ELECTRON MOTION IN A STATIONARY AXIALLY SYMMETRIC MAGNETIC FIELD VARYING LINEARLY ALONG THE AXIS

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We shall consider a stationary, axially symmetric magnetic field with linear axial distribution and non-zero initial intensity:

$$H(z) = H_0 + H_1 z \tag{1}$$

where H_0 and H_1 are constants. The field distribution (1) corresponds to a magnetic field h with the following components in cylindrical system of coordinates (r, ψ, z):

$$\left.\begin{aligned} h_r &= -\frac{H_1 r}{2} \\ h_\psi &= 0 \\ h_z &= H_0 + H_1 z \end{aligned}\right\} \tag{2}$$

(This is the so-called "cusp" magnetic field [1]).

1. The rigorous solution of the paraxial ray equation

Let us consider the paraxial motion of high velocity electrons with charge $-e$ and rest mass m in magnetic field (2). The Gaussian system of units is to be used. The electron source is situated at the $(r = r_0, \psi = \psi_0 = 0, z = z_0 = 0)$ point. The electron velocity is given by

$$v = \frac{\sqrt{\dfrac{2e}{m_0}\varphi_r}}{1 + \dfrac{e\varphi}{m_0 c^2}} = \sqrt{\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\psi}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} = \text{const}\,. \tag{3}$$

where $\varphi =$ const. is the electric potential, c is the velocity of light, and

$$\varphi_r = \varphi\left(1 + \frac{e\varphi}{2m_0c^2}\right) = \text{const}\,. \tag{4}$$

is the relativistic potential [2]. In special case of non-relativistic motion one can assume $\varphi_r \approx \varphi$ and $v \approx \sqrt{\frac{2e}{m_0}\varphi}$.

Since we consider the paraxial case, it may be assumed that

$$v \approx \frac{dz}{dt} = \dot{z} \tag{5}$$

Hence

$$\dot{r} = \frac{dr}{dt} \approx v\frac{dr}{dz} \quad \text{and} \quad \dot{\psi} = \frac{d\psi}{dt} \approx v\frac{d\psi}{dz} \tag{6}$$

The initial conditions at $z = 0$ are as follows:

$$\left.\begin{aligned} &r(0) = r_0 && \psi(0) = \psi_0 = 0 \\ &\left.\frac{dr}{dz}\right|_{z=0} = r_0' = \frac{\dot{r}_0}{v} && \left.\frac{d\psi}{dz}\right|_{z=0} = \psi_0' = \frac{\dot{\psi}_0}{v} \end{aligned}\right\} \tag{7}$$

The paraxial ray equation for the case of non-zero initial value of the axial magnetic field intensity and arbitrary initial conditions may be written in complex form as follows [2]:

$$\frac{d^2u}{dz^2} + \frac{eH^2(z)}{8m_0c^2\varphi_r}u = 0 \tag{8}$$

where

$$u(z) = r(z)\ \exp\ [i\chi(z)] \tag{9}$$

and

$$\chi(z) = \psi(z) - \frac{1}{c}\sqrt{\frac{e}{8m_0\varphi_r}}\int_0^z H(z)\,dz \tag{10}$$

($i = \sqrt{-1}$ is the imaginary unit).

At the initial point we have from (9), (10) and (7):

$$\chi(0) = \psi(0) = 0 \tag{11}$$

$$\left.\frac{d\chi}{dz}\right|_{z=0} = \psi_0' - \frac{1}{c}\sqrt{\frac{e}{8m_0\varphi_r}}H(0) \tag{12}$$

$$u(0) = r(0)\ \exp\ [i\chi(0)] = r_0 \tag{13}$$

and

$$\left.\frac{du}{dz}\right|_{z=0} = \left[\left.\frac{dr}{dz}\right|_{z=0} + ir(0)\left.\frac{d\chi}{dz}\right|_{z=0}\right]\exp[i\chi(0)] =$$

$$= r_0' + ir_0\left[\psi_0' - \frac{1}{c}\sqrt{\frac{e}{8m_0\varphi_r}}H(0)\right] \tag{14}$$

