DEPENDENCE OF MAXIMUM SOUND-PHASE FAULT CURRENTS ON NETWORK CONDITIONS

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As is known, the limits imposed on the applicability of conventional distance relays to the protection of solidly-earthed-neutral high-voltage networks greatly depends on what megnitude of fault currents will flow in the sound phases of the line to be protected in the event of a single-phase-to-earth fault under a given service condition of the network [1].

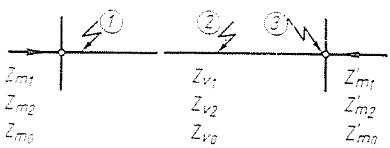

Fig. 1

The determination of the maximum sound-phase fault current often encounters difficulties. As is known, the sound-phase fault current is composed of two parts, *viz.* of a component flowing in the unloaded line (termed "fault component of the soundphase fault current"), and of a component depending on the line load (termed "load component of the sound-phase fault current"). With good approximation [1], and for practical calculations, the sum of these components can be obtained by vectorial addition of the fault component of the sound-phase fault current (no-load component) and the load current which has flown in the line before the occurrence of the fault. The maximum may simply be obtained by adding the maxima of the two components assumed to be of identical vector directions. The value of the maximum load current is usually given (*e.g.*, taken as equal to the thermal overload capacity of the line), whereas the fault component is dependent on several factors.

For the transmission line represented as an example in Fig. 1, the variables are the reduced supply impedances of the networks adjoining the line terminals, *i.e.* Z_{m1}, Z_{m2}, Z_{m0} and Z'_{m1}, Z'_{m2}, Z'_{m0}, as well as the locations of the fault (1), (2) or (3). The judgement of the dependence on the fault location is the easier task: the maximum sound-phase fault current is usually associated with the line-end faults, *i.e.* with faults ① or ③, because the zero-sequence currents defined by the concentrated zero-sequence impedances of transformers installed at the substations are always predominant over those permitted to flow by the $Z_0/Z_1 \approx 3$ impedance ratio of the transmission lines [1]. Thus, it only remains to be determined under which network conditions will the maximum fault component of the fault current flow in the sound phases.

1. Two-line substation

In order to introduce and demonstrate the application of the method, let a simple two-line substation of Fig. 2 be assumed, and let the case be found in which the sound-phase fault current will be maximum. Let the line investigated be A and let the two distinct network conditions be examined,

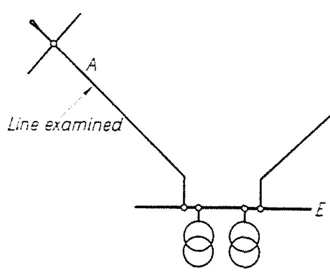

Fig. 2

first with Line B disconnected, then with the same line connected. For the sake of simplicity, the phase angles of the impedances involved are taken as equal throughout, the positive- and negative-sequence impedances are also taken as equal, the network is assumed to be unloaded* and, finally, the supply impedances behind lines A and B are taken independent of each other. As explained in Section F. 1 of the Appendix, the following rule can be established for the case of a single-phase fault occurring at Station E:

The sound-phase fault current for the protection of Line A is higher with Line B disconnected than that with Line B connected, if the ratio of the zero-sequence impedance to the positive-sequence driving point impedance is higher with Line B connected than with the same line disconnected, (1)

i.e.

$$h_A > 1, \text{ if } \frac{Z_0}{Z_1} > \frac{Z_0'}{Z_1'} \tag{2}$$

* According to [1], it is expedient to perform the investigations assuming the network unloaded and considering the loads additionally.

where

$$h_A = \frac{I'_{A\,\mathrm{sph}\,Z}}{I_{A\,\mathrm{sph}\,Z}} = \frac{\text{sound-phase fault current in Line A after disconnecting Line B}}{\text{sound-phase fault current in Line A without disconnecting Line B}}$$

Z_1 = resultant positive- (and negative-) sequence driving point impedance of station E

Z_1' = ditto, yet after disconnecting Line B,

Z_0 = resultant zero-sequence driving point impedance of Station E,

Z_0' = ditto, yet after disconnecting Line B.

2. Station with any number of lines

The theorem laid down in Section 1 can be extended to the case of stations having any number of lines.

