A SHORT SURVEY OF DETERMINISTIC OPTIMIZATION TECHNIQUES BASED ON INTEGRAL CRITERIA

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The main purpose of control systems is to eliminate the effect of the disturbances $d(t)$ and to influence the controlled variable $c(t)$ in such a way that the latter should approach or follow the constant or changeable input $r(t)$ with the highest possible accuracy (Fig. 1). This general requirement can be decom-

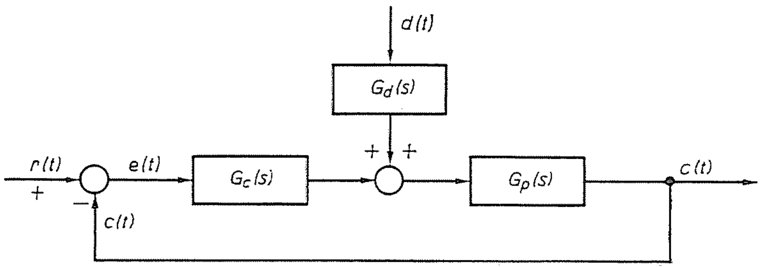

Fig. 1. Simple control system. Here $r(t)$ is the reference input, $e(t)$ is the error, $c(t)$ is the controlled variable, $d(t)$ is the disturbance, $G_c(s)$, $G_p(s)$, $G_d(s)$ are the transfer functions of the controller, plant and disturbance, respectively

pssed to several partial demands. The first of them is the *minimization* of the control *deviation* or *error* $e(t)$ under steady state conditions. It is well known that changes in the reference input $r(t)$ according to unit step-functions can be followed by a type-0 (so-called static) control system only with an error of finite magnitude, while by a type-1 (or astatic) control system it can be followed without any error in the steady-state. Control systems of type-1 may follow the ramp-function input (the constant-velocity input) with a finite error, but they are unable to follow the parabolic input (the constant-acceleration input). Finally, control systems of type-2 follow the ramp-function input without any error and the constant-acceleration input with a finite error.

In addition to the so-called steady-state requirements there are also several dynamic requirements playing highly important roles. The first dinamic demand relating to control systems is that of *stability*. When the system is deviated from the equilibrum state it must be restored to it. It is also well known that the increase of the loop gain decreases the steady-state error but at the same time the risk of unstable operation becomes more probable. In order to meet the conflicting requirements compensating elements must be built into the control system.

In addition to the provisions for stable operation, so-called *quality requirements* also arise, such as e.g. for comparatively small overshoot, short settling time, low number of oscillations, etc. It is self-evident that these dynamic requirements can be fulfilled only at the expense of compromises, as the merit of the control process is generally opposed by the costs of the controller.

It should also be considered that the development of the most advantageous control process depends highly on the means of application. Thus, e.g. when the rate of flow of a medium is regulated, even considerable overshoots can often be permitted which, in case of voltage regulation are, on the other hand, not permissible. In the control process of a copying jig lathe, only aperiodic positional changes may come into question, even at the expense of the increase of settling time.

The most advantageous setting of the controller greatly depends also on whether the effect of the disturbances must be balanced possibly quickly or the driving signal (command) must be followed at the highest possible rate. In the latter case, the quality of the disturbance also considerably influences the selection of the controller parameters. With short duration disturbances e.g. it is the purpose to amplify the proportional (P) effect, with long-duration disturbances; on the other hand, the integrating (I) effect.

In the solution of the optimization problem, it is advisable to set out from the transient processes of the control system, i.e. from the time response behaviour. The comparatively close relationship between the time domain and the frequency domain must, however, be taken into consideration. Thus, the conditions for the optimization can often be determined by the parameters of the frequency domain.

1. Integral criteria of optimization

The quality requirements for the transient control processes are more or less conflicting. Compromises can be made by means of the so-called integral criteria. A dynamic control process is called optimal if a certain pre-selected integral criterion attains a *minimum* value. The generalized form of the integral criterion is the minimization of the functional:

$$I = \int_0^\infty F\big(x(t),\ t\big)\, dt = \min \tag{1}$$

Here F stands for a certain function of two variables at least: of the time t and of a suitably selected signal $x(t)$. For the latter, we may select e.g. the weighting function of the closed-loop system:

$$x(t) = w(t) \tag{2}$$

or the difference between the transient response of unit step function and its steady-state value, that is, the transient component of the transient response:

$$x(t) = c(t) - c(\infty) = \int_0^t w(t)\,dt - \int_0^\infty w(t)\,dt \tag{3}$$

or the error (being the difference of the reference signal and the controlled variable):

$$x(t) = e(t) \tag{4}$$

or when examining the effect of the disturbance the controlled variable itself is:

$$x(t) = c(t) \tag{5}$$

There are, of course, also further possibilities for selection.

