A NEW MAP METHOD FOR SIMPLIFYING FUNCTIONS GIVEN BY *EXCLUSIVE OR* AND *AND* AND INVERTER OPERATIONS

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1. Introduction

The following theorems are used in the simplification:

Theorem 1.

$$F(x_1 \dots x_n) = f_1 + \dots + f_k = f_1 \oplus \dots \oplus f_k$$

where:

$$f_i = \text{fundamental product}$$

$$f_1 + \dots + f_k = \text{canonical sum}$$

Theorem 2.

$$f_i \oplus f_i = 0$$

Theorem 3.

$$f_i \oplus 0 = f_i$$

2. The method of simplification

The simplification is performed by using the function map. The n-variable function map is conformable to the n-variable Karnaugh map. The function map is defined by the following properties:

a) There is a square on the function map for every possible combination of variables, and there is a combination of variables for every square on the function map.

b) The left and right edges and the top and bottom edges of the not more than four-variable function map are adjacent.

c) The desired value of the switching function is in every square.

d) The group of 2^i squares, where i variables occur in all possible combinations and the other variables are constant, can be combined and these constant variables define the group.

Proof: Let g be the product of the constant variables, and let $x_1 \dots x_i$ be the i variables occurring in all possible combinations.

The algebraic form of the 2^i terms is:

$$f = gx_1 \ldots x_i \oplus \ldots \oplus g\bar{x}_1 \ldots \bar{x}_i$$

As the set of two elements [0, 1] and the set of two associated operations $[\oplus, \cdot]$ form a Boolean algebra, each of the operations distributes over the other. Thus:

$$f = g(x_1 \ldots x_i \oplus \ldots \oplus \bar{x}_1 \ldots \bar{x}_i)$$

As all the terms in the brackets are expanded, theorem 1. is valid:

$$f = g\,(x_1 \ldots x_i + \ldots + \bar{x}_1 \ldots \bar{x}_i)$$

As all the fundamental products of an i-variable function are in the brackets, the value of the function in brackets is 1. Thus:

$$f = g$$

e) Each 1-square of the function map must be considered $(2k - 1)$ times, where k is a positive integer.

Proof: This statement can be proved by multiple applying Theorems 2, 3.

f) Each 0-square can be regarded as a 1-square considered $2k$ times, where k is a nonnegative integer.

Proof: This statement can be proved by multiple applying Theorem 2.

g) Each optional square can be regarded as a 1-square considered as often as desired.

Proof: This statement results from the two statements above.

The right way of forming groups on the function map defined above is: Each group should be as large as possible, and the number of the groups should be as few as possible.

3. Examples of simplification

The switching functions to be simplified are defined by Karnaugh maps.

3.1. *Example*

The Karnaugh map is shown in Fig. 1. The function map is shown in Fig. 2. The simplified function is:

$$F = x_1 \oplus x_2\bar{x}_3$$

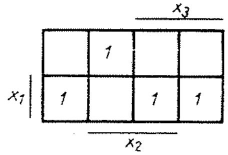

Fig. 1

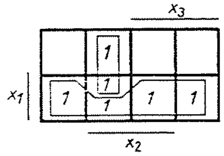

Fig. 2

3.2. *Example*

The Karnaugh map is shown in Fig. 3. The function map is shown in Fig. 4. The simplified function is:

$$F = x_1 \oplus x_2 \oplus \bar{x}_3$$

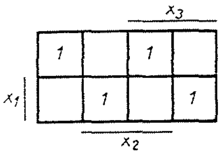

Fig. 3

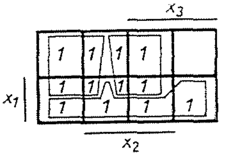

Fig. 4

3.3. *Example*

The Karnaugh map is shown in Fig. 5. The function map is shown in Fig. 6. The simplified function is:

$$F = x_1\bar{x}_2 \oplus x_2x_3$$

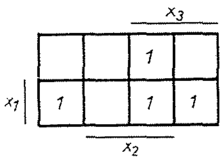

Fig. 5

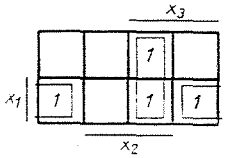

Fig. 6

3.4. *Example*

The Karnaugh map is shown in Fig. 7. The function map is shown in Fig. 8. The simplified function is:

$$F = \bar{x}_3 \oplus x_1\bar{x}_2$$

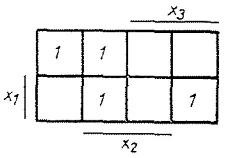

Fig. 7

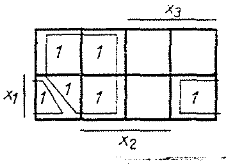

Fig. 8

3.5. *Example*

The Karnaugh map is shown in Fig. 9. The function map is shown in Fig. 10. The simplified function is:

$$F = \bar{x}_1\bar{x}_3 \oplus \bar{x}_2\bar{x}_3$$

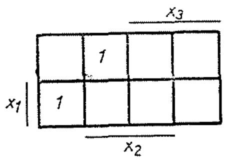

Fig. 9

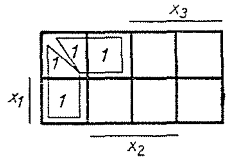

Fig. 10

3.6. *Example*

The Karnaugh map is shown in Fig. 11. The function map is shown in Fig. 12. The simplified function is:

$$F = x_2 \oplus x_3$$

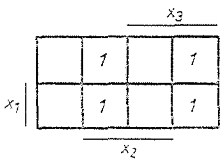

Fig. 11

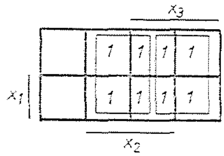

Fig. 12

3.7. *Example*

The Karnaugh map is shown in Fig. 13. The function map is shown in Fig. 14. The simplified function is:

$$F = \bar{x}_2 \oplus \bar{x}_1 x_3$$

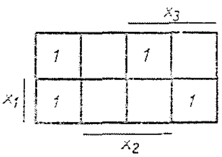

Fig. 13

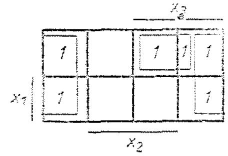

Fig. 14

Summary

This paper reports on a new simple map method for simplifying switching functions given by EXCLUSIVE OR and AND and inverter operations.

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