If out of n lines $(n-1)$ are disconnected, *i.e.* only the line investigated remains connected to the station, the equivalent impedance of the sole dis-

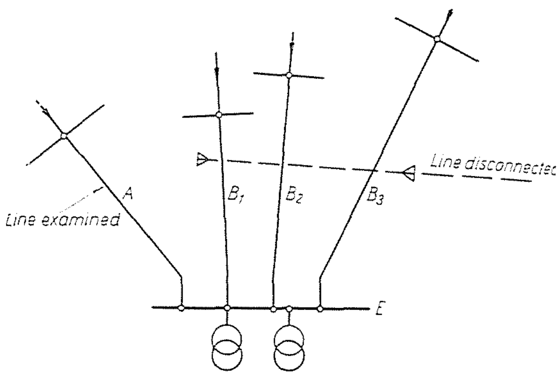

Fig. 3

connected supply (B) considered in Fig. 2, Fig. 12 and in Section F. 1 of the Appendix shall be replaced by the overall resultant supply impedance of the $(n-1)$ lines, as seen from Station E and, thus, the statement formulated at the end of Section 1 is also valid for the case of Fig. 3 ! Thus, the problem reduced to the case of two lines:

Line B (*i.e.* the supply from Line B) is replaced by the group of lines $B_1-B_2-B_3$.

If, however, the disconnection of less than (n—1) lines out of n lines is considered (Fig. 4), the problem can only be reduced to the case of three lines. The three groups and their respective impedances are as follows:

A — the supply impedance of the network connected through the investigated line,

B — the resultant supply impedance of the lines to be disconnected, as seen from station E,

C — the resultant supply impedance of lines remaining connected, as seen from Station E.

Further,

E — the impedances of transformers installed at Station E.

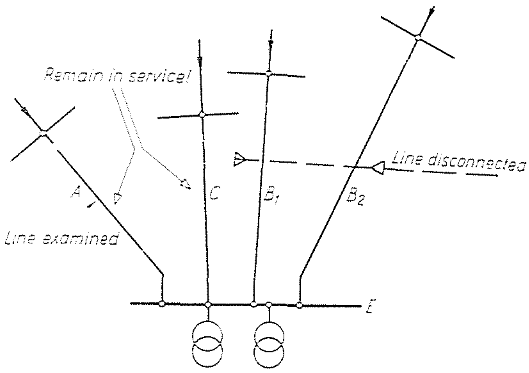

Fig. 4

As detailed in Section F. 2 of the Appendix, the results obtained are formally identical with those deduced for the two-line station. Thus, for the single-phase earth fault in Station E the following can be stated:

For any one arbitrarily chosen (investigated) line connected to a station with any number of lines, the maximum sound-phase fault current occurs with the combination of connected and disconnected lines, in which the ratio of the resultant zero-sequence impedance to the positive-sequence driving point impedance of the station is minimum, (3)

i.e.:

$$h_A = \frac{I^{ACE}_{A\,\mathrm{sph}\,Z}}{I^{ABCE}_{A\,\mathrm{sph}\,Z}} > 1, \quad \text{if} \quad \frac{Z_0^{ABCE}}{Z_1^{ABCE}} > \frac{Z_0^{ACE}}{Z_1^{ACE}}. \tag{4}$$

The upper indices refer to the connected impedances, *i.e.* to those which contribute to the resultant.

According to Section F. 3 of the Appendix, other relations between the impedance ratios can also be found, *viz.* those of the complementary impedances.

By making use of these relations, the following statements can also be made, without requiring further explanation:
First, expressed by means of formulae:

If

$$\frac{Z_0^{ABCE}}{Z_1^{ABCE}} > \frac{Z_0^{ACE}}{Z_1^{ACE}} . \tag{5}$$

then, according to F. 3

$$\frac{Z_0^{B}}{Z_1^{B}} > \frac{Z_0^{ACE}}{Z_1^{ACE}} \tag{6}$$

Thus, considering (3) and (4), the same rule expressed in words, is as follows:
For single-phase earth faults of a station, when investigating one of several lines having independent equivalent supply impedances, a higher sound-phase fault current will then be obtained, by connecting a further line, if the zero-/positive

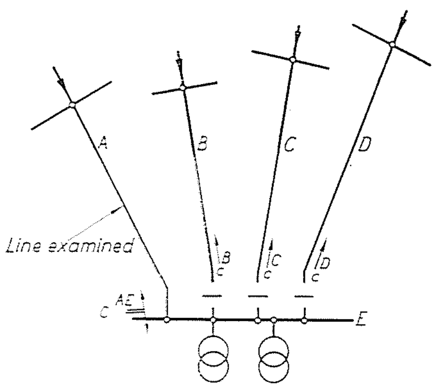

Fig. 5

sequence impedance ratio of the additionally connected line, as viewed from the station (for this additionally connected line only) *is lower than the resultant zero-/ positive sequence impedance ratio of the station determined for the case of having only the investigated line connected.* (7)

As soon as a higher impedance ratio is found with these additional lines, *no further line* needs to be connected for determining the maximum sound-phase fault current.