Note that the integral criterion contains simultaneously also the effect of signal $x(t)$ and of time t. The function to be integrated shall be selected intentionally to characterize suitably, on one hand, the quality of the transient process (e.g. by considering both the overshoot and settling time) and, on the other hand, to assume a comparatively simple form to give possibly simple relations with the system parameters. The requirements here mentioned are also more or less conflicting. Therefore, it is not surprising that integral criteria of the most varied shapes are found in the technical literature. Soviet workers, KHARKEVICH [1], FELDBAUM [2], KRASOVSKY [3] have, in the first place, suggested linear integral criteria weighted with certain powers of time:

$$\begin{aligned} I_{10} &= \int_0^\infty x(t)\,dt \\ I_{11} &= \int_0^\infty t x(t)\,dt \\ &\vdots \\ I_{1m} &= \int_0^\infty t^m x(t)\,dt \end{aligned} \tag{6}$$

and also the following generalized quadratic integral criteria:

$$\begin{aligned} I_{2(0)} &= \int_0^\infty x^2(t)\,dt \\ I_{2(1)} &= \int_0^\infty \left[x^2(t) + \tau_1^2 \dot{x}^2(t)\right] dt \\ I_{2(n)} &= \int_0^\infty \left[x^2(t) + \tau_1^2 \dot{x}^2(t) + \ldots + \tau_n^{2n} \left(\overset{(n)}{x}(t)\right)^2\right] dt \end{aligned} \tag{7}$$

Though the linear criteria can be used primarily for the evaluation of aperiodic processes, the quadratic integral criteria are suitable for the analysis of both aperiodic and oscillatory processes.

In addition to the generalized quadratic integral criteria, the simple quadratic criteria are similarly often used; these criteria often occur in their shape weighted by time (time-multiplied criteria) [4, 5, 6, 7, 8]:

$$\begin{aligned} I_{20} &= \int_0^\infty x^2(t)\, dt \\ I_{21} &= \int_0^\infty t\, x^2(t)\, dt \\ I_{22} &= \int_0^\infty t^2 x^2(t)\, dt \\ &\quad \cdots\cdots \end{aligned} \tag{8}$$

The most obvious criteria would be the integral criteria for absolute values, suitable for the evaluation of both the aperiodic and overshooting processes. Unfortunately, the mathematical treatment of the absolute values is difficult and therefore research workers rather use the quadratic criteria in mathematical analysis. With analogue computers, it is, however, easy to analyze the control system, based on the absolute value criterion. The form of the absolute-value criteria is:

$$\begin{aligned} I_{a0} &= \int_0^\infty |x(t)|\, dt \\ I_{a1} &= \int_0^\infty t|x(t)|\, dt \\ I_{a2} &= \int_0^\infty t^2 |x(t)|\, dt \\ &\quad \cdots\cdots \end{aligned} \tag{9}$$

Many other criteria can, of course, be similarly designed.

In connection with the integral criteria, there are particularly two questions which are worthy of attention. The first question is as follows: what kind of relationship occurs between a certain integral criterion and the parameters of the transfer function (e.g. the coefficients or time constants, gain factors). The significance of raising this question is that the relations of the control system are usually specified in the complex-frequency (or operator) domain and though the return to the time range is theoretically simple, in practice this proves to be laborious and the direct relationship with the system parameters often becomes blurred. The second question, justified by the consideration of the high number of the integral criteria, is, whether there exists an optimum criterion and if so, which one it is. First, the second question is dealt with by limiting this discussion to some of the more important criteria.

2. The ideal integral criterion

The integral criterion expressing the requirement for a short settling time and small overshoot in the most manageable dimensioning condition must be considered as ideal. The comparison could be performed based on the different transient processes; it is, however, the most purposeful to select as base the second-order control system and to presume a unit-step input and to take the control deviation or the error for signal $x(t)$.

