# ELECTRICAL PARAMETERS OF DOUBLE-CIRCUIT THREE-PHASE HIGH VOLTAGE AERIAL LINES 

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The electrical parameters of double-circuit three-phase aerial lines and the methods used for their computation are well known. In the following discussion the effect of disposition on these parameters will be examined.

In this course of the discussion the disposition shown in Fig. 1 will be dealt with, but it is worth-while to note that the same analysis can be used in the case of a different disposition too, and further it is to be remembered that it is independent of the voltage level of the system.

In the double-circuit system of Fig. 1 symmetry about the vertical axis is generally used (la). It is a well known fact and can also easily be shown that the case of central symmetry (lb), where the corresponding conductors are placed farther than in case symmetry about the vertical axis, results in decreased positive-sequence inductance.

This statement can be justified by the equation used to determine the reactance of double-circuit systems for positive phase sequence currents, consisting of two parts. Its main part gives the reactance of one of the systems in the case when the other one is removed, while its additional part is due to the presence of the other system. This latter part can be given by the equation

$$
X_{m}=j 0.0241 \log \frac{d_{a^{\prime} b^{\prime \prime}} \cdot d_{a^{\prime} c^{\prime \prime}} \cdot d_{b^{\prime} a^{\prime \prime}} \cdot d_{b^{\prime} c^{\prime \prime}} \cdot d_{c^{\prime} a^{\prime \prime}} \cdot d_{c^{\prime} b^{\prime \prime}}}{d_{a^{\prime} a^{\prime \prime}}^{2} \cdot d_{b^{\prime} b^{\prime \prime}}^{2} \cdot d_{c^{\prime} c^{\prime \prime}}^{2}} \quad \text { ohm } / \mathrm{km}
$$

where indices $a^{\prime} b^{\prime}$ and $c^{\prime}$ are used to denote the conductors of one system, while $a^{\prime \prime}, b^{\prime \prime}$ and $c^{\prime \prime}$ that of the other one; $d_{a^{\prime} b}, d_{a^{\prime} c^{\prime}} \ldots$ are distances between the corresponding conductors.

The distances in the numerator in both cases of symmetry are practically the same, while the $d_{a^{\prime} a^{\prime \prime}}$, and $d_{c^{\prime} c^{\prime \prime}}$ distances of the denominator are much larger in case of central symmetry. This means that $X_{m}$, and therefore the complete inductive reactance when central symmetry is used, is less than that of the other version (1a).

The decrease of inductance is about $7 \%$, but can be as high as $10 \%$. resulting also in an increased positive-sequence capacitance.

The equation $v=1 / \sqrt{L C}$ is well known, where $L$ is the positive-sequence inductance in $\mathrm{Hy} / \mathrm{km}$; $C$ the positive-sequence capacitance in $\mathrm{F} / \mathrm{km}$; and $v=$ $=300,000 \mathrm{~km} / \mathrm{sec}$ is the velocity of light. What all of this adds up to is that the capacitance is increased by about the same amount as by which the inductance is decreased (when a lossless conductor is regarded and neglecting the fact, that $L$ is defined not by the symmetrical voltage but by the symmetrical current system). The maximal shunt capacitance belongs to the minimal series inductance. This fact is important, when the reactive energy balance of a network


Fig. 1
is taken into account. On the other hand, in case of minimal series inductance and maximal shunt capacitance the minimal characteristic impedance is reached (neglecting the losses):

$$
Z=\sqrt{\frac{L}{C}} \mathrm{ohm}
$$

The surge impedance loading of the line is

$$
P_{T}=\frac{U^{2}}{Z}
$$

where $P_{T}$ is the surge impedance loading in MW, $U$ the line voltage in $k V$ and $Z$ the characteristic impedance in ohm. The decreased characteristic impedance results in increased surge impedance loading and at the same time the power, which can economically be transmitted on the serial line, is also increased. Surge impedance loading can be increased by about $5-10 \%$, when central symmetry is used. If a given electric power is to be transmitted, this disposition results in better stability, reactive power balance and voltage conditions.

