SOME NEW TIME VARYING SMOOTHING CIRCUITS FOR DIRECT READING NOISE MEASUREMENT

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In a previous article [1] and a letter [2] it was shown that the time requirement of direct reading noise measurement may be effectively reduced by using time varying smoothing network. The usual measuring arrangement is shown in Fig. 1. If the RC network consists of time-invariant components

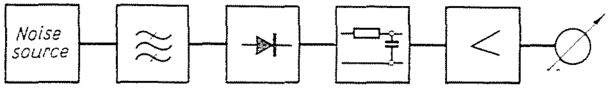

Fig. 1. Block diagram of a direct reading noise measuring instrument

the output response for a transient noise input reaches only 86.5 per cent of the steady-state value assuming equal permissible errors due to finite measuring time and non-ideal smoothing [1]. Using time varying RC network (increasing time constant *vs.* time) the approximation of steady-state response may be as good as 99 per cent.

In the early experiments an indirectly heated thermistor was used as time varying resistance. The response time, however, was limited by the thermal time constant of the thermistor.

It was also pointed out that the continuous resistance variation may be approximated by a stepped function. The circuit, built for the realization of this principle, contains parallel connected resistors, each having a transistor connected in series. During the measuring period, the originally saturated transistors cut off one after another and disconnect the resistors (Fig. 2). This circuit has provided good experimental results in spite of the fact that in the design the optimum number and shape of steps were unknown.

For the analysis a mathematical model was developed. The rectified input noise voltage could be regarded as the series of independent samples with the number of $k = 2\ BT$, where B was the bandwidth of the pre-detector filter, T was the time duration of the measurement.

These samples may be simulated by a chain of random numbers having rectified Gaussian probability density, i.e. the negative values of the original

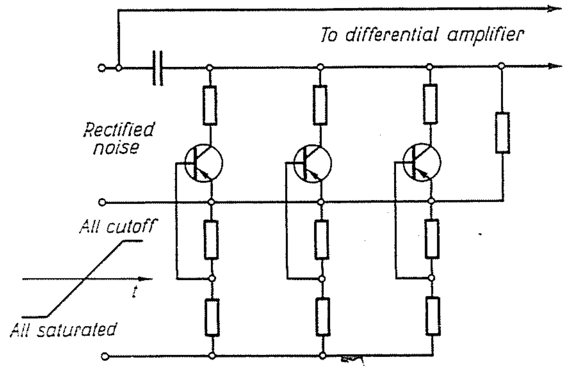

Fig. 2. Circuit for stepwise increasing time constant

distribution having been turned into positive ones by using the absolute value operation (in case of full-wave linear rectification) or by squaring (square-law detection). The upper part of Fig. 3 shows the random samples.

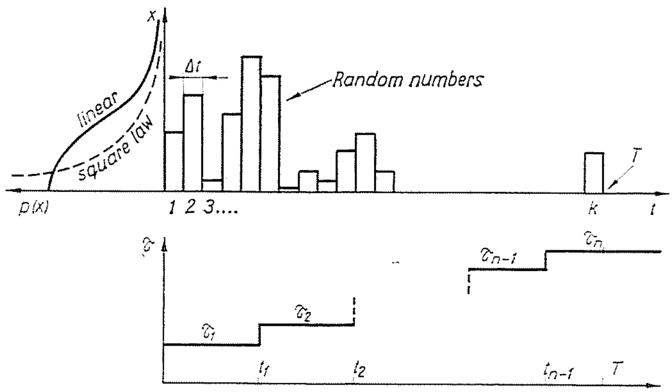

Fig. 3. Random numbers (upper curve). Stepwise increasing time constant (lower curve)

