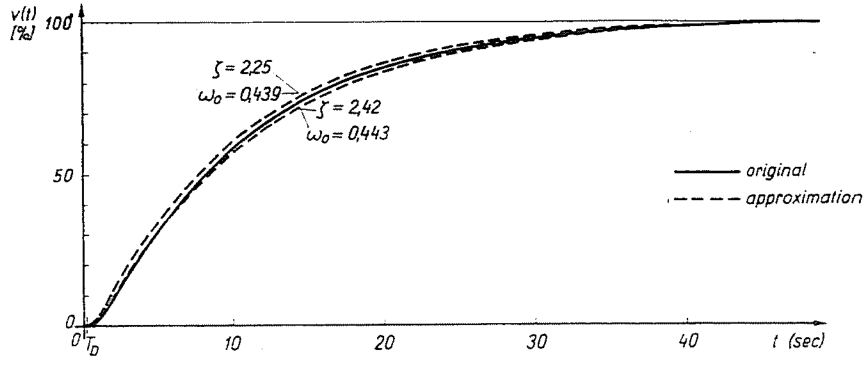

Fig. 13. Approximation of a system or process of third order. $T_a = 0.1s$; $T_b = 1s$; $T_c = 10s$. Unit-step response of the original process and of the approximative model obtained on the basis of the t_1/t_j quotients

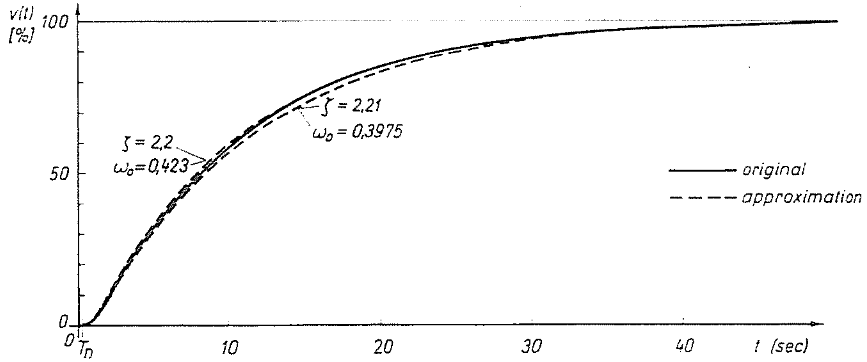

Fig. 14. Approximation of a system or process of third order. $T_a = 0.1s$; $T_a = 1s$; $T_c = 10s$ Unit-step response of the original process and of the approximative model obtained on the basis of the t_2/t_j quotients

The original and the approximation curves $v(t)$ are shown in Figs 13. 14, 15, the BODE-diagrams in Figs 16, 17, 18. On the basis of these diagrams it may be established that the gain margin of the approximation curve is only 90% of that of the original curve, so we are now on the safe side.

On the basis of both examples referring to systems with three time constants it may be established that the best approximation is given by the models of second order with dead time obtained from Fig. 6 with the help of the quotients t_3/t_j. The closer the time constants of the unknown systems are to each other, the better the approximation. But care is bidden by the fact that in such cases the gain margin may appear to be higher than it is in reality.

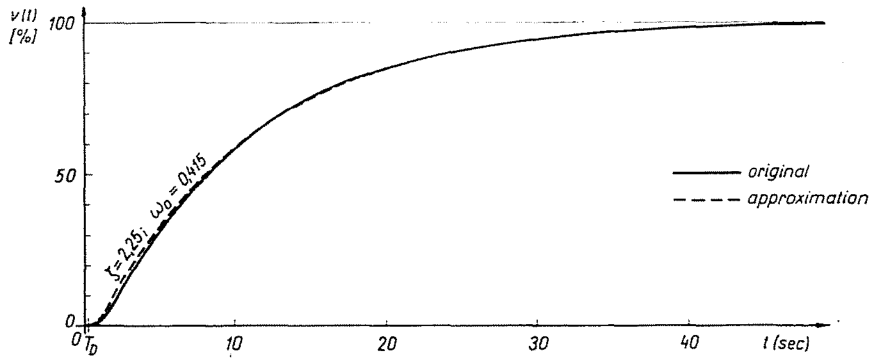

Fig. 15. Approximation of a system or process of third order. $T_a = 0.1s$; $T_b = 1s$; $T_c = 10s$. Unit-step response of the original process and of the approximative model obtained on the basis of the t_1/t_j quotients

