DETERMINATION OF ELEMENT SENSITIVITY WITHOUT DERIVATION BY STATE-VARIABLE ANALYSIS

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BYCHOVSKY [1, 2] determined the element sensitivity of linear systems as the product of two transfer functions.

In the following, a generalisation of Bychovsky's method will be given for active networks, parasitic elements and any kind of transfer quantities. With slight modifications, the results can be compared to the formulas obtained by LEEDS and UGRON [3, 4] derived for node analysis.

On account of its general character, the method to be described offers several alternatives of simplification. They can be utilized with high efficiency in state-variable analysis.

1. Generalisation of Bychovsky's method

Be Q (without discrimination) the Laplace transform of the branch current or voltage of a network. Similarly, independent current or voltage sources are marked W without discrimination. Let us consider now a branch marked i incorporating the particular element whose parameter is x. The derivation of the transfer function is to be performed with respect to x. The parameter x may be an impedance, admittance, an R, L or C element or the controlling constant of any type of source.

Theorem: if the element in the branch i fulfils the relationship

$$Q_i = xc_i(p)\, Q_l \tag{1}$$

where Q_l is the current or voltage in any branch of the network and c_i is a constant independent of current and voltage (it may be dependent on frequency !), then the sensitivity with respect to x or the transfer function Q_k/W_j can be obtained from the product of two other functions, using the relationship (Fig. 1)

$$S_x = \frac{\partial}{\partial x}\,\frac{Q_k}{W_j} = \frac{1}{x}\,\frac{Q_i}{W_j}\,\frac{Q_k}{W_i} \tag{2}$$

Transfer function

$$T_{ij} = \frac{Q_i}{W_j}\bigg|_{W_i=0} \tag{3}$$

can be measured at i-port with input excitation assumed.

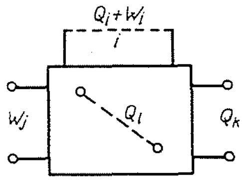

Fig. 1

Transfer function

$$T_{ki} = \frac{Q_k}{W_i}\bigg|_{W_j=0} \tag{4}$$

can be measured — with the input excitation cancelled ($W_j = 0$) — by inserting in i-port a generator of W_i in such a way that its value is added to electrical quantity Q_i. If Q_i is a voltage, W_i will be a voltage source connected in series with element i. If Q_i is a current, W_i will be a current source connected in parallel with element i.

Relationship (2) can be confirmed in the following manner.

Let parameter x in relationship (1) be varied by a value of Δx. As a result, each current and voltage of the network will vary by a ΔQ quantity (Fig. 1). Variations of quantities Q in the network will be related with those in quantities

$$\Delta Q_x = \Delta xc_i(Q_l + \Delta Q_l) \cong \Delta xc_i Q_l \tag{5}$$

associated with the variation Δx. The value of ΔQ_x is part of the total variation

$$\Delta Q_i = \Delta xc_i Q_l + xc_i \Delta Q_l \tag{6}$$

occurring in element i, whereas the second component is produced by the former (Fig. 2).

The variation at the output produced by ΔQ_x will be, on account of the superposition principle,

$$\Delta Q_k = \Delta Q_x \frac{Q_k}{W_i} \tag{7}$$

where Q_k/W_i is a transfer function between i as an input port and the output. Owing to the linearity involved, this can be measured by means of a source W_i (of any arbitrary rating) inserting in place of ΔQ_x.

It is evident from (5) and (6) that, since ΔQ_x is part of the total variation ΔQ_i, W_i will be added to the quantity Q_i measured on element i. Consequently, if Q_i is a voltage, then W_i will be a voltage source connected in series, if Q_i

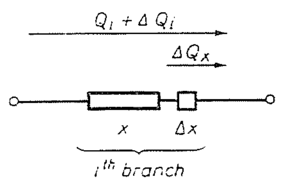

Fig. 2

is a current, then W_i will be a current source in parallel connection. Using (1) and (5), the formula of

$$\Delta Q_k \cong \Delta x \frac{Q_i}{x} \frac{Q_k}{W_i} \tag{8}$$

will be obtained from (7). Dividing (8) by input excitation W_i and transposing, the relationship

$$\frac{\Delta Q_k}{\Delta x W_j} \cong \frac{1}{x} \frac{Q_i}{W_j} \frac{Q_k}{W_i} \tag{9}$$

will be obtained.

Also, owing to the linearity involved, the quantity Q_i/W_j will be a transfer function independent of the degree of excitation. Finally, performing the transition $\Delta x \rightarrow 0$, expressions (5), (8), (9) will be turned into equalities, leading to relationship (2) to be proved by (9).