It is quite natural, however, that the use of central symmetry has its drawbacks too. Corona losses will be increased, but their growth may be less than the decrease of ohmic losses due to the improvement in voltage conditions.

After having compared the two types of symmetry, an analysis of the current and voltage symmetry in both cases, with and without conductor transposition, will be attempted.

It is assumed that the input and output currents of the system are of positive phase sequence (Fig. 2).

For this system we have the following equations:

$$
\left.\begin{array}{ll}
I_{a_{1}}=I_{a}^{\prime}+I_{a}^{\prime} & u_{a}^{\prime}=u_{a}^{\prime \prime} \\
I_{b_{1}}=I_{b}^{\prime}+I_{b}^{\prime \prime} & u_{b}^{\prime}=u_{b}^{\prime \prime} \\
I_{c_{1}}=I_{c}^{\prime}+I_{c}^{\prime \prime} & u_{c}^{\prime}=u_{c}^{\prime \prime}
\end{array}\right\} \text { series voltage drop }
$$

The conductors are numbered so that the numbers belong to the conductors and not to their geometrical position $\left(a^{\prime} \div 1, b^{\prime} \div 2, c^{\prime} \div 3, c^{\prime \prime} \div 4, b^{\prime \prime} \div 5\right.$,


Fig. 2
$\left.a^{\prime \prime} \div 6\right)$. The use of this method results in equations of the same form for both types of symmetry, but it is to be remembered when the conclusions are drawn.

In the analysis the matrix form will be used. The current and voltage matrices are accordingly:

$$
\begin{array}{ll}
\mathbf{I}^{\prime}=\left[\begin{array}{c}
I_{a}^{\prime} \\
I_{b}^{\prime} \\
I_{c}^{\prime}
\end{array}\right], & \mathbf{I}^{\prime \prime}=\left[\begin{array}{c}
I_{a}^{n} \\
I_{b}^{n} \\
I_{c}^{n}
\end{array}\right], \\
\mathbf{u}^{\prime}=\left[\begin{array}{c}
u_{a}^{\prime} \\
u_{b}^{\prime} \\
u_{c}^{\prime}
\end{array}\right], & \mathbf{u}^{\prime \prime}=\left[\begin{array}{c}
u_{a}^{\prime \prime} \\
u_{b}^{\prime \prime} \\
u_{c}^{\prime \prime}
\end{array}\right] .
\end{array}
$$

The impedance matrices are:

$$
\begin{array}{cl}
\mathrm{Z}^{\prime}=\left[\begin{array}{lll}
\left(r+j x_{s}\right) & j x_{12} & j x_{13} \\
j x_{21} & \left(r+j x_{s}\right) & j x_{23} \\
j x_{31} & j x_{32} & \left(r+j x_{s}\right)
\end{array}\right], \quad \mathrm{Z}^{n}=\left[\begin{array}{lll}
\left(r+j x_{s}\right) & j x_{65} & j x_{64} \\
j x_{56} & \left(r+j x_{5}\right) & j x_{54} \\
j x_{46} & j x_{45} & \left(r+j x_{s}\right)
\end{array}\right], \\
\mathrm{Z}_{m}=\left[\begin{array}{lll}
j x_{16} & j x_{15} & j x_{14} \\
j x_{26} & j x_{25} & j x_{24} \\
j x_{36} & j x_{35} & j x_{34}
\end{array}\right], & \mathrm{Z}_{m i t}=\left[\begin{array}{lll}
j x_{16} & j x_{26} & j x_{36} \\
j x_{15} & j x_{25} & j x_{35} \\
j x_{14} & j x_{24} & j x_{34}
\end{array}\right],
\end{array}
$$

The voltage drop in the system without conductor transposition is:

$$
\begin{aligned}
\mathbf{u}^{\prime} & =\mathbf{Z}^{\prime} \mathbf{I}^{\prime}+\mathbf{Z}_{m} \mathbf{I}^{\prime \prime} \\
\mathbf{u}^{\prime \prime} & =\mathbf{Z}^{\prime} \mathbf{I}^{\prime \prime}+\mathbf{Z}_{m:} \mathbf{I}^{\prime}
\end{aligned}
$$

