ELIMINATION OF ERRORS DUE TO THE LENGTH OF ELECTRIC RESISTANCE TYPE STRAIN GAUGES

By

GY. SZTOPA

Department for Technical Mechanics, Technical University, Budapest

(Received January 29, 1969)

Presented by Prof. Dr. A. Bosznay

1. Preface

Calculation and measurement methods have been sought for eliminating the errors caused by the non-zero length of electrical resistance type strain gauges on linearly flexible straight bars of either symmetric or asymmetric constant cross section, made of homogeneous and isotropic materials.

As proved by experience and the findings of the Research Institute for Measuring Techniques (Méréstechnikai Kutató Intézet), the larger the bonded surface and the better protected the applied strain gauge, the less is the probability and magnitude of errors due to bond imperfections or to slipping.

To prevent the said errors, particularly in outdoor measurements, it is advisable to use strain gauges with large tags, in spite of the fact that strain gauges of as little as 2—3 mm base length are currently available.

In the specific case when the side a of the surface area occupied by the resistance wire of the strain gauge — closely approximated by a parallelogram — is much smaller than its length l_0 (Fig. 1), the "nominal" specific strain ε_{bx} measured by the gauge can be obtained from the following relationship, neglecting the effect of the wire lengths normal to the direction of l_0:

$$\varepsilon_{bx} = \frac{1}{l_0} \int_0^{l_0} \varepsilon_x \, dx \tag{1}$$

where

ε_x is specific strain of the longitudinal axis of the gauge in direction x, pertaining to a point of the coordinate x as a function of x.

To define ε_x, first the matrix $\mathbf{U}$ of the stress tensor pertaining to an x point of coordinate x must be written down.

The elements of $\mathbf{U}$ are the functions of the unknown forces $\mathbf{F}_1 \ldots \mathbf{F}_n$ acting on the bar and regarded to be concentrated forces, the $\mathbf{f}_1 \ldots \mathbf{f}_m$ intensities of the distributed forces and of the $\mathbf{M}_1 \ldots \mathbf{M}_k$ moments, regarded to be concentrated. The functionality of the elements of the matrix $\mathbf{U}$ can be obtained on the basis of either the theory of elasticity or the elementary relations of the strength of materials, the degree of accuracy depending on the accuracy aimed at in the functionality relating to ε_x.

From the matrix $\mathbf{U}$, on the basis of the correlation between the strain and stress tensor, the specific elongation ε_x in the direction x, pertaining to the point of coordinate x, can be determined as a function of place.

Substituting ε_x back into (1) and performing the integration, a functionality is obtained between the rated specific strain ε_{bx} obtained by measurement and the unknown $\mathbf{F}_1 \ldots \mathbf{F}_n$, $\mathbf{f}_1 \ldots \mathbf{f}_m$, $\mathbf{M}_1 \ldots \mathbf{M}_k$ and the known length of the gauge l_0, the elasticity modulus E and the Poisson factor m.

$$\varepsilon_{bx} = \varepsilon_{bx}(\mathbf{F}_1 \ldots \mathbf{F}_n ; \mathbf{f}_1 \ldots \mathbf{f}_m ; \mathbf{M}_1 \ldots \mathbf{M}_K ; E ; m ; l_0) \tag{2}$$

Be g the number of unknown quantities in the so-called correction equation (2).

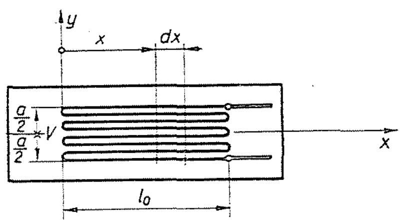

Fig. 1. Strain gauge with a resistance wire over an area approximable by a parallelogram

To determine the g unknown quantities, a g number of equations are necessary. They are produced in the following way.