The chain of these samples was applied to the input of a time varying $R(t)C$ network with stepwise increasing time constant (Fig. 3, lower curve). It can easily be shown that the output response of the network at $t = T$ on the jth input step in the range of $0 < t < t_1$ is

$$y_j = \Delta x_j \left[1 - \exp\left(- \frac{t_1 - (j-1)\,\Delta t}{\tau_1} - \frac{t_2 - t_1}{\tau_2} - \ldots - \frac{t_n - t_{n-1}}{\tau_n} \right) \right] = \\ = \Delta x_j \left[1 - C(j) \right]$$

where $\Delta x_j = x_j - x_{j-1}$. Similar equations can be written for $t_1 < t < t_2$, etc. which have decreasing number of fractions in the exponent. The resulting response at $t = T$ may be written as

$$y = \sum_{j=1}^{k} y_j = (x_1 - 0)(1 - C_1) + (x_2 - x_1)(1 - C_2) + \ldots + (x_k - x_{k-1})(1 - C_k) = -C_1 x_1 - C_2(x_2 - x_1) - \ldots - C_k(x_k - x_{k-1}) + x_k$$

The time varying response coefficients C_j may be considered as weight factors of the random steps. Fig. 4 shows $C_j(t)$ for continuous linear increase of time

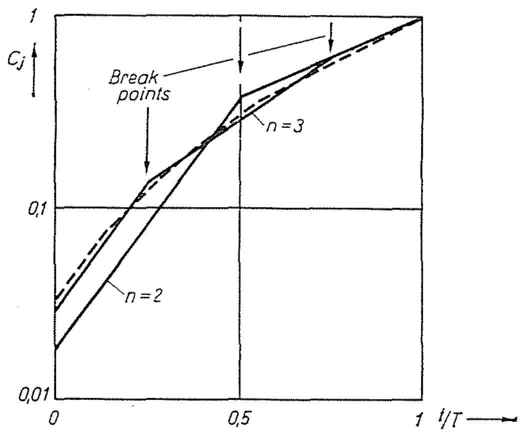

Fig. 4. Weight factors $C(j)$ *vs* time

constant as well as for the three- and two-step approximation. Better approximating curves are not represented since they follow the continuous one very closely.

Both C_j-s and y-s were computed by the Imperial College IBM 7090 Computer during a recent visit by the author. One run contained $k = 200$ samples and it was repeated 50 times with different sets of random numbers. Then the mean and standard deviation of these fifty responses were computed.

This computation was carried out on several networks having different stepwise time constant variations. The linear function was approximated by 5, 4, 3, and 2 steps. Generally no significant difference was found either in mean or in standard deviation among the various cases.

The mean value of the outputs was 0.77, which is 3.5 per cent less than the theoretical steady state mean value 0.798 of the full-wave rectified Gaussian distribution originally having zero mean and unity standard deviation. This is in excellent agreement with the differential equation assuming continuous linear increase of the time constant [1]:

$$v_{out}[T] = V_{in}\left[1 - \left(1 + \frac{R_1}{R_0}\right)^{-(R_0/R_1)(T/\tau_0)}\right]$$

where

$$R(t) = R_0 + R_1 \frac{t}{T}$$

$$\tau_0 = R_0 C$$

and in the present case $R_0/R_1 = 1/2$ and $T/\tau = 6$.

The superiority of the time varying network over the time-invariant one was again proved. The computer simulation was carried out on a time invariant network having the same smoothing capacity as the time varying one after reaching the steady state. The output was 0.691, by 13.2% less than the steady state value owing to the slow transient response. The theoretical difference from the simple equation

$$v_{\text{out}} = V_{\text{in}} \left[1 - e^{-T/(R_0+R_1)C}\right] = V_{\text{in}} (1 - e^{-2})$$

is 13.5%.