E.g., considering Fig. 5, let the ratio of the equivalent impedance of the investigated Line A to the resultant supply impedance of the station be:

$$C^{AE} = \frac{Z_0^{AE}}{Z_1^{AE}} = 0.9 .$$

The impedance ratios of the other lines *disconnected from the station* are as follows (Fig. 5):

$$C^B = \frac{Z_0^B}{Z_1^B} = 2.8\,,$$
$$C^C = 2.1\,,$$
$$C^D = 0.8\,.$$

In this particular case the maximum sound-phase fault current is obtained for Line A, if beside Line A only Line D is connected.

The statements outlined above can also be formulated by using practical data in the following way:

The zero-/positive-sequence impedance ratio of transmission lines usually lies in the vicinity of $\frac{Z_0}{Z_1} \cong 3$. If a transformer with earthed neutral is connected to the end of such a transmission line, for this terminal station the zero-/positive-sequence impedance ratio is generally much lower than 3, and often lies around 1, and may be even lower than 1 (as in the case of $C^{AE} = 0.9$ in the example). The own impedance ratio of each of the other lines is composed of the equivalent impedance of the station joined to the remote end of the line and of the impedance of the line itself. Apart from very short lines, the impedance ratio of the respective line will be decisive, thus its impedance ratio will be near to 3, but due to the smaller ratio of the remote-end station will usually be somewhat lower than that (such as $C^B = 2.8$ and $C^C = 2.1$ in the example). In the case of unusually short transmission lines the *impedance ratio of the remote-end station* becomes dominant, which is generally much lower than 3, thus the resultant impedance ratio (remote-end station + line) will also be close to, or may even be less than, 1 ($C^D = 0.8$ in the example). Thus, the case of a very short line can be regarded as if one could "see over" to the remote end of the line, and instead of the line impedance ratio that of the station became effective.

According to the above, for the protection of a transmission line *the maximum sound-phase fault current is obtained, if the line in question is the only line feeding the station (E) located at its end, further, if all (or a reasonable maximum number of) transformers are connected in Station E, and if lines with consumer loads only* ($C = 0$!, because $Z_1 \to \infty$) *and very short transmission lines are connected to the busbars.* These latter very short lines are only to be considered, if one "sees over them", *i.e.* their ratio C is lower than the joint impedance ratio of the investigated line + Station E.

The critical network condition can thus be determined in the following way:

a) The investigated line, all station transformers and the lines supplying consumer loads only (= radial lines with distribution transformers at their remote ends) is to be connected to the busbars of Station E in question. The connected lines are also to include those which are — under normal service conditions of the network — not feeder lines supplying consumer load only, but which may reasonably become such due to disconnecting all other supply lines from the remote-end station. Further, from the busbars of Station E all lines are to be disconnected for which the disconnected condition may reasonably be assumed. Now, the value of ratio C_1 is to be determined.

b) For the disconnected lines and supply lines, respectively, the value of ratio C_x as viewed from Station E is to be determined. The supply line having the lowest value of C_x — if this is lower than the resultant value determined according to *a)* — is to be connected and a new C_2 ratio is to be calculated for the station.

c) If any among the C_x values is found to be lower then C_2, the line giving the lowest ratio is to be connected and again a new C_3 ratio is to be determined.

d) The procedure given under *c)* is to be carried on until no such disconnected supply is left which would give a lower ratio than the resultant C_e finally obtained for the station. This network condition will be the one with which the maximum sound-phase fault current will flow in the investigated line.

It should be noted that after having found the reasonable minimum service condition as outlined under *a)*, there is generally no need for connecting any further lines, because the C_x found according to *b)* will usually not be lower than C_1. Thus *the method suggested in the paper* for finding the service condition, with which maximum sound-phase fault currents are associated, is a rapid procedure.

3. Station with two interdependent supply lines

In Sections 1 and 2 it was assumed that the lines feeding Station E are independent of each other. Although this "pure" case is of rare occurrence in practice, with minor neglections *the supplies may be considered as being independent in a great majority of the cases.*

If two lines connected to a station are closely interconnected (Figs 6 and 7), the statements of Sections 1 and 2 can no longer be considered as valid. It should be noted, however, that even for a considerable part of these cases the stated theorems remain applicable, *e.g.* when in the network configuration of Fig. 6 behind x and y the generating capacities are considerable, or the two lines of Fig. 7 can be treated as one line, etc.

