A TWO-DIMENSIONAL FIELD INVESTIGATION OF ELECTROSTATIC QUADRUPOLE LENSES COMPOSED OF CYLINDRICAL SURFACES

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1. Introduction

For focusing of charged particles quadrupole lenses are commonly used. Quadrupole lenses form transverse fields, i.e. their fields are roughly perpendicular to the direction of motion of the charged particle beam. Thus, their convergence is much stronger than that of axially symmetric lenses. A system of quadrupole lenses may be fully equivalent to a lens of revolution [1]. It is also possible to accomplish an achromatic compound quadrupole lens formed by electrostatic and magnetic fields imposed on each other [2]. (As well known, it is impossible to construct achromatic lenses with rotational symmetry.) Compensation of spherical aberration is also possible in quadrupole lenses. These advantages open up new vistas for increasing the resolving power of electron-optical devices by using quadrupole lenses.

An ideal electrostatic quadrupole lens consists of four identical infinitely long hyperbolic electrodes placed at equal distances from the axis of the system, which coincides with the beam axis. Oppositely placed electrodes are connected with each other, while neighbouring ones have different voltages. Thus the whole lens has two planes of symmetry and two planes of antisymmetry.*

It is impossible, however, to put such an ideal lens into practice. Commonly used electrodes are of nearly hyperbolic form, but quadrupole lenses with cylindrical or planar electrodes are also used because of their simpler construction.

2. Short description of the lenses examined

One of the simplest quadrupole lens types consists of two coaxial cylinders, the internal cylinder being partly cut out. In this work the electrostatic field distribution of such quadrupole lenses is examined by means of a two-

* A plane, with regard to which the field pattern is geometrically symmetrical but at whose both sides the field strengths are of opposite direction with respect to the plane, is called a plane of *antisymmetry*.

dimensional resistance network analogue. Some results presented here had been published previously in the authors' short communications [3, 4].

Two types of lenses have been studied. In case of the first one four identical parts are symmetrically cut out from the internal cylinder. The remaining four parts form a concave cylindrical quadrupole lens. The external cylinder

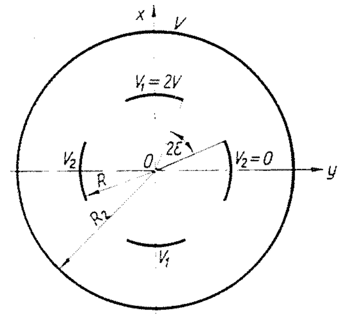

Fig. 1

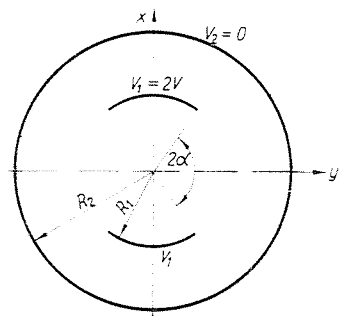

Fig. 2

is held at an average potential V; so the whole system possesses two antisymmetry planes as well as two planes of symmetry. In case of a long lens the influence of the ends is not to be considered and the field distribution practically does not depend on z-coordinate (two-dimensional model). The whole system may be characterized by its cross section shown in Fig. 1.

The second lens type may be derived from the first one by removing an oppositely placed pair of internal electrodes (Fig. 2). The remaining internal electrodes are connected and a potential difference is applied between these electrodes and the external cylinder. This system in general case possesses only two planes of symmetry ($x0z$ and $y0z$) and no plane of antisymmetry.

3. The experimental method

For measuring the electrostatic potential distributions of the lenses a two-dimensional resistance network was used. The network's linear dimensions are 120 cm and 200 cm in directions x and y, respectively. It has got 44×76 intersection points and contains 6568 resistances of 1 ± 0.005 kΩ. Values of the resistances had been doubled on one of the network's boundaries, which is coincident with the y-axis (Fig. 1 and 2). It makes possible to use this boundary as an axis of symmetry and simulate only a half of the system. (The network analogue belongs to the Department of Theoretical Electricity, Budapest Polytechnical University.)

Accuracy of data obtainable by measurements on the network has been established by simulating potential distribution of plane condensers. It is proved to be better than 0.1%.

Cylindrical electrode surfaces may be represented by parts of circles in a two-dimensional model. They were approximated by broken lines, connecting every neighbouring intersection point situated near to the electrode's contour. The difference between these broken lines and the exact circles, being about 2% in average, did not exceed 3.8%. The error arising from this difference was estimated by measuring the potential distribution of a cylindrical condenser formed by the electrodes in case of $\alpha = 0$, $R_1/R_2 = 0.8$ (Fig. 2). The greatest deviation of the potential from the calculated value was found in the vicinity of the electrodes. The error did not exceed 4%. (There is a possibility to approximate the electrode contours better by passing over some intersection points so that the maximum difference may be less than 1%. But in this case an additional error arises from the field penetration through the omitted points. This error exceeds the error arising from the rougher approximation of the electrode contours. This was the reason why the method of connecting every neighbouring intersection point was used in our experiments.)