The fact that even the roughest approximation provides good results, offers the opportunity to use the most simple circuit. In contrast of Fig. 2 it consists of one fixed and one shunting resistor only. The moment of switch-

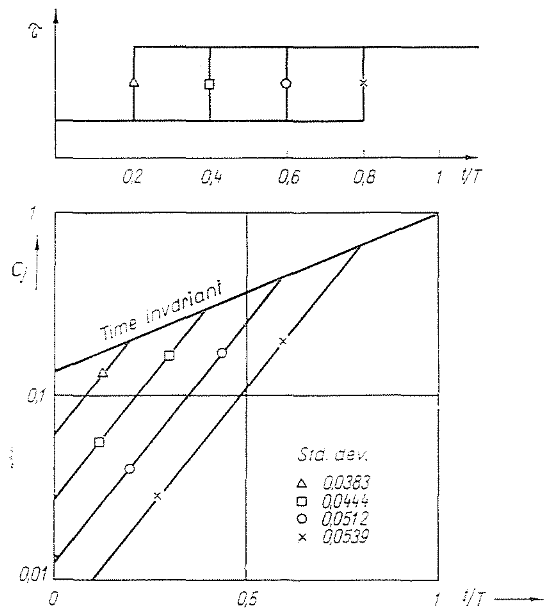

Fig. 5. Time constant and weight factor *vs* time (two steps)

ing, however, is important. Fig. 5a shows τ *vs. t*, Fig. 5b C_j *vs. t*. If the switching is too late, the C_j weight factors are low for a considerably long period and the response at $t = T$ depends mainly on the final values of the random input.

Therefore, the standard deviation increases (Fig. 5). On the other hand, early switching results in a slow transient response, since in this case the C_j *vs.* t curve hardly differs from the same curve of time invariant circuit. The best choice of switching point is (0.4...0.6) T.

The switching device is in general a bipolar transistor. The offset, caused by the nonzero saturation voltage, limits the useful range of this circuit to high levels. The range may be extended to low levels by using a field effect transistor for switching. FETs, however, permit the continuous resistance variation, too. Let us investigate a low level time varying network in the following.

Time varying smoothing network with FET

In some cases of noise measurements, e.g. using tunnel-diode square law detector [3] or Hall-correlator [4] for rectifying noise, the output voltage is very small. The network following the rectifier should be able to smooth

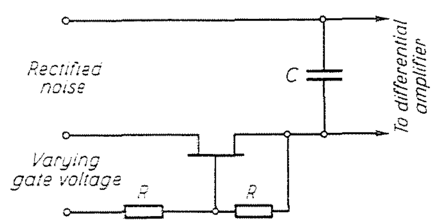

Fig. 6. Time varying smoothing network using FET

the fluctuations effectively and it should have short initial response time and neither DC nor AC spurious offset.

The circuit of Fig. 6 satisfies these requirements. The RC network is composed of a fixed capacitor and the channel resistance of a junction or MOS field-effect transistor. The channel resistance can be controlled by the gate voltage. To avoid the separate gate voltage supply the source of the FET is grounded. Thus, the output of the circuit is floating (similarly to the circuit of Fig. 2) which does not cause additional difficulties, however, since the low voltage at the output require the use of a differential amplifier anyway.

The two equal, high resistors provide a negative feedback to remove the even order nonlinearities of the channel voltage-current characteristic in the vicinity of the origin [5]. To explain this, let us approximate the depletion layer width by

$$v = w\left(\frac{u}{U_p}\right)^n$$

where w is the channel width, u the actual voltage across the pn junction and U_p the pinch-off voltage ($n = 1/2$ for abrupt junction). The current-voltage characteristic of the FET may be expressed as

$$I_D = g_0 \left\{ U_{DS} \left[1 - \frac{1}{n+1} \left(\frac{-U_{GS} + U_{DS}}{U_p} \right)^n + \frac{(-U_{GS})}{n+1} \left[\left(\frac{-U_{GS}}{U_p} \right)^n - \right.\right.\right.$$
$$\left.\left.\left. - \left(\frac{U_{GS} + U_{DS}}{U_p} \right)^n \right] \right] \right\}$$

Because of the 50% negative feedback, $-U_{GS}$ must be replaced now by $(-U_{GS} - U_{DS}/2)$. Thus,

$$I_D = g_0 \left\{ U_{DS} \left[1 - \frac{1}{n+1} \left(\frac{-U_{GS} + U_{DS}/2}{U_p} \right)^n \right] + \right.$$
$$\left. + \frac{(-U_{GS} + U_{DS})}{n+1} \left[\left(\frac{-U_{GS} - U_{DS}/2}{U_p} \right)^n - \left(\frac{-U_{GS} + U_{DS}/2}{U_p} \right)^n \right] \right\}$$