Two interdependent supplies — provided the generator voltages are equal — can in all cases be reduced to a Y connection (Fig. 8—11). For the network configurations thus obtained — with the neglections mentioned in

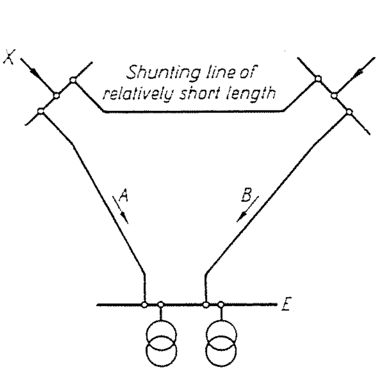

Fig. 6

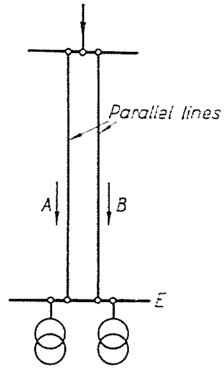

Fig. 7

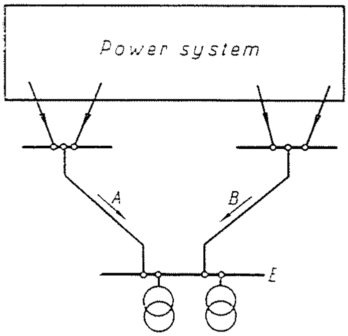

Fig. 8

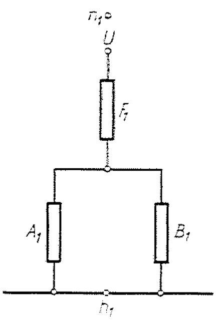

Fig. 9

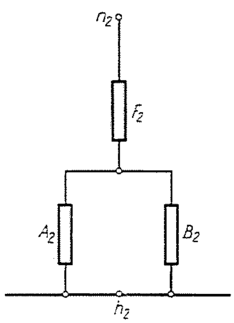

Fig. 10

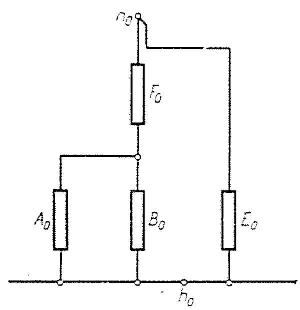

Fig. 11

Section 1 — it can also be found when the higher sound-phase fault current will occur. According to F. 4 of the Appendix, in the event of an earth fault in Station E the sound-phase fault current will be higher with Line B disconnected, than with both lines in operation, thus

$$h_{AY} = \frac{I'_{A\,\mathrm{sph}\,Z}}{I_{A\,\mathrm{sph}\,Z}} > 1\,, \tag{8}$$

if

$$\frac{Z_0}{Z_1} > \frac{\frac{Z_0'}{Z_1'}(a_1 + 2b_0) + (2a_1 - 2b_1)}{\frac{Z_0'}{Z_1'}(a_0 - b_0) + (2a_0 + b_1)}, \tag{9}$$

where Z_0, Z_1, Z_0' and Z_1', are equal to the respective quantities given in Section 1, and a_1, b_1, a_0 and b_0 are the combinations consisting of the impedances of Figs 9, 10 and 11.

Their values can be calculated from Equations F39, F40, F44 and F45.

Unfortunately, Relation (8) fails to give such a simple answer to the problem, as *e.g.* (2) or (6). Still, it gives the answer. Obviously, it is expedient to use Relations (2) and (6), respectively, if with reasonable neglections the supplies can be considered as independent.

If on the right-hand side of Relation 9 the constants are substituted by $F_1 = F_2 = F_0 = 0$ (making thereby the supplies independent of each other), the relation reduces to (2).

4. Conclusion

By means of the method described in Sections 1, 2 and 3 — without performing the short circuit-calculation — the actual connection of the network can be predetermined, from which the maximum sound-phase fault current for computing the distance-protection settings of the line in question is to be calculated. This method renders lengthy variational short-circuit calculations unnecessary for this purpose, the performance of the short-circuit calculation being required for only one predetermined network configuration.

For practical purposes, of course, the simple method of Section 2 may be recommended, and it is advisable, even at the expense of suitable neglections, to use the simple form, i.e. to make the supplies of the station in question independent of each other, possibly by applying a suitable grouping of the lines.

Finally, it is worth mentioning that the described method lends itself to be used in theoretical considerations, as well as in computing the relay settings of new lines added to existing networks. When used for the former purpose, the advantage lies in the applicability of simple relations (Section 2), while for the latter the node-point and supply impedances referred to in the paper can easily be determined from the available short-circuit map of the existing network.

5. Appendix

F.1. Two-line station

A) The following assumptions are made in the calculations:

a) Faults occur with no load on the system (thus all generated voltages are taken to be equal and in phase).

b) The phase angles of impedances are equal.

c) The positive- and negative-sequence impedances are equal.

d) The supplies from A and B (and the networks behind them) are independent of each other.