Let us substitute now the axial field distribution (1) into paraxial ray equation (8). We obtain

$$\frac{d^2u}{dz^2} + (k_0 + k_1 z)^2 u = 0 \tag{15}$$

where

$$k_0 = \frac{H_0}{2c}\sqrt{\frac{e}{2m_0\varphi_r}} \tag{16}$$

and

$$k_1 = \frac{H_1}{2c}\sqrt{\frac{e}{2m_0\varphi_r}} \tag{17}$$

We have the following initial conditions from (13) and (14):

$$u(0) = r_0 \tag{18}$$

and

$$\left.\frac{du}{dz}\right|_{z=0} = r_0' + ir_0(\psi_0' - k_0) \tag{19}$$

since $H(0) = H_0$ in this case.

Equation (15) has the following exact solution [3]:

$$u(z) = \sqrt{\frac{k_0}{k_1} + z}\left\{C_1 J_{1/4}\left[\frac{(k_0 + k_1 z)^2}{2k_1}\right] + C_2 J_{-1/4}\left[\frac{(k_0 + k_1 z)^2}{2k_1}\right]\right\} \tag{20}$$

where J_p is the Bessel function of 1st kind and p-th order; C_1 and C_2 are arbitrary constants. If $k_0 = 0$ and $\psi_0' = 0$, this solution corresponds to that given in [4].

Differentiating (20) with respect to z we have

$$\frac{du}{dz} = \left[\frac{(k_0 + k_1 z)^3}{k_1}\right]^{1/2}\left\{C_1 J_{-3/4}\left[\frac{(k_0 + k_1 z)^2}{2k_1}\right] - C_2 J_{3/4}\left[\frac{(k_0 + k_1 z)^2}{2k_1}\right]\right\} \tag{21}$$

Substituting now the initial conditions (18) and (19) into (20) and (21), and carrying out the necessary calculations one can obtain the values of the constants as follows:

$$C_1 = \frac{\pi}{2}\sqrt{\frac{k_0}{2k_1}}\left\{k_0 r_0 J_{3/4}\left(\frac{k_0^2}{2k_1}\right) + [r_0' + ir_0(\psi_0' - k_0)]J_{-1/4}\left(\frac{k_0^2}{2k_1}\right)\right\} \tag{22}$$

and

$$C_2 = \frac{\pi}{2}\sqrt{\frac{k_0}{2k_1}}\left\{k_0 r_0 J_{-3/4}\left(\frac{k_0^2}{2k_1}\right) - [r_0' + ir_0(\psi_0' - k_0)]J_{1/4}\left(\frac{k_0^2}{2k_1}\right)\right\} \tag{23}$$

Using (22) and (23), we can rewrite (20):

$$u(z) = \frac{\pi}{2k_1}\sqrt{\frac{k_0(k_0 + k_1 z)}{2}}\left\{k_0 r_0\left[J_{3/4}\left(\frac{k_0^2}{2k_1}\right)J_{1/4}(\zeta) + J_{-3/4}\left(\frac{k_0^2}{2k_1}\right)J_{-1/4}(\zeta)\right] + \right.$$

$$\left. + [r_0' + ir_0(\psi_0' - k_0)]\left[J_{-1/4}\left(\frac{k_0^2}{2k_1}\right)J_{1/4}(\zeta) - J_{1/4}\left(\frac{k_0^2}{2k_1}\right)J_{-1/4}(\zeta)\right]\right\} \tag{24}$$

where

$$\zeta = \frac{(k_0 + k_1 z)^2}{2k_1} \tag{25}$$

Expression (24) is the exact solution of Equation (15) with initial conditions (18) and (19). This solution corresponds to that given in [3] for the charged particle motion in a time-dependent uniform magnetic field.

Our next task is to return to the variables r and ψ. It follows from (9), (10), (1), (16) and (17) that

$$r = \sqrt{uu^*} = \sqrt{[Re(u)]^2 + [Im(u)]^2} \tag{26}$$

and

$$\psi = k_0 z + \frac{k_1 z^2}{2} + \operatorname{arc\,tg}\frac{Im(u)}{Re(u)} \tag{27}$$

where u^* is the conjugate of the complex number u, $Re(u)$ and $Im(u)$ are its real and imaginary parts, respectively.

