SOME PROBLEMS OF NON-EQUILIBRIUM PLASMA FLOW

By

Cs. Ferencz

Department of Theoretical Electricity, Polytechnical University, Budapest
(Received December 1, 1967)

Presented by Prof. Dr. K. Simonyi

Examination of processes in plasmas has an extraordinary importance in these days, both from telecommunicational and energetical aspects. From the examination of the phenomena it appears that the so-called cold plasma, produced by non-equilibrium ionization, plays a fundamental role both as a research aid and as an applied medium. A new device of power production is the MHD generator, where, however, the flow-conditions, the energy transfer phenomena, etc. are difficult to investigate in the equilibrium plasma of high temperature, designed as a final solution. From experimental-technical aspects it is much simpler to measure first in cold plasma. Furthermore, the transformation of the kinetic energy into electric power in plasmas, can not only be used to produce electricity, but also kinetic energy for use in space engines with very high exhaust velocities, that is, with very high specific impulses. (Without such engines interplanetary voyages would not be imaginable at all.) Since the economical use of power, the efficiency, is of a decisive importance, the exhausted gases must leave the engine cold, and neutral also, for other purposes. It is very advantageous to use only non-equilibrium ionization for most types of rocket engine. In addition, in fundamental physical experiments on plasma properties, the applied ionization is most frequently non equilibrial, being easy to control and regulate.

The problem is now to examine the variation of the number of charged particles in stationary, weakly ionized, cold plasma flow, exhausted by some ionizator at a medium pressure ranging between 10^{-4} to 10^{2} Hgmm, neglecting external forces and, for the sake of exact calculations simplifying as far as possible and as admissible.

These investigations serve to give aspects for designing a closed flow cycle of cold plasma, mainly the allowable distance between the ionizator and the space of measuring and the losses at the walls. The most important of these aspects is to give the solution in more generally valid form, and not under several special boundary conditions for a single material only.

1. Basic assumptions for the solution

Let us suppose the following:

1.1. No external forces affect the flowing medium, that is, there are no external electric, magnetic or gravitational forces. (If eventually we wish to take into account these forces afterwards, then we have to start again from the basic equations, in supposing, that $F \neq 0$. It should be noted, that under Earth conditions, the role of gravitational forces can be by all means neglected.)

1.2. Let us simplify the interactions among the particles: there is no necessity of investigating the energy transfer among the different kinds of particles, as we neither extract from, nor feed in, the gas any energy. So it may be supposed, that the energy dispersion occurs already in the ionizator itself. Let us consider the spots, which limit this energy dispersion as the end of the ionizator. The effect of internal friction can be neglected along the walls, and inside the medium it ensures only the uniform flow velocities of neutral and charged particles. The flow velocity is supposed to be constant throughout the cross-section.

1.3. The state parameters effect the constants of the functions describing the different phenomena. Since in the whole gas the number of particles does not vary, and we are to omit the energy losses, it may be supposed, that in the space where the measurements take place, the ionization and recombination do not affect the value of the state parameters.

1.4. The variation of the Debye length may be neglected, as the modification of its value, depending on the particle number, does not change the state of the plasma, within given limits. — The plasma flow, we are to deal with, can be located near to the 50% ionization limit in the EM zone of the Kantarovitz—Petschek diagram.

1.5. The degree of ionization is single.

1.6. In the space of measuring there is no internal energy-conversion, the flow velocity is constant; the plasma had been accelerated earlier.

Let us examine the variation of the charged number of particles in the plasma under the assumptions made above.

2. Mathematical formulation of the problem

Let us start from the general transport equation which can be deduced from the Boltzmann equation.

2.1. The fundamental equations

It is equally possible to use the transport equation both for the charged and for the neutral particles. As for the external force, $F = 0$, the continuity equation is of the same form for all the three kinds of particles (electrons, positive ions, and neutral particles), that is:

$$\frac{\partial}{\partial t}(n_s\langle\Theta\rangle_s) + \overline{\nabla}(n_s\langle\Theta\overline{w}\rangle_s) = \sum_{e,i,0}\int \Theta_s \left(\frac{\partial f_s}{\partial t}\right)_c d\overline{w} \tag{1}$$

where n is the particle number, Θ — the examined parameter of particles (mass, charge, number, etc.), the subscript s denotes the kind of particles, w — the velocity of the particle, $\langle\ \rangle$ — the average value, f — the distribution function, t — time, c — collision index. In examining now the particle number, therefore Θ equals 1, so we obtain:

$$\frac{\partial n_s}{\partial t} + \overline{\nabla}(n_s\langle\overline{w}\rangle_s = \sum_{e,i,0}\int \Theta_s \left(\frac{\partial f_s}{\partial t}\right)_c d\overline{w} \tag{2}$$

Under the given assumptions, the collisions produce only ionization and recombination; the average velocity is composed of the average drift velocity ($\overline{v}$), the diffusion velocity ($\overline{v}_D$) and the velocity due to the Coulomb field ($\overline{v}_{cb}$), resulting from the separation of electrons and ions. So we obtain:

$$\begin{aligned}
\sum_{e,i,0}\int \Theta_e \left(\frac{\partial f_e}{\partial t}\right)_c d\overline{w} &= n^*_{ei} - Rn_e n_i \\
\sum_{e,i,0}\int \Theta_i \left(\frac{\partial t_i}{\partial t}\right)_c d\overline{w} &= n^*_{ei} - Rn_e n_i \\
\sum_{e,i,0}\int \Theta_0 \left(\frac{\partial f_0}{\partial t}\right)_c d\overline{w} &= Rn_e n_i - n^*_{ei}
\end{aligned} \tag{3}$$

as the degree of ionization is single. (n^* is the ionization, R is the recombination coefficient.)

$$\begin{aligned}
\langle\overline{w}\rangle_e &= \overline{v} + \overline{v}_{cbe} + \overline{v}_{De} = \overline{v}_e + \overline{v}_{De} \\
\langle\overline{w}\rangle_i &= \overline{v} + \overline{v}_{cbi} + \overline{v}_{Di} = \overline{v}_i + \overline{v}_{Di} \\
\langle\overline{w}\rangle_0 &= \overline{v} + \overline{w}_{D0}\,.
\end{aligned} \tag{4}$$

So the initial equation system is of the following form:

$$\begin{aligned}
\frac{\partial n_e}{\partial t} &= n^*_{ei} - Rn_e n_i + D_e \Delta n_e - \operatorname{div}(n_e\overline{v}_e) \\
\frac{\partial n_i}{\partial t} &= n^*_{ei} - Rn_i n_e + D_i \Delta n_i - \operatorname{div}(n_i\overline{v}_i) \\
\frac{\partial n_0}{\partial t} &= Rn_e n_i - n^*_{ei} + D_0 \Delta n_0 - \operatorname{div}(n_0\overline{v})
\end{aligned} \tag{5}$$

2.2. General discussion

Before neglecting the time dependence and the effect of ionization, some — being very important in the following — analyses must be carried out.

It may be seen, that under the simplifying assumptions we made, n_0 does not effect explicitly n_e and n_i in Eqs (5). As n_0 is not of interest in the following, the equation for n_0 will be omitted.

In the case of $\bar{v}_e \neq \bar{v}_i$ charges in the plasma will be separated. Let us examine this possibility.

Because of the charge separation an internal electric field, $\bar{E}_b$ arises. In the lack of an external field, the permittivity (ε) can be considered scalar and constant. It can be supposed, that under the given circumstances the internal magnetic field is negligible. So we obtain:

$$\bar{B}_b \cong 0$$

$$\operatorname{rot} \bar{E}_b \cong 0$$

$$\operatorname{div} \bar{E}_b = \frac{e}{\varepsilon}(n_i - n_e)$$

$$\bar{E}_b = -\operatorname{grad} U = -\frac{e}{4\pi\varepsilon} \operatorname{grad} \int_V \frac{n_i - n_e}{r} dV \tag{6}$$

$$\bar{v}_e = \bar{v} - \mu_e \cdot \bar{E}_b$$

$$\bar{v}_i = \bar{v} + \mu_i \cdot \bar{E}_b$$

where e is the electron charge, U — the scalar potential, r — the distance between the observed field point and the charge $(n_i - n_e)$, V — the volume, μ — the mobility.

Using Eqs (6), the first two equations of Eqs (5) can be written in the following form:

$$\frac{\partial n_e}{\partial t} = n_{ei}^* - Rn_e n_i + D_e \Delta n_e - n_e \operatorname{div} \bar{v} + \frac{\mu_e e}{\varepsilon}(n_e n_i - n_e^2) - $$
$$- (\operatorname{grad} n_e)\left[\bar{v} + \frac{\mu_e e}{4\pi\varepsilon} \operatorname{grad}\left(\int_V \frac{n_i - n_e}{r} dV\right)\right]$$

$$\frac{\partial n_i}{\partial t} = n_{ei}^* - Rn_e n_i + D_i \Delta n_i - n_i \operatorname{div} \bar{v} + \frac{\mu_i e}{r}(n_i n_e - n_i^2) - \tag{7}$$
$$- (\operatorname{grad} n_i)\left[\bar{v} - \frac{\mu_i e}{4\pi\varepsilon} \operatorname{grad}\left(\int_V \frac{n_i - n_e}{r} dV\right)\right]$$

In analysing Eqs (7), — (for the use in further studies) the following can be established:

2.21. The plasma always tends to get into a neutral state. On the right side of Eqs (7), the fifth and sixth terms show, that with time n_e and n_i will approach each other. It depends on the relative velocity of this process to other phenomena, whether we may use later the $n_e = n_i$ equality or not.

