INVESTIGATIONS ON SIMULTANEOUS OPERATION OF A CLUSTER OF CIRCUIT-BREAKERS

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In designing the protective scheme of electrical systems the following problem has been encountered: What is the probability of the required operation sequence in case of simultaneous application of a cluster of circuit-breakers in different network configurations? The arising questions are as follow:

What is the probability of at least one of two circuit-breakers in series (Fig. 1) breaking up to the time t,

— if the frequency functions of the breaking times of both circuit-breakers are identical;

— if the frequency functions of the breaking times of both circuit-breakers are not identical, but lag in time?

What is the relation between the number of the circuit-breakers in series (Fig. 2) and the probability of at least one of them breaking up to the time t, supposed that the frequency functions of the breaking times of the circuit-breakers are identical?

In case of two circuit-breakers in series (Fig. 3) what is the probability of A breaking earlier than B,

— if the frequency functions of the breaking times of both circuit-breakers are identical;

— if their frequency functions are not identical?

What is the probability of one of a cluster of circuit-breakers A in series breaking earlier than B (Fig. 4), supposed that the frequency functions of the breaking times are identical?

These questions can be answered by applying the principles of probability theory.

1. The first problem is the following: how can the resultant breaking time be influenced by applying two circuit-breakers in series? The arrangement of both circuit-breakers is illustrated in Fig. 1.

Let us examine the probability of at least one of the circuit-breakers breaking within the time t. The following presumptions are herewith adopted:

The operations of both circuit-breakers are independent.

The frequency function of the breaking times is of normal distribution.

By "breaking time" we mean the time from the event inducing the operation of the circuit-breaker to the ignition of the arc. (This definition departs from the standard definition, but it is more convenient for calculations.)

The frequency function of the breaking times of circuit-breaker A is $f_A(t)$.

The frequency function of the breaking times of circuit-breaker B is $f_B(t)$.

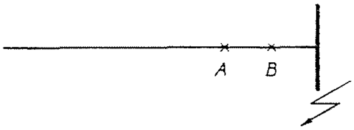

Fig. 1

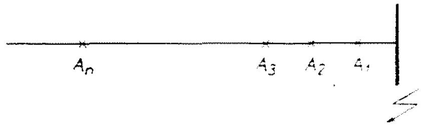

Fig. 2

The corresponding distribution functions are: $F_A(t)$; $F_B(t)$.

The question is: what will the combined frequency function of both circuit-breakers be like? It is required to know, what is the probability of the event that circuit-breaker A breaks at the time t without B having broken earlier.

The probability of A breaking in the time interval $t < \vartheta < t + dt$ is:

$$P[A(t,t + dt)] = f_A(t)dt$$

Fig. 3

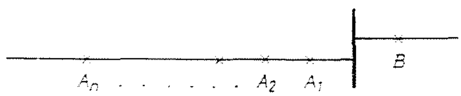

Fig. 4

The probability of B not breaking before the time t is:

$$P[\bar{B}(\tau < t)] = 1 - F_B(t)$$

The probability of the simultaneous occurrence of both events can be calculated under the presumption that the events are independent:

$$P[A(t,t + dt) \cap \bar{B}(\tau < t)] = f_A(t) \cdot [1 - F_B(t)]\, dt \tag{1.1}$$

Inverting the preceding sequence: the probability of B breaking in the time interval $t < \vartheta < t + dt$ without A having broken before the time t is:

$$P[B(t) \cap \bar{A}(\tau < t)] = f_B(t) \cdot [1 - F_A(t)]\, dt \tag{1.1a}$$

The probability of the simultaneous ignition of the arcs in both circuit breakers is zero. As a consequence, the probability of one of the circuit-breakers breaking in the time interval $t < \vartheta < t + dt$ earlier than the other is:

$$f_{AB}(t)dt = f_A(t)[1 - F_B(t)]\, dt + f_B(t)[1 - F_A(t)]dt \tag{1.2}$$

If the frequency functions of the breaking times of both circuit-breakers are identical, this frequency function takes a simpler form:

$$f_{AA}(t)dt = 2f_A(t)\, [1 - F_A(t)]\, dt \tag{1.3}$$

Let us examine the distribution functions of frequency functions (1.2) and (1.3):

$$F_{AB}(t) = \int_{-\infty}^{t} f_{AB}(\tau)\, d\tau = \int_{-\infty}^{t} f_A(\tau)\, d\tau + \int_{-\infty}^{t} f_B(\tau)\, d\tau - \int_{-\infty}^{t} f_A(\tau)\, F_B(\tau)\, d\tau -$$

$$- \int_{-\infty}^{t} f_B(\tau)\, F_A(\tau)\, d\tau$$

since:

$$f(t) = \frac{dF(t)}{dt}$$

and:

$$\int_{-\infty}^{t} f_A(\tau)\, F_B(\tau)\, d\tau = \int_{-\infty}^{t} \frac{dF_A(\tau)}{d\tau} F_B(\tau)\, d\tau = F_A(t)\, F_B(\tau) - \int_{-\infty}^{t} \frac{F_B(\tau)}{d\tau} F_A(\tau)\, d\tau$$

Hence:

$$F_{AB}(t) = F_A(t) + F_B(t) - F_A(t)\, F_B(t) \tag{1.4}$$

If frequency functions of both circuit-breakers are identical, then the distribution function is:

$$F_{AA}(t) = 2F_A(t) - F_A^2(t) \tag{1.5}$$

The question was, how the resultant breaking time could be influenced by applying two or more circuit-breakers in series. The resultant breaking time can be expressed by the expected value of the distribution, therefore the expected value of the combined distribution function of two or more circuit-breakers must be found.

The frequency function of a single circuit-breaker is illustrated in Fig. 6. Connecting two circuit-breakers with identical frequency functions in series, their combined frequency function $f_{AA}(t)$ decreases to the value $- 0.5\, \sigma$

as illustrated in the figure; this value is characteristic of the decrease of the resultant breaking time. If the frequency functions of both circuit-breakers are not identical, the combined frequency function is $f_{AB}(t)$ (Fig. 7). Its expected value plotted against the shift between the expected values of both distribution

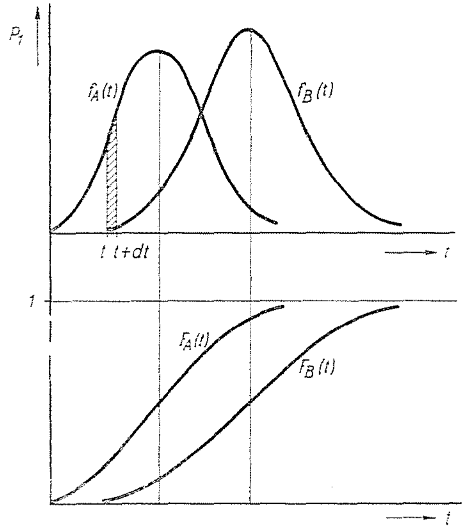

Fig. 5

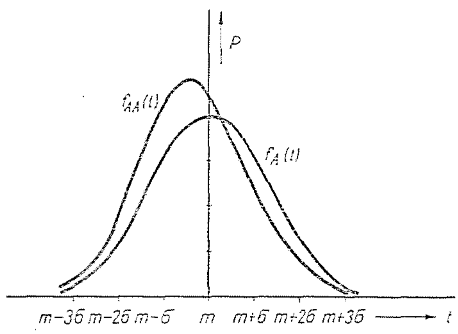

Fig. 6

functions is shown in Fig. 8. This latter has been drawn under the presumption that the dispersions of both distributions are identical, their expected values, however, are different. The difference between both expected values — the lag — has been expressed the following way: by how much is the expected value for the higher-rate circuit-breaker (B) exceeded by that for the other one (A).

2. The second problem is the following: how can the resultant breaking time or rather the expected value of the breaking time be influenced by con-

necting a cluster of circuit-breakers with identical distribution functions in series?

The frequency function for two circuit-breakers is described by relation (1.3).