2. Location of excitations and outputs

Assume that the sensitivities are to be computed with respect to R, L, C and to parameters α, β of the controlled sources.

In the case of a controlled voltage source, relationship (1) will take the form of

$$U_\alpha = \alpha Q \tag{10}$$

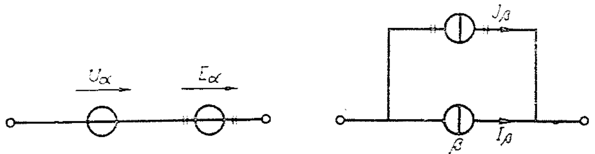

Fig. 3

and, in the case of current source,

$$I_\beta = \beta Q. \tag{11}$$

(The branch voltages and currents are marked U and I, respectively, together with the controlled sources. The independent sources are distinguished by E and J. Furthermore, the external sources initially not included in the network are placed between marks $\#\bigcirc\#$.)

The externally connected sources are shown in Fig. 3. It may be noted that, as is evident from relationships (10) and (11), the type of source control is entirely irrelevant. Components R, L, C may be equally used in conjunction with voltage and current sources.

Studying the conditions from the viewpoint of capacitance, with a series voltage source applied, requirement (1) will take the form of

$$U_C = \frac{1}{pC} I_C \tag{12}$$

and, in the case of a current source,

$$I_C = pCU_C. \tag{13}$$

Deriving by $x = \dfrac{1}{C}$ in Eq. (12), relationship (2) will be turned into

$$\frac{\partial}{\partial 1/C} \frac{Q_k}{W_j} = C \frac{U_C}{W_j} \frac{Q_k}{E_C}. \tag{14}$$

On the other hand, in accordance with the rule of indirect derivation,

$$\frac{\partial}{\partial C} \frac{Q_k}{W_j} = -\frac{1}{C^2} \frac{\partial}{\partial 1/C} \frac{Q_k}{W_j}. \tag{15}$$

Replacing (14) in (15), the expression

$$\frac{\partial}{\partial C} \frac{Q_k}{W_j} = -\frac{1}{C} \frac{R_C}{W_j} \frac{Q_k}{E_C} \tag{16}$$

will be obtained. Accordingly, derivation by the reciprocal quantity will cause only a sign change. That negative sign is taken into account by connecting source E_c with opposite polarity relative to U_c.

Fig. 4 shows the arrangements necessary for calculating derivatives with respect to three elements. Observing the rules of signs given in Figs 3 and 4 the sensitivities with respect to $x = \alpha, \beta, R, L, C$ will be given uniformly by Eq. (2).

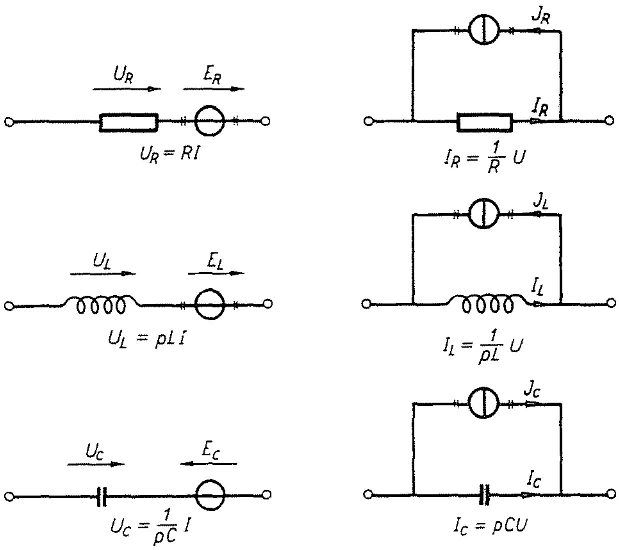

Fig. 4

Example 1

Let us determine sensitivity with respect to C of a two-port transfer function shown in Fig. 5.

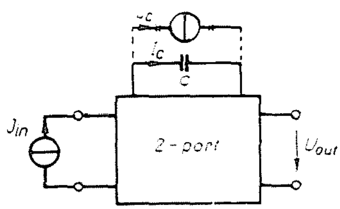

Fig. 5

Condition (1) is fulfilled by the relationship

$$I_C = pCU_C \tag{17}$$

which means that the W_i inserted is a current source J_c connected in parallel with the capacitance. From Eq. (2), the sensitivity is

$$S_C = \frac{\partial}{\partial C} \frac{U_{\text{out}}}{I_{\text{in}}} = \frac{1}{C} \frac{I_C}{J_{\text{in}}} \frac{U_{\text{out}}}{J_C}. \tag{18}$$

Example 2

Let us determine sensitivity with respect to R of the one-port impedance shown in Fig. 6.