When the conductors are transposed, the voltage drop is (Fig. 3): First section of transposition:

$$
\begin{aligned}
\mathbf{u}^{\prime} & =\frac{1}{3}\left(\mathbf{Z}^{\prime} \mathbf{I}^{\prime}+\mathbf{Z}_{m} \mathbf{I}^{\prime \prime}\right) \\
\mathbf{u}^{\prime \prime} & =\frac{1}{3}\left(\mathbf{Z}^{\prime} \mathbf{I}^{\prime \prime}+\mathbf{Z}_{m t} \mathbf{I}^{\prime}\right)
\end{aligned}
$$



Fig. 3
Second section of transposition:

$$
\begin{aligned}
& \mathbf{u}^{\prime}=\frac{1}{3}\left[\left(\Omega^{2} \mathbf{Z}^{\prime} \boldsymbol{\Omega}\right) \mathbf{I}^{\prime}+\left(\Omega^{2} \mathbf{Z}_{m} \Omega\right) \mathbf{I}^{\mu}\right] \\
& \mathbf{u}^{\prime \prime}=\frac{1}{3}\left[\left(\Omega^{2} \mathbf{Z}^{\prime \prime} \Omega\right) \mathbf{I}^{\prime \prime}+\left(\Omega^{2} \mathbf{Z}_{m t} \boldsymbol{\Omega}\right) \mathbf{I}^{\prime}\right]
\end{aligned}
$$

where

$$
\Omega=\left[\begin{array}{lll}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{array}\right]
$$

Third section of transposition:

$$
\begin{aligned}
& \mathbf{u}^{\prime}=\frac{1}{3}\left[\left(\Omega \mathbb{Z}^{\prime} \Omega^{2}\right) \mathbb{I}^{\prime}+\left(\Omega \mathbf{Z}_{m} \Omega^{2}\right) \overline{\mathbf{I}}^{\prime \prime}\right] \\
& \mathbf{u}^{\prime \prime}=\frac{1}{3}\left[\left(\Omega \mathbb{Z}^{\prime \prime} \Omega^{2}\right) \mathbf{I}^{\prime \prime}+\left(\Omega \mathbb{Z}_{m i} \Omega^{2}\right) \mathbb{I}^{\prime}\right]
\end{aligned}
$$

The total voltage drop is:

$$
\begin{gathered}
\mathbf{u}^{\prime}=\frac{\mathbf{Z}^{\prime}+\left(\Omega^{2} \mathbf{Z}^{\prime} \boldsymbol{\Omega}\right)+\left(\boldsymbol{\Omega} \mathbf{Z}^{\prime} \Omega^{2}\right)}{3} \mathbf{I}^{\prime}+\frac{\mathbf{Z}_{m}+\left(\Omega^{2} \mathbf{Z}_{m} \boldsymbol{\Omega}\right)+\left(\Omega \mathbf{Z}_{m} \Omega^{2}\right)}{3} \mathbf{I}^{\prime \prime} \\
\mathbf{u}^{\prime \prime}=\frac{\mathbb{Z}^{\prime \prime}+\left(\Omega^{2} \mathbf{Z}^{\prime \prime} \Omega\right)+\left(\Omega \mathbb{Z}^{\prime \prime} \Omega^{2}\right)}{3} \mathbf{I}^{\prime \prime}+\frac{\mathbb{Z}_{m t}+\left(\Omega^{2} \mathbf{Z}_{m t} \boldsymbol{\Omega}\right)+\left(\Omega \mathbb{Z}_{m i} \Omega^{2}\right)}{3} \mathbf{I}^{\prime} \\
\mathbf{u}^{\prime}=\mathbf{A} \mathbf{I}^{\prime}+\mathbf{B \mathbf { I } ^ { \prime \prime }} \\
\mathbf{u}^{\prime \prime}=\mathbf{C l}^{\prime \prime}+\mathbf{D I ^ { \prime }}
\end{gathered}
$$