Since we take out electrons from the plasma during measurements, and under operation of the MHD generators, we have to provide the experimental devices with electron emitters.

As for electrons and ions move together, it is necessary to take into account the ambipolar diffusion.

2.22. The recombination is slow compared with the trend of being neutral. In examining the second and fifth terms of the right side of Eqs (7), we find:

$$n_e n_i \left(\frac{\mu_{e,i}}{\varepsilon} - R \right)$$

Let us estimate the two terms in brackets, according to the measured data [1]:

$$R \sim 10^{-17} \quad \mathrm{m^3/s}$$

$$\langle \mu \rangle_{e,i} \sim 50. \quad \mathrm{m^2/s.}\,V.$$

$$e = 1{,}6. \quad 10^{-19}\, As$$

$$\varepsilon = 8{,}86. \quad 10^{-12}\, As/Vm.$$

As the plasma is not too dense we may assume that ε_r does not alterate the orders of magnitude, so we obtain:

$$\frac{\mu_{e,i} e}{\varepsilon} \sim 10^{-6} \quad \mathrm{m^3/s},$$

that is

$$\frac{\mu_{e,i}\, e}{\varepsilon} \gg R\,.$$

So we may calculate in the following as

$$n_e \cong n_i = n$$

$$\bar{v}_e \cong \bar{v}_i = \bar{v}$$

which means ambipolar diffusion.

2.23. In thin plasmas we find the diffusion to be more decisive than the recombination. In dense plasmas it is just reversed. In using the data given by von ENGEL [1] for estimating the orders of magnitude:

$$D \sim 1\mathrm{m^2/s}, \quad R \sim 10^{-17}\,\mathrm{m^3/s}\,.$$

In thin plasma $n_1 \sim 10^{16}$ 1/m^3, and in "dense" one $n_2 \sim 10^{20}$ 1/m^3, so we have

$$Rn_1^2 \ll D\Delta n_1$$

and

$$Rn_2^2 \gg D \cdot \Delta n_2 .$$

This result can be used for controlling our further examinations too, as for it must be obtained from those as well.

2.3. The final equation and ist comparison with the works of other authors. In accordance with our aim, we want to examine stationary processes outside the ionizator when the flow velocity does not vary.

If we want to study the processes in ionizators or nozzles of varying velocities, then we have to start again from Eqs (7), but we may not neglect so many members (e.g. n_{ei} and $n \cdot \operatorname{div} \bar{v}$).

From our earlier statements we get

$$n_e = n_i = n$$

$$\frac{\partial n}{\partial t} = 0$$

$$n^* = 0$$

$$\bar{v} = \text{constant}$$

$$D_i = D_e = D_a \equiv D$$

and applying now the starting assumptions, the final equation runs as follows:

$$D \cdot \Delta n - \bar{v} \cdot \operatorname{grad} n - Rn^2 = 0 \tag{8}$$

In the following we want to analyse this equation under different boundary conditions.

2.31. Such examinations are very important in the case of plasma rocket engines. On the other hand, the experiments clearing up the basic similar laboratory types of phenomena on plasma-flow are nowadays rather developing. Both these two facts show themselves in the studying of literature. There were very few data to be found for studying similar topics and these ones rather on the rocket engines.

Chuan [2] indicated in his report "Plasma Heating Research" on the Plasma-Conference of the US Air Force on the Maryland University in 1958 that they began to investigate together with preliminary experiments the effect of diffusion, recombination and the escape of electrons in plasma, to make possible the examination of processes in the nozzle.

They started by supposing that $n_e = n_i$, from the equation

$$\frac{dn_e}{dt} = D \cdot \Delta n_e - Rn_e^2 - \beta n_e$$

They did not communicate any results or publications, neither occurred any references to be found later on.

On the Conference on Aerospace Electro-Technology in 1964 BROMBERG and FREE [3] reported measurements on Cs rocket engines. They took into account the effect of drift velocity and recombination in the theoretical approximation. The calculations were made only for one single initial condition, so they are not valid generally.

3. One-dimensional solution of the problem

Eq. (8) describing the problem is a second-ordered, non-linear differential equation of more variables. As it is very difficult to solve it generally, let us start with as simple boundary conditions, as possible. For the first we omit

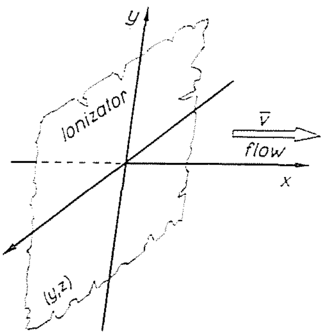

Fig. 1

the effect of the walls on the plasma in flow. So we suppose, that the plasma flows from a plane ionizator of infinitely large surface (Fig. 1), normally to the plane of the ionizator — of (y, z) plane in Cartesian coordinates — with a constant velocity $\bar{v} = v\bar{i}$ in the space without bounding walls. Let us now derive the number of charged particles along the X-axis.

Eq. (8) in Cartesian coordinates sounds:

$$D\left(\frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} + \frac{\partial^2 n}{\partial z^2}\right) - v \cdot \bar{i}\left(\frac{\partial n}{\partial x}\bar{i} + \frac{\partial n}{\partial y}\bar{j} + \frac{\partial n}{\partial z}\bar{k}\right) - R \cdot n^2 = 0$$

Using the assumptions and carrying out the arithmetical operations:

$$D\frac{d^2 n}{dx^2} - v\frac{dn}{dx} - Rn^2 = 0 \tag{9}$$

The boundary conditions for Eq. (9) are the following:

$n(x = 0) = n_0$, the initial number of charged particles at the ionizator,

$n(x = \infty) = 0$, as it takes infinitely long time for the particles to reach the infinity, therefore the recombination is perfect.

3.1. The transformation process based on special solutions

As for also Eq. (9) cannot be solved in a simple way, we begin with looking for special solutions for it.

3.11. Special solutions. There are two simplified forms for Eq. (9). For the first case we choose the flow-velocity of the medium to be zero. In the second case we choose that velocity to be so big, that we may neglect the diffusion beside it.

3.11.1. Let the flow-velocity of the plasma ($\bar{v}$) be zero. So we have for Eq. (9):

$$D \frac{d^2 n}{dx^2} - Rn^2 = 0 \tag{10}$$

Eq. (10) can already be solved with the well-known methods, and after two steps of integrating and in applying the boundary conditions, we have as final result:

$$n = \frac{n_0}{1 + \sqrt{\frac{2R}{3D} n_0} \cdot |x| + \frac{R}{6D} n_0 |x|^2} \tag{11}$$

3.11.2. Let the diffusion be negligible beside the flow, then we have for the equation to be solved:

$$v \frac{dn}{dx} + Rn^2 = 0 \tag{12}$$

the solution of which is

$$n = 0 \qquad x < 0$$

$$n = \frac{n_0}{1 + \frac{R}{v} n_0 x} \qquad x \geqq 0 \tag{13}$$

Later on we can give the range of validity for Eqs (12) and (13) only then, if we find some possibility of comparing the flow and diffusion velocities. We need Eq. (9) converted into a more suitable form.

3.12. Solution using the method of phase-plane. It is well known, that the method of phase-plane may often be very well used to solve the second order, non-linear differential equations of constant coefficients. We throw our Eq. (9) into a usable form.

Let be

$$y(n) = \frac{dn}{dx} \tag{14}$$

the new function to be examined. Then we have

$$\frac{d^2 n}{dx^2} = y \frac{dy}{dn}$$

and the new variables are $y, \frac{dy}{dn}$ and n. Let the plane (y, n) be called phase-plane. Then after sketching the plots

$$\frac{dy}{dn} = \text{constant} = \beta$$

one can give the function $y(n)$; by integrating it furthermore we have the result we wanted: $n = n(x)$.

The new form for Eq. (9) is:

$$Dy \frac{dy}{dn} - v \cdot y - Rn^2 = 0 \tag{15}$$

so we have to determine the plots of

$$y = \frac{R}{D \frac{dy}{dn} - v} n^2 \tag{16}$$

which can be done by drawing (16) taken $\beta =$ constant.

It is easily to be seen that if the diffusion is small, $D\beta \ll v$, then $y \cong - \frac{R}{v} n^2$, and if the drift velocity is small $D\beta \gg v$, then $y \cong \frac{R}{D\beta} n^2$. So a comparison between the diffusion and the drift mentioned in 3.11 can be carried out.

3.13. Critical velocity and characteristic length. We have to determine that range of velocities, where both diffusion and drift are significant. Let us modify Eq. (16) into the following form:

$$y = \frac{R}{D} \frac{n^2}{\frac{dy}{dn} - \frac{v}{D}} \tag{17}$$

The effect of diffusion and drift is of the same order if

$$\frac{dy}{dn} \cong -\frac{v}{D} \tag{18}$$

Let that velocity be the critical velocity $= v_0$. It follows then that

$$dy \cong -\frac{v_0}{D}\, dn \tag{19}$$

If we put (19) into Eq. (17), we have

$$y \cong -\frac{R}{2v_0}\, n^2 \tag{20}$$

The recombination is proportional to n^2. Based on this statement and Eq. (19) we may accept

$$y \sim -\frac{v_0}{D}\, n \tag{21}$$

as valid approximation.