Using (26), (27) and (24), we obtain the rigorous expressions which, together with (25), exactly determine the paraxial trajectory of an electron moving in the magnetic field (2):

$$r(z) = \frac{\pi}{2k_1}\sqrt{\frac{k_0(k_0+k_1z)}{2}}\Bigg\{r_0^2(\psi_0'-k_0)^2\Bigg[J_{-1/4}\left(\frac{k_0^2}{2k_1}\right)J_{1/4}(\zeta)-$$

$$-J_{1/4}\left(\frac{k_0^2}{2k_1}\right)J_{-1/4}(\zeta)\Bigg]^2+\Bigg[k_0r_0\left\langle J_{3/4}\left(\frac{k_0^2}{2k_1}\right)J_{1/4}(\zeta)+\right.$$

$$\left.+J_{-3/4}\left(\frac{k_0^2}{2k_1}\right)J_{-1/4}(\zeta)\right\rangle+r_0'\left\langle J_{-1/4}\left(\frac{k_0^2}{2k_1}\right)J_{1/4}(\zeta)-\right.$$

$$\left.-J_{1/4}\left(\frac{k_0^2}{2k_1}\right)J_{-1/4}(\zeta)\right\rangle\Bigg]^2\Bigg\}^{1/2} \tag{28}$$

and

$$\psi(z) = k_0z+\frac{k_1z^2}{2}+$$

$$+\operatorname{arc\,tg}\frac{r_0(\psi_0'-k_0)}{r_0'+k_0r_0\dfrac{J_{3/4}\left(\frac{k_0^2}{2k_1}\right)J_{1/4}(\zeta)+J_{-3/4}\left(\frac{k_0^2}{2k_1}\right)J_{-1/4}(\zeta)}{J_{-1/4}\left(\frac{k_0^2}{2k_1}\right)J_{1/4}(\zeta)-J_{1/4}\left(\frac{k_0^2}{2k_1}\right)J_{-1/4}(\zeta)}} \tag{29}$$

If the electron enters the magnetic field parallel to the z-axis, the initial velocity is $v_0 = \dot{z}_0$, so

$$r_0' = \psi_0' = 0 \tag{30}$$

Substituting (30) into (28) and (29) we obtain the trajectory as follows:

$$r(z) = \frac{\pi k_0^{3/2}}{2k_1}r_0\sqrt{\frac{k_0+k_1z}{2}}\Bigg\{\Bigg[J_{-1/4}\left(\frac{k_0^2}{2k_1}\right)J_{1/4}(\zeta)-$$

$$-J_{1/4}\left(\frac{k_0^2}{2k_1}\right)J_{-1/4}(\zeta)\Bigg]^2+\Bigg[J_{3/4}\left(\frac{k_0^2}{2k_1}\right)J_{1/4}(\zeta)+$$

$$+J_{-3/4}\left(\frac{k_0^2}{2k_1}\right)J_{-1/4}(\zeta)\Bigg]^2\Bigg\}^{1/2} \tag{31}$$

and

$$\psi(z) = k_0z+\frac{k_1z^2}{2}-$$

$$-\operatorname{arc\,tg}\frac{J_{-1/4}\left(\frac{k_0^2}{2k_1}\right)J_{1/4}(\zeta)-J_{1/4}\left(\frac{k_0^2}{2k_1}\right)J_{-1/4}(\zeta)}{J_{3/4}\left(\frac{k_0^2}{2k_1}\right)J_{1/4}(\zeta)+J_{-3/4}\left(\frac{k_0^2}{2k_1}\right)J_{-1/4}(\zeta)} \tag{32}$$