The frequency function for three circuit-breakers starting from relations (1.3) and (1.5) is the following:

$$f_{3A}(t) = 3f_A(t)\left[1 - F_{AA}(t)\right] = 3f_A(t)\left\{1 - \left[2F_A(t) - F_A^2(t)\right]\right\} \qquad (2.1)$$

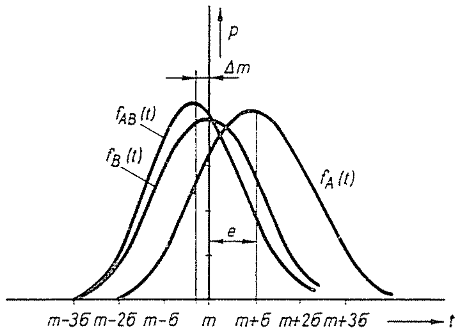

Fig. 7

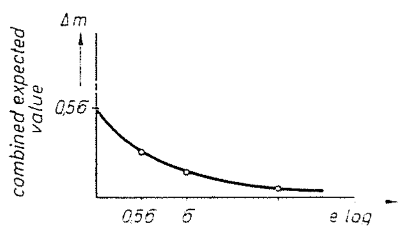

Fig. 8

The corresponding distribution function is:

$$F_{3A}(t) = \int_{-\infty}^{t} 3f_A(\tau)\left[1 - 2F_A(\tau) + F_A^2(\tau)\right] d\tau = 3F_A(t) - 3F_A^2(t) + F_A^3(t) \qquad (2.2)$$

In case of four circuit-breakers:

$$f_{4A}(t) = 4f_A(t)\left\{1 - \left[3F_A(t) - 3F_A^2(t) + F_A^3(t)\right]\right\} \qquad (2.3)$$

$$F_{4A}(t) = \int_{-\infty}^{t} 4f_A(\tau)\left\{1 - \left[3F_A(\tau) - 3F_A^2(\tau) + F_A^3(\tau)\right]\right\} d\tau =$$

$$= 4F_A(t) - 6F_A^2(t) + 4F_A^3(t) - F_A^4(t)$$

In case of n circuit-breakers:

$$f_{nA}(t) = nf_A(t)\Big\{1 - \Big[\binom{n-1}{1} F_A(t) - \binom{n-1}{2} F_A^2(t) + \ldots$$
$$\ldots + (-1)^{(k-1)} \binom{n-1}{k} F_A^k(t) + \ldots \pm \binom{n-1}{n-1} F_A^{(n-1)}(t)\Big]\Big\} \tag{2.4}$$

The freqency function curves showing the expected values of the resultant distribution in function of the number of the circuit-breakers in series, are illustrated in Fig. 9. It can be stated that the breaking time decreases with the number of the circuit-breakers in series.

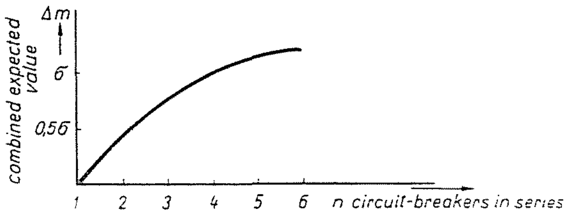

Fig. 9

The rate of the breaking time decrease caused by connecting a cluster of circuit-breakers in series, can be determined on the basis of the preceding investigations.

3. The third problem can be put as follows: what is the probability of a distinguished one out of two circuit-breakers (Fig. 3.) breaking earlier than the other one?

Relation (1.1) gives the probability of circuit-breaker A breaking in the time interval $t < \vartheta < t + dt$ without B having broken before the time t. From this relation the probability of B breaking earlier than A in the full time spectrum can be obtained:

$$P[A \cap \bar{B}] = \int_{-\infty}^{\infty} f_A(t)\,[1 - F_B(t)]\,dt \tag{3.1}$$

This integration can easily be performed if the frequency functions of both circuit-breakers are identical:

$$P[A \cap \bar{B}] = \int_{-\infty}^{\infty} f_A(t)[1 - F_A(t)]\,dt = \Big[F_A(t) - \frac{1}{2} F_A^2(t)\Big]_{-\infty}^{\infty} = \frac{1}{2}$$

If the frequency functions of both circuit-breakers are not identical, the integral is tedious to calculate, so it is advisable to apply a graphical method or a computer.