where

$$
\begin{aligned}
& \mathbf{A}=\frac{1}{3}\left(\mathbf{Z}^{\prime}+\Omega^{2} \mathbf{Z}^{\prime} \Omega+\Omega \mathbf{Z}^{\prime} \Omega^{2}\right) \\
& \mathbf{B}=\frac{1}{3}\left(\mathbf{Z}_{m}+\Omega^{2} \mathbf{Z}_{m} \Omega+\Omega \mathbf{Z}_{m} \Omega^{2}\right) \\
& \mathbf{C}=\frac{1}{3}\left(\mathbf{Z}^{\prime \prime}+\Omega^{2} \mathbb{Z}^{\prime \prime} \Omega+\Omega \mathbf{Z}^{\prime \prime} \Omega^{2}\right) \\
& \mathbf{D}=\frac{1}{3}\left(\mathbb{Z}_{m t}+\Omega^{2} \mathbf{Z}_{m!} \Omega+\Omega \mathbf{Z}_{m t} \Omega^{2}\right)
\end{aligned}
$$

For the system the following equations can be written:

$$
\begin{gathered}
\mathrm{u}^{\prime}=\mathbf{u}^{\prime \prime} \\
\mathrm{AI}^{\prime}+\mathrm{BI} \mathbf{I}^{\prime \prime}-\mathrm{CI}^{\prime \prime}-\mathrm{DH}^{\prime}=0 \\
(\mathrm{~A}-\mathrm{D}) \mathrm{I}^{\prime}+\left(\mathrm{B}-\mathrm{C}^{\prime} \mathbf{I}^{\prime \prime}=0\right.
\end{gathered}
$$

The current distribution between the systems:

$$
\begin{aligned}
& \mathbf{I}=\mathbf{I}^{\prime}+\mathbf{I}^{\prime \prime} \rightarrow \mathbf{I}^{\prime \prime}=\mathbf{I}-\mathbf{I}^{\prime} \\
& \mathrm{I}^{\prime}=\mathrm{I}-\mathrm{I}^{\prime} \\
& (A-B+\mathbb{C}-D) \mathbb{I}^{\prime}=(\mathbb{C}-B) \mathbb{I} \\
& (A-D) I=I^{\prime \prime}(A-D-B+C) \\
& \left.I^{\prime}=(A-B+\mathbb{C}-D)^{-1}(C-B) \mathbb{I}\right] \\
& B^{\prime \prime}=(A-B+C-D)^{-1}(A-D) I
\end{aligned}
$$

A. Symmetry about the vertical axis
(Fig. 4a)

1. The conductors are unsymmetrically spaced and transposed.

$$
\begin{aligned}
& \mathbb{Z}^{\prime}=\mathbb{Z}^{\prime \prime} \rightarrow \mathbf{A}=\mathbb{C} \\
& \mathbb{Z}_{m}=\mathbf{Z}_{m t} \rightarrow \mathbf{B}=\mathbf{D} \\
& \mathbf{C}-\mathbb{B}=\mathbf{A}-\mathbf{D}
\end{aligned}
$$

From this follows that $\mathbf{I}^{\prime}=\mathbf{I}^{\prime \prime}=\mathbf{I} / 2$, which means that the current is distributed equally between the two systems. There are no negative or zerosequence current components.
2. The conductors are spaced according to Fig. $4 b$ and transposed.

It follows from case No. 1 that $\mathbf{I}^{\prime}=\mathbf{I}^{\prime \prime}=\mathbf{I} / 2$. There are only positivesequence current components.