In comparison of Eq. (20) with Eq. (21) we may define the critical velocity as

$$v_0 = \sqrt{\frac{RD}{2}\, n_0} \tag{22}$$

where n_0 is the initial number of particles. In the environment of this drift velocity the diffusion and the drift are comparable, otherwise one of them is negligible. It can be seen, that the critical velocity depends on the state of the medium, that means, that like the sound velocity, it characterizes the medium — plasma — which determines the processes.

After defining v_0, let us investigate our two special results in a detailed manner.

If $D = 0$, then Eq. (13) gives

$$n = \begin{cases} 0 & x < 0 \\ \dfrac{n_0}{1 + \dfrac{R}{v} n_0 x} & x \geqq 0 \end{cases}$$

Introducing here the ratio v_0/v, we have

$$n = \frac{n_0}{1 + \dfrac{v_0}{v} \sqrt{\dfrac{2R}{D} n_0 \cdot x}} \tag{23}$$

Keeping in sight the factor $\left(\sqrt{\frac{2R}{D} n_0}\ x\right)$ we can convert the result of the case $v = 0$ (11)

$$n = \frac{n_0}{\left(1 + \frac{1}{\sqrt{12}} \sqrt{\frac{2R}{D} n_0} |x|\right)^2} \tag{24}$$

Let be

$$w = \frac{1}{\sqrt{12}} \frac{v}{v_0} \text{ and } N = n/n_0 \tag{25}$$

Let us define furthermore the "characteristic distance" as:

$$\zeta = \sqrt{\frac{R}{6D} n_0} \cdot x \tag{26}$$

here ζ means the same distance for each medium, where N has the same magnitude and from where the actual distance can be calculated as

$$x = \zeta \sqrt{\frac{6D}{Rn_0}}$$

Using Eqs (25) and (26), both the special cases, and the equation to be solved can be transformed into new variables N and ζ, with w as a parameter.

If $v = 0$, and $\zeta \gtreqless 0$, we obtain:

$$N = \frac{1}{(1 + |\zeta|)^2} \tag{27a}$$

and

$$\frac{d^2 N}{d\zeta^2} - 6N^2 = 0 \tag{27b}$$

if $D = 0$,

$$\left.\begin{array}{ll} \zeta < 0 & N = 0 \\ \zeta \geq 0 & N = \dfrac{1}{1 + \zeta/w} \end{array}\right\} \tag{27c}$$

and

$$6w \frac{dN}{d\zeta} - 6N^2 = 0 \tag{27d}$$

Finally the new transformed form of the equation to be solved:

$$\frac{d^2 N}{d\zeta^2} - 6w \frac{dN}{d\zeta} - 6N^2 = 0 \tag{28}$$

the boundary conditions are $\zeta = 0$, $N = 1$, and $\zeta = \infty$, $N = 0$.

Therefore the problem can be solved in general, independently of the given parameters of the medium.

3.2. *Solution of the transformed equation and discussion of the solution*

Eq. (28) can be solved e.g. using the phase-plane method. Let be

$$Y = \frac{dN}{d\zeta} \tag{29}$$

then from (28) follows:

$$Y = \frac{6}{\dfrac{dy}{dN} - 6w} N^2 \tag{30}$$

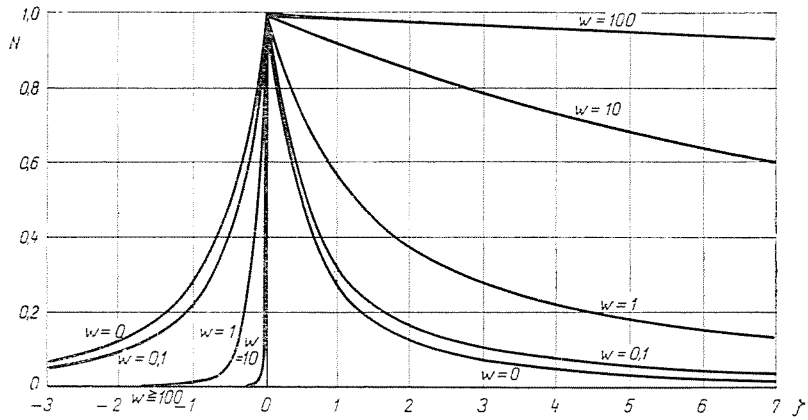

Fig. 2

After sketching the diagrams, belonging to $\frac{dy}{dN}$ = const., on the plane (Y, N), the $Y = Y(N)$ functions can be given. It makes much easier to plot $Y(N)$ after drawing the phase lines of the two special solutions. By integrating $Y(N)$ we obtain the final result: $N = N(\zeta)$. The result is represented on Fig. 2 using the parameter values of $w = 0$; 0,1; 1; 10 and 100 which is valid for the plasma flow if it meets the boundary conditions — approximately with a certain accuracy, of course — independently of the material and state of plasma.

Discussion: In examining the validity of the result (2), one has to check first the conditions of Part 1. These conditions are rather valid in thin, cold plasma flow. The roughest approximation among these is to neglect the viscosity and the profile of velocity (the laminarity or turbulence of the flow).

So in that case, in first approximation we may accept and so we can use the result as a design base for determining the maximum allowable value of the distance between the ionizator and the useful space, and the examined variation of the particle number in the space of measuring.

If the cross-section of the plasma in ζ is large, the charged particle number hardly varies in the transversal direction in the middle of the flow. That means, that the distribution along the axis well approaches the one plotted in Fig. 2. We can get more accurate information from the two-dimensional examination.

The value of v_0 shows, what order of velocity magnitude we need to reach any given number of charged particles somewhere in the flow, that is, we have data for the design of the accelerating part — e.g. Laval nozzle.

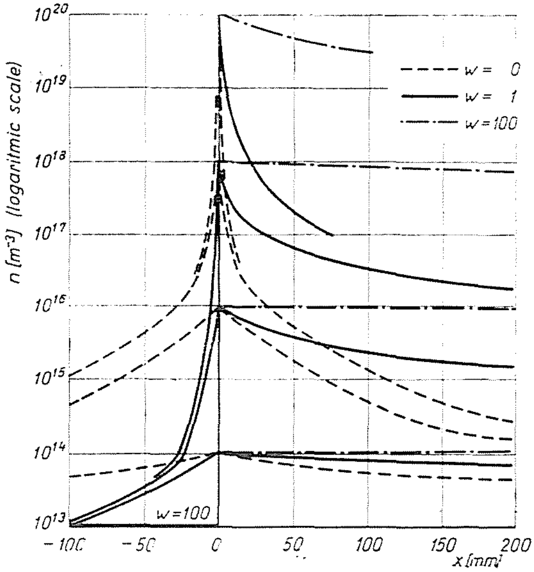

Fig. 3

As for the material to be used: the value of $x_0 = \sqrt{\dfrac{6D}{Rn_0}}$ belonging to $\zeta = 1$ gives the order of dimensions in the equipment, or it contains indirectly a hint to the material — D/R — and the ionization degree, n_0, at a certain pressure and density.

The data of ENGEL [1] give e.g. for hydrogen (H_2) if $T_e \cong T_i = 600$ K°, than $D_a \cong 0.1$ m²/s, $R = 2 \cdot 10^{-12}$ m³/s. So, if the ionization is feeble, $n_0 \sim$ $\sim 10^{18}$ m^{-3} $= 10^{12}$ cm^{-3}; so $v_0 = 316$ m/s and $\zeta = 1820 \cdot x^m$, that is, $x =$ $= 0.55\,\zeta$ [mm]. So it can be seen, that the hydrogen even at great velocities does not drift in large scale from the ionizator. In case of mercury (Hg) $T \cong 600$ K°, $T_i \cong 500 - 600$ K°, $T_e \cong 1000 - 1200$ K°, and let be the pressure in the flow $p \cong 0.01$ atm. (Under such conditions a very well handled cycle of mercury vapour can be realized.) According to the data of [1] $D_a \cong$ $\cong 1.4 \cdot 10^{-3}$ m²/s, $R \cong 10^{-15}$ m³/s, furthermore $v_0 = 8.35 \cdot 10^{-10} \cdot (n_0)^{1/2}$ m/s and $x = 2.9 \cdot 10^6 (n_0)^{-1/2}$m, where $[n_0] = 1/m^3$. Let us observe the flow in more detailed:

$n_0 =$	10^{14}	10^{16}	10^{18}	10^{20}	$[m^{-3}]$
$x =$	290ζ	29ζ	2.9ζ	0.29ζ	[mm]
$v_0 =$	0.00835	0.0835	0.835	8.35	[m/s]

The plots of particle number vs. distance, — that is, the curves of $n(x)$ — can be seen on Fig. 3 with $w = 0$, 1 and 100 as parameters. It is clearly to be seen that in thin plasma the diffusion, in dense plasma the recombination dominates (par 2.23).

It is very important, that the outdrift of the mercury (see Fig. 3) seems to be in order of about 10 cm. It is also interesting that the $n_w(x)$ curves approach each other assimptotically, so it is no use of increasing the value of n_0, taking a given value of the parameter w (!). So n_0 can be optimalized.

Up to now the most important result of our consideration is the transformation process which comes by introducing the critical velocity and characteristic length. This makes it possible to concentrate the parameters of the medium into the variables, and to obtain equations which are easily solvable for any material flow and are valid generally, limited only by the boundary conditions.