In special case of *zero initial field intensity* we have

$$H_0 = k_0 = 0 \tag{33}$$

and after necessary operations we obtain from (28), (29) and (25):

$$r(z) = \frac{\Gamma(1/4)}{2\sqrt{2}k_1^{1/4}}\sqrt{z}\left\{r_0^2\psi_0'^2 J_{1/4}^2\left(\frac{k_1 z^2}{2}\right) + \left[r_0' J_{1/4}\left(\frac{k_1 z^2}{2}\right) + \right.\right.$$
$$\left.\left. + \frac{\sqrt{2k_1}}{\pi} r_0 \Gamma^2(3/4)\, J_{-1/4}\left(\frac{k_1 z^2}{2}\right)\right]^2\right\}^{1/2} \tag{34}$$

and

$$\psi(z) = \frac{k_1 z^2}{2} + \operatorname{arc\,tg} \frac{r_0 \psi_0'}{r_0' + r_0 \dfrac{\sqrt{2k_1}}{\pi}\Gamma^2(3/4) \dfrac{J_{-1/4}\left(\frac{k_1 z^2}{2}\right)}{J_{1/4}\left(\frac{k_1 z^2}{2}\right)}} \tag{35}$$

where Γ (p) is the gamma function.

If the azimuthal initial velocity of the electrons is zero ($\psi_0' = 0$) in such a field, all the electrons with different values of r_0 and r_0' will move in a plane which is rotating around the z-axis. In this case we obtain from (35):

$$\psi(z) = \frac{k_1 z^2}{2} \tag{36}$$

independently of the initial conditions. Otherwise, in this case we need not use the complex variable u because $\chi \equiv 0$. Then we can consider the motion in a meridian plane common for all the electrons. In this plane negative values of r may occur formally while those are evidently positive when rotation of the plane is taken into consideration.

If the electron enters the field parallel to the z-axis, we obtain from (34) and (30):

$$r(z) = r_0 \frac{k_1^{1/4}\Gamma(3/4)}{\sqrt{2}} \sqrt{z} J_{-1/4}\left(\frac{k_1 z^2}{2}\right) \tag{37}$$

Using (36) and (37), it is easy to find a simple expression for the projection of the trajectory to the r, ψ plane:

$$r = r_0 \Gamma(3/4)\left(\frac{\psi}{2}\right)^{1/4} J_{-1/4}(\psi) \tag{38}$$

Expression (38) entirely coincides with the corresponding one given in [3] for charged particles moving in linearly time-dependent uniform magnetic field and having zero initial velocity.

The *uniform magnetic field* is another special case derived from (1) when

$$H_1 = k_1 = 0 \tag{39}$$

The solution of the paraxial equation becomes very simple:

$$r(z) = \sqrt{\left(r_0 \cos k_0 z + \frac{r_0}{k_0} \sin k_0 z\right)^2 + r_0^2 \left(\frac{\psi'_0}{k_0} - 1\right)^2 \sin^2 k_0 z} \tag{40}$$

and

$$\psi(z) = k_0 z + \operatorname{arc\,tg} \frac{r_0 \left(\frac{\psi'_0}{k_0} - 1\right) \sin k_0 z}{r_0 \cos k_0 z + \frac{r'_0}{k_0} \sin k_0 z} \tag{41}$$

From these expressions one can easily obtain all the well-known formulae regarding to the motion of charged particles in uniform magnetic field.

2. Electron-optical properties

The solution (20) of the paraxial ray equation may be written as follows:

$$u(z) = C_1\, u_1(z) + C_2\, u_2(z) \tag{42}$$

where

$$u_1(z) = \sqrt{\frac{k_0}{k_1} + z}\;\; J_{1/4}(\zeta) \tag{43}$$

and

$$u_2(z) = \sqrt{\frac{k_0}{k_1} + z}\;\; J_{-1/4}(\zeta) \tag{44}$$

$\zeta(z)$ is determined by (25).

The position z_{bn} of the n-th electron-optical image is determined by the following equation [2, 4]:

$$\frac{u_1(z_0)}{u_2(z_0)} = \frac{u_1(z_{bn})}{u_2(z_{bn})} \tag{45}$$

where z_0 is the object position. (Our condition $z_0 = 0$ is quite suitable because an arbitrary value of H_0 may be chosen.)

