LOCALIZATION OF STEREO SOUND USING TWO LOUDSPEAKERS

By

F. Takács

Department of Wireless Telecommunications, Polytechnical University, Budapest

Presented by Prof. Dr. I. Barta

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The use of a dual-channel stereo system using two loudspeakers permits to realize a sound source, which seems to be operating from a direction differing from those of the two loudspeakers. The realization of such a virtual sound source is based on a correlation between the signals of the two loudspeakers. This correlation may be either:

1. The two loudspeakers create the same sound pressure at the place of observation but with a time lag T between them. In this case the direction of the virtual sound source depends on the time lag T.

2. The two loudspeakers radiate the same sound pressure function but with different amplitudes. In this case the direction of the virtual sound source depends on the ratio of the two signal amplitudes.

In the following discussion the stereo sound reproduction system based on the second principle will be studied. The direction of the virtual sound source as a function of the ratio (and of the time lag) of the signals was first determined experimentally by de Boer [1].

By making use of the mechanism of directional hearing the relationship between the signals and the direction of the virtual sound source was first established by Clark, Dutton and Vanderlyn [3], for sinusoidal sounds. For sound sources radiating nonsinusoidal, arbitrary time functions, Leakey gave equations to determine the direction of the sound source [5].

In his paper, Leakey made use of the fact well known from the physiology of hearing that the direction of the sound source is determined by the time difference between the two sound pressure time functions reaching the two ears. He assumed that for low frequencies this time difference is determined in the hearing centre as that between the instantaneous values of sound pressures reaching the ears. In case of stereo sound reproduction using two loudspeakers, both ears hear the sum of the signals of the two loudspeakers. Expanding into series the sound pressures created at the ears by the loudspeakers, it can be shown that the intensity difference between the loudspeakers results in a time difference between the resultant signals reaching the ears. The direction of the virtual sound source (Fig. 1) deducted from this time difference is given by the equation:

$$\sin\beta = \frac{X-Y}{X+Y}\sin\beta_h \tag{1}$$

where X and Y are proportional to the signal of the left and right loudspeakers, respectively.

The ratio of the sound pressures of the two ears was not studied by LEAKEY, but his equations assume it to be unity irrespective of the direction of the virtual sound source, consequently the pressure amplitudes would be equal in the two ears, according to his results.

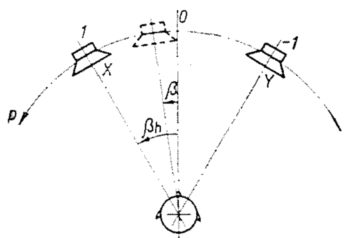

Fig. 1. Stereo sound reproduction system using two loudspeakers

The same equation was deducted by the quoted and other authors [7] for loudspeakers radiating sinusoidal signals. However, this method cannot be applied to determine the direction of sound sources at frequencies over 1 kHz, since at higher frequencies the phase difference between the two ears exceeds 360° for sound sources placed sideways to the head. Therefore the hearing centre cannot determine the direction of the sound source from the phase difference between two sinusoidal signals or in case of arbitrary time function from the time difference between the instantaneous values of the two signals.

LEAKEY supposed that the hearing centre evaluated the time difference between the envelopes of the two signals reaching the ear for high frequencies. He further presumed that the signals reaching one of the ears from the two loudspeakers add up according to a square-law. He also considered the shielding effect of the head resulting in a lower sound pressure intensity in the ear being opposite to the loudspeaker in comparison to the one lying on the same side. Using these three assumptions he deducted the following expression for the direction of the virtual sound source for high frequencies:

$$\sin\beta = \frac{m^2(X^4-Y^4)}{(X^2+m^2Y^2)(m^2X^2+Y^2)}\sin\beta_h \tag{2}$$

where m stands for the ratio of sound pressures in the shaded ear to the directly exposed one. LEAKEY has also proved the above equation to be valid between the two loudspeakers in the frequency band ranging from 300 Hz to 4 kHz by way of measurements. He determined that the shielding effect of the head may be considered by assuming $m = 0.56$ (5 dB).

Good quality sound reproduction systems, however, cover a considerably wider frequency range than the one referred to above. Consequently the trend to extend the examinations especially towards the high frequencies is readily justified. It is also to be examined how is it possible to shift the virtual sound source beyond the line connecting the two loudspeakers. Since the direction of the virtual sound source depends on the difference of the signals of the two loudspeakers, this difference-signal lends itself as the independent variable. In order to decrease the number of variables, the sum of the signals of the two loudspeakers is designated by

$$S = X - Y \tag{3}$$

$$M = X + Y \tag{4}$$

Considering the data available in the literature [6], [8], [9] the shielding effect of the head is considerably high even below 1000 Hz. Therefore, using LEAKEY's method instead of Eq. (1) an expression has been derived (see Appendix I), taking into consideration the shielding effect of the head in the low frequency range too. To describe the direction of the virtual sound source, the following symbol is introduced:

$$p = \frac{\sin\beta}{\sin\beta_h} \tag{5}$$

This provides a convenient means to problems comprising an arbitrary large listening system having optional listening angle (β_h). The meaning of p is given in Fig. 1, where the directions for significant p values are plotted.