3.3. Approximation possibilities of the effect of the wall

In most cases we must not neglect the effect of the wall in our experimental equipments. The walls in reality promote the recombination, moreover, in many cases (e.g. metallic walls) they can be regarded as perfect absorbent. It often occurs that the wall increases the recombination factor to be relatively large to the free path (generally it becomes $l \sim 2\,\lambda$, where λ is the free path). Let us try to approximate this phenomenon in one-dimension.

In this approximation we suppose, that nothing changes parallel the walls. This assumption may be valid in many cases (positive column, etc.) but it is not admissible in the plasma flow. It means that our results are only informatory, although they are rather convenient for estimating the orders of magnitude in calculations and giving aspects for the design. At the same time they can be very useful in other experimental fields. Of course, we could get accurate results only from two-dimensional examinations.

Let us investigate two possibilities for approximation:

a) Supposing that the plasma ray is small compared to the dimensions of the equipment (e.g. the diameter of the tube which contains the flow), the situation can be regarded as if ionization existed only in the axis, the median plane, etc., and the plasma gets at the perfectly absorbing wall ($n_{\text{wall}} = 0$) by the aid of diffusion, while being recombined. — It does not give any information on distribution in the ray, but often can be useful for estimating the losses, etc.

b) Let us suppose in the approximative examination of distribution inside the plasma ray, that the losses of the plasma in diffusing and recombining inside the fully ionized walls will be compensated by an ionization dispersed in the whole space. — That means in fact the examination of the ionizator space, but the flow also can be regarded as if it were excitation — which drifts particles into the cross-section — being proportional to n in our case.

3.31. Effect of the wall standing in front of the plasma ionizator. Our examination will be based on the approximation *a)*. We regard the phenomenon as a plane problem, that is, we start from Eq. (9), when there is no drift between the ionizator and the wall perpendicular to the wall, so $v = 0$.

The boundary conditions are as follows: The number of the particles is known at the ionizator: $n(x = 0) = n_0$. The wall at the place of $x = a$ is perfectly absorbent, that means: $n(x = a) = 0$. The differential equation to be solved is the same as Eq. (10). Using its transformed form Eq. (27b) (see 3.2), and applying the boundary conditions, we have

$$\frac{d^2 N}{d\zeta^2} = 6N^2$$

under the conditions of

$$\begin{aligned} \zeta &= 0 \qquad & N &= 1 \\ \zeta &= \alpha \qquad & N &= 0 \end{aligned} \tag{31}$$

where

$$\alpha = \sqrt{\frac{Rn_0}{6D}}\, a$$

The general solution — before applying the boundary conditions —

$$N = \frac{1}{|\zeta|^2} \tag{32}$$

Furthermore, it is known that if $\alpha = \infty$,

$$N = \frac{1}{(1 + |\zeta|)^2}.$$

The solution of general validity, which contains the constants of the boundary conditions too:

$$N = \frac{1}{\left(\frac{1}{C_1} + |\zeta|\right)^2} - C_2 \tag{33}$$

The two equations for the boundary conditions:

$$\zeta = 0, \quad N = 1 \qquad C_1^2 - C_2 = 1$$

$$\zeta = \alpha, \quad N = 0 \qquad \frac{1}{\left(\frac{1}{C_1} + \alpha\right)^2} - C_2 = 0 \tag{34}$$

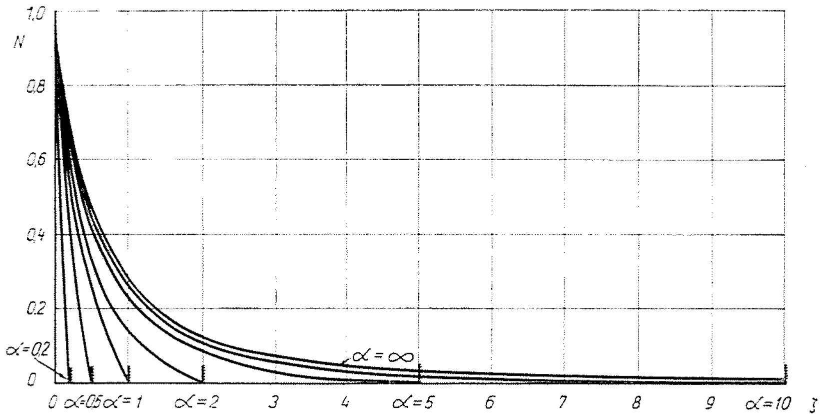

Fig. 4

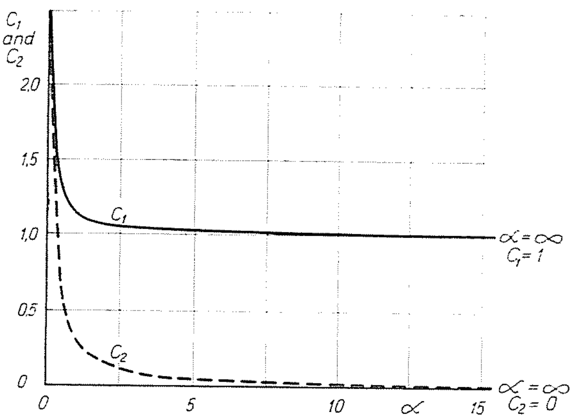

Fig. 5

If $\alpha \to \infty$, we obtain $C_1 = 0$, $C_2 = 0$.

We have made the calculations for the values of $\alpha = 0.1;\ 0.2;\ 0.5;\ 1;\ 2;\ 5$ and 10. The functions $N(\zeta)$, which describe the presence of the wall, can be seen in Fig. 4. These are even functions of ζ. The curves of $C_1(\alpha)$ and $C_2(\alpha)$ can be seen in Fig. 5.

This result gives a very good help in dimensioning the diameter of the flow channel and the electrode distances.

3.31.1. The recombination factor is greater beside the walls, than near to the ionizator. The walls increase the recombination factor in addition to its being perfectly absorbent in a depth of one-two free paths. The phenomenon is not completely clarified yet; the supposition has presented itself partly as an explanation on the results of measurements, partly as based on the physical conception (the particle being in the distance λ from the wall may "freely" escape). We neglect the occasional charge accumulation beside the wall.

The boundary conditions are, as earlier: $n(x = 0) = n_0$, $n(x = a) = 0$, furthermore: the recombination factor is R_1, if $0 \leqq x \leqq a - d$ and $R_2 > R_1$, if $a - d \leqq x \leqq a$. The diffusion coefficient let be in both spacepart D, and the effect of the walls presents itself in a layer of thickness of d.

Starting from our initial Eq. (10)

$$D_1 \frac{d^2 n^{(1)}}{dx^2} - R_1 \cdot n^{(1)^2} = 0$$

$$D_2 \frac{d^2 n^{(2)}}{dx^2} - R_2 \cdot n^{(2)^2} = 0 \tag{35}$$

As for generally $d < a$, let be

$$\zeta = \sqrt{\frac{R_1 n_0}{6 D_1}} x$$

$$N = \frac{n}{n_0} \tag{36}$$

in the transformation. After transformation

$$\frac{d^2 N^{(1)}}{d\zeta^2} - 6 N^{(1)^2} = 0$$

$$\frac{d^2 N^{(2)}}{d\zeta^2} - 6 \left(\frac{A}{B} N^{(2)}\right)^2 = 0 \tag{37}$$

where

$$A = \sqrt[+]{\frac{D_1}{D_2}} \text{ and } B = \sqrt[+]{\frac{R_1}{R_2}} \tag{38}$$

So we have

$$N^{(1)} = \frac{1}{\left(\frac{1}{C_{11}} + |\zeta|\right)^2} - C_{21}$$

$$N^{(2)} = \frac{1}{\left(\frac{1}{C_{12}} + \frac{A}{B} |\zeta|\right)^2} - C_{22} \tag{39}$$

The two solutions fit together continuously, that is

$$\begin{array}{lll} \zeta = 0 & N^{(1)} = 1 & \\ \zeta = \beta & N^{(1)} = N^{(2)} & 0 \leqq \beta \leqq \infty \\ \zeta = \beta & \dfrac{dN^{(1)}}{d\zeta} = \dfrac{dN^{(2)}}{d\zeta} & \text{(the particle flow is continuous)} \\ \zeta = \alpha & N^{(2)} = 0 & \end{array} \tag{40}$$

where — in a well known manner:

$$\alpha = \sqrt{\frac{R_1 n_0}{6D_1}}\, a, \quad b = a - d, \quad \beta = \sqrt{\frac{R_1 n_0}{6D_1}} \cdot b \tag{41}$$

We can examine different special cases by the aid of the system of equations.

3.31.11. The plasma diffuses through two space part of different recombination to the "wall in infinite distance". In this case $\alpha = \infty$, $N_2 = 0$, that is, $C_{22} = 0$. So the equations:

$$\begin{gathered} 1 = C_{11}^2 - C_{21} \\ \frac{1}{\left(\dfrac{1}{C_{11}} - \beta\right)^2} - C_{21} = \frac{1}{\left(\dfrac{1}{C_{12}} + \dfrac{A}{B}\beta\right)^2} \\ \frac{1}{\left(\dfrac{1}{C_{11}} + \beta\right)^3} = \frac{A}{B} \frac{1}{\left(\dfrac{1}{C_{12}} + \dfrac{A}{B}\beta\right)^3} \end{gathered} \tag{42}$$

It is advisable to determine the coefficients by using a computer.