Specific strains of $\varepsilon_{bx_1} \ldots \varepsilon_{bx_j}$ are measured by g strain gauges attached in appropriately selected points of the bar. Their specific strains yield a system of g equations of the (2) type.

$$\left.\begin{array}{l} \varepsilon_{bx_1} = \varepsilon_{bx_1}(\mathbf{F}_1 \ldots \mathbf{F}_n ; \mathbf{f}_1 \ldots \mathbf{f}_m ; \mathbf{M}_1 \ldots \mathbf{M}_K ; E ; m ; l_0) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \varepsilon_{bx_j} = \varepsilon_{bx_j}(\mathbf{F}_1 \ldots \mathbf{F}_n ; \mathbf{f}_1 \ldots \mathbf{f}_m ; \mathbf{M}_1 \ldots \mathbf{M}_K ; E ; m ; l_0) \end{array}\right\} \tag{3}$$

It follows from the above reasoning that the $\mathbf{F}_1 \ldots \mathbf{M}_k$ values obtained by solving the system of equations do not contain any more the error due to the length l_0 of the gauge.

The system of equations (3) permits to use strain gauges of any length, since the correction equations help to eliminate the source of error due to the dimension l_0.

This fact is particularly favourable from the aspect of the second paragraph of the Preface.

By a simple transformation, the right sides of Eqs (3) can be divided into sums of two terms each.

Namely be ε_{Vx} the specific strain in direction x pertaining to the end V of the gauge of length l_0 (Fig. 1). The specific strain pertaining to point ε_{Vx} is not, generally, identical with ε_{bx} measured by the gauge in the direction x viz. with the nominal specific strain. Let the difference between ε_{bx} and ε_{Vx} be ε_{kx} the specific correction strain in direction x

$$\varepsilon_{bx} = \varepsilon_{Vx} + \varepsilon_{kx} \tag{4}$$

Also ε_{kx} is a function of $\mathbf{F}_1 \ldots \mathbf{F}_n$; $\mathbf{f}_1 \ldots \mathbf{f}_m$; $\mathbf{M}_1 \ldots \mathbf{M}_k$; E; m; and l_0.

In the expression of ε_{kx} the unknown quantities $\mathbf{F}_1 \ldots \mathbf{F}_n$; $\mathbf{f}_1 \ldots \mathbf{f}_n$; $\mathbf{M}_1 \ldots \mathbf{M}_k$ may also be eliminated. This renders it possible to write down the functionality of ε_{kx} with the known quantities E; m; and l_0. This functionality includes the geometry of the tested bar.

$$\varepsilon_{kx} = \varepsilon_{kx}(E\,; m\,; l_0) \tag{5}$$

Since in possession of ε_{kx}, on the basis of the values ε_{Vx} measured according to (5) the specific strains pertaining to point ε_{Vx} can be determined, on hand of the well known methods the strain or stress tensor of point V can also be defined.

2. Examination of the deformation of a strain gauge attached to a cantilever prismatic beam

To support the theory outlined in the Preface, let us now examine the cantilever beam shown in Fig. 2, with the following assumptions:

a) its cross section is a narrow rectangle with a cross-sectional area A;

b) the beam depth $2c$ is of the same order as the length l;

c) the system of forces of the resultant $\mathbf{F}_1$ acting along the right side end cross-section of the beam is distributed according to the parabolic law of shear stress variation;

d) the resultant of the system of forces uniformly distributed along the cross-section of the beam at its right-side end, perpendicular to the cross-sectional plane is $\mathbf{F}_2$, and

e) a uniformly distributed vertical system of forces of intensity f acts along the beam top.

Let us follow now the deformation of a strain gauge of length l_0 attached parallel with the longitudinal axis of the bar, starting out from point A on the beam surface (Fig. 2).