From considerations given in Appendix I, the ratio of the two ear pressures at low frequencies is given by Eq. (6):

$$\frac{|J|}{|B|} = \frac{1 + m - (1 - m)\dfrac{S}{M}}{1 + m + (1 - m)\dfrac{S}{M}} \tag{6}$$

The time difference between the pressure functions of the two ears is:

$$T_2 = \frac{4m\dfrac{S}{M}}{(1 + m)^2 - (1 - m)^2\dfrac{S}{M}}\, T_1 \tag{7}$$

Consequently in stereo sound reproduction both pressure and time differences exist between the sound fields before the two ears. The time difference defines the direction of the virtual sound source as:

$$p_1 = \frac{T_2}{T_1} = \frac{4m\frac{S}{M}}{(1+m)^2 - (1-m)^2\left(\frac{S}{M}\right)^2} \tag{8}$$

At high frequencies the ratio of the mean pressure values of the two ears is given by:

$$\frac{|\bar{J}|}{|\bar{B}|} = \sqrt{\frac{1+m^2-(2-2m^2)\frac{S}{M}+(1+m^2)\left(\frac{S}{M}\right)^2}{1+m^2+(2-2m^2)\frac{S}{M}+(1+m^2)\left(\frac{S}{M}\right)^2}} \tag{9}$$

The time difference between the envelope functions of the sound pressures becomes:

$$T_2 = \frac{8m^2\frac{S}{M}\left[1+\left(\frac{S}{M}\right)^2\right]}{(1+m^2)^2-[2-12m^2+2m^4]\left(\frac{S}{M}\right)^2+(1+m^2)^2\left(\frac{S}{M}\right)^4} T_1 \tag{10}$$

The relative direction derived from the time difference results in:

$$p_2 = \frac{T_2}{T_1} = \frac{8m^2\frac{S}{M}\left[1+\left(\frac{S}{M}\right)^2\right]}{(1+m^2)^2-[2-12m^2+2m^4]\left(\frac{S}{M}\right)^2+(1+m^2)^2\left(\frac{S}{M}\right)^4} \tag{11}$$

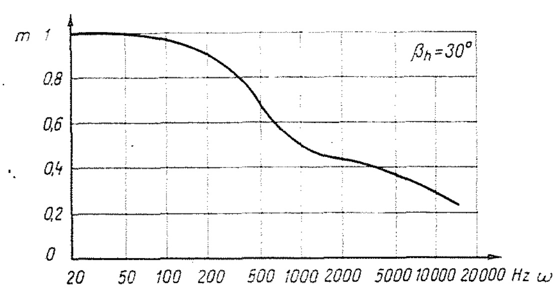

Fig. 2. The ratio of the sound intensities of the two ears as a function of frequency in case o. sound source placed at an angle of 30° to the symmetry plane of the head

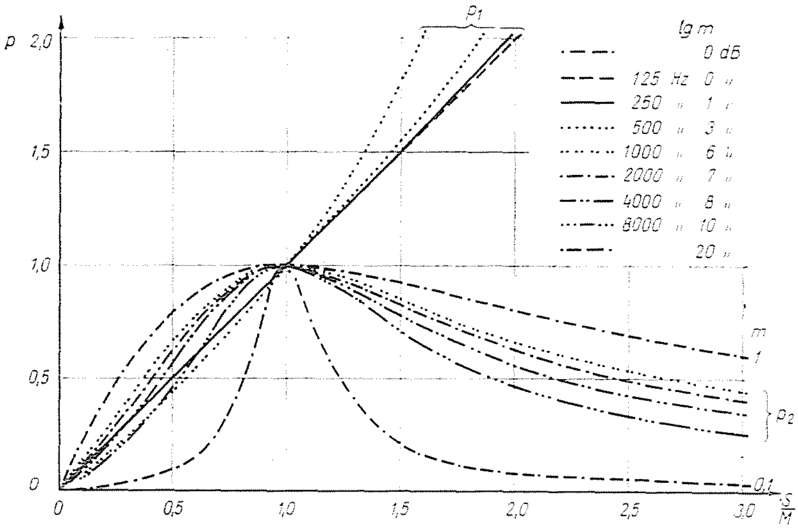

Fig. 3. The relative direction of the virtual sound source as a function of intensity difference and of frequency

The m values in the equations, as determined by W. SCHIRMER for sound sources including an angle of 30° are shown in Fig. 2. It is seen that the intensity difference between the two ears is far from being constant at high frequencies, as LEAKEY supposed it and is not negligible even at a frequency as low as 500 Hz. The relative direction of the virtual sound source is plotted for seven different frequencies in Fig. 3. At frequencies of 0.125, 0.25, 0.5 and 1 kHz Eq. (8) has been used, while at 1, 2, 4 and 8 kHz the relative direction has been determined by using Eq. (11). The figure represents p values corresponding to positive S/M values only, for negative S/M values the same value of p belongs but with negative sign. It is also apparent from Fig. 3 that directions corresponding to p values higher than unity can also be realized, according to the low frequency model. It should be noted, however, that the range of p values derived from Eq. (5) is limited. It is evident that the maximum value of p is determined by the maximum value of $\sin \beta$, therefore:

$$p_M = \frac{1}{\sin \beta_h} \tag{12}$$

The denominator becomes unity, when

$$\beta = \frac{\pi}{2}$$

which means that on the basis of the low frequency model it is possible to position virtual sound sources along a horizontal half circle in front of the observer. In case of sound sources for which from Eq. (8)

$$p_1 > p_M$$

Eq. (5) yields no direction whatever.