3.31.12. In the vicinity of the wall the recombination factor increases, but the diffusion constant does not alter.

In this case $A = 1$, and

$$\beta = \alpha - \delta = \sqrt{\frac{R_1 n_0}{6\, D_1}}\,(a - d)$$

Let us introduce $\dfrac{\delta}{\alpha}$ as a new variable which is:

$$0 \leqq \frac{\delta}{\alpha} \leqq 1.$$

Our equations containing the boundary conditions are the following:

$$C_{21} = C_{11}^2 - 1$$

$$\frac{1}{\alpha^2\left[1 + \frac{1}{\alpha C_{11}} - \left(\frac{\delta}{\alpha}\right)\right]^2} - C_{21} = \frac{1}{\alpha^2\left[\frac{1}{B} + \frac{1}{\alpha C_{12}} - \frac{1}{B}\left(\frac{\delta}{\alpha}\right)\right]^2} - C_{22}$$

$$\left[1 + \frac{1}{\alpha C_{11}} - \left(\frac{\delta}{\alpha}\right)\right]^3 = B\left[\frac{1}{B} + \frac{1}{\alpha C_{12}} - \frac{1}{B}\left(\frac{\delta}{\alpha}\right)\right]^3 \qquad (43)$$

$$\left(\frac{1}{C_{12}} + \frac{\alpha}{B}\right)^2 = C_{22}$$

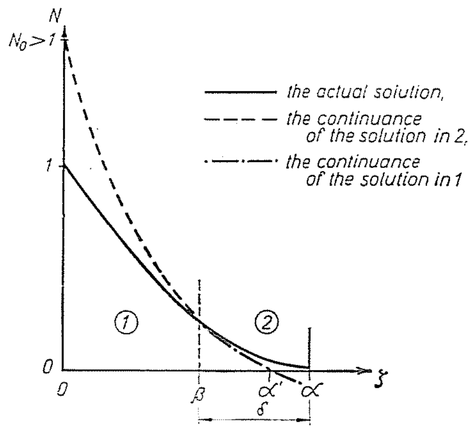

Fig. 6

In a particular case it is advisable to calculate the C_{ik}-s on a computer. Let us examine the solution qualitatively. The $N(\zeta)$ is plotted qualitatively on Fig. 6. It is to be seen that a new "effective place of the wall" may be defined, α', based on the curve $N^{(1)}$. That means the variation of $N^{(1)}(\zeta)$ in case of $R_2 > R_1$ because of the boundary layer in such a manner, as if the wall would be on the place $(\alpha - \delta) \leqq \alpha' \leqq \alpha$! This is a very useful estimation for design and for evaluate the measurements where α' can be very well determined, it is also sufficient, because already Eq. (33) gives also an accurate description on the space outside the boundary layer.

If necessary, the accurate diagram can be calculated by solving Eq. (43), and then α' can be determined, because

$$N^{(1)}(\zeta = \alpha') = 0 \qquad (44)$$

so

$$\alpha' = \frac{1}{\sqrt{C_{11}^2 - 1}} - \frac{1}{C_{11}} \tag{45}$$

If the use of α' is very important to design, it is also sufficient to determine C_{11}.

3.32. Examination of excitations dispersed in the space. Our examination will be based on the approximation *b)*. The effect of the flow, which drifts particles into the space perpendicularly to the one-dimensional cross-section, will be regarded as excitation. The results are very useful for ionizator examinations under the conditions seen in Chapter 1, and the flow cross-section can be as accurately calculated, as the successfully approximation is done by supposing an excitation instead of drift effect of the flow.

Let us start from Eqs (5), omitting the equation for n_0, and supposing that $n_e = n_i = n$. So we obtain:

$$\frac{\partial n}{\partial t} = n^* - Rn^2 + D \cdot \Delta n - \mathrm{div}(n \cdot \bar{v}) \tag{46}$$

In the cross-section plane there is no flow, so $\bar{v} = 0$, and examining only stationary states, we obtain, that $\frac{\partial n}{\partial t} = 0$. It follows that

$$n^* + D \cdot \Delta n - R \cdot n^2 = 0 \tag{47}$$

In the following let us examine only one-dimensional flow, that is:

$$D \frac{d^2 n}{dx^2} - Rn^2 + n^* = 0 \tag{48}$$

After using the transformation in 3.13, and that

$$\zeta = \sqrt{\frac{R \cdot n_0}{6D}} \cdot x \quad \text{and} \quad N = \frac{n}{n_0}$$

we obtain

$$\frac{1}{6} \frac{d^2 N}{d\zeta^2} - N^2 + \frac{n^*}{R \cdot n_0^2} = 0 \tag{49}$$

It should be noted, that the third term on the left side of Eq. (49) is the ratio of excitation and initial recombination (it is generally the maximum) on the place $x = 0$.

Let this term be the so called relative excitation:

$$\Theta = \frac{n^*}{R \cdot n_0^2} \tag{50}$$

Now let us characterize the type of excitation by Θ. We have to solve the equation:

$$\frac{d^2 N}{d\zeta^2} - 6N^2 + 6\Theta = 0 \tag{51}$$

under the following boundary conditions:

$$n(x=0) = n_0, \text{ that is } \zeta = 0, N = 1$$

$$n(x=a) = 0, \text{ that is } \zeta = \alpha, N = 0,$$

the wall is at $x = a$.

Let the excitation be proportional to the number of particles:

$$n^* = \gamma \cdot n$$

and

$$\Theta - \frac{\gamma}{Rn_0} N = \vartheta N \tag{52}$$

The solution will be even to the $\zeta = 0$ axis, just as we have seen it in Chapter 3.31.

We are looking for the solution by the aid of the phase-plane method. By using Eq. (29) we obtain

$$Y \frac{dy}{dN} - 6(N^2 - \vartheta N) = 0 \tag{53}$$

After integrating between the limits $1 \to N$ and $Y_0 \to Y$, we have that the slope of the variation in the particle number on the point $\zeta = \alpha$ is the following:

$$Y_\alpha = \frac{dN}{d\zeta}\bigg|_{\zeta=\alpha} = \pm 2 \sqrt{\frac{3}{2}\vartheta - 1} = Y_\alpha(\vartheta). \tag{54}$$

The losses at the walls will be determined by Y_α, so we have the conception, also physically correct, that for ensuring a given n_0 particle number it is necessary to have a certain ϑ excitation to compensate the losses, and *vice-versa*: supposing a fixed ϑ, there can be only a certain number n_0. So, if the excitation does not exceed a critical level, it cannot supply the recombination losses, and we have $n \equiv 0$.

From Eq. (53) it follows, that

$$Y = \pm 2 \sqrt{N^2 \left(N - \frac{3}{2}\vartheta\right) - \left(1 - \frac{3}{2}\vartheta\right)} \tag{55}$$

As the solution has no meaning if the polinom under the square-root becomes negative, ϑ may not have any values, by which the polynom has any zeros in $0 < N < 1$. Furthermore in using that inside the walls $n > 0$, we have $\left|\left[\frac{dN}{d\zeta}\right]_{\zeta=\alpha}\right| > 0$. The analysis shows that one of the zeros is $N_0 = 1$, independently of the possible positions of zeros can be seen in Fig. 7.

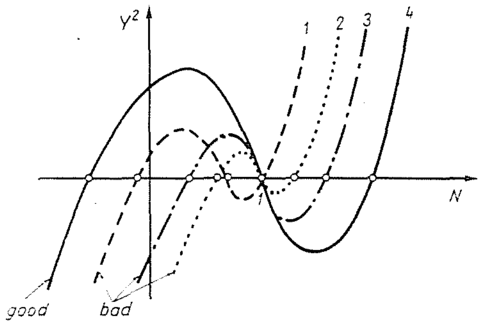

Fig. 7

The examination shows that the condition

$$\vartheta \geqq 1 \tag{56}$$

must be fulfilled. So the "critical excitation" which exist by any other excitation-form distributed in the space too, has an only analytically different form, though it can be determined in a similar way. In this case it is:

$$\vartheta_k = 1, \qquad \text{that is} \qquad \gamma_{\min} = Rn_0 \tag{57}$$

Eq. (53) parameterized with ϑ already can be solved. The solution $N(\zeta)$ may be seen in Fig. 8. We produce always the same relative particle number in our task ($N = 1$) with varying α on the place $\zeta = 0$. So to insure $N = 1$, it is necessary to have a strictly given ϑ relative excitation; ϑ plotted against α can be seen in Fig. 9.