Fig. 4
3. The conductors are unsymmetrically spaced and there is no transposition used.

$$
\begin{gathered}
\mathbb{Z}^{\prime}=\mathbb{Z}^{\prime \prime} \rightarrow \mathbf{A}=\mathbb{C} \\
\mathbb{Z}^{\prime}=\mathbb{Z}_{m t} \rightarrow \mathbf{B}=\mathbf{D} \\
\mathbb{C}-\mathbf{B}=\mathbf{A}-\mathbf{D}
\end{gathered}
$$

from which it follows that

$$
\mathbb{I}^{\prime}=\mathbb{I}^{\prime \prime}=\frac{\mathbf{I}}{2}
$$

and this means that the current is distributed equally between the two systems and there are no negative and zero-sequence current components.
4. The conductors are spaced according to Fig. 4b, and there is no transposition used.

It follows from case No. 3, that

$$
\mathbb{I}^{\prime}=\mathbb{I}^{\prime \prime}=\frac{\mathbb{I}}{2}
$$

There are only positive-sequence current components.

## B. Central symmetry

(Fig. 5a)

1. The conductors are unsymmetrically spaced and transposed.

$$
\begin{aligned}
& \mathbb{Z}^{\prime} \neq \mathbb{Z}^{\prime \prime} \\
& \mathbb{Z}_{m} \neq \mathbb{Z}_{m i}
\end{aligned}
$$

It can be shown that

$$
\begin{aligned}
& \mathbf{A}=\mathbf{C} \\
& \mathbf{D}=\mathbf{B}_{i}
\end{aligned}
$$

from which we have

$$
\begin{aligned}
& \mathbf{I}^{\prime}=\frac{1}{2}\left(\mathbf{C}-\frac{\mathbf{B}}{2}-\frac{\mathbf{B}_{i}}{2}\right)^{-1}(\mathbf{C}-\mathbf{B}) \\
& \mathbf{I}^{\prime \prime}=\frac{1}{2}\left(\mathbf{C}-\frac{\mathbf{B}}{2}-\frac{\mathbf{B}_{i}}{2}\right)^{-1}\left(\mathbb{C}-\mathbf{B}_{i}\right)
\end{aligned}
$$

There are no negative and zero-sequence current components.


Fig. 5
2. The conductors are spaced according to Fig. 5b, and transposed.

$$
\begin{gathered}
\mathbb{Z}^{\prime}=\mathbb{Z}^{\prime \prime} \rightarrow \mathbb{A}=\mathbb{C} \\
\mathbb{Z}_{m}=\mathbb{Z}_{m i} \rightarrow \mathbb{D}=\mathbb{D} \\
\mathbf{C}-\mathbf{B}=\mathbf{A}-\mathbb{D}
\end{gathered}
$$

from which follows that

$$
\mathbb{I}^{\prime}=\mathbb{I}^{\prime \prime}=\frac{\mathbf{I}}{2}
$$

There are only positive-sequence current components.
3. The conductors are unsymmetrically spaced and there is no transposition used.

$$
\begin{aligned}
& \mathbb{Z}^{\prime} \neq \mathbb{Z}^{\prime \prime} \\
& \mathbb{Z}_{m} \neq \mathbb{Z}_{m t} \\
& \mathbb{I}^{\prime} \neq \mathbb{I}^{\prime \prime}
\end{aligned}
$$

It can be shown that in this case components of all three phase-sequences are generated.
4. The conductors are spaced according to Fig. $5 b$ and there is no transposition used.

$$
\begin{gathered}
\mathbf{Z}^{\prime}=\mathbf{Z}^{\prime \prime} \rightarrow \mathbf{A}=\mathbf{C} \\
\mathbf{Z}_{m}=\mathbf{Z}_{m t} \rightarrow \mathbf{D}=\mathbf{B} \\
\mathbf{C}-\mathbf{B}=\mathbf{A}-\mathbf{D}
\end{gathered}
$$

from which follows that

$$
\mathbf{I}^{\prime}=\mathbf{I}^{\prime \prime}=\frac{\mathbf{I}}{2}
$$

and there are no negative and zero-sequence current components. On the basis of the above analysis the conclusion can be drawn, that when symmetry about the vertical axis is used there is no unsymmetry in the currents even in the case of unsymmetrically spaced conductors without transposition. When central symmetry is used, some geometrical limitations and conductor transposition are needed in order to get equal currents in the two systems, but even in the case of unsymmetrically spaced conductors there are no negative and zero-sequence current components. Equal distribution of currents can be attained also without conductor transposition, but some geometrical limitations must be observed. Using symmetry about the diagonal and conductors unsymmetrically spaced, when no transposition is used, with $\mathbb{I}^{\prime} \neq \mathbb{I}^{\prime \prime}$ there will be negative and zero-sequence current components generated.