Suppose the frequency functions of both circuit-breakers differ only by their expected values, i.e. the expected value of the breaking time of one of the circuit-breakers is lower or higher than that of the other. Such a case is illustrated in Fig. 10, in which the expected value of the frequency function of circuit-breaker B is by σ higher than that of A. The function

$$f_A(t)\,[1 - F_B(t)] = f_{res}(t)$$

is illustrated in the mentioned figure; the integral of this function results in the required probability.

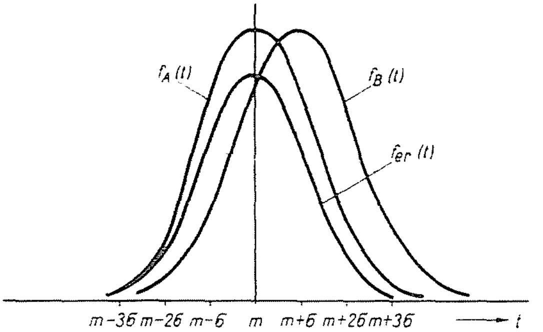

Fig. 10

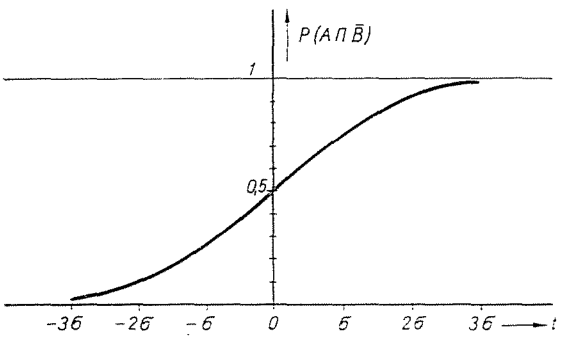

Fig. 11

The probability of circuit-breaker A breaking earlier than B illustrated in Fig. 11; the curve is plotted against the expected value for circuit-breaker B.

4. The problem as put in the prefatory part, is the following: what is the probability of one of the circuit-breakers A_i in series (Fig. 4) breaking earlier than circuit-breaker B?

In Paragraph 3 the case was dealt with when cluster A consisted of a single circuit-breaker. In this case the wanted probability is:

$$P[A \cap \bar{B}] = \int_{-\infty}^{\infty} f_A(t)\,[1 - F_B(t)]\,dt$$

If instead of A there are two circuit-breakers in series, the wanted probability is:

$$P[A_1 \cup A_2 \cap \bar{B}] = \int_{-\infty}^{\infty} f_{A_1 A_2}(t)\,[1 - F_B(t)]\,dt \tag{3.2}$$

in which according to (1.2):

$$f_{A_1 A_2}(t) = f_{A_1}(t)\,[1 - F_{A_2}(t)] + f_{A_2}(t)\,[1 - F_{A_1}(t)]$$

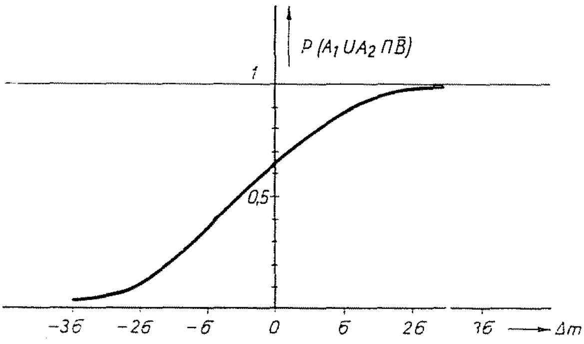

Fig. 12

If the frequency functions of circuit-breakers A_1 and A_2 are identical, then according to relation (1.3):

$$P[A_1 \cup A_2 \cap \bar{B}] = \int_{-\infty}^{\infty} 2 f_A(t)\,[1 - F_A(t)]\,[1 - F_B(t)]\,dt \tag{3.3}$$

If also circuit-breaker B has the same frequency function, then:

$$P[A_1 \cup A_2 \cap \bar{B}] = \int_{-\infty}^{\infty} 2 f_A(t)\,[1 - F_A(t)]^2\,dt = -\frac{2}{3}\left[\left\{1 - F_A(t)\right\}^3\right]_{-\infty}^{\infty} = \frac{2}{3}$$

Should the frequency function of circuit-breaker B be different from that of circuit-breakers A, then it is again advisable to integrate by a graphical method or by means of a computer.