All the distances were expressed in "network units". A network unit is equal to the distance between two neighbouring intersection points.

4. An electrostatic quadrupole lens composed of four concave cylindrical electrodes and an external cylindrical chamber

As we have shown in Fig. 1, the internal electrodes of our first lens are formed by four identical parts of a cylinder with radius R. These electrodes are held at potentials $V_1 = 2V$ and $V_2 = 0$, respectively. The gaps between the electrodes are measured by their angular distance 2ε. These internal electrodes are surrounded by a coaxial external cylindrical chamber with radius R_2, held at the average potential V, which is equal to the potential on the axis (z) of the system [4].

In a two-dimensional approximation the potential distribution of this lens on the x-axis may be expressed by the following series:

$$\frac{\varphi(x,0)}{V} = 1 + K_2 \frac{x^2}{R^2} + K_6 \frac{x^6}{R^6} + K_{10} \frac{x^{10}}{R^{10}} + \dots \tag{1}$$

where the coefficients K_n are constants which depend on the values of 2ε and R_2/R. The symmetry properties are taken into account by the fact that $n = 2 + 4i$ $(i = 0, 1, 2, \dots)$.

Coefficients K_n had been calculated by BERNARD [5] for the case when the external chamber is removed and the potential distribution in the gaps between the electrodes is assumed to be linear. These assumptions are acceptable only for small values of the angle 2ε. Using the method of calculation given in [5] one can obtain the coefficients as functions of 2ε:

$$K_2 = \frac{4}{\pi} \frac{\sin 2\varepsilon}{2\varepsilon} \tag{2}$$

and

$$K_6 = -\frac{4}{3\pi} \frac{\sin 6\varepsilon}{6\varepsilon} \tag{3}$$

(We note that in [5] the value of K_6 is wrong.)

When the gaps between the internal electrodes are not very small, the potential distribution of the lens is sufficiently effected by the boundary conditions outside the electrode system. Practically these conditions are determined by the geometry and potential of the vacuum chamber in which the lens is arranged. In case of a metallic coaxial cylindrical chamber, held at the average potential V, no change will occur in conditions of symmetry.

Thus, the external boundary of the system was simulated on the resistance network analogue by a circle with radius $R_2 = 35$ network units. This is near to the greatest possible radius on this network. The value of this radius was constant during the whole experiment.

Radius R of the internal electrodes was altered so that the value of R_2/R varied in the interval $1 \leq R_2/R \leq 3.5$. The value of the angle 2ε varied in the interval $2°20' \leq 2\varepsilon \leq 90°$.

The potential was measured at every intersection point on the x- and y-axes, and also along the straight lines linking the ends of neighbouring electrodes. Potential along the lines at 45° angles to both x and y axes was measured for checking the symmetry properties and was found equal to the potential V of the lens centre.

Plots of the potential distribution in the gaps between the electrodes are given in Fig. 3 in case of $R_2/R = 1.25$. (Here w is the distance from the end of the electrode held at zero potential.) Curve 1 refers to $2\varepsilon = 26°$, curve 2 to $2\varepsilon = 48°$, curve 3 to $2\varepsilon = 86°$. As it is obvious from Fig. 3, the potential distribution is quite different from linear (dotted line) and this difference is growing with the increase of the angular distance between the electrodes.

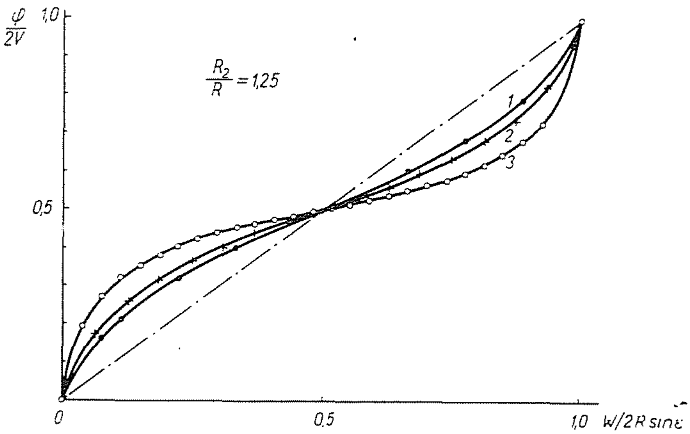

Fig. 3

Coefficients K_2, K_6, and K_{10} were calculated on the basis of potential measurements along the axis in the interval $0 \leq x/R \leq 0.8$. For this interval terms of higher than 10th order were neglected in series (1). The calculations of the coefficients at each value of R_2/R and ε were carried out by solving a system of 3 linear algebraic equations obtained from (1) at three different values of x/R and φ/V, and also by using the method of least squares for systems of equations containing all measured values of x/R and φ/V. The results are presented in Figs 4 and 5.