Let the coordinates of point A by l_1, and y_1. At a point of the bar of optional coordinates x, y, the following stresses will arise:

Due to tension

$$\left.\begin{aligned} \sigma_{2x}(x, y) &= \frac{F_2}{A} \\ \sigma_{2y}(x, y) &= 0 \\ \tau_{xy} &= 0 \end{aligned}\right\} \tag{6}$$

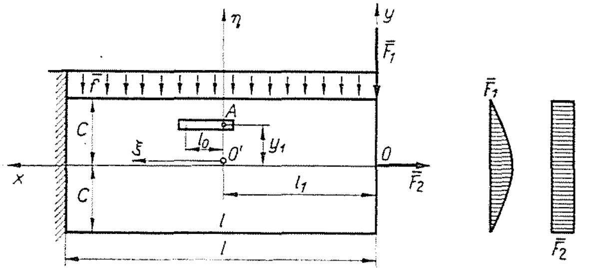

Fig. 2. Cantilever beam with a strain gauge of length l_0 at point A

Due to bending:

a) due to F_1:

$$\left.\begin{aligned} \sigma_{1x}(x, y) &= \frac{F_1 xy}{I} \\ \sigma_{1y}(x, y) &= 0 \\ \tau_{xy} &= \frac{F_1}{2I}(c^2 - y^2) \end{aligned}\right\} \tag{7}$$

b) due to f:

$$\left.\begin{aligned} \sigma_{px}(x, y) &= \frac{fx^2 y}{2I} - \frac{f}{2I}\left(\frac{2}{3}y^3 - \frac{2}{5}c^2 y\right) \\ \sigma_{py}(x, y) &= -\frac{f}{2I}\left(c^2 y - \frac{1}{3}y^3 + \frac{2}{3}c^3\right) \\ \tau_{xy} &= \frac{fx}{2I}(c^2 - y^2) \end{aligned}\right\} \tag{8}$$

In the above correlation I denotes the inertia moment of the cross-section referred to the bending axis.

Summing up the corresponding stresses:

$$\sigma_x = \frac{F_2}{A} + \frac{F_1 yx}{I} + \frac{fx^2 y}{2I} - \frac{f}{2I}\left(\frac{2}{3}y^3 - \frac{2}{5}c^2 y\right) \tag{9}$$

$$\sigma_y = \sigma_{2y} + \sigma_{1y} + \sigma_{py} = -\frac{f}{2I}\left(c^2 y - \frac{1}{3}y^3 + \frac{2}{3}c^3\right) \tag{10}$$

$$\tau_{xy} = \frac{F_1}{2I}(c^2 - y^2) + \frac{fx}{2I}(c^2 - y^2) \tag{11}$$

In accordance with the relationship of the strain and stress matrices as assumed in the Preface, and taking into consideration that $\sigma_z = 0$,

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \tag{12}$$

where

E modulus of elasticity,

ν reciprocal of the Poisson factor.

Substituting (9) and (10) into (12), the nominal specific elongation $\varepsilon_{b\xi}$ measured by the gauge will be:

$$\left.\begin{aligned}\varepsilon_{bx} = \varepsilon_{b\xi} = \frac{1}{E}\Bigg[&\frac{F_2}{A} + \frac{F_1 l_1}{I} y_1 + \frac{f l_1^2 y_1}{2I} + \\ &+ l_0\left(\frac{F_1 y_1}{2I} + \frac{f l_1 y_1}{2I} + \frac{f y_1}{6I} l_0\right) - \\ &- \frac{f}{2I}\left(\frac{2}{3} y_1^3 - \frac{2}{5} c^2 y_1\right) + \frac{\nu f}{2I}\left(c^2 y_1 - \frac{1}{3} y_1^3 + \frac{2}{3} c^3\right)\Bigg]\end{aligned}\right\} \tag{13}$$

(13) may be split to the sum of the specific elongation pertaining to point ε_{Ax} and the correction strain ε_{kx}.

