LIMIT-CYCLE ANALYSIS OF RELAY CONTROL SYSTEMS

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1. Introduction

The purpose of the present article is to compare the limit-cycle calculation methods of relay control systems. Without pretending to completeness some analytical methods are treated here. These methods are the classical differential equation method, the Laplace-transformation method, the state-space method, and the variations of the latter such as the canonical forms and the phase-space method. For the sake of briefness only a very simple example is examined enabling to illustrate the advantages and disadvantages of the various methods.

2. The statement of the problem

The relay control system [e.g. 1—7] to be investigated is depicted in Fig. 1 or in Fig. 2. Here the plant $G(s)$ is linear. The poles and zeros of the plant are assumed to be simple, that is of multiplicity one. Furthermore, ideal-

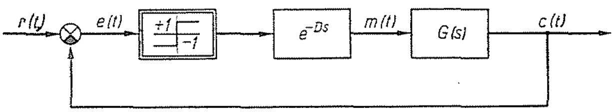

Fig. 1

ized relay with symmetrical characteristic is assumed, but the imperfections of the relay are taken into consideration by a pure dead time D, where D is the time which elapses between relay excitation $e(t)$ and the actual polarity reversal in the actuating variable $m(t)$ (Fig. 3). For the sake of simplicity the input reference variable $r(t)$ is assumed to be zero. Thus, both configurations lead to the same results, and $e(t) = -c(t)$, where $c(t)$ is the controlled variable.

By the way it is mentioned, that the relay with hysteresis shows somewhat similar effects like the relay with dead time, the only difference being that in the first case the time delay D changes with the amplitude of oscillations, while in the second case D is constant. For the sake of simplicity we shall not specially include the dead zone effect, although this also could be done.

As a consequence of the latter restrictions, the actuating variable $m(t)$ can only assume two values, which are normalized e.g. to ± 1.

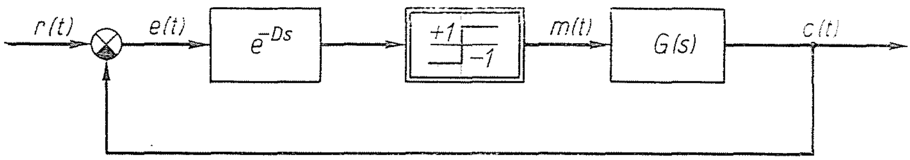

Fig. 2

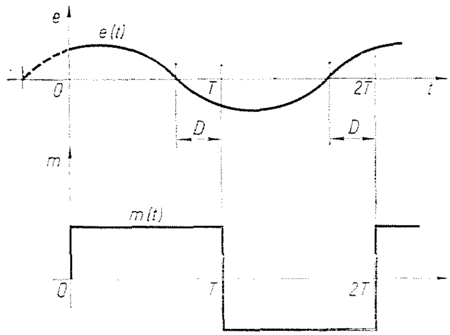

Fig. 3

3. Conditions for limit cycles

Let us assume that a limit cycle of period $2T$ and of half period T exists. Hence the following relations must be valid:

$$\begin{aligned} e(t) &= -e(t+T) \\ m(t) &= -m(t+T) \\ c(t) &= -c(t+T) \end{aligned} \tag{1}$$

Beside these relations also we have

$$e(t)\Big|_{t=kT-D} = 0 \text{ for } k = \text{integer} \tag{2}$$

$$\dot{e}(t)\Big|_{t=kT-D} = \begin{cases} < 0 \text{ for } k = \text{odd} \\ > 0 \text{ for } k = \text{even} \end{cases} \tag{3}$$

where the dot, as common, means the derivative according to the time.

4. The differential-equation method

The first method to be shown is the differential-equation method [e.g. 8]. Let us assume the differential equation of the plant in the following form:

$$\sum_{i=0}^{n} a_i \left(\frac{d}{dt}\right)^i c(t) = \sum_{j=0}^{m} b_j \left(\frac{d}{dt}\right)^j m(t) = f(t) \tag{4}$$

where $(d/dt)^0 c(t) = c(t)$, $(d/dt)^0\, m(t) = m(t)$.