In Fig. 4 coefficient K_2 versus 2ε is plotted for several values of R_2/R (curves 2, 3 and 4 refer to $R_2/R = 3.5$; $R_2/R = 1.95$; and $R_2/R = 1.25$, resp.) Curve 1 is calculated from (2). It can be seen that for low values of the angle 2ε all the curves coincide with each other. When the angular distance 2ε between electrodes increases the experimental curves are significantly different from the curve 1, even if $R_2/R = 3.5$. As the value of R_2/R tends to 1, this difference increases and the values of K_2 decrease.

In Fig. 5 K_6/K_2 is given as a function of 2ε, for the same values of R_2/R as in Fig. 4. (Notations are the same as in Fig. 4.) Curve 1 is plotted on the basis of formulae (2) and (3). As we can see, for low values of 2ε the experimental curves are close to the calculated one. When the value of 2ε increases, K_6/K_2 is significantly greater than its calculated value. The difference between them rapidly increases as R comes closer to R_2 because of the greater influence of the external cylinder.

In the limiting case of $R = R_2$ the possible minimum angular distance between two neighbouring electrodes on the network was $2\varepsilon = 2°20'$. That is a lens composed of a single cylinder which is cut in four identical parts. In this

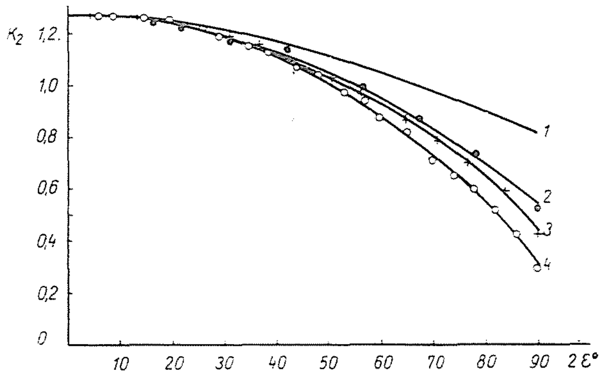

Fig. 4

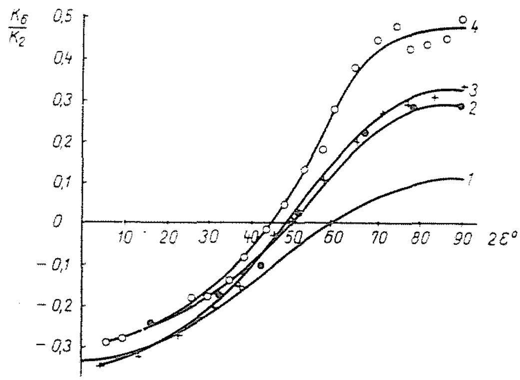

Fig. 5

case expressions (2) and (3) may be considered as exact formulae and we obtain: $K_2 = 1.273$; $K_6 = -0.423$. On the basis of our measurements we found these coefficients to be $K_2 = 1.27$; $K_6 = -0.43$. As we can see, experimental data are in very good agreement with the calculated ones.

From the point of view of compensating lens aberrations the case of $K_6 = 0$ is specially important. This case occurs, as it follows from (3), for $2\varepsilon = 60°$. But we can see from Fig. 5 that the value of 2ε at which the coefficient K_6 is actually cancelled depends on R_2/R and for $R_2/R \leq 3.5$ we have: $2\varepsilon = 45° - 50°$. In this case the value of the coefficient K_{10} varies in the interval $-0.2 < K_{10} < -0.1$.

All these experimental results were obtained in case of highly thin electrodes. It is obvious that the greater the value of 2ε the more effective must be the influence of electrode thickness on the potential distribution. We have measured the potential distribution of a lens with electrode thickness equal to 0.1 R. As a result of finite thickness the values of K_2 increased a little while the curve of K_6/K_2 was displaced in the direction of higher values of ε. Consequently, we found $K_6 = 0$ for $2\varepsilon = 52°-53°$.

5. An electrostatic quadrupole lens composed of two parts of a cylinder and an external cylindrical chamber

If we remove the two oppositely placed internal electrodes held at zero potential and apply this potential to the external cylinder, we get the second lens type. So, the two identical parts of an internal cylinder with radius R_1 are held at a potential $V_1 = 2V$ while the potential of the coaxial external cylindrical chamber with radius R_2 is $V_2 = 0$. (From practical point of view it is convenient to place the chamber at ground potential.) The dimensions of the two symmetrically placed parallel slits are measured by the angle 2α (Fig. 2). This system had been partly studied in [3] and [6].