The slope of the $I_D - U_{DS}$ curve gives the differential conductance of the FET:

$$\frac{dI}{dU_{DS}} = g_0 \left[1 - \frac{1}{2} \left(\frac{-U_{GS} + U_{DS}/2}{U_p} \right)^n - \frac{1}{2} \left(\frac{-U_{GS} - U_{DS}/2}{U_p} \right)^n \right]$$

If

$$\frac{U_{DS}}{2} \ll |-U_{GS}|$$

then

$$\frac{dI}{dU_{DS}} \cong g_0 \left[1 - \left(\frac{-U_{GS}}{U_p} \right)^n \right]$$

i.e. independent of both the magnitude and polarity of U_{DS}. This latter equation may be used as first approximation for MOS transistors, too, substituting $n = 1$. Without negative feedback this equation would change to

$$\frac{dI}{dU_{DS}} = g_0 \left[1 - \left(\frac{-U_{GS} + U_{DS}}{U_p} \right)^n \right]$$

which is nonsymmetrical in function of U_{DS}.

The useful voltage range of junction FET is about ± 200 mV. MOSFETs, however, offer a considerably greater linear range. To obtain this, the substrate must not be connected to the source — as is usually — since the drain-substrate junction opens at reverse bias and shunts the channel. Fig. 7 shows the solution: the substrate is connected to the midpoint of a high resistance voltage-

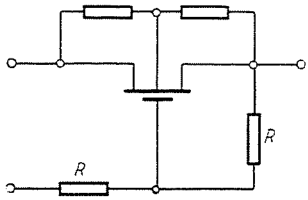

Fig. 7. MOSFET configuration

divider. Fig. 8 and Fig. 9 are oscillograms of the voltage-current characteristics near the origin. Fig. 9 represents ten times larger voltage and current swings than Fig. 8. The curvatures are remarkable: the middle curve bends down

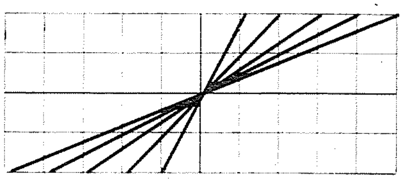

Fig. 8. $I-U$ curves with different gate voltages. Horizontal: 200 mV/cm, vertical: 100 μA/cm

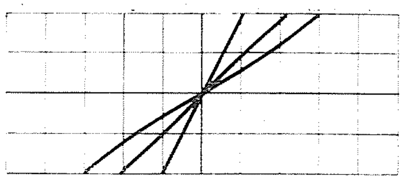

Fig. 9. $I-U$ curves. Horizontal: 2 V/cm, vertical: 1 mA/cm

(according to the normal MOS behaviour) but the one representing the highest resistance tends upwards. This phenomenon is caused by the substrate voltage divider, not negligible if the channel resistance is high.

The permissible rms noise voltage at the output of the detector (Fig. 1) can be estimated according to Fig. 10. The amplitude probability density of linear and square-law detected Gaussian noise are represented in the upper part of Fig. 10a and b. The averages (equal to the steady state output voltage of the time-varying smoothing network) are also drawn. It is obvious that the linear part of the V—I characteristic must be greater than or at least equal to the average output since in stationary state the maximal negative deviation — with maximal probability — is equal to the average in both cases. At the same positive deviation the amplitude probability density is 0.22 for linear detected Gaussian noise and 0.027 for square-law. Therefore, the neces-