Condition (1) is fulfilled by

$$U_R = RI_R. \tag{19}$$

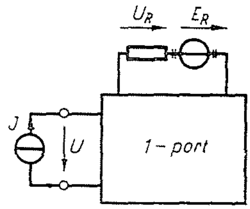

Fig. 6

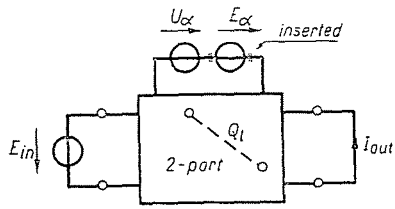

Fig. 7

Accordingly, the source to be connected is a voltage source. According to (2), the sensitivity is

$$S_R = \frac{\partial}{\partial R} \frac{U}{J} = \frac{1}{R} \frac{U_R}{J} \frac{U}{E_R}. \tag{20}$$

Example 3

Let us determine the sensitivity of the two-port transfer function shown in Fig. 7 with respect to parameter α of the controlled voltage source.

Relationship

$$U_\alpha = \alpha U_l \tag{21}$$

fulfils the condition of (1), so that

$$S_\alpha = \frac{\partial}{\partial \alpha} \frac{I_{out}}{E_{in}} = \frac{1}{\alpha} \frac{U_\alpha}{E_{in}} \frac{I_{out}}{E_\alpha}. \tag{22}$$

It should be noted here that the determination of each transfer function presupposes the "cut-out" of all other sources not involved in the measurement (short-circuited or open-circuited in the case of a voltage or current source, respectively).

3. Determination of sensitivities from the transfer matrix of the network

An analysis of relationship (2) will show that the determination of a component sensitivity requires the product of two transfer functions, i.e. one from the input to the component being tested, and the other between the component and the output. It appears to be convenient to construct a network in which the outputs represent just the electrical quantities Q developing at the elements (provided with tolerances) — be voltages or currents — and each element incorporates an excitation corresponding to Q_i (current or voltage source).

This may be written in matrix form as

$$\mathbf{q} = \mathbf{T}\mathbf{w} \tag{23}$$

where $\mathbf{q}$ and $\mathbf{w}$ are column vectors obtained from Q_i and W_i, respectively; $\mathbf{T}$ is a matrix with dimensions corresponding to the number of elements provided with tolerances. Its components are rational fractional functions of complex frequency p.

Written in details,

$$\begin{bmatrix} Q_1 \\ \vdots \\ Q_i \\ \vdots \\ Q_j \\ \vdots \\ Q_k \\ \vdots \\ Q_N \end{bmatrix} = \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ k \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccc} & \phantom{T_{ki}} & & j & \end{array} \\ \begin{bmatrix} & & & \vdots & \\ & & & \vdots & \\ & & & T_{ij} & \\ & & & \vdots & \\ & & & \vdots & \\ & & & \vdots & \\ \cdots & T_{ki} & \cdots & \boxed{T_{kj}} & \\ & & & \vdots & \\ & & & \vdots & \end{bmatrix} \end{array} \begin{bmatrix} W_1 \\ \vdots \\ W_i \\ \vdots \\ W_j \\ \vdots \\ W_k \\ \vdots \\ W_N \end{bmatrix} \tag{24}$$

Assume that the principal transmission is a transfer function

$$T_{kj} = \left. \frac{Q_k}{W_j} \right|_{W_l=0,\ l \neq j} \tag{25}$$

in the jth column of the kth row. Now with reference to (2), the sensitivity related to parameter x_i, fulfilling condition (1), will be obtained from the relationship

$$S_{x_i}^{kj} = \frac{\partial}{\partial x_i} \frac{Q_k}{W_j} = \frac{1}{x_i} \frac{Q_i}{W_j} \frac{Q_k}{W_i} \tag{26}$$

which is the product of two transfer functions

$$S_{x_i}^{kj} = \frac{1}{x_i} T_{ij} T_{ki} \tag{27}$$

in the kth line and in the jth column.

Obviously, in a generalized form, transfer matrix **T** will include — assuming the above excitations and outputs — all sensitivities of any transfer function in the network related to any element.