First we determine the solution of the homogeneous equation with $f(t) = 0$:

$$c_h(t) = \sum_{i=1}^{n} K_i e^{p_i t} \tag{5}$$

where $p_i (i = 1, 2, \ldots, n)$ are the roots of the characteristic equation

$$\sum_{i=0}^{n} a_i s^i = 0 \tag{6}$$

and K_i are undetermined constants for the time being.

Then, we determine a particular solution of the inhomogeneous ($f(t) \neq 0$) differential equation, assuming

$$c_p(t) = \sum_{i=1}^{n} K_i(t) e^{p_i t} \tag{7}$$

The time-variable coefficients $K_i(t)$ are simple integrals

$$K_i(t) = \int_0^t \dot{K}_i(t)\, dt \tag{8}$$

of the derivatives $\dot{K}_i(t)$ calculated from the equation system:

$$\begin{aligned} \dot{K}_1(t) e^{p_1 t} + \dot{K}_2(t) e^{p_2 t} + \ldots + \dot{K}_n(t) e^{p_n t} &= 0 \\ \dot{K}_1(t) p_1 e^{p_1 t} + \dot{K}_2(t) p_2 e^{p_2 t} + \ldots + \dot{K}_n(t) p_n e^{p_n t} &= 0 \\ \cdots\cdots\cdots\cdots & \\ \dot{K}_1(t) p_1^{n-1} e^{p_1 t} + \dot{K}_2(t) p_2^{n-1} e^{p_2 t} + \ldots + \dot{K}_n p_n^{n-1} e^{p_n t} &= f(t) \end{aligned} \tag{9}$$

Now, the complete solution is

$$c(t) = c_h(t) + c_p(t) \tag{10}$$

Finally, the constant coefficients K_i can be determined from the initial conditions as the solutions of the following equation system:

$$\begin{aligned} \sum_{i=1}^{n} K_i &= c(0) - c_p(0) \\ \sum_{i=1}^{n} p_i K_i &= \dot{c}(0) - \dot{c}_p(0) \\ \cdots\cdots\cdots\cdots & \\ \sum_{i=1}^{n} p_i^{n-1} K_i &= c^{(n-1)}(0) - c_p^{(n-1)}(0) \end{aligned} \tag{11}$$

As we search for periodic quasi-stationary solution the initial conditions $c(0)$, $\dot{c}(0)$ ... $c^{(n-1)}(0)$ must be adjusted in a manner to fullfil the conditions 1) of the limit cycle:

$$\begin{aligned} c(0) &= -c(T) \\ \dot{c}(0) &= -\dot{c}(T) \\ &\dots\dots\dots \\ c^{(n-1)}(0) &= -c^{(n-1)}(T) \end{aligned} \tag{12}$$

4.1 *Illustrative example*

Let be the differential equation of the plant:

$$\ddot{c}(t) + \dot{c}(t) = m$$

where $m = +1$ for the first semi-period.

From Eq. (6) the roots (or the poles of the transfer function $G(s)$) are $p_1 = 0$, $p_2 = -1$, thus according to Eq. (5):

$$c_h(t) = K_1 + K_2\, e^{-t}$$

Then from Eq. (9):

$$\dot{K}_1(t) = 1,\ \dot{K}_2(t) = -e^t$$

and on the basis of Eq. (8):

$$K_1\,(t) = t, \quad K_2\,(t) = 1 - e^t$$

Following Eqs (7), (10):

$$c(t) = t + e^{-t} - 1 + K_1 + K_2\, e^{-t}$$

or taking into consideration Eq. (11):

$$c(t) = t + e^{-t} - 1 + c(0) + \dot{c}(0)(1 - e^{-t})$$

Finally, on the basis of Eqs (12), we have

$$c(0) = -\frac{T}{2} + \tanh\frac{T}{2}$$

$$\dot{c}(0) = -\tanh\frac{T}{2}$$

5. The Laplace-transform method

Laplace-transforming the differential equation (1) of the plant, and performing some algebraic manipulations, we get $C(s)$ that is the transform of the output:

$$C(s) = \mathscr{L}[c(t)] \tag{13}$$

and the final solution $c(t)$ can be obtained very quickly after using the inverse Laplace transformation technique:

$$c(t) = \mathscr{L}^{-1}[C(s)] \tag{14}$$

It is worth to mention that the initial conditions are taken into consideration automatically. All these circumstances will be visualized by the same example.