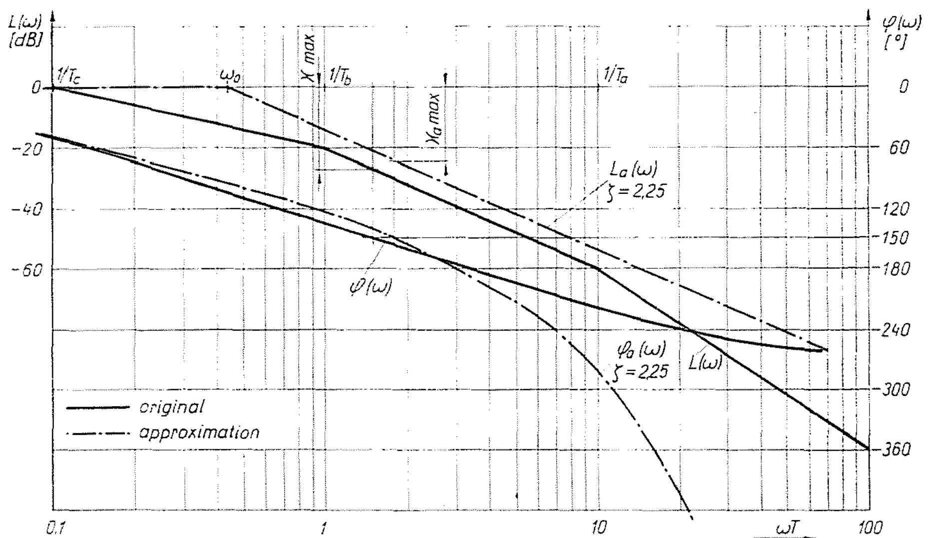

Fig. 16. Approximation of a system or process of third order. $T_a = 0.1s$; $T_b = 1s$; $T_c = 10s$. BODE-diagrams of the original process and of the approximative model obtained on the basis of the t_1/t_j quotients

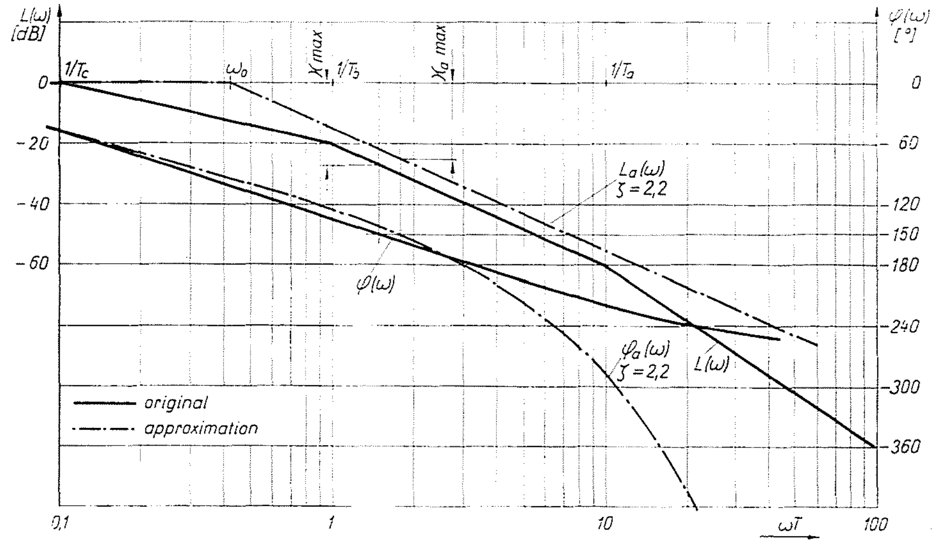

Fig. 17. Approximation of a system or process of third order. $T_a = 0.1s$; $T_b = 1s$; $T_c = 10s$. BODE-diagrams of the original process and of the approximative model obtained on the basis of the t_2/t_j quotients