In case of non-zero initial magnetic field the value of the electron-optical magnification M_n is given by

$$M_n = \frac{r(z_{bn})}{r_0} = (-1)^n \frac{u_1(z_{bn})}{u_1(z_0)} \tag{46}$$

where n is the serial number of image.

Substituting (43) and (44) into (45) and (46) we obtain the first order electron-optical characteristics of the magnetic field (2):

$$\frac{J_{1/4}(\zeta_{bn})}{J_{-1/4}(\zeta_{bn})} = \frac{J_{1/4}\left(\frac{k_0^2}{2k_1}\right)}{J_{-1/4}\left(\frac{k_0^2}{2k_1}\right)} = A = \text{const.} \tag{47}$$

and

$$M_n = (-1)^n \sqrt{1 + \frac{k_1 z_{bn}}{k_0}} \; \frac{J_{1/4}(\zeta_{bn})}{J_{1/4}\left(\frac{k_0^2}{2k_1}\right)} \tag{48}$$

where

$$\zeta_{bn} = \frac{(k_0 + k_1 z_{bn})^2}{2k_1} \tag{49}$$

In special case of *zero initial field intensity* we have

$$J_{1/4}\left(\frac{k_1 z_{bn}^2}{2}\right) = 0 \tag{50}$$

and

$$M_n = \frac{(-1)^n}{\sqrt{2}} k_1^{1/4} \Gamma(3/4) \sqrt{z_{bn}}\, J_{-1/4}\left(\frac{k_1 z_{bn}^2}{2}\right) \tag{51}$$

For *large values of* ζ we can use the well-known asymptotic formula

$$J_p(\zeta) = \sqrt{\frac{2}{\pi\zeta}} \cos\left(\zeta - \frac{p\pi}{2} - \frac{\pi}{4}\right) \tag{52}$$

It follows from (43), (44) and (52) that in case of $\zeta \gg 1$ the trajectory may be calculated by means of the following expressions:

$$u_1(z) = \frac{2}{\sqrt{\pi(k_0 + k_1 z)}} \left(\sin\frac{\pi}{8} \cos\zeta + \cos\frac{\pi}{8} \sin\zeta\right) \tag{53}$$

and

$$u_2(z) = \frac{2}{\sqrt{\pi(k_0 + k_1 z)}} \left(\cos\frac{\pi}{8} \cos\zeta + \sin\frac{\pi}{8} \sin\zeta\right) \tag{54}$$

Using these formulae together with (47) and (48) we obtain *asymptotic expressions* for the image position and magnification:

$$\zeta_{bn} = \operatorname{arctg} \frac{A \cos \frac{\pi}{8} - \sin \frac{\pi}{8}}{\cos \frac{\pi}{8} - A \sin \frac{\pi}{8}} \tag{55}$$

and

$$M_n = \frac{2(-1)^n}{\sqrt{\pi} J_{1/4}\left(\frac{k_0^2}{2k_1}\right)} \frac{\sin \frac{\pi}{8} \cos \zeta_{bn} + \cos \frac{\pi}{8} \sin \zeta_{bn}}{\sqrt{\frac{k_0^2}{k_1} + k_0 z_{bn}}} \tag{56}$$

If the initial field intensity is zero, the asymptotic formulae become much simpler:

$$\frac{k_1 z_{bn}^2}{2} = \left(n - \frac{1}{8}\right)\pi \tag{57}$$

and

$$M_n = \frac{\Gamma(3/4)}{\sqrt{\pi} k_1^{1/4} \sqrt{z_{bn}}} \tag{58}$$

3. Examples

We have calculated various electron trajectories with $r_0 = 1$ cm, $\psi_0' = 0$ and r_0' being a parameter, for two different field forms.