5.1 Illustrative example

The Laplace transform of Eq. (4) is in our case

$$C(s) = \frac{M(s) + s\,c(0) + c(0) + \dot{c}(0)}{s^2 + s}$$

according to Eq. (13).

It is mentioned that

$$\mathscr{L}[\ddot{c}(t)] = s^2C(s) - s\,c(0) - \dot{c}(0)$$

and

$$\mathscr{L}[\dot{c}(t)] = s\,C(s) - c(0)$$

By the way, the transfer function of the plant is

$$G(s) = \frac{1}{s(s+1)}$$

Taking $M(s) = \mathscr{L}[m(t)] = 1/s$, because of $m(t) = 1(t)$ for the first half-period. (Without any consequence we can assume $T \to \infty$ in this step.) Then utilizing Eq. (14):

$$c(t) = t - 1 + e^{-t} + c(0)e^{-t} + [c(0) + \dot{c}(0)](1 - e^{-t})$$

which leads to the same result as in Example 4.1.

6. The state-space method

Recently the state-space methods came into fashion [4—7 etc.]
The system equation of the plant is given in the form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}m \tag{15}$$

where $\mathbf{x}$ and $\mathbf{b}$ are $n \times 1$ matrices (column vectors) and $\mathbf{A}$ is an $n \times n$ matrix. Th output variable is given by

$$c(t) = \mathbf{c}^T \mathbf{x}(t) \tag{16}$$

where $\mathbf{c}^T$ is an $1 \times n$ matrix (a row vector). The solution of Eq. (15) is

$$\mathbf{x}(t) = \mathbf{\Phi}(t)\mathbf{x}(0) + \boldsymbol{\varphi}(t) \tag{17}$$

where $\mathbf{\Phi}(t)$ is the *transition* matrix, while $\boldsymbol{\varphi}(t)$ is the *distribution* matrix. The former can be obtained by the SYLVESTER expansion theorem

$$\mathbf{\Phi}(t) = \sum_{i=1}^{n} \left(e^{p_i t} \prod_{\substack{j=1 \\ j \neq i}} \frac{\mathbf{A} - p_j \mathbf{I}}{p_i - p_j} \right) \tag{18}$$

for the case of stationary (time-invariant) system matrix $\mathbf{A}$.

The latter can be obtained from the following integral expression

$$\boldsymbol{\varphi}(t) = \int_0^t \mathbf{\Phi}(t - \tau)\, \mathbf{b}\, m(\tau)\, d\tau \tag{19}$$

On the other hand Laplace-transforming Eq. (15) the result is

$$s\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\mathbf{X}(s) + \mathbf{b}M(s)$$

After some algebraic manipulations we obtain

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\, \mathbf{x}(0) + (s\mathbf{I} - \mathbf{A})^{-1}\, \mathbf{b}M(s)$$

Hence the following relations are valid:

$$\mathbf{\Phi}(t) = \mathscr{L}^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}] \tag{20}$$

and

$$\boldsymbol{\varphi}(t) = \mathscr{L}^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}\, \mathbf{b}M(s)] \tag{21}$$

The limit-cycle condition can be expressed as

$$\mathbf{x}(0) = -\mathbf{x}(T) = -\mathbf{\Phi}(T)\mathbf{x}(0) - \boldsymbol{\varphi}(T) \tag{22}$$

or

$$\mathbf{x}(0) = -(\mathbf{I} + \mathbf{\Phi}(T))^{-1}\boldsymbol{\varphi}(T) \tag{23}$$

In the following chapters we examine some important special cases.