$$\varepsilon_{kx} = \frac{l_0}{E}\left(\frac{F_1 y_1}{2I} + \frac{f l_1 y_1}{2I} + \frac{f y_1}{6I} l_0\right) \tag{14}$$

$$\left.\begin{aligned}\varepsilon_{Ax} = \frac{1}{E}\Bigg[&\frac{F_2}{A} + \frac{F_1 l_1}{I} y_1 + \frac{f l_1^2 y_1}{2I} - \\ &- \frac{f}{2I}\left(\frac{2}{3} y_1^3 - \frac{2}{5} c^2 y_1\right) + \frac{\nu f}{2I}\left(c^2 y_1 - \frac{1}{3} y_1^3 + \frac{2}{3} c^3\right)\Bigg]\end{aligned}\right\} \tag{15}$$

The relationship (15) for ε_{kx} includes the quantities depending on the geometry of the tested bar, the modulus of elasticity, F_1 and f from among the characteristics of the external loads and the length l_0 of the strain gauge. In spite of the smallness of the gauge length l_0 other numerical quantities in the expression for ε_{kx} may cause it to be of an order of magnitude which is either comparable to ε_{Ax} or non negligible in relation to it. This fact should be taken into consideration depending on the accuracy requirements and possibilities.

The problem may be, e.g. to define by extensometry the values of the unknown loads f, F_1 and F_2 acting on the beam in Fig. 2.

To define the three unknown quantities, three equations are necessary. Therefore optionally three gauges are attached on any chosen points of the bar surface parallelly with the x axis (Fig. 3).

From the nominal specific strains ε_{bx} of the gauges of known lengths l_0, attached at points A, B, and C, the correction equations may be set up on the basis of (14):

$$\left.\begin{aligned} \varepsilon_{bx_A} &= \varepsilon_{bx_A}(f, F_1, F_2, l_0) \\ \varepsilon_{bx_B} &= \varepsilon_{bx_B}(f, F_1, F_2, l_0) \\ \varepsilon_{bx_C} &= \varepsilon_{bx_C}(f, F_1, F_2, l_0) \end{aligned}\right\} \tag{16}$$

yielding the values of the unknown f, F_1 and F_2.

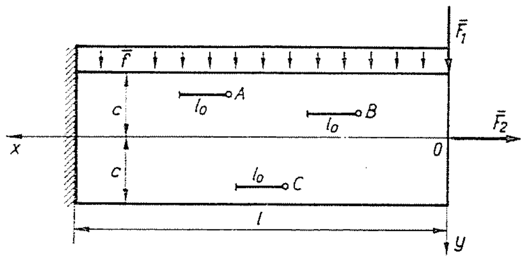

Fig. 3. Cantilever beam with strain gauges at points A, B and C

By this procedure the source of error due to the gauge length l_0 could be eliminated, provided the assumptions in the Preface are regarded to be valid.

3. Examination of the strain gauge deformation on a pierced plate under tension

Let us examine a plate under tension with a relatively small circular hole $\varnothing\ 2b$ drilled in its midline (Fig. 4).

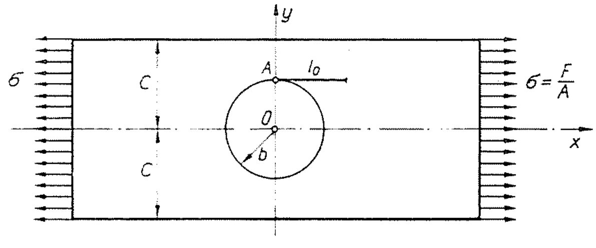

Fig. 4. Tensile plate with circular hole and strain gauge at point A

As known, stress peaks tend to arise at the hole, with values varying in function of the relationship b/c. Strain gauges are seldom used in such cases because, due to the finite non-zero length of the gauges, the peak stress values cannot be defined from the nominal specific strains measured by the gauge alone, without the application of some correction formula.

Author wanted to establish to what extent strain gauges of basic length l_0 (7 to 20 mm) lend themselves to define stresses at points under high stress gradients, or stress peaks, with the intermediary of correction formulae not yet known in literature.