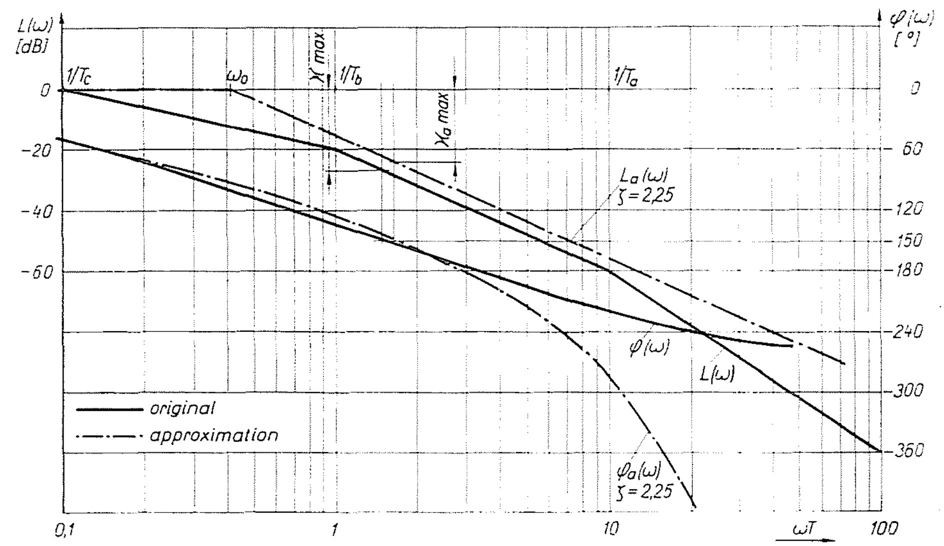

Fig. 18. Approximation of a system or process of third order. $T_a = 0.1s$; $T_b = 1s$; $T_c = 10s$ BODE-diagrams of the original process and of the approximative model obtained on the basis of the t_3/t_j quotients

Example 3

The previous examples represented investigations of overdamped systems. Let us examine now an underdamped system given by the transfer function of

$$G(s) = \frac{1}{(1 + 2\zeta sT + s^2 T^2)^2}$$

with $\zeta = 0.7$ and $T = 2$ sec. The dead time is determined from the unit-step response as $T_D = 2$ sec. Table 3 contains the values ω_0 and ζ determined on the basis of the t_i/t_j quotients from Figs 4, 5, 6. The original and the approximation $v(t)$ curves are shown in Figs 19, 20, 21 and the BODE-diagrams in Figs 22, 23, 24. These figures show that good approximations are obtained, with the exception of the one based on the quotient t_1/t_5.

The study of the unit-step response in the vicinity of the maximum overshot shows that the best approximations are obtained on the basis of the quotients t_2/t_j and t_3/t_j. From the BODE-plots it is also seen that the gain margin is greatest in the case of an approximation obtained on the basis of the quotient t_3/t_j. The original curve $\varphi(\omega)$ is approximated best by the curve $\varphi_a(\omega)$ in this case.

Table 3

i	$t_i[s]$	j	$t_j[s]$	t_i/t_j	$t_j\omega_0$	$\omega_0[1/s]$	ζ
1	1.55	5	4.4	0.3524	1.8	0.409	1.02 *
1	1.55	6	5	0.31	2.02	0.404	0.91
1	1.55	7	5.7	0.272	2.228	0.4	0.87
1	1.55	8	6.45	0.2405	2.55	0.3955	0.8
1	1.55	9	7.4	0.2092	2.94	0.3972	0.74 *
2	2.45	5	4.4	0.557	1.56	0.3543	0.76 *
2	2.45	6	5	0.49	1.76	0.352	0.687
2	2.45	7	5.7	0.43	2	0.3508	0.68
2	2.45	8	6.45	0.38	2.25	0.349	0.65
2	2.45	9	7.4	0.3312	2.56	0.346	0.64 *
3	3.15	5	4.4	0.416	1.5	0.341	0.67 *
3	3.15	6	5	0.63	1.7	0.34	0.63
3	3.15	7	5.7	0.553	1.91	0.335	0.625
3	3.15	8	6.45	0.485	2.2	0.341	0.62
3	3.15	9	7.4	0.426	2.5	0.3379	0.6 *