7. The phase-space method

The first special case of the general state-space method is the phase-space method. In the latter case the $\mathbf{A}$ and $\mathbf{b}$ matrices are of a special form:

$$\mathbf{A}_0 = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ 0 & 0 & 0 & \dots & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 & \dots & -\alpha_{n-1} \end{bmatrix} \quad \mathbf{b}_0 = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ 1/a_n \end{bmatrix} \tag{24}$$

(where $\alpha_i = a_i/a_n$ $(i = 0, 1, \dots, n-1)$.

The simplest way to obtain the phase-variable form leads through the substitution $c = x_1, \dot{c} = x_2, \dots c^{(n-1)} = x_n$ into the differential equation (1).

7.1 Illustrative example

Performing the substitution $c = x_1$, $\dot{c} = x_2$, which can be illustrated by the block diagram of the plant depicted in Fig. 4, we obtain

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -x_2(t) + m(t)$$

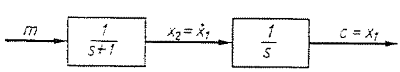

Fig. 4

By the way, these differential equations of the first order can be solved directly. On the other hand, according to (24) and (16):

$$\mathbf{A}_0 = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, \quad \mathbf{b}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{c}^T = [1, 0]$$

For the case of $m = 1$ in the first half-period the transition matrix is

$$\mathbf{\Phi}(t) = \begin{bmatrix} 1 & 1 - e^{-t} \\ 0 & e^{-t} \end{bmatrix}$$

as obtained from Eq. (18) or Eq. (20). The distribution matrix (column vector) is:

$$\boldsymbol{\varphi}(t) = \begin{bmatrix} t - 1 + e^{-t} \\ 1 - e^{-t} \end{bmatrix}$$

according to Eq. (19) or (21). Using Eq. (23) the initial condition matrix (column vector) is

$$\mathbf{x}(0) = \begin{bmatrix} -\dfrac{T}{2} + \tanh\dfrac{T}{2} \\ \\ -\tanh\dfrac{T}{2} \end{bmatrix}$$

while applying Eq. (17) and Eq. (16):

$$c(t) = t - 1 + e^{-t} - \frac{T}{2} + \tanh\frac{T}{2} - (1 - e^{-t})\tanh\frac{T}{2} =$$

$$= t - 1 - \frac{T}{2} + e^{-t}\left(1 + \tanh\frac{T}{2}\right)$$

which is the same result as obtained in the previous examples.

8. Methods based on canonical forms

Often are used matrix transformations, which result canonical forms. Let us introduce a nonsingular matrix transformation by the relations

$$\mathbf{x}(t) = \mathbf{L}\mathbf{y}(t); \; \mathbf{y}(t) = \mathbf{L}^{-1}\,\mathbf{x}(t) \tag{25}$$

Thus, Eqs. (15) and (16) can be transformed as

$$\dot{\mathbf{y}}(t) = \mathbf{L}^{-1}\,\mathbf{A}\mathbf{L}\mathbf{y}(t) + \mathbf{L}^{-1}\,\mathbf{b}\,m(t) \tag{26}$$

$$c(t) = \mathbf{c}^T\,\mathbf{L}\,\mathbf{y}(t) \tag{27}$$

The transformation is called canonical if

$$\mathbf{L}^{-1}\,\mathbf{A}\,\mathbf{L} = \mathbf{S}$$

where $\mathbf{S} = \text{diag}\,[p_1, p_2, \ldots, p_n]$ and p_i $(i = 1, 2, \ldots, n)$ are the roots of the characteristic equation or the poles of the transfer function $G(s)$ of the plant.

The transformation (25), (28) is not unique. The first possibility is to take $\mathbf{L} = \mathbf{V}$ in case of $\mathbf{A}=\mathbf{A}_0$ where $\mathbf{V}$ is the VANDERMONDE-matrix:

$$\mathbf{V} = \begin{bmatrix} 1 & 1 & \ldots 1 \\ p_1 & p_2 & \cdots p_n \\ & \cdot\;\cdot\;\cdot & \\ p_1^{n-1} & p_2^{n-1} & \ldots p_n^{n-1} \end{bmatrix}$$