According to the p_2 curves derived from the high frequency model it is possible to realize a virtual sound source only on the line connecting the two

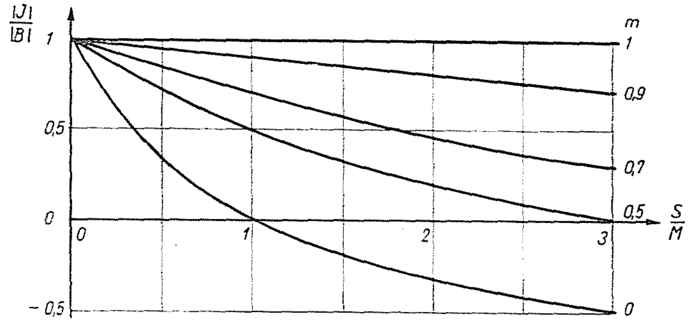

Fig. 4. The ratio of the sound pressures of the two ears on the basis of the low frequency model

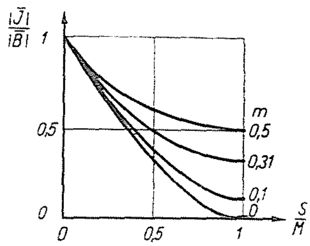

Fig. 5. The ratio of the sound pressures of the two ears on the basis of the high frequency model

loudspeakers. It can easily be shown from Eq. (11) that substituting S/M by its reciprocal the value of the equation remains the same. This means that the direction of the virtual sound source is indifferent for a phase-change between the two channels. This could also be deducted from the initial assumptions. The direction of the sound source of 1 kHz has been determined by both methods, this frequency being just on the border line of the two listening models.

The ratios of the sound pressures of the two ears described by Eqs. (6) and (9) are plotted in Figs 4 and 5, resp. The diagrams show that the pressures of the two ears differ from each other in case of virtual sound sources out of the symmetry plane ($S/M \neq 0$). Although the ratio of the two pressures differs from that created by the actual sound sources (actually they are equal in the trivial cases of $S/M = -1, 0, +1$ only), it is clear that the ratio of the pressures created by a stereo and a mono sound source, resp., are nearly the same at the two ears.

Measurements have been made in order to determine the direction of the virtual sound source as a function of frequency. The measurements of the subjective directional hearing have been carried out in an echoless chamber as shown in Fig. 6. The two stereo loudspeakers and the observer were positioned

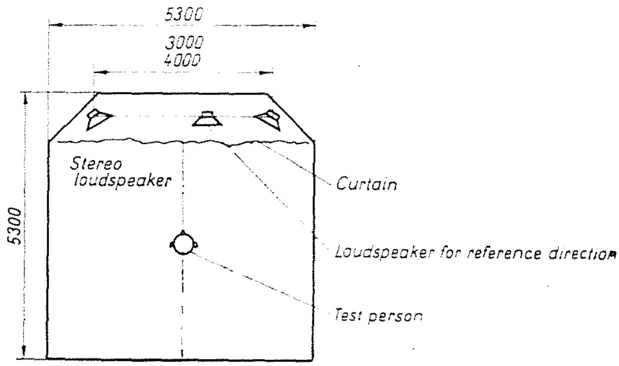

Fig. 6. The positioning of the two loudspeakers and the observer for the determination of the direction of the virtual sound source

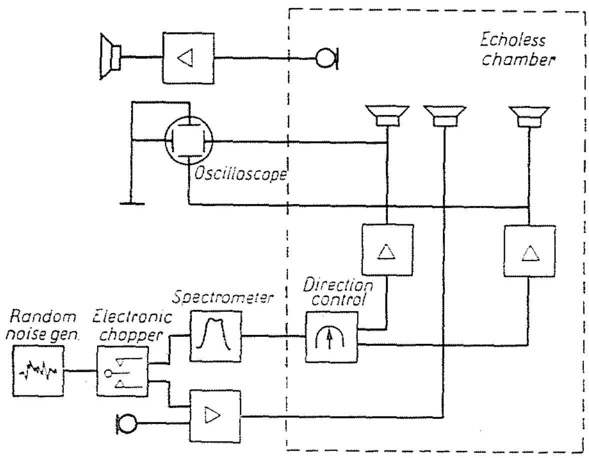

Fig. 7. Measuring setup for the determination of the direction of the virtual sound source

so as to form an equilateral triangle. In order to ensure perfectly unbiased observing circumstances, there was a curtain in front of the loudspeakers.