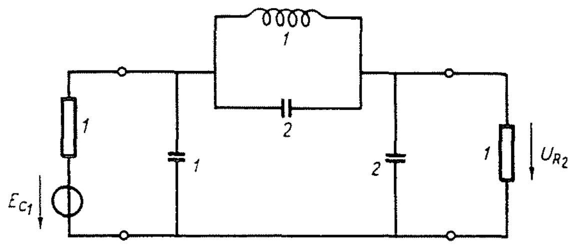

Fig. 8

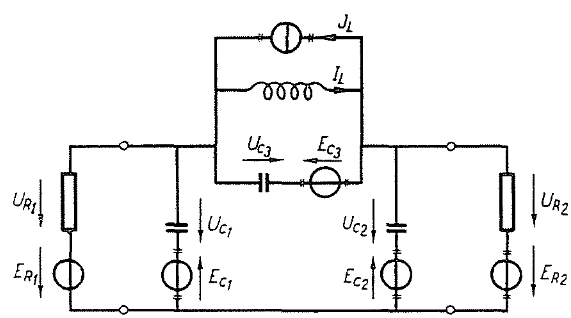

Fig. 9

In general, the method illustrated in (24) is particularly useful when the method of analysis is likely to produce, beside the principal transmission, the side transmission as a "byproduct".

Example 4

Let us determine the sensitivities of the circuit arrangement shown in Fig. 8 with different types of sources.

a) A possible generator circuit arrangement is shown in Fig. 9.

The pertaining transfer matrix is

$$
\mathbf{T}=\frac{1}{8p^3+7p^2+4p+2}\cdot
\begin{array}{c|cccccc|}
 & E_{C_1} & E_{C_2} & J_L & E_{R_1} & E_{R_2} & E_{C_3} \\
\hline
U_{C_1} & & & & 4p^2+p+1 & & \\
U_{C_2} & & & & 2p^2+1 & & \\
I_L & & & & 2p+1 & & \\
U_{R_1} & & & & -8p^3-3p^2-3p-1 & & \\
U_{R_2} & -2p^3-p & -6p^3-2p^2-2p & -p^2-p & 2p^2+1 & -8p^3-4p^2-3p-1 & 2p^3+2p^2 \\
U_{C_3} & & & & 2p^2+p & & \\
\hline
\end{array}
\tag{28}
$$

Hence, for example, the sensitivity by C_3 is

$$
S_{C_3}=\frac{1}{C_3}\,\frac{(2p^2+p)(2p^3+2p^2)}{(8p^3+7p^2+4p+2)^2}\,. \tag{29}
$$

b) An alternative circuit arrangement is shown in Fig. 10.

The number of sources can be reduced by taking into account that, from Figs 8 and 9,

$$
\frac{U_{R_2}}{E_{R_1}}=\frac{I_{R_2}}{J_{C_1}} \tag{30}
$$

so that the current transmission of Fig. 10 equals the voltage transmission of Fig. 8. Accordingly, a single source can be used for the "excitation" of several components.

The transfer matrix for Fig. 10 is

$$
\mathbf{T}=\frac{1}{8p^3+7p^2+4p+2}\cdot
\begin{array}{c|ccc|}
 & I_{C_1} & I_{C_2} & I_{C_3} \\
\hline
I_{C_1} & -4p^3-p^2-p & & \\
I_{C_2} & -4p^3-2p & & \\
I_L & -2p-1 & & \\
I_{R_1} & 4p^2+p+1 & & \\
I_{R_2} & 2p^2+1 & 3p^2+p+1 & p^2+p \\
I_{C_3} & 4p^3+2p^2 & & \\
\hline
\end{array}\,. \tag{31}
$$

The rest of sensitivities by C_3 of this circuit arrangement are

$$S_{C_3} = \frac{1}{C_3} \frac{(p^2 + p)(4p^3 + 2p^2)}{(8p^3 + 7p^2 + 4p + 2)^2} \tag{32}$$

identical with (29).

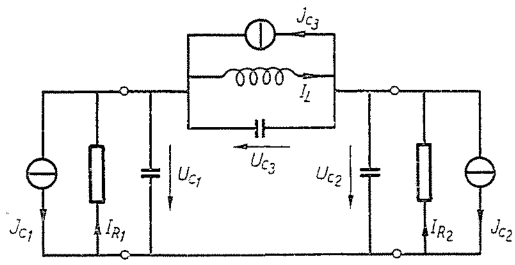

Fig. 10

Note. It follows from Fig. 4 that, with several components parallelly connected, a single current source can be used for the excitation of each component. On the other hand, with voltage sources connected in the parallel branches, the number of outputs can be reduced (the output will be the common voltage). In the case of series components, the opposite of the above statement will apply.