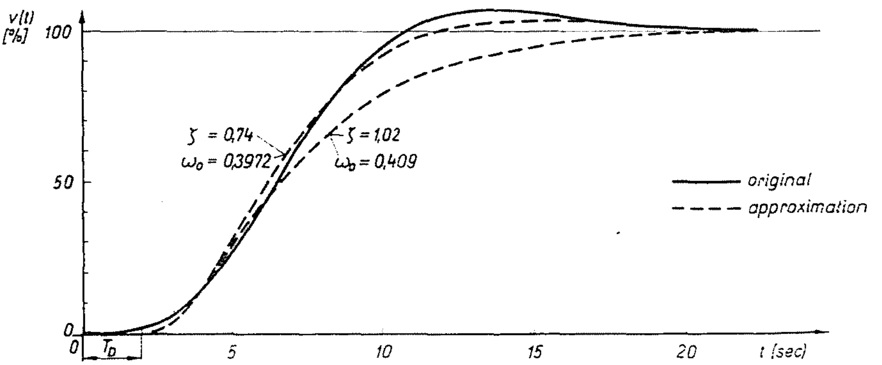

Fig. 19. Approximation of a system or process of fourth order. Unit-step response of the original process and of the approximative model obtained on the basis of the t_1/t_j quotients

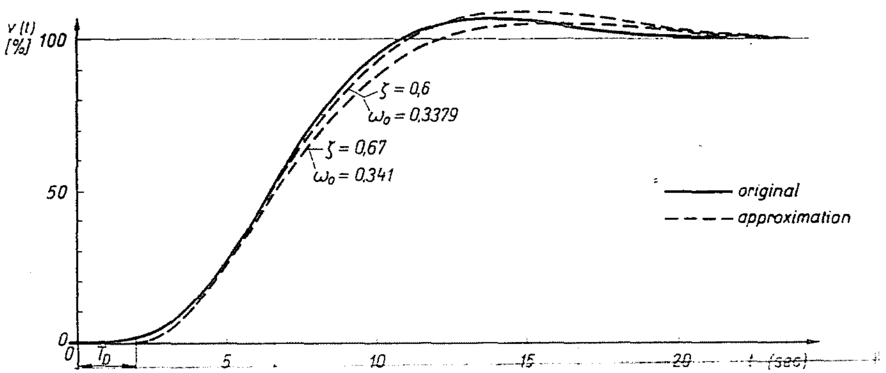

Fig. 20. Approximation of a system or process of fourth order. Unit-step response of the original process and of the approximative model obtained on the basis of the t_2/t_j quotients

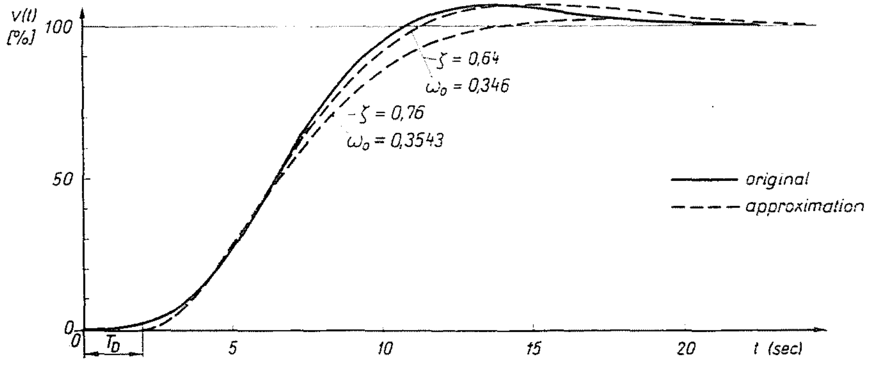

Fig. 21. Approximation of a system or process of fourth order. Unit-step resopnse of the original process and of the approximative model obtained on the basis of the t_3/t_j quotients