It is quite natural, however, that in the cases discussed above, the presence of mostly positive-sequence current components does not in any way indicate that the same is true for the various voltage components too.

## Appendix

As an example, the Hungarian section of the Sajószöged-Munkács serial line will be examined.

$$
\begin{aligned}
U_{n i} & =220 \mathrm{l} V \\
q & =350 \mathrm{~mm}^{2}(\mathrm{ACSR}) \\
l & =120.3 \mathrm{~km}
\end{aligned}
$$

The disposition of the double-circuit three-phase system is shown in Fig. 6.
The inductive reactance of one system is:

$$
\begin{gathered}
x=\frac{0.145}{2} \log \frac{\sqrt{G M D_{a b} G M D_{a c} G M D_{b c}}}{\sqrt{G M R_{a}^{*} G M R_{b}^{*} G M R_{c}^{*}}}= \\
=0.0725 \log \frac{\sqrt[12]{D_{a^{\prime} b^{\prime}} D_{a^{\prime} b^{\prime \prime}} D_{a a^{\prime \prime} b^{\prime}} D_{a^{\prime \prime} b^{\prime \prime}} D_{a^{\prime} c^{\prime}} D_{a^{\prime \prime} c^{\prime}} D_{a^{\prime} c^{\prime \prime}} D_{a^{\prime \prime} c^{\prime \prime}} D_{b^{\prime} c^{\prime}} D_{b^{\prime} c^{\prime \prime}} D_{b^{\prime \prime} c^{\prime}} D_{b^{\prime \prime}}}}{\sqrt[2: 2]{G M R^{2} D_{a^{\prime} a^{\prime \prime}}^{2} \cdot G M R^{2} D_{b^{\prime} b^{\prime \prime}}^{2} \cdot G M R^{2} D_{z^{\prime} c^{\prime \prime}}^{2}}}
\end{gathered}
$$

$x_{1}=0.4384 \mathrm{ohm} / \mathrm{km}$ (symmetry about the vertical axis)
$x_{2}=0.404 .1 \mathrm{ohm} / \mathrm{km}$ (central symmetry)
In case of central symmetry, when compared to the case of symmetry about the vertical axis the inductive reactance is decreased by

$$
\frac{(0.4384-0.4041) 100}{0.4384}=7.83 \%
$$



Fig. 6

Using the assumptions mentioned in the discussion the capacity can be determined:

$$
\begin{gathered}
v=\frac{1}{\sqrt{L C}} \rightarrow c=\frac{1}{L v^{2}} \\
c_{1}=\frac{1}{\frac{0.4384}{314}\left(3 \cdot 10^{5}\right)^{2}}=7.97 \cdot 10^{-9} \mathrm{~F} / \mathrm{km} \text { (symmetry about vertical axis) } \\
c_{2}=\frac{1}{\frac{0.4041}{314}\left(3 \cdot 10^{5}\right)^{2}}=8.65 \cdot 10^{-9} \quad \mathrm{~F} / \mathrm{km} \text { (central symmetry) }
\end{gathered}
$$

The characteristic impedance:

$$
Z=\sqrt{\frac{L}{C}}
$$

$$
\begin{aligned}
& Z_{1}=\sqrt{\frac{1.395 \cdot 10^{-3}}{7.97 \cdot 10^{-9}}}=418 \mathrm{ohm} \quad \text { (symmetry about the vertical axis) } \\
& Z_{2}=\sqrt{\frac{1.285 \cdot 10^{-3}}{8.65 \cdot 10^{-9}}}=385 \mathrm{ohm} \quad \text { (central symmetry) }
\end{aligned}
$$