The measuring setup is shown in Fig. 7. A random noise generator (Brüel Type 1402) was used as a signal source. The output signal from the generator was connected to an electronic chopper, which switched the generator output alternately to two cables. One of these was connected to an AF spectrometer (Brüel Type 2112), which served as an output control as well as an indicator for setting the spectrum of the measuring signal. The output of the spectrometer drove the direction control unit, from which the two stereo loudspeakers were driven via two power amplifiers. With the use of the direction control unit, the ratio of the two driving voltages could continuously be varied over a wide range and thereby the direction of the sound source could be shifted arbitrarily.

The outputs of the two power amplifiers were also connected to the horizontal and vertical inputs of an oscilloscope. In view of the fact that the amplitude and phase responses of the two amplifiers were the same throughout the

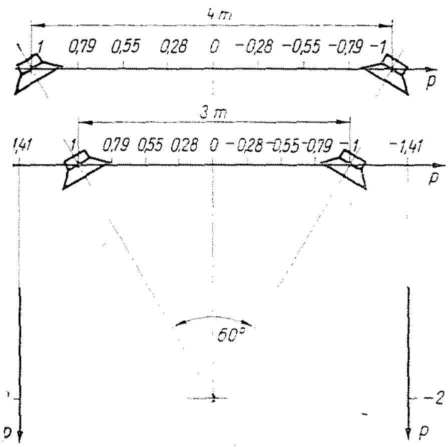

Fig. 8. The directions given by the reference loudspeaker for the determination of the directions

whole audio frequency band within 0.1 dB and within a few degrees, the display on the oscilloscope was a tilted line corresponding to the setting of the direction control unit. By using a special external graticule, it was possible to read directly the S/M values off the oscilloscope screen.

The two loudspeakers were specially selected and thoroughly tested ones. The difference between their sound pressure responses was within 2 dB, and it became 4 dB around 200 Hz. As to their phase responses it was impossible to detect any difference with the measuring setup available.

The second output of the electronic chopper (which output was energized while the first output was inoperative) was driving an other audio frequency amplifier. The output of this latter one was connected to a third loudspeaker, which was used to provide for the reference direction. In order to facilitate a continuous measurement, there was a two way intercom system in use. The channel linking the outside with the echoless chamber was switched off during the actual measurements.

The measurements themselves were carried out as follows. The direction to be observed was set by the third loudspeaker radiating a wide band noise.

The positionings of this loudspeaker and the relative directions calculated from Eq. (5) are shown in Fig. 8.

The observing person was listening alternately to the wide band signal of the reference loudspeaker and to the third octave bandwidth signals of the two stereo loudspeakers, as the chopper energized the two systems one after the other. The observer's task was to set manually the direction control until the virtual sound source radiating the third octave bandwidth signal was heard from the same direction as that of the reference loudspeaker.

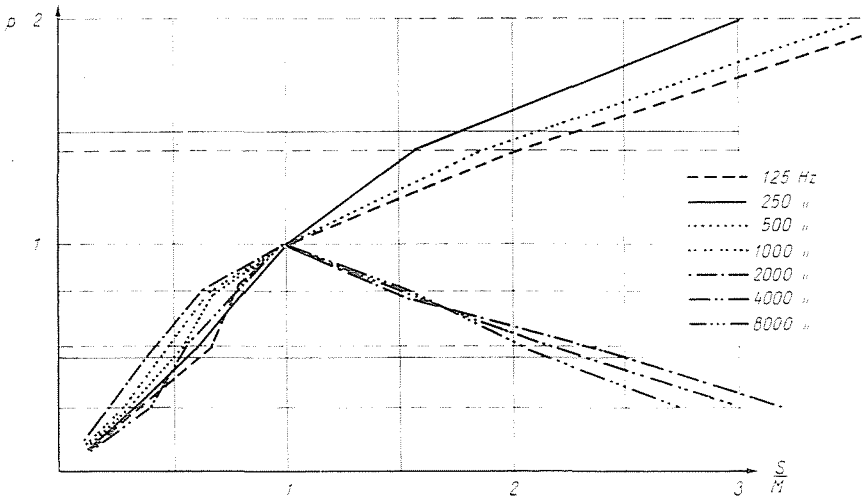

Fig. 9. The measured average direction of the virtual sound source at basewidths 3 and 4 m

The experimenting persons were asked to keep their heads in the symmetry plane of the listening system and neither to move nor turn it while setting the direction.

Fig. 9 shows the results obtained with 3 and 4 m basewidths, resp. For values $S/M > 1$ the smaller basewidth only was used.

Comparing Figs 3 and 9 it is easy to conclude that in the frequency bands below 500 Hz and above 2000 Hz the calculated and the measured directions of the virtual sound source between the two loudspeakers reasonably coincide. It has been proved that at low frequencies a virtual sound source can be realized beyond the line connecting the two loudspeakers, but in this case there is a significant difference between the measured and the calculated values. The greatest differences between measured and calculated results occurred at 1000 Hz. It is noted that the measured curve does not correspond to that of either the low or the high frequency models and around 1000 Hz there could generally be found no realizable direction for values $S/M > 1$.