In the first case $k_0 = 0.2/\text{cm}$ and $k_1 = 0.1/\text{cm}^2$. Electron trajectories are calculated from (28) and (29) and plotted in Fig. 1 and Fig. 2 for 5 different

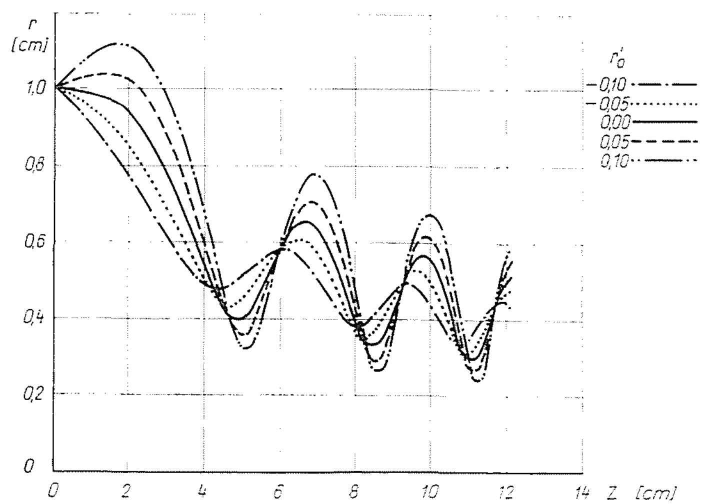

Fig. 1

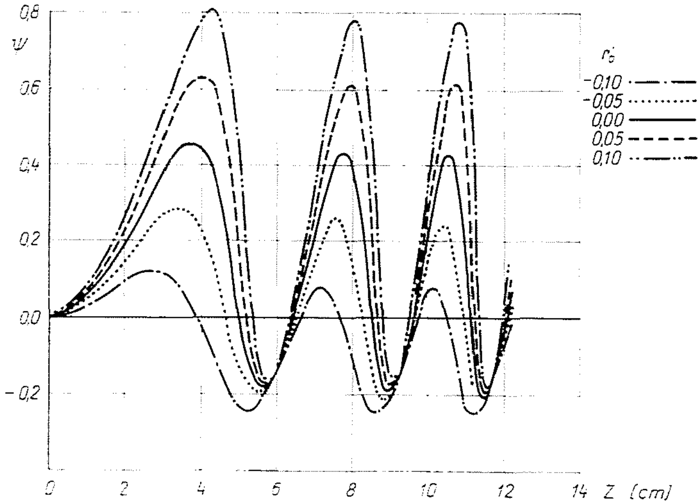

Fig. 2

Table 1

Order of image n	Image position z_{bn}(cm)	Magnification M_n
1	5.99	0.578
2	9.24	0.489
3	11.75	0.441
4	13.87	0.412
5	15.74	0.393
6	17.43	0.372
7	18.98	0.358
8	20.43	0.346

values of r_0'. (The notation of the trajectories is given in the figures.) One can see that in the plotted interval ($0 \leq z \leq 12$) 3 electron-optical images are formed. We have also calculated the image positions and magnifications from (47) and (48) (Table 1). Since these values for the 4th image have practically coincided with those calculated from the asymptotic formulae, the further image characteristics have been calculated from (55) and (56). It can be seen from Table 1 that the successive images become smaller and smaller.

The second example is given for the case of zero initial field intensity: $k_0 = 0$; $k_1 = 0.1/_{\text{cm}^2}$. These trajectories are calculated from (34) and (36). Since in this case $\psi(z)$ is a very simple parabolic function which is independent of the initial conditions, only $r(z)$ is plotted (Fig. 3). Values of r_0' are the same