The total surge impedance loading of the two systems is

$$
P_{T}=2 \frac{U^{2}}{Z}
$$

$$
\begin{aligned}
& P_{T 1}=2 \cdot \frac{220^{2}}{418}=232 \mathrm{MW} \quad \text { (symmetry about the vertical axis) } \\
& P_{T 2}=2 \cdot \frac{220^{2}}{385}=255 \mathrm{MW} \quad \text { (central symmetry) }
\end{aligned}
$$

The increase in the surge impedance loading of the central symmetric disposition, compared to the symmetry about the vertical axis disposition is

$$
\frac{(255-232) 100}{232}=9.9 \%
$$

The computation of corona:

## Corona losses:

The corona loss can be given by the equation

$$
P_{s}=\frac{20.96 \cdot 10^{-6} \cdot f \cdot U_{n}^{2} \cdot \varphi_{c}^{\prime}}{\left(\lg \frac{2 F}{d}\right)^{2}} \mathrm{~kW} / \mathrm{km} / 3 \text { phase }
$$

where

$$
\begin{aligned}
& f=\text { frequency } \\
& U_{n}=\text { nominal line voltage in } \mathrm{kV} \\
& F=\text { mean phase spacing in } \mathrm{cm} \\
& d=\text { diameter of the conductor in cm }
\end{aligned}
$$

$$
\varphi_{c}^{\prime}=f\left(\frac{U_{n}}{U_{0}}\right) \quad \text { (from the diagram of Fig. 7) }
$$

The corona limit voltage (line voltage) is:

$$
U_{0}=\frac{\sqrt{3} \cdot 4860 \cdot V^{2 / 3} \cdot m\left[\lg \frac{2 F}{k d_{t}}+(i-1) \lg \frac{2 F}{d-k d_{t}}\right]}{\frac{200}{k d_{t}}+\frac{100(t-1)}{d-k d_{i}}} \mathrm{kV}
$$

where $d_{t}=$ diameter of a wire in the conductor in cm $t=$ number of wires in the outer layer

$$
k=1-\frac{\cos \frac{\pi}{t}}{\frac{\pi}{2}+\frac{\pi}{t}} ; \left.\quad \frac{t}{k}\left|\frac{12}{0.472}\right| \frac{16}{0.445}\left|\frac{18}{0.436}\right| \frac{24}{0.417} \right\rvert\, \frac{28}{0.410}
$$

$m$ is usually $0.87-0.9$

$$
\left.v=\frac{17.9 x(\text { air pressure in } m m \mathrm{Hg})}{25.4\left[491+\frac{9 x(\text { temp. in }}{}{ }^{\circ} \mathrm{C}\right)}{ }_{5}^{5}\right] \quad \text { meteorological factor }
$$

$\left(v=1\right.$ at 760 mm Hg and $25^{\circ} \mathrm{C}$ )


The necessary data:

$$
\begin{aligned}
d_{t} & =2.9 \mathrm{~mm} \\
t & =24 \\
G M R & =1.057 \mathrm{~cm}
\end{aligned}
$$

$F_{1}=1120 \mathrm{~cm}$ (symmetry about the vertical axis)

$$
\begin{aligned}
{\left[x_{1}\right.} & =0.145 \log \frac{F}{G M R}=0.145 \log \frac{F}{1.057}=0.4348 \rightarrow F \\
F_{2} & =653 \mathrm{~cm} \text { (central symmetry) } \\
v & =1 \\
m & =0.9 \\
k & =0.417
\end{aligned}
$$