The overall transfer function of the closed-loop system is thus (by selecting $\omega_0 = 1$):

$$W(s) = \frac{1}{s^2 + 2\zeta s + 1} \tag{10}$$

And the transform of the error occurring on the effect of the unit step:

$$X(s) = \frac{1}{s} - \frac{1}{s^2 + 2\zeta s + 1}\,\frac{1}{s} = \frac{s + 2\zeta}{s^2 + 2\zeta s + 1} \tag{11}$$

By applying this method, GRAHAM and LATHROP [5] have made comparisons for the following eight cases:

$$\begin{aligned}
&(A) \quad \int_0^\infty x(t)\,dt \\
&(B) \quad \int_0^\infty |x(t)|\,dt \\
&(C) \quad \int_0^\infty t\,x(t)\,dt \\
&(D) \quad \int_0^\infty |t\,x(t)|\,dt \\
&(E) \quad \int_0^\infty x^2(t)\,dt \\
&(F) \quad \int_0^\infty t\,x^2(t)\,dt \\
&(G) \quad \int_0^\infty t^2 x^2(t)\,dt \\
&(H) \quad \int_0^\infty t^2\,|x(t)|\,dt
\end{aligned} \tag{12}$$

The values of each of the integral criteria as functions of the damping factor ζ of the second-order system are demonstrated in Fig. 2.

Criteria A [9] and C [7] are unsuitable as both yield minima for case $\zeta = 0$ which would correspond to undamped oscillation. Particuarly criteria

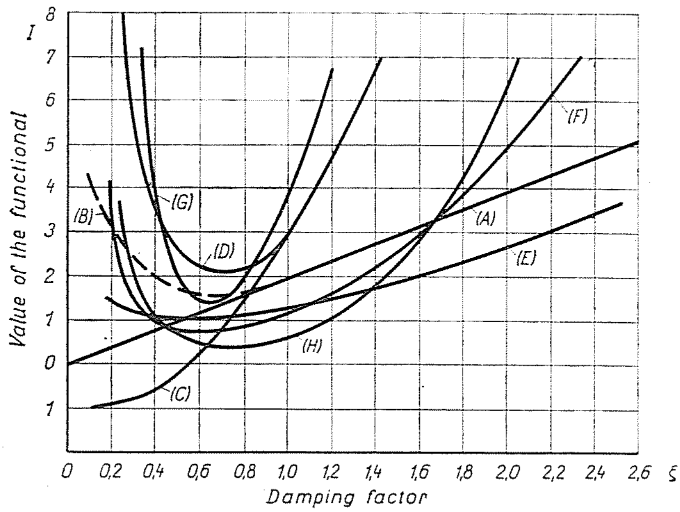

Fig. 2. The values of integral criteria in function of the damping factor ζ of the second-order control system

$(A)\ \int x(t)\,dt;\quad (C)\ \int t\ x(t)\ dt;\quad (E)\ \int x^2(t)\,dt;\quad (G)\ \frac{1}{10}\int t^2 x^2(t)\,dt$

$(B)\int |x(t)|\,dt;\quad (D)\ \int |t\ x(t)|\,dt;\quad (F)\ \frac{1}{10}\int t\,x^2(t)\,dt;\quad (H)\ \frac{1}{10}\int t^2|x(t)|\,dt$

D [5] and G [5] seem to be suitable, both assuming sharp minima for the value $\zeta = 0.7$ considered to be the most advantageous for second-order systems. Criterion H has already shown a flatter minimum around $\zeta = 0.7$ and criteria B, E and F [10] assume flatter minima for a value of ζ smaller than 0.7. Criteria F, G and H being comparatively complicated were left out of further investigations and criteria B, D and E were subjected to further comparisons based on the third-order system. Criterion D proved to be the most advantageous. In technical literature, this criteria is usually called criterion "ITAE" (integral of time-multiplied absolute value of error). Based on the forementioned criterion, GRAHAM and LATHROP [5] specified the coefficients of the optimum system, up to the 8th order systems. These coefficients are summed up in Table 1.

Without aiming at completeness, in the following the relations between a few integral criteria and the system parameters are examined, thus the first subject-matter raised at the end of the preceeding paragraph is dealt with.