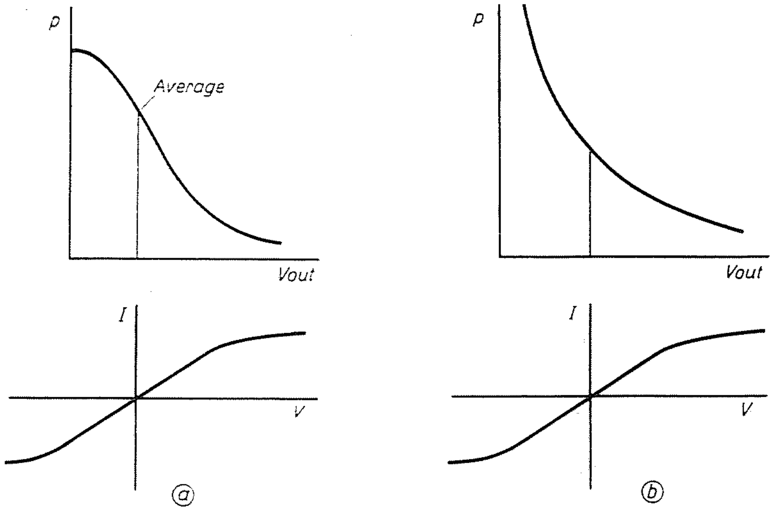

Fig. 10. Amplitude density and $I-U$ curves

sary linear range is twice or manifold of the average voltage, depending on the required accuracy.

Finally let us investigate the required gate voltage waveform. The channel resistance depends on U_{GS} as

$$R = \frac{1}{g_0} \frac{1}{\left[1 - \left(\frac{-U_{GS}}{U_p}\right)^n\right]}$$

which is a highly nonlinear (convex) function. If

$$U_{GS} = U_{GS0}\left[1 - e^{-t/\vartheta}\right] + U'_{GS}$$

where U'_{GS} provides the initial channel resistance, the resulting R *vs.* t function is almost linear, since $U_{GS}(t)$ is a concave one. Fig. 11 shows $R(t)$, the variation of R is 1 : 4.

Fig. 12a shows the output voltage waveform of a time varying RC integrator realized by a junction FET. For comparison, in Fig. 12*b* and *c*

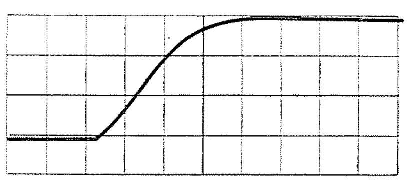

Fig. 11. Channel resistance *vs* time, when the gate voltage varies exponentially. Horizontal: 1 sec/cm, vertical: 1 kΩ/cm

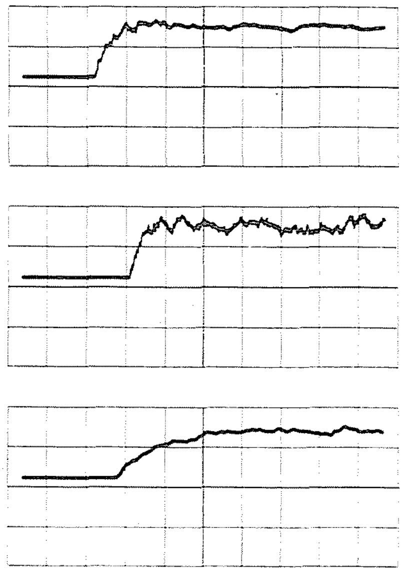

Fig. 12. Noise response of *a)* time varying, *b)* time invariant, initial equivalent, *c)* time invariant, steady-state equivalent networks

the initial-, and steady state equivalent time invariant responses are shown. The time varying circuit connects the two advantages: fast initial response and effective smoothing.

Summary

In a previous letter and article it was shown that the time requirement of direct reading noise measurement may be effectively reduced by using time varying smoothing network. For example, stepwise increasing time constant can be obtained when the resistor of a simple RC network is replaced by a chain of resistors switched on or off by switching transistors. The mathematical model of this circuit was constructed, and the noise input was simulated by random numbers. The output was calculated by an electronic computer. It has been shown that even two steps of time constant give fairly good results.

In some cases of noise measurement the voltage to be smoothed is very small. For this purpose the RC network is composed of a fixed capacitor and the channel resistance of a junction or MOS field effect transistor. The channel resistance can be controlled by the gate voltage. FETs were investigated both theoretically and experimentally to fulfil the requirements.

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