After this integration has been done, Fig. 12 illustrates the probability of circuit-breaker A_1 or A_2 breaking earlier than B, supposed that the frequency functions of circuit-breakers A are identical, and the expected value of the breaking time of circuit-breaker B is by the time m longer but its dispersion is the same as that of circuit-breakers A.

The probability of at least one of the circuit-breakers A breaking earlier than B can be calculated in a similar way, under the presumption that the frequency functions of all circuit-breakers are identical. This is nothing else but an improvement of relation (3.4):

$$P[A_1 \cup A_2 \cup \ldots \cup A_n \cap \bar{B}] = \frac{n}{1+n} \tag{3.5}$$

Other possibilities also present themselves for the solution of the problems raised; the principles of these can be found in the Appendix.

Normal distribution of the breaking times was the presumption underlying our investigations. This presumption can, however, be considered only as an approximation, as the value of the breaking time cannot be negative. It would be more correct to take a reduced normal distribution into consideration, this would, however, make calculations more difficult. How correct this approximation be, can be judged only after determination of the actual distribution, by means of measurements. If the dispersion is small as compared with the expected value, then the approximation is correct. Parameters of the normal distribution (expected value, dispersion) are not known even by accepting this presumption. Consequently, in case of an actual application, investigations are to be made for the determination of the distribution of breaking times. The investigations commenced here are to be continued in this direction.

Appendix I

The problems put in the prefactory part can be solved by the application of an other principle, too. The train of thought of this kind of solution is presented in the following paragraphs.

Problem I: What is the probability of at least one of n circuit-breakers in series breaking before the time t?

The opposite of the event that at least one of a cluster of circuit-breakers breaks is that none of them breaks:

$$[\overline{A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n}] = \bar{A}_1 \cap \bar{A}_2 \cap \ldots \cap \bar{A}_n$$

The probability of these events is:

$$P[A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n] = 1 - P = [\bar{A}_1 \cap \bar{A}_2 \cap \ldots \cap \bar{A}_n]$$

Let the distribution function $F_i(t)$ of the breaking time of the circuit-breaker A_i be known.

Accepting the presumption that the breakings by each of the circuit-breakers are independent events, the distribution function of the event $[\bar{A}_1 \cap \bar{A}_2 \cap \ldots \cap \bar{A}_n]$ is:

$$F[\bar{A}_1 \cap \bar{A}_2 \cap \ldots \cap \bar{A}_n](t) = \\ = [1 - F_{A_1}(t)] \cdot [1 - F_{A_2}(t)] \cdot \ldots \cdot [1 - F_{A_n}(t)] \tag{F.1}$$

and from this relation:

$$F[A_1 \cup A_2 \cup \ldots \cup A_n](t) = \\ = 1 - [1 - F_{A_1}(t)] \cdot [1 - F_{A_2}(t)] \cdot \ldots \cdot [1 - F_{A_n}(t)] \qquad \text{(F.2)}$$

This latter relation is nothing else but a special case of the general probability theorem of JORDÁN.

This theorem presents namely the probability of at least k occurring out of n events. In our case $k = 1$.

For two circuit-breakers this relation is written as follows:

$$F_{[A \cup B]}(t) = 1 - [1 - F_A(t)] \cdot [1 - F_B(t)] = F_A(t) + F_B(t) - F_A(t) F_B(t) \quad \text{(F.3)}$$

This relation is identical with R. (1.4).

By means of relation (F.3) questions 1 and 2 of the prefatory part can be solved. Questions 3 and 4 can be solved besides the way described above, also by applying the following principle:

Problem II: What is the probability of a distinguished one (A) out of two circuit-breakers in series breaking earlier than the other one (B)?