It is also a result of the measurements that the directional feeling cannot be described by two curves, since it is a continuous function of frequency. For directions falling between the two loudspeakers, the variance of the directional feeling is especially high in the band from 700 Hz to 4000 Hz. This causes uncertainty in the localization of sound sources of wide frequency spectra, since the observed direction especially from half-left or half-right directions greatly differs for the frequency components of the source and varies with the sound pitch of the source. This is all the more undesirable, since the above mentioned

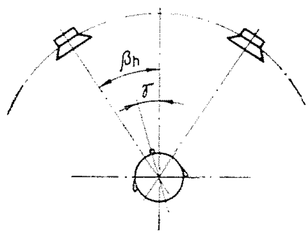

Fig. 10. The observer positioned at an angle γ to the symmetry plane of the sound reproduction system

frequency band contains the major part of the frequency spectra of musical sound sources (instruments, soloists).

In order to justify the corrections of the personal errors the experiments included measurements, in which the observer sat just opposite one and than the other of the stereo loudspeakers. By simple geometrical reasoning (see Appendix II), the observed direction of the virtual sound source, including a γ angle with the middle axis, is the following (Fig. 10):

$$\sin (\beta - \gamma) = \frac{S}{M} \sin \beta_h \cos \gamma - \cos \beta_h \sin \gamma \tag{13}$$

The measurements were made with the observer sitting in front of each loudspeaker. The results obtained using 250, 1000, 2000 and 8000 Hz third octave noises are shown in Fig. 11, together with the function of the relative direction calculated by Eq. (13). Studying this curve it is to be expected that upon turning the head from the symmetry plane of the listening system, the virtual sound source moves in the same direction as the head did. An exception is the relative direction belonging to values $S/M = +1$ or -1. It is, however, self-evident, because the observed direction must naturally be independent of the position of the head, if either the left, or the right hand loudspeaker alone operates ($p = 1$, or $p = -1$).

Fig. 11 also shows the curves for 1, 2 and 8 kHz third octave noises. It is seen that the virtual sound sources are shifted especially in the middle region. It is also to be noted that sound sources of different frequencies are shifted differently from their original position. The shift is at its greatest at 2 kHz mean frequency third octave noises, which value coincides with the frequency at which the previous measurements also showed the greatest deviations from the average measured results.

The results of the calculations and measurements can be summarized as follows. The intensity stereo sound reproduction can be divided into three

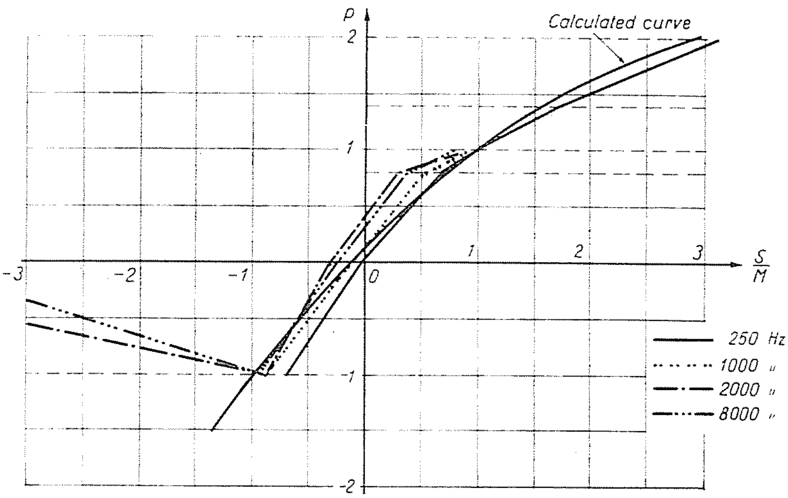

Fig. 11. The relative direction of the virtual sound source, when the observer sits opposite to the left stereo loudspeakers

frequency bands as far as the localization of the virtual sound source is concerned:

1. *Frequency band below 500 to 600 Hz.* The interpretation range of the relative intensity difference corresponding to virtual sound sources having discernible direction, is

$$-\frac{1}{\sin\beta_h} < \frac{S}{M} < \frac{1}{\sin\beta_h}$$

The lower and upper limits, in conventional listening systems, depending on the observers, are:

$$-(4 \div 2.5) < \frac{S}{M} < (2.5 \div 4)$$

The direction of the virtual sound so urcein this range means a half circle before the observer, which can be expressed in terms of relative directions as follows:

$$-(3 \div 2) < p < (3 \div 2)$$

or expressed in directional angles of the virtual sound source:

$$-90^\circ < \beta < 90^\circ$$

2. *Frequency band 600 Hz to 1500 Hz.* The interpretation range of the relative intensity difference for sound sources with a definite direction, is:

$$-1 < \frac{S}{M} < 1$$

The range of values of the relative directions yielding the above interpretation range is:

$$-1 < p < 1$$

that is

$$-\beta_h < \beta < \beta_h$$

In the above two frequency bands there is no virtual sound source having a definite direction, for relative intensity difference values beyond the interpretation range. For relative intensity difference values beyond the interpretation range, the observer hears a stereo sound with no definite direction causing a very queer feeling.