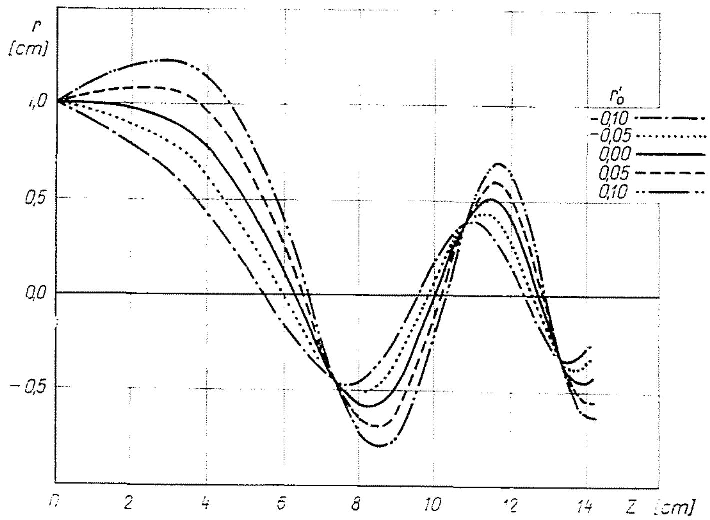

Fig. 3

Table 2

Order of image n	Image position z_{bn}(cm)	Magnification M_n
1	7.46	0.448
2	10.87	0.372
3	13.45	0.335
4	15.61	0.311
5	17.51	0.294
6	19.22	0.280
7	20.79	0.270
8	22.25	0.261

as in the previous example. As we have mentioned earlier, appearance of negative values of r is to be considered only formal. In the plotted interval ($0 \leq z \leq 14$) 3 images are formed. The image characteristics are also calculated from (50) and (51) (Table 2). Asymptotic formulae (57) and (58) give practically the same results for $n \geq 2$.

Other examples have been calculated by different authors using numerical [5 — 8] and approximative [1, 9] methods.

4. Concluding remarks

Long magnetic lenses are commonly used in beta-ray spectrometers. Thus, the rigorous solution of the trajectory equation has a practical importance. On the basis of the very close analogy between charged particle motion in time-dependent uniform magnetic field and that in non-uniform stationary magnetic field with corresponding distribution [10], the paraxial equation can be solved for some other cases too. These cases will be covered in the next papers of the author.

The solution given in this paper may easily be applied for an electrostatic field. Introducing the variable

$$R(z) = r(z)\, \varphi^{1/4}(z) \tag{59}$$

where $\varphi(z)$ is the axial potential, we can write [2] the paraxial equation in form of

$$\frac{d^2R}{dz^2} + \frac{3}{16}\left(\frac{\frac{d\varphi}{dz}}{\varphi}\right)^2 R = 0 \tag{60}$$

This equation has the same form as Equation (15) if

$$\frac{\sqrt{3}}{4}\,\frac{\frac{d\varphi}{dz}}{\varphi} = k_0 + k_1 z \tag{61}$$

Thus, the solution (24) for u is directly applicable as a solution for R if

$$\varphi(z) = \varphi_0 \exp\left[\frac{4}{\sqrt{3}}\left(k_0 z + \frac{k_1 z^2}{2}\right)\right] \tag{62}$$

where φ_0 is the potential on the axis at the initial point $z = 0$, k_0 and k_1 are arbitrary constants. For the special case of $k_0 = 0$ this solution had been obtained in [11].

On the basis of the solution presented in this paper we are able to suggest *a new magnetic lens model*. As a matter of fact, every magnetic field distribution can be approximated by a broken line (Fig. 4). Since the trajectory is known at every linear segment of the magnetic field distribution, the whole electron-optical analysis of the given magnetic lens can be carried out with great accuracy by linking together the series of trajectory segments. The thorough description of this new method and its application to a short magnetic lens is shortly to be published by the author.

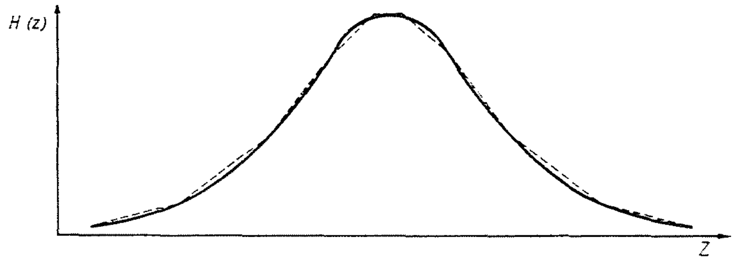

Fig. 4

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Summary

A rigorous solution of the paraxial equation of relativistic electron trajectories in axially symmetric magnetic field of the form $H(z) = H_0 + H_1 z$ is given for arbitrary initial conditions. Electron-optical properties are investigated. Special cases as well as numerical examples are given. The results obtained serve also as bases for some generalizations and a new magnetic lens model.

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