Table 1

Coefficients of optimal systems based on the time-weighted absolute value criterion (Integral of time-multiplied absolute value of error)

n	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
	$W(s) = \frac{a_0\,\omega_0^n}{a_n s^n + a_{n-1}\,{}_0 s^{n-1} + \ldots + a_1\,{}_0^{n-1} s + a_0\,{}_0^n}$								
1								1.00	1.00
2							1.00	1.40	1.00
3						1.00	1.75	2.15	1.00
4					1.00	2.10	3.40	2.70	1.00
5				1.00	2.80	5.00	5.50	3.40	1.00
6			1.00	3.25	6.60	8.60	7.45	3.95	1.00
7		1.00	4.475	10.42	15.08	15.54	10.64	4.58	1.00
8	1	5.20	12.80	21.60	25.75	22.20	13.30	5.15	1.00
	$W(s) = \frac{a_1\,\omega_0^{n-1} s + a_0\,\omega_0^n}{a_n s^n + s_{n-1}\,\omega_0 s^{n-1} + \ldots + a_1\,\omega_0^{n-1} s + a_0\,\omega_0^n}$								
2							1.00	3.20	1.00
3						1.00	1.75	3.25	1.00
4					1.00	2.41	4.93	5.14	1.00
5				1.00	2.19	6.50	6.30	5.24	1.00
6			1.00	6.12	13.42	17.16	14.14	6.76	1.00
	$W(s) = \frac{a_2\,\omega_0^{n-2} s^2 + a_1\,\omega_0^{n-1} s\, a_0\,\omega_0^n}{a_n s^n + a_{n-1}\,\omega_0 s^{n-1} + \ldots + a_1\,\omega_0^{n-1} s + a_0\,\omega_0^n}$								
3						1.00	2.97	4.94	1.00
4					1.00	3.71	7.88	5.93	1.00
5				1.00	3.81	9.94	13.44	7.36	1.00
6			1.00	3.93	11.68	18.56	19.30	8.06	1.00

3. Evalution of integral criterion I_{10}

The linear integral criterion:

$$I_{10} = \int_0^\infty x(t)\,dt = \min \tag{13}$$

is the most simple criterion. Unfortunately, this criterion is not suitable for the evaluation of oscillating processes because the positive and negative area (Fig. 3) are subtracted from each other.

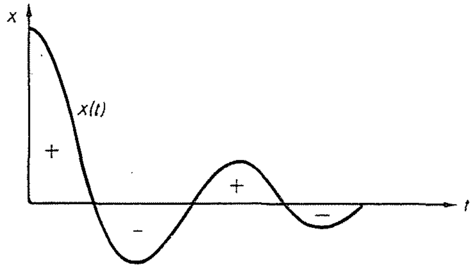

Fig. 3. Linear integral criterion. If oscillations arrive, positive and negative area are subtracted from each other

Obviously, the integral criterion can also be expressed in the following form:

$$I_{10} = \lim_{s \to 0} \int_0^\infty x(t)\ e^{-st}\, dt = \lim_{s \to 0} X(s) = X(0)$$

which will considerably simplify the evaluation.

Example 1

If

$$W(s) = \frac{\omega_0^2}{s^2 + 2\zeta\, \omega_0\, s + \omega_0^2}$$

and

$$R(s) = \frac{1}{s}$$

then the transform of the change in the controlled variable caused by the unit step reference input change $r(t) = 1(t)$ is

$$C(s) = \frac{\omega_0^2}{s^2 + 2\zeta\, \omega_0 + \omega_0^2}\, \frac{1}{s}$$

and the transform of the error signal:

$$X(s) = E(s) = R(s) - C(s) = \frac{s + 2\zeta\, \omega_0}{s^2 + 2\zeta\, \omega_0\, s + \omega_0^2}$$

The value of the linear integral will be

$$X(0) = \frac{2\zeta}{\omega_0}$$

This will assume the minimum value $X(0) = 0$ if $\zeta \to 0$. Thus, according to this criterion the oscillatory system without damping would be optimal which, from the physical point of view, is nonsense.

Example 2

Let the transfer function of the closed-loop system be of the following shape:

$$W(s) = \frac{b_m s^m + \ldots + b_1 s + b_0}{a_n s^n + \ldots + a_1 s + a_0} \qquad (n > m)$$

The transform of the error occurring on the effect of unit step can be conveyed into the following shape:

$$X(s) = \frac{a_n s^n + \ldots + (a_1 - b_1) s + a_0 - b_0}{a_n s^n + \ldots + a_1 s + a_0} \; \frac{1}{s}$$