To establish stress peak at point A of the hole in direction x (Fig. 4) the deformation of the strain gauge of length l_0 attached parallel with the longitudinal axis of the beam and strating from point A was examined.

$$\varepsilon_b = \frac{1}{l_0} \int_0^{l_0} \varepsilon_x \, dx \tag{17}$$

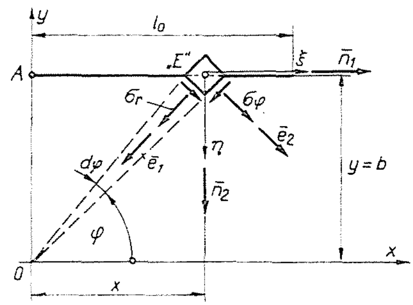

Fig. 5. Stresses arising in point E of the longitudinal axis of a strain gauge of length l_0

The nominal specific strain ε_b of the strain gauge was found to be

Expressed in terms of polar coordinates the following relationships are known:

$$\left.\begin{aligned}
\sigma_r &= \frac{\sigma}{2}\left[\left(1-\frac{b^2}{r^2}\right)+\left(1+3\frac{b^4}{r^4}-4\frac{b^2}{r^2}\right)\right]\cos 2\varphi \\
\sigma_\varphi &= \frac{\sigma}{2}\left(1+\frac{b^2}{r^2}\right)-\frac{\sigma}{2}\left(1-3\frac{b^4}{r^4}\right)\cos 2\varphi \\
\tau_{r\varphi} &= -\frac{\sigma}{2}\left(1-3\frac{b^4}{r^4}+2\frac{b^2}{r^2}\right)\sin 2\varphi
\end{aligned}\right\} \tag{18}$$

These stresses, pertaining to point E of coordinate x on the longitudinal axis of the gauge, are shown in Fig. 5.

Find the function ε_x parallel to the x axis, to define the relationship (17).

Matching to point E the ξ, η system seen in Fig. 5. in accordance with the relationship of strain and stress matrices, and corresponding to the plane stress state

$$\varepsilon_\xi = \frac{1}{E}(\sigma_\xi - \nu\sigma_\eta) \tag{19}$$

The stress matrix **F** characterising the stress state at in plane point E is:

$$F = \begin{bmatrix} \sigma_r & \tau_{r\varphi} \\ \tau_{r\varphi} & \sigma_\varphi \end{bmatrix} \tag{20}$$

Now find functions σ_ξ and σ_η for Eqs. (20); $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors of system (ξ, η) while $\mathbf{e}_1$ and $\mathbf{e}_2$ are unit vectors designating the direction of σ_r and σ_φ.

Thus:

$$\left.\begin{aligned} \sigma_\xi &= n_1 . F . n_1 \\ \sigma_\eta &= n_2 . F . n_2 \end{aligned}\right\} \tag{21}$$

Substituting (21) back into (19), a function is obtained for ε_ξ which includes the polar coordinates r and φ.

$$\left.\begin{aligned} \varepsilon_\xi = \frac{\sigma}{2E}\Bigg\{&\left[1 - \frac{b^2}{r^2} + 1 + 3\frac{b^4}{r^4} - 4\frac{b^2}{r^2}\cos 2\varphi\right]\left[\cos^2\varphi - \right. \\ &\left. - \nu\sin\varphi\right] + \left[1 + \frac{b^2}{r^2} - (1 + 3\frac{b^4}{r^4}\cos 2\varphi)\right]\left[\sin^2\varphi - \right. \\ &\left. - \nu\cos^2\varphi\right] + \left[-\left(1 - 3\frac{b^4}{r^4} + 2\frac{b^2}{r^2}\right)\sin 2\varphi\right][-\sin 2\varphi - \\ &- \nu\sin 2\varphi] \end{aligned}\right\} \tag{22}$$

In accordance with (17), $\varepsilon_\xi = \varepsilon_x$ must be integrated along a line of length l_0.