giving

$$\dot{\mathbf{y}}(t) = \mathbf{V}^{-1}\;\mathbf{A}_0\mathbf{V}\,\mathbf{y}(t) + \mathbf{V}^{-1}\,\mathbf{b}m(t) \tag{28}$$

$$c(t) = \mathbf{c}^T\,\mathbf{V}\mathbf{y}(t) \tag{29}$$

The second possibility is to choose $\mathbf{L}$ in a manner that*

$$\mathbf{L}^{-1}\,\mathbf{b} = (1, 1, \ldots, 1)^T = \mathbf{e}^T \tag{30}$$

This transformation leads to the canonical form of LUR'E [9, 10] very often utilized in control engineering:

$$\dot{\mathbf{z}}(t) = \mathbf{S}\mathbf{z}(t) + \mathbf{e}m \tag{31}$$

$$c(t) = \mathbf{c}^T\,\mathbf{L}\mathbf{z}\,(t) \tag{32}$$

where for the sake of distinction instead of $\mathbf{y}$, the new variable is $\mathbf{z}$.

It is worth to mention that the canonical forms can also be obtained directly from the transfer function $G(s)$. Let us write $G(s)$ in a form

$$\frac{C(s)}{M(s)} = G(s) = K\,\frac{(s-z_1)\,(s-z_2)\ldots(s-z_m)}{(s-p_1)\,(s-p_2)\ldots(s-p_n)} \tag{33}$$

* Vector $\mathbf{e}$ has nothing to do with error $e(t)$.

where p_i ($i = 1, 2, \ldots, n$) are the poles and z_j ($j = 1, 2, \ldots, m$) are the zeros (the latter having nothing to do with the components of vector $\mathbf{z}$).

Taking the partial fraction expansion of $G(s)$ assuming separate poles and expressing the Laplace transform of the controlled variable:

$$C(s) = \sum_{i=1}^{n} \frac{r_i}{s - p_1} M(s) \tag{34}$$

where $r_i (i = 1, 2, \ldots, n)$ are the residues of $G(s)$

$$r_i = \lim_{s \to p_i} (s - p_i) G(s) \tag{35}$$

ntroducing the transform of new variables by

$$Y_i(s) = \frac{r_i}{s - p_1} M(s) \tag{36}$$

or

$$sY_i(s) = p_i Y(s) + r_i M(s)$$

The inverse Laplace-transformation of the latter gives

$$\dot{y}_i(t) = p_i y_i(t) + r_i m(t); \quad (i = 1, 2, \ldots, n) \tag{37}$$

while taking also (34) and (36) into consideration

$$c(t) = \sum_{i=1}^{n} y_i(t) \tag{38}$$

which are the expanded versions of Eqs. (26) and (27). On the other hand defining the transforms of new variables as $Z_i(s) = Y_i(s)/r_i$ or

$$Z_i(s) = \frac{1}{s - p_i} M(s) \tag{39}$$

the final result is

$$\dot{z}_i(t) = p_i z_i(t) + m(t); \qquad (i = 1, 2, \ldots, n) \tag{40}$$

and

$$c(t) = \sum_{i=1}^{n} r_i z_i(t) \tag{41}$$

These equations give the LUR'E canonical form of the state variables that is Eqs (31), (32) in expanded form.

8.1 Illustrative example

Starting from the phase-variable form

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} m(t)$$

$$c(t) = [1,\ 0]\ \mathbf{x}(t)$$

and taking the transformation matrices according to (29):

$$\mathbf{V} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \text{and} \quad \mathbf{V}^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

Eqs. (26) and (27) lead with $\mathbf{L} = \mathbf{V}$ to:

$$\dot{\mathbf{y}}(t) = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \mathbf{y}(t) + \begin{bmatrix} -1 \\ 1 \end{bmatrix} m(t)$$

$$c(t) = [1,\ 1]\ \mathbf{y}(t)$$

The final solution is, of course, the same as previously although the transition matrix and distribution matrix are different from those of Example 7.1 being

$$\mathbf{\Phi}(t) = \begin{bmatrix} 1 & 0 \\ 0 & e^{-t} \end{bmatrix}; \qquad \boldsymbol{\varphi}(t) = \begin{bmatrix} t \\ e^{-t} - 1 \end{bmatrix}$$

while the initial conditions are related to each other by

$$y_1(0) + y_2(0) = x_1(0) = c(0);\ x_2(0) = -\dot{x}_1(0) = -\dot{c}(0)$$

or

$$y_1(0) = x_1(0) + \dot{x}_1(0) = c(0) + \dot{c}(0);\ y_2(0) = -\dot{x}_1(0) = -\dot{c}(0)$$