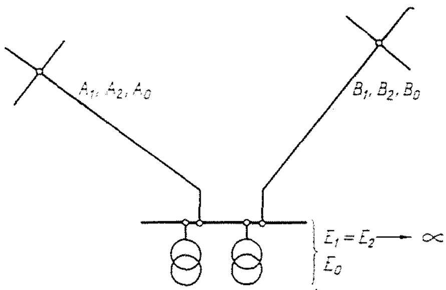

Fig. 12

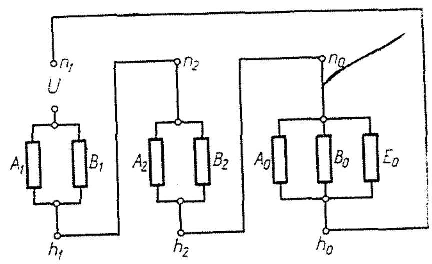

Fig. 13

With these assumptions the equivalent sequence network for the single-phase-to-earth short-circuit occurring at the busbars of Station E shown in Fig. 12 is represented in Fig. 13.

The sequence currents flowing in the system on the occurrence of a phase-to-earth short-circuit are as follows:

$$I_1 = I_2 = I_0 = \frac{U}{\frac{A_1 B_1}{A_1 + B_1} + \frac{A_2 B_2}{A_2 + B_2} + \frac{A_0 B_0 E_0}{A_0 B_0 + B_0 E_0 + A_0 E_0}} \qquad \text{(F. 1)}$$

The currents flowing in Line A are

$$I_{A1} = I_1 \frac{B_1}{A_1 + B_1}, \qquad \text{(F. 2)}$$

$$I_{A2} = I_1 \frac{B_2}{A_2 + B_2}, \qquad \text{(F. 3)}$$

$$I_{A0} = I_1 \frac{B_0 E_0}{A_0 B_0 + B_0 E_0 + A_0 E_0}. \qquad \text{(F. 4)}$$

According to the assumption made under *c)*

$$A_2 = A_1 \quad \text{and} \quad B_2 = B_1 , \tag{F. 5}$$

further [1] the fault component of the sound-phase current being

$$I_{A\,\mathrm{sph}\,Z} = I_{A0} - I_{A1} , \tag{F. 6}$$

we obtain

$$I_{A\,\mathrm{sph}\,Z} = I_{A0} - I_{A1} = I_1 \left[\frac{B_0 E_0}{A_0 B_0 + B_0 E_0 + A_0 E_0} - \frac{B_1}{A_1 + B_1} \right] \tag{F. 7}$$

Substituting F.1. into F.7.:

$$I_{A\,\mathrm{sph}\,Z} = \frac{U}{2 \dfrac{A_1 B_1}{A_1 + B_1} + \dfrac{A_0 B_0 E_0}{A_0 B_0 + B_0 E_0 + A_0 E_0}} \cdot$$

$$\cdot \left[\frac{B_0 Z_0}{A_0 B_0 + B_0 E_0 + A_0 E_0} - \frac{B_1}{A_1 + B_1} \right]. \tag{F. 8}$$

Simplifying F.8:

$$I_{A\,\mathrm{sph}\,Z} = \frac{U}{2Z_1 + Z_0} \cdot \left[\frac{Z_0}{A_0} - \frac{Z_1}{A_1} \right], \tag{F. 9}$$

where

$$Z_1 = \frac{A_1 B_1}{A_1 + B_1}$$

is the *resultant positive- (and negative)* sequence driving point impedance of Station E,

$$Z_0 = \frac{A_0 B_0 E_0}{A_0 B_0 + A_0 E_0 + B_0 E_0}$$

is the *resultant zero-sequence* driving point impedance of Station E,

A_1 is the positive- (and negative) sequence impedance of the supply from A as viewed from Station E

A_0 is the zero-sequence impedance of the supply from A as viewed from Station E.

B) If from Station E shown in Fig. 12 Line B is disconnected, the sound-phase fault current can similarly be obtained: (F.10)

$$I'_{A\,\mathrm{sph}\,Z} = \frac{U}{2Z'_1 + Z'_0} \cdot \left[\frac{Z'_0}{A_0} - \frac{Z'_1}{A_1} \right]. \tag{F. 11}$$

In accordance with the interpretation shown in Fig. 12, it can be seen that when disconnecting Line B,

$$Z_1' = A_1 , \tag{F. 12}$$

thus

$$I'_{A\,\mathrm{sph}\,Z} = \frac{U}{2A_1 + Z_0'} \cdot \left[\frac{Z_0'}{A_0} - 1\right]. \tag{F. 13}$$

C) The ratio of the two currents (F.9 and F.13), for Line A will be

$$h_A = \frac{I'_{A\,\mathrm{sph}\,Z}}{I_{A\,\mathrm{sph}\,Z}} = \frac{2Z_1 + Z_0}{2A_1 + Z_0'} \cdot \frac{\frac{Z_0'}{A_0} - 1}{\frac{Z_0}{A_0} - \frac{Z_1}{A_1}}. \tag{F. 14}$$