3. *Frequency band above 1500 Hz.* The interpretation range of the relative intensity difference is unlimited:

$$-\infty < \frac{S}{M} < \infty$$

The range of values of the relative direction is:

$$-1 < p < 1$$

or

$$-\beta_h < \beta < \beta_h$$

In this frequency band it is impossible to create a stereo sound without directional feeling with real values of the relative intensity difference.

The interpretation ranges and the range of values for the three frequency bands are shown in Fig. 12.

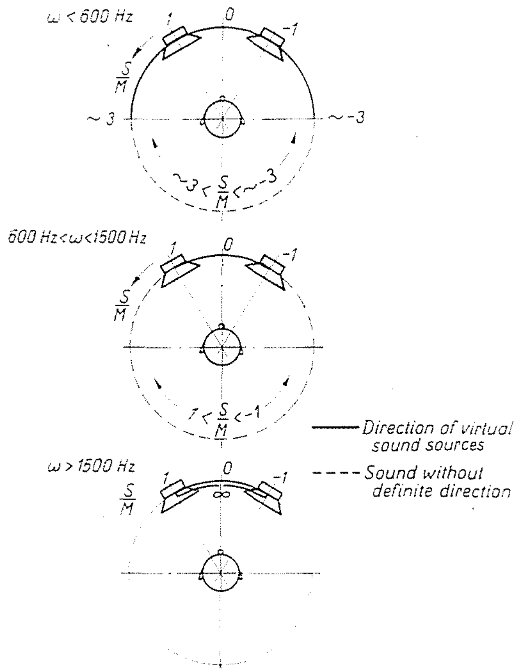

Fig. 12. The direction of the virtual sound sources having well definable direction in cases of low, middle and high frequencies

Appendix I

In order to determine the resulting direction of the virtual sound sources due to the intensity difference of the loudspeakers, let the sound pressure functions produced in the left ear by the left loudspeaker and in the right ear by the right loudspeaker be denoted by $X \cdot f(t)$, and by $Y \cdot f(t)$, respectively. In other words, the signals radiated by each loudspeaker have the same waveform, differing in intensity only (Fig. 13).

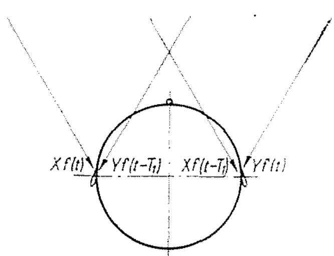

Fig. 13. The paths of the sounds reaching the ear in case of two channel stereo sound reproduction

Let us first consider the sound pressure function developing in front of the left ear. The two loudspeakers will be regarded as point sources, located far from the observer. The head of the observer is situated in the symmetry plane of the two loudspeakers.

The resulting sound pressure function can be realized as the sum of the sound pressure functions of the two loudspeakers. Since the right hand loudspeaker is somewhat farther from the left ear than the left hand one, the sound pressure caused by the right hand loudspeaker appears somewhat later and — because of the shielding effect of the head — is attenuated in front of the left ear. Let T_1 denote this time lag and m the attenuation of the sound pressure. So the sound pressure function before the left ear is following:

$$B = X \cdot f(t) + m \cdot Y \cdot f(t - T_1)$$

By expanding the second term on the right into Taylor series, we get

$$B = X \cdot f(t) + m \cdot Y \cdot f(t) - m \cdot Y \cdot T_1 f'(t) + m \cdot Y \cdot \frac{T_1^2}{2!} f''(t) \mp \ldots =$$

$$= (X + mY)\left[f(t) - \frac{mY}{X + mY} T_1 f'(t) + \frac{mY}{X + mY} \frac{T_1^2}{2!} f''(t) \mp \ldots\right]$$

Comparing the function

$$f\left(t - \frac{mY}{X + mY} T_1\right)$$

expanded into series with the term within the square brackets, it can be proved that they differ only from the third term onwards, since

$$f\left(t - \frac{mY}{X + mY} T_1\right) = f(t) - \frac{mY}{X + mY} T_1 f'(t) + \frac{(mY)^2}{(X + mY)^2} \frac{T_1^2}{2!} f''(t) \mp \ldots$$

The higher order terms may be neglected as compared to the first two ones, if

$$\max\left[T_1 f'(t)\right] \ll \max\left[\frac{T_1^2}{2!} f''(t)\right] \ll \max\left[\frac{T_1^3}{3!} f'''(t)\right]$$

that is

$$\omega_M \frac{T_1}{2} \ll 1$$

where ω_M is the highest frequency in the Fourier spectrum of $f(t)$.