If $a_0 \neq b_0$, then $I_{10} = \infty$ which is understandable from a physical point of view as in this case

$$r(\infty) = \lim_{s \to 0} s\, R(s) = 1$$

and, at the same time,

$$c(\infty) = \lim_{s \to 0} s\, C(s) = \lim_{s \to 0} W(s) = \frac{b_0}{a_0} \neq 1$$

thus

$$x(\infty) = r(\infty) - c(\infty) = 1 - \frac{b_0}{a_0} \neq 0$$

would exist. The condition for the application of criterion I_{10} is thus that the controlled variable would comply with the reference input and the control deviation be consequently equal to zero in the steady-state. This condition is fulfilled if $a_0 = b_0$, in this case

$$X(s) = \frac{c_{n-1} s^{n-1} + \ldots + c_1 s + c_0}{a_n s^n + \ldots + a_1 s + a_0}$$

and

$$I_{10} = X(0) = \frac{c_0}{a_0} = \frac{a_1 - b_1}{a_0}$$

This assumes minimum value if

$$c_0 = a_1 - b_1 = 0$$

4. Evaluation of integral criterion $\mathbf{I}_{1m}$

The integral criterion weighted with a certain power of time is

$$I_{1m} = \int_0^\infty t^m x(t)\, dt = \min \tag{14}$$

According to a rule of the Laplace transforms:

$$\int_0^\infty t^m x(t)\, e^{-st}\, dt = (-1)^m \frac{d^m}{ds^m} X(s) \tag{15}$$

and thus the integral in question can be evaluated on base of the following formula:

$$I_{1m} = \lim_{s \to 0} \left[(-1)^m \frac{d^m}{ds^m} X(s) \right] \tag{16}$$

It is also mentioned here that the integral weighted with a power of time is suitable for the calculation of the so-called equivalent time-constants:

$$T_{e1m} = \sqrt[m]{\frac{\int_0^\infty t^m x(t)\, dt}{\int_0^\infty x(t)\, dt}} = \sqrt[m]{\frac{(-1)^m \dfrac{d^m X(s)}{ds^m}}{X(s)}}\Bigg|_{s=0} \tag{17}$$

provided that there is a positive quantity below the root-sign.

Example 3

In Example 1 it could be seen that

$$X(s) = \frac{s + 2\zeta\, \omega_0}{s^2 + 2\zeta\, \omega_0\, s + \omega_0^2}$$

As

$$(-1) \frac{d}{ds} X(s) = \frac{s^2 + 4\zeta\, \omega_0\, s + (4\zeta^2 - 1)\, \omega_0^2}{(s^2 + 2\zeta\, \omega_0\, s + \omega_0^2)^2}$$

therefore,

$$I_{11} = \int_0^\infty t\, x(t)\, dt = \frac{4\zeta^2 - 1}{\omega_0^2}$$

attaining its minimum of value $-1/\omega_0^2$ when $\zeta = 0$. (On the other hand, when $\zeta = 0.5$, then $I_{11} = 0$.)

Example 4

Based on the results of Examples 1 and 3 the equivalent time constant is

$$T_{e1m} = \frac{4\zeta^2 - 1}{\omega_0^2} : \frac{2\zeta}{\omega_0} = \frac{4\zeta^2 - 1}{2\zeta\omega_0}$$

Physical interpretation can be given only if $\zeta > 0.5$.

5. The generalized quadratic integral criterion

The most simple form of the generalized quadratic integral criterion [13] is

$$I_{2(1)} = \int_0^\infty (x^2 + \tau_1^2 \dot{x}^2)\, dt \tag{18}$$

By completing this term to full quadratic expression and by partial integration, the following form can be obtained:

$$I_{2(1)} = \int_0^\infty (x + \tau_1 \dot{x})^2\, dt - \int_0^\infty 2\tau_1 x \dot{x}\, dt = \int_0^\infty (x + \tau_1 \dot{x})^2 \; dt + \tau_1 x^2(0) \tag{19}$$

provided that $x(\infty) = 0$. The integral can also be expressed in the following form:

$$I_{2(1)} = \int_0^\infty (x + \alpha_1 \dot{x})^2 \; dt + C_0 \tag{20}$$

This obviously attains its minimum if the transient process $x(t)$ comes the closest to the solution

$$x(t) = x_0 \; e^{-\frac{t}{\tau_1}}$$

of the differential equation

$$\alpha_1 \dot{x} + x = 0$$

determined by the initial condition

$$x(0) = x_0$$

In case of complete equality,

$$I_{2(1)} = C_0 = \tau_1 x^2(0)$$

The generalized n-order quadratic integral criterion [13, 14, 15] is:

$$I_{2(n)} = \int_0^\infty \left(x^2 + \tau_1 \dot{x}^2 + \ldots + \tau_n^{2n} (\overset{(n)}{x})^2\right) dt \tag{21}$$