If the value of h_A is higher than unity, then by disconnecting Line B the sound-phase fault current of Line A will increase in the case of a single-phase-to-earth fault at E. *When will h_A be higher than unity?*

$$h_A = \frac{Z_0 + 2 \cdot Z_1}{Z_0' + 2 \cdot Z_1'} \cdot \frac{\frac{1}{A_0} Z_0' - \frac{1}{A_1} Z_1'}{\frac{1}{A_0} Z_0 - \frac{1}{A_1} Z_1}. \tag{F. 15}$$

With the assumption given under *b)* the phase angles of all elements in F.15 are equal, thus the fractions are all real quantities, so the relation

$$h_A \geqq 1$$

can be interpreted, thus

$$\frac{Z_0 + 2Z_1}{Z_0' + 2Z_1'} \cdot \frac{\frac{1}{A_0} Z_0' - \frac{1}{A_1} Z_1'}{\frac{1}{A_0} Z_0 - \frac{1}{A_1} Z_1} \geqq 1 ,$$

or, rewritten into another form:

$$\frac{\frac{Z_0}{Z_1} + 2}{\frac{Z_0'}{Z_1'} + 2} \cdot \frac{\frac{Z_0'}{Z_1'} \cdot \frac{1}{A_0} - \frac{1}{A_1}}{\frac{Z_0}{Z_1} \cdot \frac{1}{A_0} - \frac{1}{A_1}} \geqq 1 . \tag{F. 16}$$

According to [1] — based on practical cases — on the occurrence of a single-phase-to-earth fault in a station a "lack of I_0" arises in the lines, *i.e.* a lower I_0 than I_1 will flow towards the fault, because a considerable amount of I_0 and no I_1 will be supplied by the local transformers of the station: Hence, the terms in sequare brackets of Eqs F.7, F.8, F.9 and F.11, F.12 will be negative.

In Equ. F.7 $I_{A\,\text{sph.}Z} = I_{A0} - I_{A1}$ and following from the above, $|I_{A0}| < |I_{A1}|$. Thus, in the unequality F.16 both the numerator and denominator of the second (right-hand-side) fraction are negative. Before solving the inequality let both the numerator and denominator be multiplied by (-1):

$$\frac{\dfrac{Z_0}{Z_1}+2}{\dfrac{Z_0'}{Z_1'}+2} \cdot \frac{\dfrac{1}{A_1}-\dfrac{Z_0'}{Z_1'}\dfrac{1}{A_0}}{\dfrac{1}{A_1}-\dfrac{Z_0}{Z_1}\dfrac{1}{A_0}} \geqq 1\,,$$

so

$$\frac{\dfrac{Z_0}{Z_1}+2}{\dfrac{1}{A_1}-\dfrac{Z_0}{Z_1}\cdot\dfrac{1}{A_0}} \geqq \frac{\dfrac{Z_0'}{Z_1'}+2}{\dfrac{1}{A_1}-\dfrac{Z_0'}{Z_1'}\dfrac{1}{A_0}}\,. \tag{F. 17}$$

From F.17 it can be seen that the condition of equality is satisfied when

$$\frac{Z_0}{Z_1} = \frac{Z_0'}{Z_1'}\,. \tag{F. 18}$$

Rewriting and reducing F.17, the following relation is obtained:

$$h_A \geqq 1, \text{ if } \frac{Z_0}{Z_1} \geqq \frac{Z_0'}{Z_1'} \tag{F. 19}$$

or in words:

The sound-phase fault current flowing in Line A, with Line B disconnected, will then be higher than with Line B connected, if the ratio of the zero-sequence impedance to the positive-sequence driving point impedance of the station is higher with Line B connected than with the latter disconnected.

F.2. Three-line station

In the network shown in figure 14. and applying the results of F.1 to this case, the sound-phase fault current flowing in Line A, with all other lines connected will be (see Equ. F. 9):

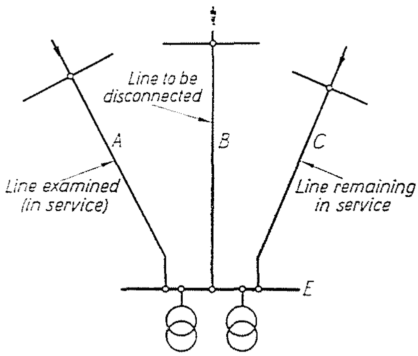

Fig. 14

$$I_{A\ \mathrm{sph.}\ Z} = \frac{U}{2Z_1 + Z_0} \cdot \left[\frac{1}{A_0} Z_0 - \frac{1}{A_1} Z_1\right], \tag{F.20}$$

where

Z_1 is the resultant positive (and negative) sequence driving point impedance of Station E,

Z_0 is the resultant zero-sequence driving point impedance of Station E.