The corona limit voltage is:

$$
\begin{aligned}
U_{01} & =\frac{\sqrt{3} \cdot 4860 \sqrt[3]{1^{2}} \cdot 0.9\left[\log \frac{1120 \cdot 2}{0.417 \cdot 0.29}+23 \log \frac{1120 \cdot 2}{2.61-0.417 \cdot 0.29}\right]}{\frac{200}{0.417 \cdot 0.29}+\frac{100 \cdot 23}{2.61-0.417 \cdot 0.29}}= \\
& =212 \mathrm{kV} \text { (symmetry about the vertical axis) }
\end{aligned}
$$

$$
U_{02}=\frac{\sqrt{3} \cdot 4860 \cdot 0.9\left[\log \frac{653 \cdot 2}{0.417 \cdot 0.29}+23 \log \frac{653 \cdot 2}{2.61-0.417 \cdot 0.29}\right]}{\frac{200}{0.417 \cdot 0.29}+\frac{100 \cdot 23}{2.61-0.417 \cdot 0.29}}=
$$

$$
=195 \mathrm{KV} \text { (central symmetry) }
$$

The corona losses are:

$$
\begin{gathered}
\frac{U_{n}}{U_{01}}=\frac{220}{212}=1.037 \rightarrow \varphi_{c}^{\prime}=0.042 \\
P_{c 1}=\frac{20.96 \cdot 10^{-6} \cdot 50 \cdot 220^{2} \cdot 0.042}{\left(\log \frac{2240}{2.61}\right)^{2}}=0.2465 \mathrm{~kW} / \mathrm{km} / \mathrm{system} \\
P_{c 2}=\frac{20.96 \cdot 10^{-6} \cdot 50 \cdot 220^{2} \cdot 0.0615}{\left(\log \frac{653 \cdot 2}{2.61}\right)^{2}}=0.4281 \mathrm{~kW} / \mathrm{km} / \mathrm{system} \\
U_{02}
\end{gathered} \frac{220}{195}=1.129 \rightarrow \varphi_{c}^{\prime}=0.0615 \mathrm{C} .
$$

The corona losses in fair weather for one system are:
$P_{c_{1}}=0.2465 \mathrm{~kW} / \mathrm{km} /$ system (symmetry about the vertical axis)
$P_{c 2}=0.4281 \mathrm{~kW} / \mathrm{km} /$ system (central symmetry).
The difference between the corona losses for the complete line (for the two systems and the total length of line):

$$
\begin{gathered}
2\left(P_{c 2}-P_{c 1}\right) \cdot 1= \\
=2(0.4281=0.2466) 120.3=43.6 \mathrm{~kW}
\end{gathered}
$$

The increase of corona losses is high in percentage, but it is negligible when compared to the power transmitted.

When radio interference is taken into account, for the corona limit voltages with $m_{1} \cdot m_{2}=0.9$ and 0.75 values and for the height equal to 0 m using the data taken from the diagram of Fig. 8 (where $m_{1}$ is the surface factor and $m_{2}$ is the meteorological factor) we have:

|  | $m_{1} \cdot m_{2}$ | $U_{01(\mathrm{~kW} \text { line })}$ |
| :--- | :--- | :--- |
| 0.9 | 520 | $U_{02(\mathrm{~kW} \text { Iine })}$ |
| 0.75 | 442 | 407 |



Fig. 8

When the central symmetry is used, corona losses are increased, while the corona limit voltage is decreased, when they are compared with the case of symmetry about the vertical axis, but even in this case radio interference is far from becoming troublesome.

These statements are further justified by the diagram data, given by the analysis of capacitive generation.

Using a digital computer, the potential distribution in the vicinity of the aerial line analysed above was determined in both cases of symmetry. Fig. 9 a and 9 b show the potential distribution in a given moment of time in case of symmetry about the axis and about the diagonal, respectively, while in Fig. 10a and 10b the maximum values of the potential are shown occurring during the time period.


Fig. 10
On the other hand, Fig. 10 clearly shows the already well known fact that due to its more advantageous potential distribution the central symmetry arrangement is more advantageousalso, when the vehicles, isolated from ground and moving under the line are taken into consideration.

## Summary

The paper deals with the electrical parameters of double-circuit three-phase aerial lines. The differencesin the parameters, advantages and disadvantages, due to the symmetry about the vertical axis and of central symmetryare clearly shown. It is attempted to determine geometrical limitations which must be observed, regarding the two possible arrangements in order to avoid the generation of negative and zero-sequence current components in the parallel system. There is an example given for a line in operation.

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