3.32.1. Let the place a of the wall, and the n_0 particle number, generated on the axis, be known. Knowing that

$$\alpha = \sqrt{\frac{Rn_0}{6D}}\, a,$$

we can get the necessary value of ϑ from Fig. 9. Of n_0 and ϑ we can calculate the necessary effective excitation:

$$\gamma = R n_0 \vartheta$$

3.32.2. If we know the place a of the wall, and the value ϑ of the excitation,

$$\vartheta = \frac{\gamma}{R n_0} = \vartheta(n_0)$$

$$\alpha = \sqrt{\frac{R n_0}{6D}}\, a = \alpha(n_0)$$

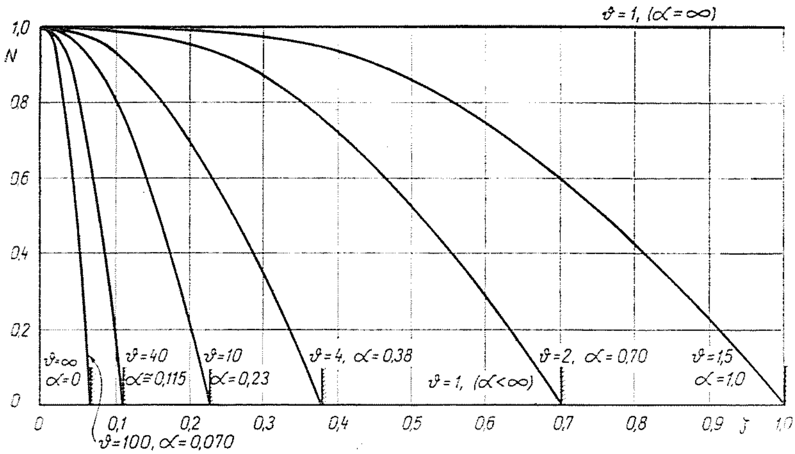

Fig. 8

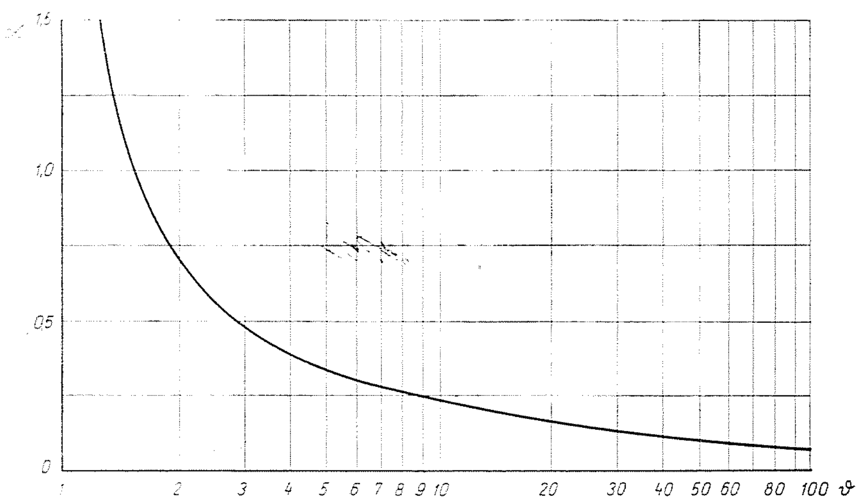

Fig. 9

From these equations the available particle number n_0 can be determined.

It simply follows that

$$\alpha = a \sqrt{\frac{\gamma}{6D}}\, \vartheta^{-1/2}$$

and so

$$\alpha = \alpha(\vartheta)$$

These last two equations determined ϑ and so

$$n_0 = \frac{\gamma}{R\vartheta}$$

is the particle number in the axis.

3.32. Discussion of the result and the methods. Im examining our qualitative and quantitative results it can be pointed out, that it is an important achievement to introduce the conception of the effective wall in calculating the increase on the recombination caused by the presence of the wall. The method is very useful to examine the ionizators, where the critical excitation — "ignition level" — is very interesting. Furthermore, it is also possible, to apply the transformation in 3.13., on equations which differ from the original one, so besides the fact that the result are independent from the material constants and other parameters, we get a very clear physical conception.

4. Two-dimensional solution of the problem with cylindrical symmetry

The flowing plasma, flow channel, nozzles, etc. have in many cases a rolling symmetry. Therefore let us investigate the rolle-symmetrical solutions.

4.1. The task to be solved

We have to solve Eq. (8) using cylindrical coordinates. Let us suppose that the gases flow along the z-axis:

$$\bar{v} = v \cdot \bar{e}_z\,. \tag{58}$$

Let be the wall of the channel at $r = a$, and the flow rolle-symmetrical. The boundary conditions are the following:

$$\begin{aligned} z = 0, \quad & r = 0, \quad n = n_0 \\ & r = a, \quad n = 0 \\ z = \infty, \quad & \qquad\quad\; n = 0 \end{aligned} \tag{59}$$

$$\frac{\partial n}{\partial \varphi} = 0$$

The fact that recombination can be greater at the walls, may be omitted owing to the introduction of the effective place of the wall:

$$D\left[\frac{\partial^2 n}{\partial z^2}+\frac{\partial^2 n}{\partial r^2}+\frac{1}{r}\,\frac{\partial n}{\partial r}\right]-v\,\frac{\partial n}{\partial z}-Rn^2=0 \tag{60}$$

Let us apply the transformation introduced in Chapter 3.13:

$$\zeta=\sqrt{\frac{Rn_0}{6D}}\,z, \qquad \varrho=\sqrt{\frac{Rn_0}{6D}}\,r$$
$$w=\frac{1}{\sqrt{12}}\,\frac{v}{v_0}, \qquad v_0=\sqrt{\frac{RDn_0}{2}} \tag{61}$$
$$N=\frac{n}{n_0}.$$

The boundary conditions have now the following form:

$$\begin{aligned} &\zeta=0, \quad \varrho=0, \quad N=1\\ &\qquad\quad\ \varrho=\alpha, \quad N=0\\ &\zeta=\infty \qquad\qquad\ N=0 \end{aligned} \tag{62}$$

and the transformed equation to be solved is:

$$\frac{\partial^2 N}{\partial \zeta^2}+\frac{\partial^2 N}{\partial \varrho^2}+\frac{1}{\varrho}\,\frac{\partial N}{\partial \varrho}-6w\,\frac{\partial N}{\partial \varrho}-6N^2=0 \tag{63}$$

The solution must be produced with the parameters w and α.

4.2. *Applicability of the one-dimensional results*

The uniformly continuous solution of Eq. (63) may exist only then if

$$\left.\frac{\partial N}{\partial \varrho}\right|_{\varrho=0}=0$$

This supposition is admissible, for we are examining the process only outside the ionizator and want only a stationary solution. If there existed any break along the axis, the grad $_{\varrho=0}\, n \neq 0$ resulted a loss in particles, which either would have been compensated by the aid of ionization or the diffusion would have obliterated the break till the stationary state was taking place.

Based upon these we can estimate the axial distribution. If the wall is not too near the axis ($\alpha \geqq 1$), we may state according to Fig. 8, that

$$\lim_{\varrho \to 0} \frac{1}{\varrho} \frac{\partial N}{\partial \varrho} \sim 0$$

and

$$\lim_{\varrho \to 0} \frac{\partial^2 N}{\partial \varrho^2} \sim 0.$$

That results the Eq. (63) in turning into the Eq. (28) in the immediate environment of the axis $\varrho = 0$, and on the axis itself. So it can be seen that the axial distribution will be given with a good approximation by the curves of Fig. 2, if the diameter of the flow channel reaches at least the order $\zeta = 2$

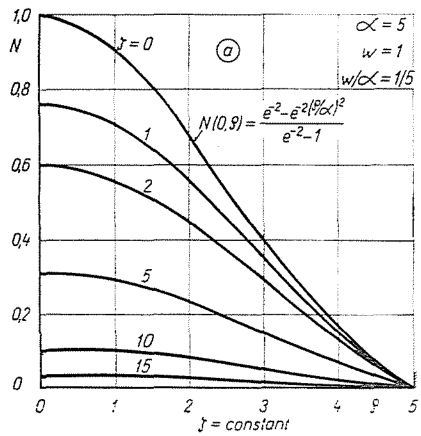

Fig. 10a

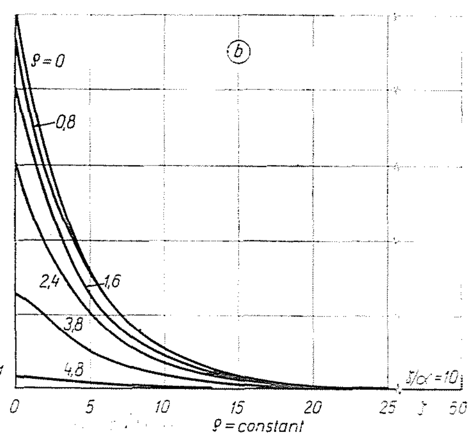

Fig. 10b

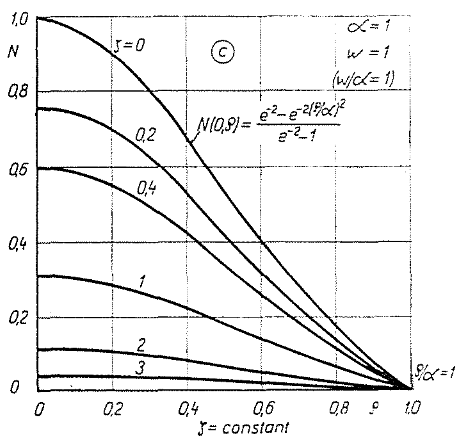

Fig. 10c

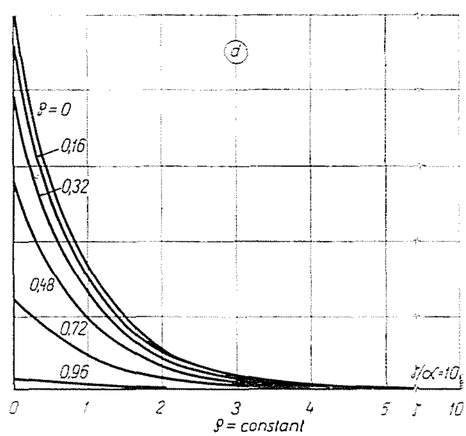

Fig. 10d

The distribution in the cross-section (ζ = const.) can also be approximated. As concerning the magnitude of the "excitation" owing to the particle drift into the cross-section — see Chapter 3.32 — we will find that the excitation is proportional to the particle number. As for the magnitude of ϑ, we find that apart from the sections of $N(\zeta)$ which vary quite steeply, the value of ϑ is very near to the critical 1, that is, the cross-sectional distribution flattens out rather quickly.