Similarly, after disconnection of Line B (according to F.11):

$$I'_{A\ \mathrm{sph.}\ Z} = \frac{U}{2Z'_1 + Z'_0} \cdot \left[\frac{1}{A_0} Z'_0 - \frac{1}{A_1} Z'_1\right], \tag{F.21}$$

Z'_1 is the resultant positive (and negative) sequence driving point impedance of station E, after disconnecting Line B (here $Z'_1 \neq A_1$!)

Z'_0 is the resultant zero-sequence driving point impedance of Station E, after disconnecting Line B.

Due to the identity of formulae shown above the conclusions which can be drawn are in principle the same as those arrived at for the case of the two-line station (see Section F.1 of the Appendix).

F.3. Conversion to the supply impedances (Generalization)

In all the cases discussed above the ratios between the resultant sequence impedances and that between partial sequence-impedances have been compared. *E.g.* (Equ. F.19):

$$\frac{Z_0}{Z_1} \geqq \frac{Z'_0}{Z'_1}. \tag{F.22}$$

Here the impedances Z_0 and Z_0', and Z_1 and Z_1', respectively, can be brought into interrelation by means of the complementary impedances:

$$\frac{1}{Z_0} = \frac{1}{Z_0'} + \frac{1}{Z_0^k} \tag{F.23}$$

and

$$\frac{1}{Z_1} = \frac{1}{Z_1'} + \frac{1}{Z_1^k}. \tag{F.24}$$

Substituting F.23 and F.24 into F.22 and rearranging the latter, the following relation is obtained:

$$\frac{\frac{1}{Z_1'} + \frac{1}{Z_1^k}}{\frac{1}{Z_0'} + \frac{1}{Z_0^k}} \geq \frac{\frac{1}{Z_1'}}{\frac{1}{Z_0'}}. \tag{F.25}$$

As can be seen, the two quantities on the right-hand side appear also on the left-hand side of the relation. If all terms in the fraction are positive (and this condition applies to the impedances), then after some multiplications, simplifications and regrouping the following relation is obtained:

$$\frac{\frac{1}{Z_1^k}}{\frac{1}{Z_0^k}} \geq \frac{\frac{1}{Z_1'}}{\frac{1}{Z_0'}}, \tag{F.26}$$

or

$$\frac{Z_0^k}{Z_1^k} \geq \frac{Z_1'}{Z_0'}. \tag{F.27}$$

Thus, when the ratio of the resultant zero-sequence to positive-sequence impedances is higher than the ratio of the corresponding partial impedances, then the ratio of the complementary impedances will also be higher than those of the partial impedances.

F.4. Two interdependent supplies

The reduced equivalent networks of the station with two interdependent supplies shown in Fig. 8 are represented in Figs 9, 10, and 11. Based on these equivalent networks, the symmetrical components of the short-circuit currents flowing in the case of a single phase-to-earth fault of Station E will be as follows:

$$I_1 = I_2 = I_0 =$$

$$= \frac{U}{F_1 + \frac{A_1 B_1}{A_1 + B_1} + F_2 + \frac{A_2 B_2}{A_2 + B_2} + \frac{\left(F_0 + \frac{A_0 B_0}{A_0 + B_0}\right) E_0}{F_0 + \frac{A_0 B_0}{A_0 + B_0} + E_0}} \cdot \quad \text{(F.28.a)}$$

The respective sequence currents, when Line B is disconnected, will be

$$I_1' = I_2' = I_0' = \frac{U}{F_1 + A_1 + F_2 + A_2 + \frac{(F_0 + A_0) E_0}{F_0 + A_0 + E_0}} . \quad \text{(F.28.b)}$$

Let the following notations be introduced for the resultant impedances referring to Station E:

Z_1 for the positive-sequence driving point impedance ($Z_2 = Z_1$, according to the conditions of Section F.1.),

Z_0 for the zero-sequence driving point impedance, and

Z_1' and Z_0' for the respective values as defined above, yet referring to the case when Line B is disconnected.

Thus

$$Z_1 = F_1 + \frac{A_1 B_1}{A_1 + B_1} = \frac{F_1 A_1 + F_1 B_1 + A_1 B_1}{A_1 + B_1} \quad \text{(F. 29)}$$

$$Z_2 = F_2 + \frac{A_2 B_2}{A_2 + B_2} = Z_1 \quad \text{(F. 30)}$$

$$Z_0 = \frac{\left(F_0 + \frac{A_0 B_0}{A_0 + B_0}\right) E_0}{F_0 + \frac{A_0 B_0}{A_0 + B_0} + E_0} \quad \text{(F. 31)}$$

and

$$Z_1' = F_1 + A_1 \quad \text{(F. 32)}$$

$$Z_2' = F_2 + A_2 = Z_1' \quad \text{(F. 33)}$$

$$Z_0' = \frac{(F_0 + A_0) E_0}{F_0 + A_0 + E_0} . \quad \text{(F. 34)}$$

Substituting these relations into F.28.a and F.28.b:

$$I_1 = I_2 = I_0 = \frac{U}{2 \cdot Z_1 + Z_0}, \tag{F. 35}$$

$$I'_1 = I'_2 = I'_0 = \frac{U}{2 \cdot Z'_1 + Z'_0}. \tag{F. 36}$$

The positive-sequence current in Line A, with all lines connected, will be

$$I_{1A} = \frac{U}{2 \cdot Z_1 + Z_0} \cdot \frac{B_1}{A_1 + B_1} = \frac{U}{2 \cdot Z_1 + Z_0} \cdot \frac{B_1}{F_1 A_1 + F_1 B_1 + A_1 B_1} \cdot Z_1 \tag{F. 37}$$

and the zero-sequence current:

$$I_{0A} = \frac{U}{2Z_1 + Z_0} \cdot \frac{Z_0}{F_0 + \dfrac{A_0 B_0}{A_0 + B_0}} \cdot \frac{B_0}{A_0 + B_0} = \frac{U}{2Z_1 + Z_0} \cdot \frac{B_0}{F_0 A_0 + F_0 B_0 + A_0 B_0} \cdot Z_0 . \tag{F. 38}$$

Using the following notations:

$$a_1 = \frac{B_1}{F_1 A_1 + F_1 B_1 + A_1 B_1}, \tag{F. 39}$$

and

$$a_0 = \frac{B_0}{F_0 A_0 + F_0 B_0 + A_0 B_0}, \tag{F. 40}$$

the sound-phase fault current in Line A, with all lines connected, will be

$$I_{A \text{ sph } U} = I_{0A} - I_{1A} = \frac{U}{2 \cdot Z_1 + Z_0} \cdot \left[a_0 Z_0 - a_1 Z_1 \right]. \tag{F. 41}$$

The positive-sequence current in Line A, with Line B disconnected.

$$I'_{1A} = I'_1 = \frac{U}{2Z'_1 + Z'_0} = \frac{U}{2Z'_1 + Z'_0} \cdot \frac{1}{F_1 + A_1} Z'_1 . \tag{F. 42}$$

and the zero-sequence current:

$$I'_{0A} = \frac{U}{2Z'_1 + Z'_0} \cdot \frac{E_0}{F_0 + A_0 + E_0} = \frac{U}{2Z'_1 + Z'_0} \cdot \frac{1}{F_0 + A_0} \cdot Z'_0 . \quad (F.43)$$

With the notations

$$b_1 = \frac{1}{F_1 + A_1} \quad (F.44)$$

and

$$b_0 = \frac{1}{F_0 + A_0}, \quad (F.45)$$

the sound-phase fault current in A with B disconnected:

$$I'_{A\,\mathrm{sph}\,Z} = \frac{U}{2 \cdot Z'_1 + Z'_0} \cdot [b_0 Z'_0 - b_1 Z'_1] . \quad (F.46)$$

The ratio of F.41 to F.46 will be

$$h_{AY} = \frac{I'_{A\,\mathrm{sph}\,Z}}{I_{A\,\mathrm{sph}\,Z}} = \frac{2Z_1 + Z_0}{2Z'_1 + Z'_0} \cdot \frac{b_0 Z'_0 - b_1 Z'_1}{a_0 Z_0 - a_1 Z_1} . \quad (F.47)$$

When will this ratio be higher than unity?

$$h_{AY} \geqq 1, \text{ if}$$

$$\frac{Z_0}{Z_1} \geqq \frac{\frac{Z'_0}{Z'_1}(a_1 + 2b_0) + (2a_1 - 2b_1)}{\frac{Z'_0}{Z'_1}(a_0 - b_0) + (2a_0 + b_1)} . \quad (F.48)^*$$

If the values $F_1 = F_2 = F_0 = 0$ are substituted into F.48 (making thereby the supplies independent of each other, see Figs 9, 10 and 11), then the relation

$$\frac{Z_0}{Z_1} \geqq \frac{Z'_0}{Z'_1} \quad (F.49)$$

already known (F.19) is again obtained.

* Similar to the procedure applied for reducing relation F. 16. in Section F. 1. of the Appendix, also here the numerator and denumerator is to be multiplied by (—1) to rearrange the inequality.

Summary

On single-phase-to-earth short-circuits occurring in solidly-earthed-neutral systems fault corrents of considerable magnitude flow in the two sound phases not directly affected by the fault. The dependence of these currents termed "sound-phase fault currents" on prevailing network conditions is discussed by the author. The results obtained and the methods proposed in the paper are suitable for practical applications and lend themselves to form the basis for drawing conclusions of general validity.

References

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