By completing it to full quadratic term and by subsequent partial integration and considering the initial conditions

$$x(0) = x_0 ; \; \dot{x}(0) = \ldots = \overset{(n)}{x}(0) = 0$$

and the end-values

$$x(\infty) = x(\infty) = \ldots = \overset{(n)}{x}(\infty) = 0$$

it can generally be conveyed to the following form:

$$I_{2(n)} = \int_0^\infty (x + \alpha_1 \dot{x} + \ldots + \alpha_n \overset{(n)}{x})^2 \, dt + C_0 \tag{22}$$

where

$$\begin{aligned} C_0 &= \alpha_1 x_0^2 \\ \tau_1^2 &= \alpha_1^2 - 2\alpha_0 \alpha_2 \\ \tau_2^4 &= \alpha_2^2 - 2\alpha_1 \alpha_3 + 2\alpha_0 \alpha_4 \\ \tau_3^6 &= \alpha_3^2 - 2\alpha_2 \alpha_4 + 2\alpha_1 \alpha_5 - 2\alpha_0 \alpha_6 \\ &\ldots\ldots \\ \tau_n^{2n} &= \alpha_n^2 \end{aligned} \tag{23}$$

The correctness of these relations which exist between the constants α and the weights τ are proved by elementary methods in reference [11], and by variational calculus in reference [12], therefore, here we will not go into details.

Everywhere in these terms, $\alpha_0 = 1$ and α_0 is figuring only in order to make the relationships symmetrical.

The lowest value of $I_{2(n)}$ is C_0. This occurs when $x(t)$ is just equal to the solution of the differential equation

$$\alpha_n \overset{(n)}{x} + \alpha_{n-1} \overset{(n-1)}{x} + \ldots + \alpha_1 \; x + \alpha_0 = 0 \tag{24}$$

with the forementioned initial conditions.

From the coefficients $\alpha_n, \ldots, \alpha_0$ of the differential equation predefined as the aim of the approximation, the coefficients $\tau_1, \ldots, \tau_n$ of the integral criterion can be determined by means of the aforedescribed formulae (23).

The integral criterion proper can be computed purposefully in the following form:

$$I_{2(n)} = \sum_{i=0}^{n} \tau_i^{2i} I_{20(i)}\,; \quad (\tau_0 = 1) \tag{25}$$

where

$$I_{20(i)} = \int_0^\infty (\overset{(i)}{x})^2\, dt \tag{26}$$

Thus, the calculation of the generalized quadratic integral criterion can be traced back to the evaluation of the unweighted, simple quadratic integrals.

Example 5

It is not difficult to demonstrate [11], that with preselected weights τ_1, τ_2, the most advantageous transient process is the solution of differential equation

$$\tau_2^2 \ddot{x} + \sqrt{\tau_1^2 + 2\tau_2^2}\, \dot{x} + x = 0$$

Example 6

Let us determine the damping factor ζ of the second-order control system to enable that the error as response on the unit step-function input may approximate to the closest possible extent the transient process defined by the exponential function $e^{-\omega_0 t}$.

Transform of the error is

$$\mathscr{L}[x(t)] = X(s) = \frac{s + 2\zeta\,\omega_0}{s^2 + 2\zeta\,\omega_0\, s + \omega_0^2}$$

and the transform of its differential quotient:

$$\mathscr{L}[x(t)] = s\;X(s) - x_0 = \frac{-\,\omega_0^2}{s^2 + 2\zeta\omega_0\, s + \omega_0^2}$$

because

$$x_0 = x(0) = s\;X(s)\,|_{s=0} = 1$$

The simple quadratic integral values are, in the given case,

$$I_{20(0)} = \frac{4\zeta^2 + 1}{4\zeta\omega_0}$$

$$I_{20(1)} = \frac{\omega_0}{4\zeta}$$

which can be obtained, for example, by the theorem of residues or by evaluating tables [see e.g. 16] based on PARSEVAL's theorem.

As presently $\tau_1 = 1/\omega_0$, therefore, according to Eq. (25),

$$I_{2(1)} = I_{20(0)} + \tau_1^2 I_{20(1)} = \frac{4\zeta^2 + 2}{4\zeta\omega_0}$$

The minimum is attained if

$$\frac{\partial I_{2(1)}}{\partial \zeta} = \frac{2\zeta^2 - 1}{2\zeta^2 \omega_0} = 0$$

that is

$$\zeta = 1/\sqrt{2} = 0.707$$

Summary

The paper gives a short survey on the optimization techniques based on integral criteria. First, the integral criteria are classified and compared to one another. Then the evaluation methods relating to the connection between the frequency domain and time domain are shown. Some results are illustrated by simple examples.

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