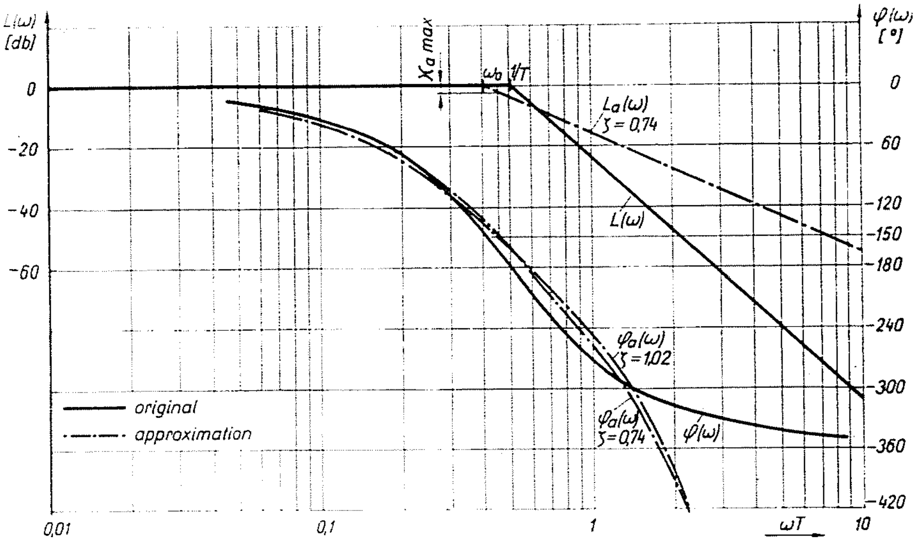

Fig. 22. Approximation of a system or process of fourth order. BODE-diagrams of the original process and of the approximative model obtained on the basis of the t_1/t_j quotients

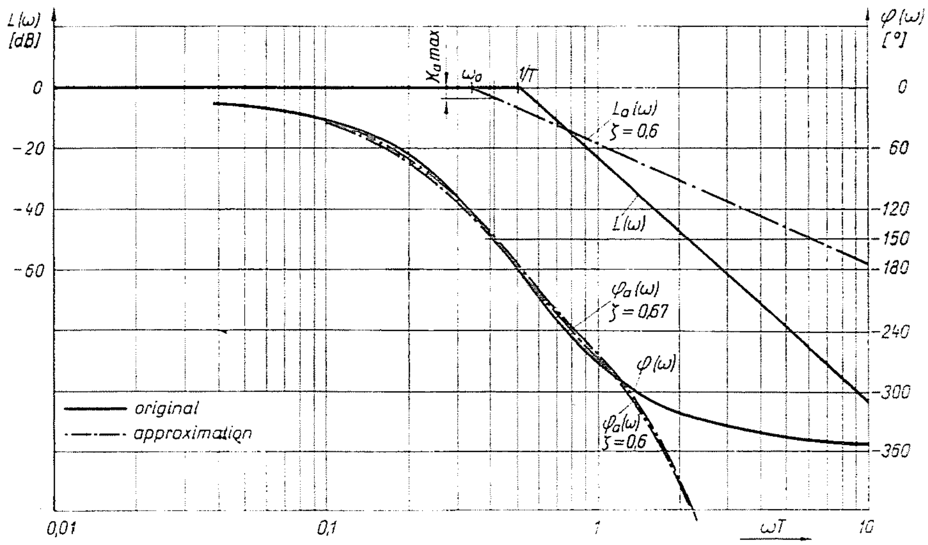

Fig. 23. Approximation of a system or process of fourth order. BODE-diagrams of the original process and of the approximative model obtained on the basis of the t_2/t_j quotients