4.3. *Solution of the problem*

The calculation-work was done on the computer (Ural 2) of the University Calculation Centre. The program — giving a solution being sufficiently correct from technical aspects — was made by the Centre itself.

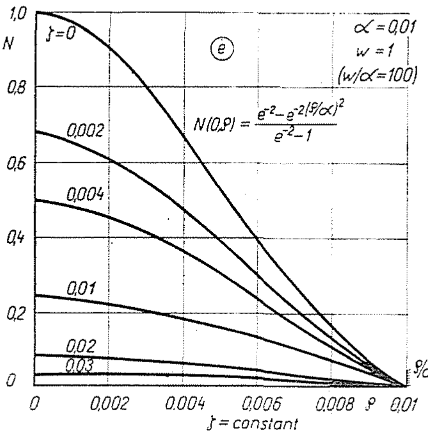

Fig. 10e

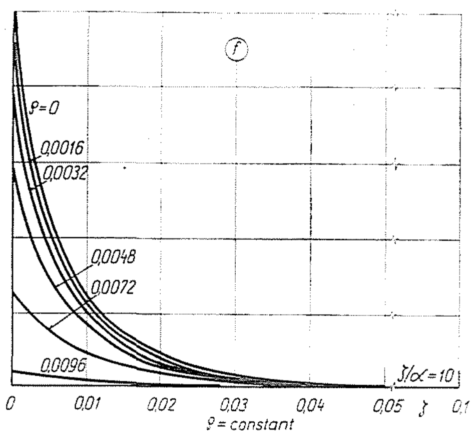

Fig. 10f

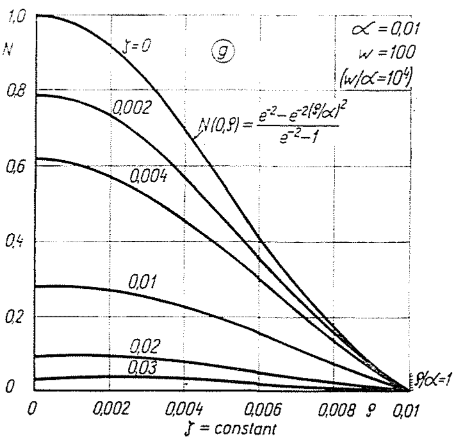

Fig. 10g

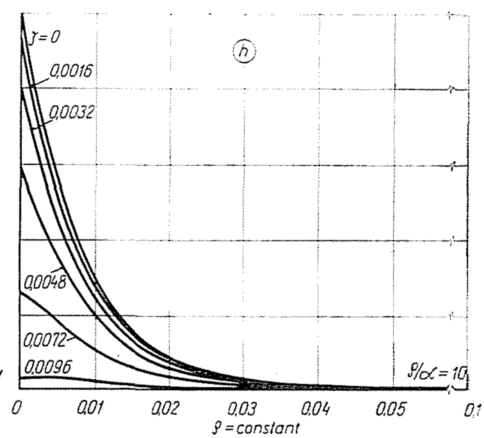

Fig. 10h

The results of the calculation can be seen in Fig. 10. Evaluating the experimental results it is advisable to choose a measured distribution as a cross-sectional profile.

Because of the small capacity of the computer Ural-2, the calculations could not be carried out by the boundary conditions $\zeta \cong \infty$; the $N = 0$ must have been adopted after a very short section (ζ be small).

We can lay down by examining the curves, that our calculations and conclusions theoretically had been correct. The practical correctness will be checked through measurements.

5. Evaluation

5.1. Comparison with measured data

As we have seen at the beginning, similar investigations are very important both for new rocket-engines and for energy supply in the space. We could not get through with measurements till the time of writing this paper, but we

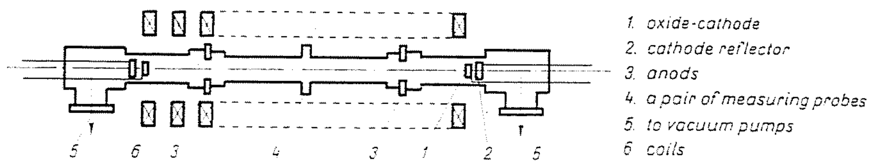

Fig. 11

have found in two cases measured data in other publications which are suitable for controlling all our work.

5.11. Comparison with the measured results of BONNAL [4]. He published the data in Fig. 12, measured in pure helium (!), as an addition to his report "Réalisation d'un plasma en régime permanent" on the Fifth Conference on Ionization Phenomena.

The measuring system can be seen in Fig. 11. The data of the discharge tube were the following: length: 80 $\sim$ 150 cm (variable), diameter: 10 cm. The diameter of the direct current discharge — the length of which was variable by pushing the electrodes — varied between 1 and 6 cm, depending on the experimental circumstances. An axial magnetic field could be generated in the plasma. Then the wall-losses of the plasma strongly decreased because of the magnetic-wall effect and the discharge widened. The pressure was also variable in the tube. The purpose of the experiments was to examine the interaction between magnetic field and plasma.

In the experiment the length of plasma was much longer, than its diameter, so the one-dimensional wall-effect (Chapter 3.3) can be used to control our calculations.

The measured data on the distribution of the particle number in helium are seen in Fig. 12. The circumstances of the measuring:

Sign of the curve	1	2	3
p [Hgmm]	$3.4 \cdot 10^{-3}$	$6.2 \cdot 10^{-3}$	$6.2 \cdot 10^{-3}$
B [Gauss]	1600	1600	400

The discharge current was $I = 2$ A; during the measurements $T_e = 6$ eV, $T_i \sim 1$ eV, and the diffusion constants:

$D_{\text{experimental}} = 2 \cdot 10^4$ cm²/s (in presence of magnetic field)
$D_{\perp}^*$ (data of SIMON — [4]) $= 4.1 \cdot 10^4$ cm²/s
$D_{\perp a} = 3.1 \cdot 10^3$ cm²/s.

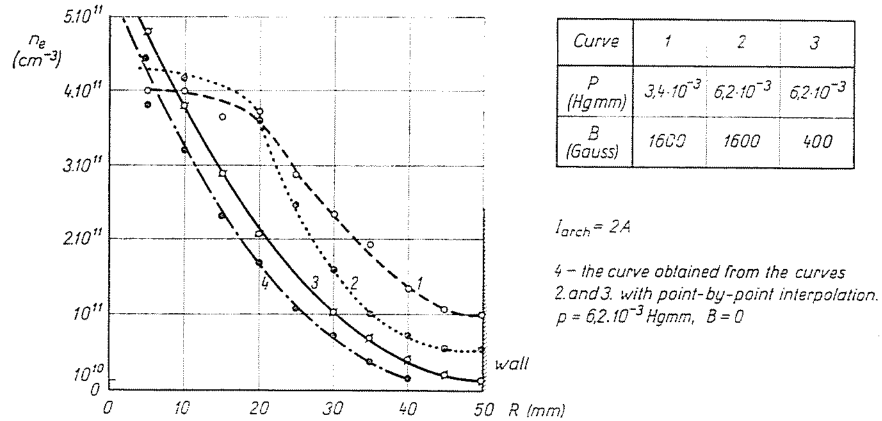

Fig. 12

Our calculations are valid for the case without external forces. As Fig. 12 shows, the effect of B is great, therefore we use plot 4, obtained by interpolation from plots 2 and 3, for controlling our calculations.

At the wall, as $B = 0$, we can suppose for plot 4 that $n = 0$. Then the free path: $\lambda \sim 0.5$ cm under a pressure $p = 6.2 \cdot 10^{-3}$ Hgmm.

Controlling the given diffusion constants by calculation based on data of ENGEL [1], we get similar orders of magnitude. Therefore we may use in our calculations the D_{1a} ambipolar diffusion in the direction of the given wall. The recombination coefficient is $R = 1.7 \cdot 10^{-8}$ cm³/s, if $p \cong 1$ Hgmm, and $T_e \cong 0.03$ eV. As the electron-temperature gets higher, the collision cross-section, and so R, also decreases if the ion-temperature remains constant. But in our case $T_e \sim T_i$, so we may suppose that the collision cross-sections for ions and electrons do not differ significantly compared to the initial values taken by the original low $T_e \sim T_i$ temperatures. The electron-ion recombination factor varies proportionally with $\sqrt{p}$ towards the decreasing magnitudes of pressure [1]. So, modifying R for the calculation only in depending of the pressure, we can use it as $R \cong 1.34 \cdot 10^{-9}$ cm^3/s for $p = 6.2 \cdot 10^{-3}$ Hgmm.