Applying, on the other hand, the transformation

$$\mathbf{y}(t) = \mathbf{R}z(t) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \mathbf{z}(t)$$

and substituting in (25) the transformation matrices becomes

$$\mathbf{L} = \mathbf{VR} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{L}^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Thus, applying Eqs (26) and (27):

$$\dot{\mathbf{z}}(t) = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \mathbf{z}(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} m(t)$$

$$c(t) = [1, -1] \; \mathbf{z}(t)$$

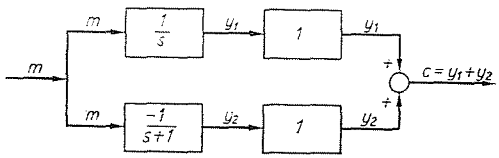

Fig. 5

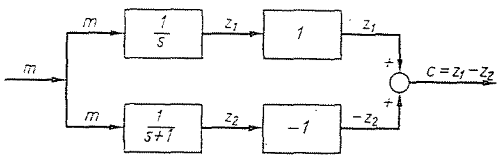

Fig. 6

The transition matrix is the same as in the first part of the example but the distribution matrix is different

$$\Phi(t) = \begin{bmatrix} 1 & 0 \\ 0 & e^{-t} \end{bmatrix}; \qquad \varphi(t) = \begin{bmatrix} t \\ 1 - e^{-t} \end{bmatrix}$$

while the initial conditions are related as

$$z_1(0) - z_2(0) = x_1(0) = c(0); \; z_2(0) = \dot{x}_1(0) = \dot{c}(0)$$

or

$$z_1(0) = x_1(0) + \dot{x}_1(0) = c(0) + \dot{c}(0); \; z_2(0) = \dot{x}_1(0) = \dot{c}(0)$$

The final result is, of course, again the same.

The previous relations can be obtained by the method outlined in connection with Eqs (37), (38) and Eqs (40), (41), respectively. For illustrative purpose, in Fig. 5 and Fig. 6 the canonical state variables of the plant $G(s)$ are shown in block diagrams.

9. The final solution of the problem

Now we return to the solution of the problem. First we remark that $e(t) = -c(t)$ if $r(t) = 0$. Thus, the periodic solution of the error is