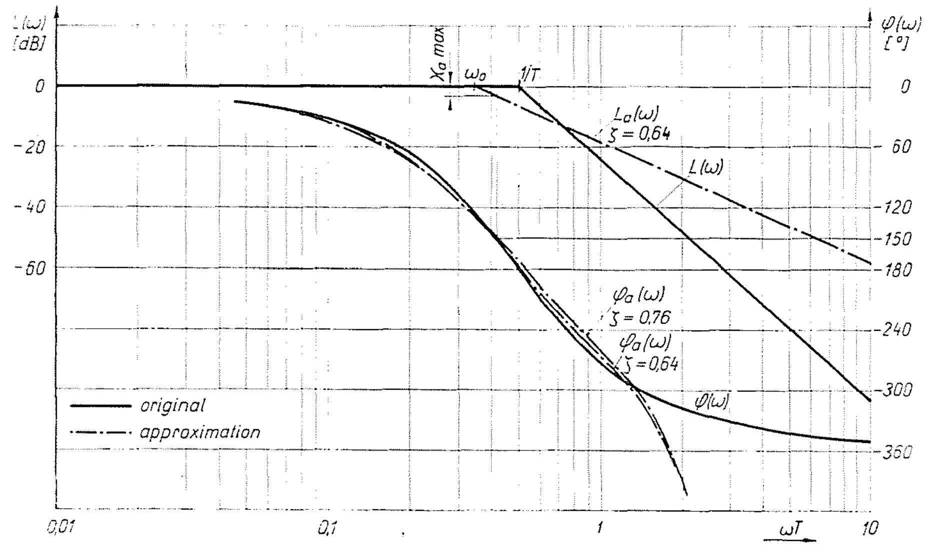

Fig. 24. Approximation of a system or process of fourth order. BODE-diagrams of the original process and of the approximative model obtained on the basis of the t_3/t_j quotients.

IV. Conclusions

1. The approximation of the unit-step response of a process depends on the relative positions of the time constants in the original system. The gain margin of the approximating model may be either smaller or greater than that of the original system as shown by the examples.

2. The best approximation (in the vicinity of the phase margin of 30°) is given by the models obtained on the basis of the quotients t_3/t_j, approximating either underdamped or overdamped processes.

3. The approximation is greatly influenced by the precision of reading of the values t_i and t_j off the unit-step response curves, a major source of error of the approximations based on the quotients t_1/t_j.

4. An error is caused also by the fact that by the construction method used for the determination of the dead time on the basis of the available unit-step response, the dead time is given with a broad scattering. This error may be considerably suppressed if it is possible to choose a low value time scale for the unit-step response.

On the basis of all the above it may be stated that the approximation is applicable in the cases of both underdamped and overdamped systems. It is mostly advisable for the investigations to be carried out with approximations obtained on the basis of the t_3/t_j quotients.

Summary

The determination of the approximative transfer function of a model function of a system applying precalculated nomograms is described. The method is based on the approximation of the process by a unit-step response of second order with dead time.

The determination of the approximation parameters is demonstrated on examples.

References

1. GBARBE, E. M.: Handbook of Automation Computation and Control Stimulation Method (22—48).
2. LASPE, CH. G.: Determining Frequency Response from Transient Response. Instruments and Control System. p. 125, December 1964.
3. GÖLDNER, K.: Zur Berechnung des Frequenzganges aus der Übergangsfunktion. Messen, Steuern, Regeln Nr. 12. p. 412—415.
4. OLDENBOURG, R. C.—SARTORIUS, H.: The Dynamics of Automatic Controls. The American Society of Mechanical Engineers, New York 276, 1948.
5. SMITH, C. L.—MURRILL, P. W.: Analytical Tuning of Underdamped Systems. ISA Journal **13**, 40.
6. MEYER, J. R.—WHITEHOUSE, G. D.—SMITH, C. L.—MURRILL, P. W.: Simplifying Process Response Approximation Instruments and Control System **40**, 76.
7. COX, J. B. et al.: A Practical Spectrum of DDC Chemical-Process Control Algorithms. ISA Journal **13**, 70.

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