Let the distribution functions of both circuit-breakers be known:

$$f_A(t) ; f_B(t)$$

In order to simplify the calculation a different designation is applied:

$$f_A(t) = f(t)$$
$$f_B(t) = g(\vartheta)$$

Investigating both frequency functions simultaneously, it is obvious that circuit-breaker A breaks earlier than B if

$$t - \vartheta < 0 \qquad \text{(F. 4)}$$

The following probability is therefore wanted:

$$P[(t - \vartheta) < 0]$$

For this purpose the frequency function of random variable $(t - \vartheta)$ must be found. This can be obtained by combining the distributions.

Let random variable $(t - \vartheta)$ be denoted by

$$\xi = t - \vartheta$$

The two frequency functions being independent in our presumption, the combined distribution of t and ϑ is:

$$h(t, \vartheta) = f(t)g(\vartheta)$$

The combined distribution function of both variables is:

$$P[\xi < z] = R(z) = \iint_E h(t, \vartheta)\, dt\, d\vartheta = \iint_E f(t)\, g(\vartheta)\, dt\, d\vartheta$$

After introducing the parameters

$$u = t - \vartheta$$
$$v = \vartheta$$

the distribution function takes the following form:

$$R(z) = \iint_E f(t)\, g(\vartheta)\, dt\, d\vartheta = \iint_H f(u + v)\, g(v)\, du\, dv =$$
$$= \int_{-\infty}^{Z} \int_{-\infty}^{\infty} f(u + v)\, g(v)\, dv\, du$$

from which the frequency function of variable z is:

$$r(z) = \int_{-\infty}^{\infty} f(u + v)\, g(v)\, dv = \int_{-\infty}^{\infty} f(u + \vartheta)\, g(\vartheta)\, d\vartheta \qquad \text{(F.5)}$$

Considering that the examinations have been made under the presumption that the distribution functions of the breaking times are normal, the combination of these normal distributions is a normal distribution itself (1):

$$r(z) = \frac{1}{\sqrt{2R\pi\sigma}} \exp\left[-\frac{(x - m)^2}{2\sigma^2} \right]$$

where

$$m = M(t) - M(\vartheta)$$

and

$$\sigma = \sqrt{D^2(t) + D^2(\vartheta)}$$

According to the curve of this frequency function, the probability of circuit-breaker A breaking earlier than B (Fig. 13) is according to relation (F.4):

$$P[A \cap \bar{B}] = \int_{-\infty}^{0} r(z)\, dz$$

The curve in Fig. 11 is to be plotted according to this relation.

If instead of a single circuit-breaker A there are a cluster of circuit-breakers in series, the problem is to be solved by writing $F_{[A_1 \cup A_2 \cup \ldots \cup A_n]}(t)$ instead of $f_A(t)$.

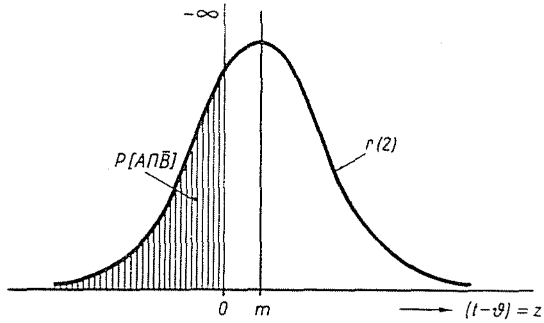

Fig. 13

Appendix II

The question arises, how the dispersion σ of a circuit-breaker operation can be stated.

If a circuit-breaker performs n breakings among identical short-circuit conditions, generally n different breaking times are obtained. There is a shortest and a longest value among them, and these two values are the limits of the so-called "range", denoted by w. If the number n of the tests is known, then the expected value of the range and the dispersion are related as:

n	2	3	4	5	6	10	50
$\frac{E(w)}{\sigma}$	1.13	1.69	2.06	2.33	2.53	3.08	4.5

Summary

In designing the protective scheme of electrical systems the following questions are often encountered:

What is the probability of one of a cluster of circuit-breakers in series breaking earlier than the others?

How does the resultant breaking time change in case of a cluster of circuit-breakers in series?

Taking certain presumptions in consideration, the paper determines the wanted probabilities for different tripping times by means of distribution functions of circuit-breakers

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