Therefore φ has to be eliminated and a coordinate x parallel l_0 introduced as a variable. This can be achieved by the following relationships of transformation:

$$\left.\begin{aligned} r\sin\varphi &= y = b \\ r\cos\varphi &= x \\ r^2 &= x^2 + b^2 \end{aligned}\right\} \tag{23}$$

Applying the transformation equations (23), function $\varepsilon_\xi(x)$ arises, whereby

$$\varepsilon_{bx} = \frac{1}{l_0}\int_0^{l_0} \varepsilon_\xi(x)\, dx \tag{24}$$

More precisely:

$$\varepsilon_{bx} = \frac{3\sigma}{E} = \frac{\sigma}{2E}[-4 + A + B + C] \tag{25}$$

In (25) the constants A, B and C stand for the following quantities:

$$\left.\begin{aligned} A &= \frac{b^2}{l_0^2 + b^2}(5 + \nu) \\ B &= -\frac{b^4}{(l_0^2 + b^2)^2}(5 + \nu) \\ C &= \frac{b^6}{(l_0^2 + b^2)^3}(4 + 4\nu) \end{aligned}\right\} \tag{26}$$

Since in (25) $\frac{3\sigma}{E}$ denotes the specific strain ε_{Kx} pertaining to point A of direction x and denoting the specific correction strain by ε_{kx} we obtain:

$$\varepsilon_{kx} = \frac{\sigma}{2E}[-4 + A + B + C], \tag{27}$$

$$\varepsilon_{bx} = \varepsilon_{Ax} + \varepsilon_{kx} \tag{28}$$

viz.: the constants A, B and C in (26) depend solely on the l_0 gauge length, the radius b of the bore and the reciprocal of the Poisson factor ν.

In consideration of (27) and (28), Eq. (25) may be written also in the following form

$$\varepsilon_{Ax} = \varepsilon_{bx}\left(1 - \frac{-4 + A + B + C}{2 + A + B + C}\right) \tag{29}$$

In the course of the checking tests three steel plates (each of a length of $l = 45$ cm, a width $2c = 6$ cm and a thickness $v = 0.5$ cm) have been prepared each pierced by circular holes of different diameters. In the first the hole diameter was $2b_1 = 0.5$ cm, in the second $2b_2 = 0.7$ cm and in the third $2b_3 = 1.3$ cm.

The experiments aimed at informing by measurements of the variations of the stress peak values arising under centric tension, depending on the hole diameter and the plate width by means of strain measurements and correction equations.

Tensile force F has been determined in tensile tester; the strain gauge stuck in point A in direction x was of type EMG 2359 TH 110 Ω and of a length $l_0 = 20$ mm.

PONOMARIOV [9], FÖPPL-SONNTAG [10] generally refer σ_{Ax} to the so-called mean σ_k stress, quotient of the actual tensile force acting on the beam by the smallest cross-sectional area A' of the pierced beam.

Table I contains the

$$\frac{\sigma_{AX}}{\sigma_K}$$

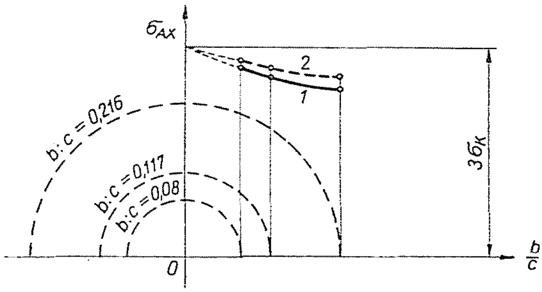

Fig. 6. Variations of σ_{Ax} as a function of b/c. The functional curve 1 is the one plotted by author, No. 2 was taken from literature

values obtained by extensometry at the three hole diameters compared to those by FÖPPL-SONNTAG [10]. Fig. 6, on the other hand, shows the strain gauge values σ_{Ax} vs. the quotient b/c compared to the FÖPPL-SONNTAG curve [10].