In view of the fact that for conventional loudspeaker arrangements T_1 is 0.3 to 0.4 msec, the above unequality means the frequency band

$$\omega_M \ll 1\,\text{kHz}$$

Consequently in this frequency band the following equation is valid:

$$B \simeq (X + mY) f \left(t - \frac{mY}{X + mY} T_1\right)$$

Similarly for the right hand side

$$J \simeq (mX + Y) f \left(t - \frac{mY}{X + mY} T_1\right).$$

Comparing the last two equations it may be concluded that the sound pressure built up in front of the ears is directly proportional to the weighted sum of the sound pressures corresponding to the respective loudspeakers and that there is a time difference between the time functions of the two pressures.

The ratio of the pressures on the two ears is

$$\frac{|J|}{|B|} = \frac{mX + Y}{X + mY}$$

and the time lag between the pressure functions

$$T_2 = \left[\frac{mX}{mX + Y} - \frac{mY}{X + mY}\right] T_1$$

If the right hand loudspeaker is silent ($Y = 0$), then the relationship between the time difference and the direction of the loudspeaker is approximately

$$T_1 \cong \frac{D}{c} \sin \beta_h$$

where D denotes the distance between the ears.

If both loudspeakers operate, then T_2 time difference may be related to the direction of a virtual sound source

$$T_2 = \frac{D}{c} \sin \beta$$

From the last two equations:

$$\frac{\sin\beta}{\sin\beta_h} = \frac{T_2}{T_1}$$

The ratio of the absolute values of the sound pressures on the two ears expressed with the sum and difference signals is as follows:

$$\frac{|J|}{|B|} = \frac{1 + m - (1 - m)\dfrac{S}{M}}{1 + m + (1 - m)\dfrac{S}{M}}$$

The time difference between both signals

$$T_2 = \frac{4m\dfrac{S}{M}}{(1 + m)^2 - (1 - m)^2\dfrac{S}{M}}\, T_1 .$$

The relative direction of the virtual sound source corresponding to this time difference is

$$p_1 = \frac{T_2}{T_1} = \frac{4m\dfrac{S}{M}}{(1 + m)^2 - (1 - m)^2\left(\dfrac{S}{M}\right)^2}$$

The approximate value of the sound pressure in front of the two ears can similarly be determined for high frequencies. If the signals of the two loudspeakers is added squared, then the sound pressure on the left ear is

$$\bar{B}^2 \simeq (X^2 + m^2 Y^2)\, g^2 \left(t - \frac{m^2 Y^2}{X^2 + m^2 Y^2}\, T_1\right)$$

and the sound pressure on the right ear is:

$$|\bar{J}| \simeq (m^2 X^2 + Y^2) \cdot g \left(t - \frac{m^2 X^2}{m^2 X^2 + Y^2}\, T_1\right)$$

where $g(t)$ is the envelope function of the sound pressure of the loudspeakers. The ratio of the sound pressures on the two ears is

$$\frac{|\bar{J}|}{|\bar{B}|} = \sqrt{\frac{X^2 + m^2 Y^2}{m^2 X^2 + Y^2}} =$$

$$= \sqrt{\frac{1 + m^2 - (2 - 2m^2)\frac{S}{M} + (1 + m^2)\left(\frac{S}{M}\right)^2}{1 + m^2 + (2 - 2m^2)\frac{S}{M} + (1 + m^2)\left(\frac{S}{M}\right)^2}} .$$

The time difference between the signals given by the envelope functions is

$$T_2 = \left(\frac{m^2 X^2}{m^2 X^2 + Y^2} - \frac{m^2 Y^2}{X^2 + m^2 Y^2}\right) T_1 =$$

$$= \frac{8m^2 \frac{S}{M}\left[1 + \left(\frac{S}{M}\right)^2\right]}{(1 + m^2)^2 - [2 - 12m^2 + 2m^4]\left(\frac{S}{M}\right)^2 + (1 + m^2)^2\left(\frac{S}{M}\right)^4} T_1$$

The relative direction of the virtual sound source due to this time lag is

$$p_2 = \frac{T_2}{T_1} = \frac{8m^2 \frac{S}{M}\left[1 + \left(\frac{S}{M}\right)^2\right]}{(1 + m^2)^2 - (2 - 12m^2 + 2m^4)\left(\frac{S}{M}\right)^2 + (1 + m^2)^2\left(\frac{S}{M}\right)^4}$$

Appendix II

In order to determine the direction of the virtual sound source when the head is turned sideways (Fig. 10), let us determine the displacement of the ears in the direction of the signals radiated by the loudspeakers. In this calculation both the change in propagation path and the shielding effect of the head will be neglected. The latter one is especially justified, since the analysis will be made for low frequencies only and the pressure difference of the two ears will not be taken into consideration. The time difference between the pressures of the two ears is practically independent of the shielding effect of the head at low frequencies.

On Fig. 14 the line connecting the two ears has been drawn for the cases, when it coincides with the symmetry axis of the listening system and when it forms an angle γ with it. In order to determine the direction of the virtual sound source, the relative time lags of the four signals reaching the ears from the two loudspeakers must be known. Let us consider first the paths of the signals to the left ear.