$$e(t) = 1 + \frac{T}{2} - t - e^{-t}\left(1 + \tanh\frac{T}{2}\right)$$

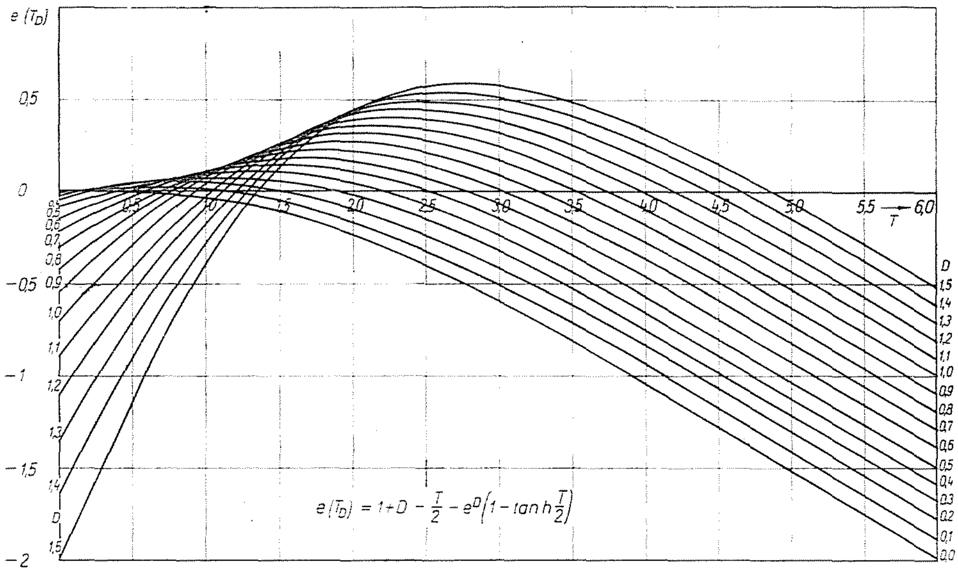

Fig. 7

At the instance $t = T - D = T_D$ the error is

$$e(T_D) = 1 - \frac{T}{2} + D - e^{D}\left(1 - \tanh\frac{T}{2}\right)$$

which according to the starting assumptions, must be

$$e(T_D) = 0$$

In Fig. 7 the diagrams of $e(T_D)$ are visualized in function of T and taking the dead time D as parameter.

Naturally we are interested only in the solutions $e(T_D) = 0$. The roots of this equation are summarized in Table 1.

Table 1

T \ D	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
T_1	0	0.0992	0.197	0.293	0.388	0.481	0.574	0.666
T_2	0	1.076	1.531	1.893	2.210	2.499	2.770	3.028

T \ D	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
T_1	0.756	0.846	0.935	1.023	1.111	1.198	1.285	1.371
T_2	3.276	3.516	3.750	3.979	4.204	4.427	4.646	4.862

According to the simplified stability criteria [10] the limit cycle is stable (or convergent) if

$$\left.\frac{de(T_D)}{dT}\right|_{T=T_i} < 0$$

and is unstable (or divergent) if

$$\left.\frac{de(T_D)}{dT}\right|_{T=T_i} > 0$$

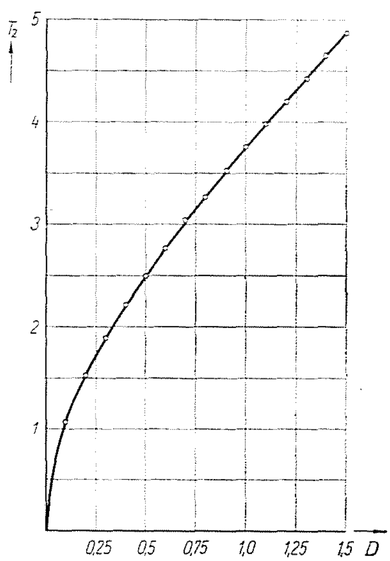

Fig. 8

where T_i means a root. Thus, in Table 1 only the values T_2 give stable limit cycles. The latter half-periods T_2 are shown in function of the dead time D in Fig. 8. From this figure we can conclude that as a rule of thumb

$$T_2 \approx \frac{1}{4}(6 + 9D); \quad (1.5 > D > 0.5)$$

can be taken for the particular relay control system investigated. On the other hand, for great D values, the asymptotic value of T_2 is:

$$T_2 \approx 2 + 2D = 2(1 + D); \ (2 < D)$$

10. Concluding remarks

In the previous treatise and examples without pretending to completeness, various methods are shown for analysing the limit cycle conditions in relay control systems. For the sake of briefness only a very simple example was taken. The main characteristics of the various methods nevertheless are thrown into relief.

We can draw the conclusion that there is no significant difference between the differential equation, the Laplace-transform, the state-variable, the phase-variable and the canonical-variable methods. According to the opinion of the author the Laplace transform method is the most advantageous.

It is also shown in Chapter 8 that Laplace-transforms can also be utilized in deriving canonical forms. In Chapter 6 it was remarked that Laplace-transforms can be applied in determining the transition and distribution matrices.

The direct time-domain methods are somewhat more complex and cumbersome. Perhaps there is a distinct advantage for state-variable methods in comparison to the differential-equation method.

Summary

In connection with limit-cycle analysis of relay control systems the methods of differential equations, Laplace-transforms, state variables, phase variables, canonical variables are compared. The same illustrative example is solved by various methods to show the advantages and disadvantages of each.

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