Table I

Values of $\frac{\sigma_{Ax}}{\sigma_k}$ as a function of b/c

2b [cm]	σ_{ax} [kp/cm²]	σ_{ax}/σ_K		$\frac{b}{c}$
		a)	b)	
0.5	2491	2.85	2.89	0.0835
0.7	2497	2.75	2.78	0.117
1.3	2603	2.54	2.60	0.216

Column a) shows authors' values. Column b) shows the values found in literature.

The applied Hungarian strain gauges were of the following types:

1. EMG 2359 TH, 110Ω; $l_{01} = 200$ mm
2. Kaliber, 110Ω; $l_{02} = 16$ mm
3. Kaliber, 115Ω; $l_{03} = 7$ mm

Denoting by h the quotient of the absolute value of the specific correction strains by that of the strain in the selected point:

$$h = \frac{|\varepsilon_{kx}|}{|\varepsilon_{Ax}|} \tag{30}$$

The percentage h as a function of different gauge lengths l_0 and of relations b/c for the three different bore diameters, have been compiled in Table II. It appears that the numerical values of the specific correction strain may amount to 41 or 65 per cent of the numerical values of the specific strain pertaining to a point, and that these values must not be disregarded in the exact definition of the specific strain pertaining to that point.

Table II

Variations in the h value, in function of b/c and l_0

l_0 [cm]	h %		
	$b/c = 0.083$	$b/c = 0.117$	$b/c = 0.216$
2	65.5	63.2	59.3
1.6	64.2	62.9	56.3
0.7	58	52.6	41

According to Table I, on the other hand, the described method lends itself to measure stresses, or stress peaks at points with high stress gradients, using strain gauges of even large basic lengths l_0, provided conditions specified in the Preface prevail.

Summary

The errors caused by the length of strain gauges can be eliminated if the function of the specific strain at an optionally selected point in the direction of the longitudinal axis of the gauge can be determined, depending on the external forces acting on the tested bar, making use of the relationship between strain and stress tensors. Integrating the function of the specific strain along the gauge length, a functionality can be created between the so-called nominal specific strain as measured by the gauge and the unknown external beam loads. Setting up as many functionalities termed correction equations as there are unknown quantities, the unknown quantities can be defined from the equations.

References

1. Filonenko, M. M.—Boroditsch: Festigkeitslehre II. Verlag Technik, Berlin, 1952.
2. Heeringer, J.: Nyúlásmérő ellenállás és méréstechnikája (Resistance-type strain gauge and the technique of its use). MTI 1954.
3. Bezuhov, N. I.: Bevezetés a rugalmasságtanba és a képlékenységtanba (Introduction to the theories of elasticity and plasticity). Tankönyvkiadó, Budapest, 1952.
4. Grave, H. F.: Elektrische Messung nicht-elektrischer Größen. Leipzig 1962, Akademische V. G.
5. Neuber, H.: Kerbspannungslehre. Springer Verlag, 1958.
6. Gröbner, W.—Hofreiter, N.: Integraltafel. Springer Verlag, 1949.
7. Mangoldt—Knopp: Einführung in die höhere Mathematik. S. Hirzel Verlag, Leipzig, 1954.
8. Sztopa, G.: Mérési módszerek szimmetrikus és aszimmetrikus keresztmetszetű rudak összetett igénybevételének ellenőrzésére (Measurement methods for compound stresses acting on bars with symmetric and asymmetric cross-section). Mérés és Automatika, 1963.6.
9. Ponomarjov, S. D.: Szilárdsági számítások a gépészetben 5. (Strength calculations in mechanical engineering). Műszaki Könyvkiadó, Budapest, 1965.
10. Föppl—Sonntag: Tafeln und Tabellen zur Festigkeitslehre. Verlag von R. Oldenbourg, München, 1951.

Dr. Gyula Sztopa, Budapest Hungary