That moment when the signal reaches the left ear from the left loudspeaker at symmetrical head positioning will be taken as the reference time. The signal will arrive from the right hand loudspeaker to the left ear by a time

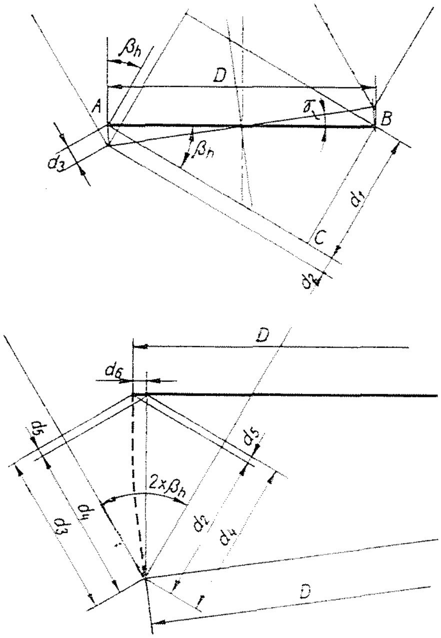

Fig. 14. For the calculation of the sound pressure of the left ear in case the head forms an angle γ with the symmetry axis of the system

lag necessary to propagate over a distance d_1. If the head is turned by an angle γ the signal of the left loudspeaker must make an excess distance d_3, while that of the right hand one an excess distance $(d_1 + d_2)$. These distances are now determined from a triangle ABC.

$$d_1 = D \sin \beta_h$$

From the lower diagram

$$d_2 = d_4 - d_5$$

$$d_3 = d_4 + d_5$$

Since

$$d_4 = \frac{D}{2} \sin \gamma \cos \beta_h$$

and

$$d_5 = d_6 \sin \beta_h = \frac{D}{2} \sin \beta_h (1 - \cos \gamma)$$

Therefore

$$d_2 = \frac{D}{2} [\sin \gamma \cos \beta_h - \sin \beta_h (1 - \cos \gamma)]$$

and

$$d_3 = \frac{D}{2} [\sin \gamma \cos \beta_h + \sin \beta_h (1 - \cos \gamma)]$$

The time lag of the signal coming from the left hand loudspeaker to the left ear is

$$T_A = \frac{d_3}{c} = \frac{D}{2c} [\sin \gamma \cos \beta_h + \sin \beta_h (1 - \cos \gamma)]$$

The time lag of the signal coming from the right hand loudspeaker to the left ear is

$$T_B = \frac{d_1 + d_2}{c} = \frac{D}{c} \left[\sin \beta_h + \frac{1}{2} (\sin \gamma \cos \beta_h - \sin \beta_h (1 - \cos \gamma)) \right]$$

Consequently the sound pressure at the left ear is

$$B = Xf(t - T_A) + Yf(t - T_B)$$

By expanding both left side terms into series and neglecting the higher order terms, we get the following expression for the sound pressure at the left ear:

$$B \simeq (X + Y) f \left(t - \frac{X \cdot T_A + Y \cdot T_B}{X + Y} \right)$$

A similar calculation results in the following equation for the time lag of the signal coming from the left hand loudspeaker to the right ear:

$$T_C = \frac{D}{c} \left[\sin \beta_h - \frac{1}{2} (\sin \gamma \cos \beta_h + \sin \beta_h (1 - \cos \gamma)) \right]$$

and the time lag for the signal coming from the right hand loudspeaker is similarly:

$$T_D = - \frac{D}{2c} [\sin \gamma \cos \beta_h - \sin \beta_h (1 - \cos \gamma)]$$

Consequently the sound pressure on the right ear is

$$J \simeq (X+Y) f\left(t - \frac{XT_C + YT_D}{X+Y}\right)$$

The time difference between the sound pressures of the two ears is

$$T = \frac{XT_C + YT_D - XT_A - YT_B}{X+Y} =$$

$$= \frac{X(T_C - T_A) + Y(T_D - T_B)}{X+Y}$$

Replacing variables X and Y by M and S, resp., we get

$$T = \frac{1}{2}\left[T_C + T_D - T_A - T_B + \frac{S}{M}(T_C + T_B - T_A - T_D)\right]$$

Substituting the four time lags:

$$T = \frac{D}{2c}\left(\frac{S}{M}\sin\beta_h \cos\gamma - \sin\gamma\cos\beta_h\right)$$

The same time difference results from using a point source, the direction of which includes an angle β with the symmetry axis of the listening system and for which the following equation is valid:

$$T = \frac{D}{c}\sin(\beta - \gamma)$$

It is possible to eliminate T from the last two equations

$$\sin(\beta - \gamma) = \frac{S}{M}\sin\beta_h \cos\gamma - \sin\gamma\cos\beta_h$$

Summary

The direction of a virtual sound source, produced by intensity stereo-sound reproduction using two loudspeakers, can be calculated from the data of the listening system by making use of the mechanism of directional hearing. Both the calculations and the tests carried out on experimental persons proved that the direction of the sound source as a function of frequency somewhat differs from the results published so far in the literature. By using low frequency signals the virtual sound source can be located outside the line connecting the two loudspeakers.

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Ferenc Takács, Budapest XI., Sztoczek u. 2—4, Hungary