The measuring circumstances approximate very well the ideal initial conditions supposed by the theoretical calculations: The effect of the external magnetic field can be eliminated by linear interpolation. No energy transfer took place in the experiment; the pressure was small, the viscosity could have been neglected; turbulence etc. did not occur. The state-parameters were constant in the whole measuring space, and there had been enough time to take their stationary values. The ionization was not very intensive ($n_{max} \cong \cong 4.5 \cdot 10^{11}$ cm^{-3}) and so the supposition of single ionization-degree was

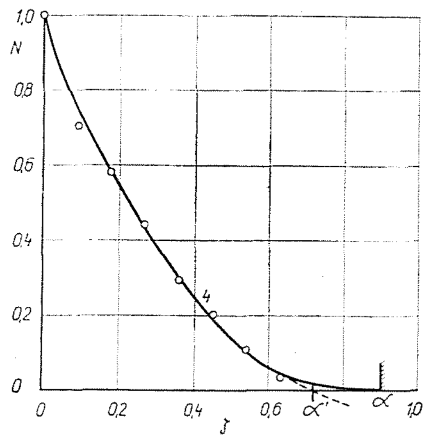

Fig. 13

rightful. In the range of the investigations there was no energy-transfer (acceleration, etc.).

Control: Based on Fig. 12 and other references in the publication the diameter of the discharge may have been 1 cm in case of $B = 0$. So the process between the place $r = 5$ mm and the wall could be regarded as if the plasma had been diffused and recombined between a "plane-ionizator" and a "plane-wall" (Chapter 3.31.). As the radius ratio is 10, the supposition of being "planes" there does not cause a rough error, and the density decreasing effect of the axial dilatation can be implied into the recombination factor besides other inaccuracies of the estimations. As for R is inaccurate, it would be of no use to modify it.

So we have $D_{\perp a} = D = 0.31$ m^2/s; $R \cong 1.34 \cdot 10^{-15}$ m^3/s; $n_0 \cong 4.5 \cdot 10^{17}$ m^{-3}. Then: $\zeta \cong 18\,x$. By using it the 4. transformed "experimental" curve can be sketched (Fig. 13). It can be seen from it that till $d \sim 1.5\,\lambda$, the recombination-increasing effect of the wall is obvious.

The effective wall-place is

$$\alpha' \cong 0.72$$

Knowing α', and using the results of Chapter 3.31, we have

$$N = \frac{1}{\left(\frac{1}{C_1} + |\zeta|\right)^2} - C_2$$

and based on Fig. 5, to $\alpha' = 0.72$ belong $C_1 = 1.18$ and $C_2 \sim 0.4$. So

$$N = \frac{1}{\left(\frac{1}{1.18} + |\zeta|\right)^2} - 0.4 \qquad \text{where} \qquad 0 \leqq \zeta \leqq 0.72.$$

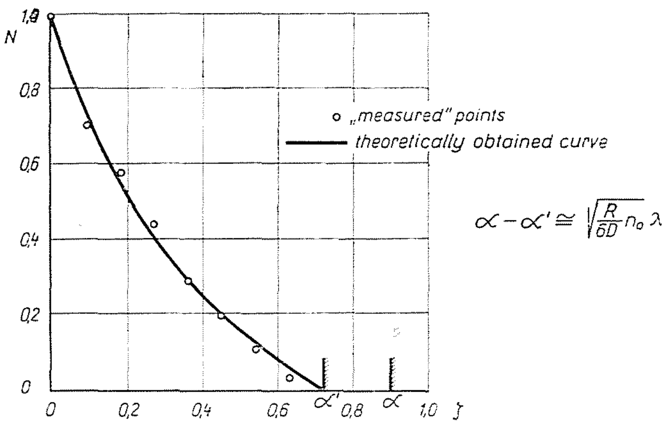

Fig. 14

This last theoretically obtained curve and the interpolated measured data can be seen in Fig. 14.

The measuring conditions and the starting theoretical results showed an extraordinary coincidence. This explains, that the measured and theoretically computed data coincide so deeply. Of course the values of the respective constants (R, n_0, etc.) are not accurate. The absence of this error can be explained perhaps by the fact, that the linear interpolation causes opposite errors.

It can be established that the idealization of the theoretical calculation is not overdone, and the parameters we have found, leading to the transformation of the equation, the effective wall, the physical parameters of the plasma and the solutions of the transformed equation are very useful.

5.12. Comparison with the measurements of BLOMBERG and FREE [3]. These authors reported their investigations on rocket engine operating with cesium, in 1964, on the Conference on Space Research in the United States (Cesium Plasma Generation — M. L. Blomberg and B. A. Free — IEEE Transaction of Aerospace Vo. As-2, No. 2. 1964, pp. 509—513).

During the measurement there was only cesium flow investigated. The authors determined the variation of the number of charged particles in the axis of the flow outside the exhaust area of rocket engines. The examinations were carried out in fully ionized plasma (!), where the effect of the n^3 ionization appeared also. Though the initial ionization was a "thermical" one (arc discharge), the conditions outside the nozzles were not equilibral anymore. The authors studied the tendency of the recombination factor, but in the publication there was a very short hint to this to be found. The conditions for measurements and theoretical calculations were very different.

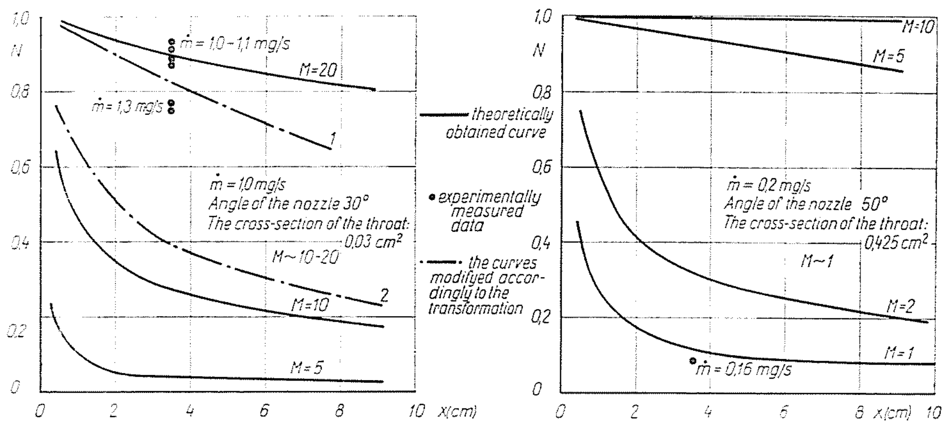

Fig. 15

Control: The theoretically calculated curves for a single material under strictly given initial conditions and several measured data can be seen in Fig. 15.

If we start with the supposition that the theoretically calculated curves are special solutions of the equation valid under the measured circumstances, then in the case of our theoretically correct calculation, the tendency showed by the modifications of the curves caused by the change in the initial conditions of the transformation, must be also correct.

The calculated and measured points coincide in Fig. 15b, there is no need of modification. But the calculated and measured initial conditions differ from each other in Fig. 15a. By modifying the theoretically obtained curves, they will be shifted towards the measured values.

Although the measuring conditions considerably differed from our initial conditions, and even the circumstances were not exactly defined, the theoretical modification brought a change with correct tendency.

5.2. Further possibilities for generalization

Examining the basic equation system (Chapter 2.1), under other simplifying conditions, we could get new results in similar manner. One of the most important of these would be to examine the effect of variation in the velocity (div $\bar{v} \neq 0$). The more detailed analysis of the collision members and the enlargening of them would give a possibility to examine the energy transfer phenomena.

The transformation (Chapter 3.13) may also be applied successfully for equations differing from the original one in their form, only we have to introduce new important parameters.

It would be advisable to investigate the connection between the parameters obtained and the process-parameters already being in use. We did not succeeded in proving the connection between the critical velocity $\bar{v}_0$ and the sound-velocity. But, by the aid of experiments one could explore similar relations. The relation between the material constants and the free path could be investigated experimentally by investigating the effective place and real place of the wall. The relative excitation makes possible the general examination of ionization.

Our results mean aid to design. We can well limit the order of dimensions in the measuring spaces, experimental equipments. We can deduce the order of magnitude by the velocity to be applied, the intensity of the necessary ionization, which is finally determined by the distance between the ionizator and the useful space.

The transformation makes it possible to use the results of the experiments with one material by the calculations of other materials. It has to be investigated what kinds of models are possible.

*

Thanks are due to professor dr. K. Simonyi (Department of Theoretical Electricity, The Polytechnical University of Budapest), to E. Csörsz, electrical engineer, and to T. Márkus, mathematician (The University Calculation Centre) for their valuable help in my work.

References

1. von Engel, A.: Ionized Gases — Clarendon Press, Oxford, 1955.
2. Conference on Ion and Plasma Research (PB 151 316). Air Research and Development Command USAF, University of Maryland, College Park (Maryland), 1958.
3. IEEE Transaction on Aerospace, Vo. As — 2.— Burlington (Massachusetts), April, 1964.
4. Proceedings of the Fifth International Conference on Ionization Phenomena in Gases (H. Maecker). North Holland Publishing Co., Amsterdam 1962.

Summary

The paper intends to determine the variation in the number of charged particles in non-equilibrium cold plasma flow under simplified conditions. Starting out of the transport equation and taking into account our assumptions on the circumstances we get the equation we want in fact to solve. The problem will be simplified to a one-dimensional one. We investigate the possibilities for the solution and introduce the critical velocity and the characteristic length. We obtain a transformation process which gives a result valid for every material by concentrating the material constants into the variables. In applying this method we can examine the effect of the wall, and the excitation, and then we can obtain the rolle-symmetrical, two-dimensional solution. Finally, we control our results and the correctness of the method using results measured by other researchers.

Dr. Csaba FERENCZ, Budapest, XI. Egri József u. 18—20. Hungary