In a two-dimensional approximation the potential distribution of this lens in the xy-plane may be expressed as follows:

$$\frac{\varphi(x,y)}{V} = K_0 + \frac{K_2}{R^2}(x^2 - y^2) + \frac{K_4}{R^4}(x^4 - 6x^2y^2 + y^4) +$$

$$+ \frac{K_6}{R^6}\left[x^6 - y^6 - 15x^2y^2(x^2 - y^2)\right] + \dots \tag{4}$$

where $R = \dfrac{R_1 + R_2}{2}$; K_n are constants characterizing a lens and depending on geometrical factors R_2/R_1 and 2α. In general case of arbitrary values of R_2/R_1 and 2α the electrostatic field of this lens possesses only two symmetry planes and no plane of antisymmetry. As it is seen from expression (1), in case of quadrupole lenses possessing also two planes of antisymmetry the series expansion of the potential distribution does not contain any terms with exponents divisible by four.

Our main problem was to find for every value of R_2/R_1 such an optimum value of the angle $\alpha = \alpha_0$ at which the coefficient K_4 in (4) vanished. In this case the potential distribution of the lens considered must be close to the potential distribution of a usual quadrupole lens in a wide interval.

The method of investigation has been described in parts 3 and 4 of this paper. The values of R_2/R_1 and α varied in very wide ranges. The possible minimum difference between two values of α on the network was: $\Delta\alpha_{min} = 57.3°/R_1$. The potential was measured along the axes of symmetry and along the diagonal lines at 45° angles to both axes ($x = y$). The coefficients were calculated from the results of measurements.

We found K_4 as a function of α for every value of R_2/R_1. In the vicinity of the optimum angle α_0 this function was practically linear and passed through

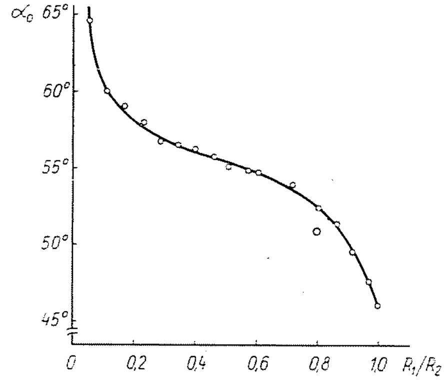

Fig. 6

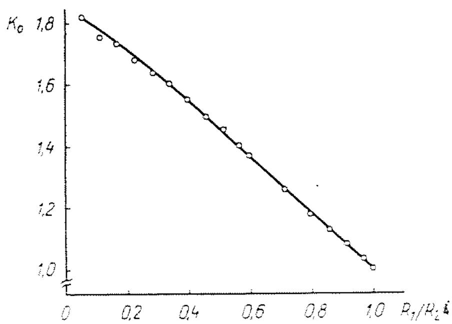

Fig. 7

zero. The resulting plot of α_0 versus R_1/R_2 is presented in Fig. 6. In the limiting case of $R_1 = R_2 = R$ (it is the well-known symmetrical lens, described in part 4) $\alpha_0 = 45° + \varepsilon = 46°\ 10'$. The value of α_0 increases as the ratio R_1/R_2 becomes smaller. The circle indicates the value of α_0 given by [6].

The potential at the centre of the lens is determined by the coefficient K_0: $\varphi(0,0) = K_0 V$. In the limiting symmetrical case $K_0 = 1$. In Fig. 7 K_0 is presented as a function of R_1/R_2 for the case $K_4 = 0$ ($\alpha = \alpha_0$). As it is seen, the potential at the centre of the lens increases as the internal electrodes are placed closer to the z axis.

It follows from (4) that in case of $K_4 = 0$ ($\alpha = \alpha_0$) the potential along the diagonal line $x = y$ in the paraxial region must be equal to the potential at the centre. We found that this condition was really satisfied for the region $0 \leq r < 0.7\, R_1$ where r is the distance of an arbitrary point on the diagonal line from the centre. We may say that in this region the lens has two planes of antisymmetry, in addition to its two symmetry planes.

The advantage of this construction is its simpler adjustment in comparison with quadrupole lenses composed of four electrodes. For this lens it is also possible to compensate the spherical aberration of the width of a linear image at several points [7]. This may be achieved for a given ratio of R_2/R_1 by choosing a suitable value of α different from α_0.

Summary

The electrostatic field distributions of two quadrupole lens types composed of cylindrical surfaces are examined by means of a two-dimensional resistance network analogue. The coefficients of the series expansions of the potential are determined as functions of the lens geometry. Special cases when some coefficients are equal to zero are thoroughly investigated.

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