4. Extreme sensitivities

The method here described can be applied for determining the sensitivities by measurement as well. For accuracy reasons, it is impractical to apply relationship (2) whenever the tested component assumes an extremely low or extremely high value. It is evident from Fig. 4 that, under extreme conditions the currents or voltages to be measured may be extremely low or suitable sources may be difficult to be connected (e.g. in the case of stray capacitance).

Relationship (2) may be modified — for applications by Fig. 4 — in such a way that the quantity to be measured on the circuit component is replaced by another quantity (in accordance with Ohm's law). A reduction by circuit component x_i can be obtained through a proper selection of sources connected into the network.

The modified circuit arrangement is shown in Fig. 11.

In Fig. 11, the sensitivities are obtained from relationship

$$S_{x_i} = \frac{\partial}{\partial x_i} \frac{Q_{\text{out}}}{W_{\text{in}}} = z \frac{Q_i}{W_{\text{in}}} \frac{Q_{\text{out}}}{W_i} . \tag{33}$$

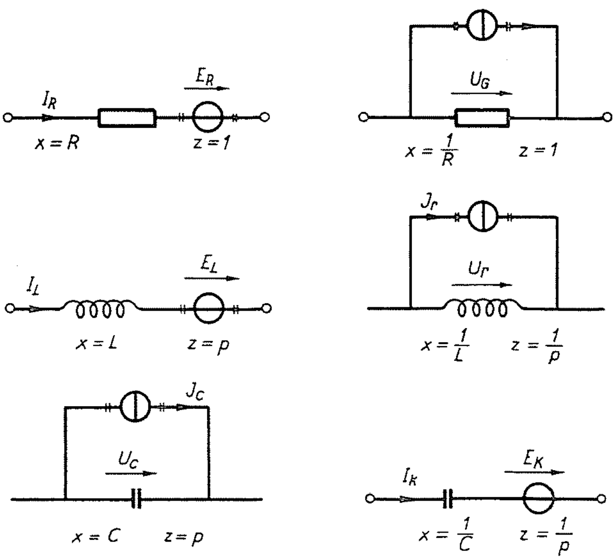

Fig. 11

In case ot low component values, it is advisable to use the left-hand side of Fig. 11, in the case of high values — the right-hand side.

5. Parasitic sensitivity

In the foregoing it has been assumed that the circuit components never get zeroed. At a zero component value, the degree of a network may decrease. Accordingly, parasitic sensitivity

$$S_x^p = \frac{\partial F(p)}{\partial x} = \frac{\partial}{\partial x} \left. \frac{Q_{\text{out}}}{W_{\text{in}}} \right|_{x=0} \tag{34}$$

is not expressed correctly by relationships (2) or (33).

To solve the problem, let us consider the so-called bilinear relationship. As it is familiar, any network function is a bilinear function of its one-port components:

$$F(p,x) = \frac{a(p) + xb(p)}{c(p) + xd(p)} \tag{35}$$

where functions a, b, c, d are polynoms of complex variable p.

Let us form now the partial derivative of F:

$$\frac{\partial F}{\partial x} = \frac{bc - ad}{(c + xd)^2} \,. \tag{36}$$

According to relationship (36):

a) a sensitivity function with respect to a one-port component includes that component only in the denominator;

b) the denominator of the sensitivity function is the square of the natural frequencies of the network.

From the foregoing it follows that the transition $x \to 0$ does not affect the numerator of the sensitivity function. It reduces only the denominator to the square of polynom $c(p)$ corresponding to the network not containing the component "x".

If that denominator were calculated directly from a network not containing x, difficulties might be encountered in determining the constant. Under such conditions, it is more practical to determine $c(p)$ in (36) by calculating the polynom of the denominator at two different values of x. Now

$$c(p) + x_1 d(p) = f_1(p) \tag{37}$$

and

$$c(p) + x_2 d(p) = f_2(p). \tag{38}$$

Solving the equations for $c(p)$

$$c(p) = f_1(p) - x_1 \frac{f_1(p) - f_2(p)}{x_1 - x_2}. \tag{39}$$

Choose x_2 to be equal to $2x_1$, then

$$c(p) = 2f_1(p) - f_2(p) \tag{40}$$

will be obtained.

6. Applications of state-variable analysis

As it is familiar, state-variable equations take the forms of

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{w}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{w}(t) \end{aligned} \tag{41}$$

Hence the transfer matrix connecting the inputs with the outputs can be given by the expression

$$\mathbf{T}(p) = \mathbf{C}(p\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}. \tag{42}$$

Even a simple analysis requires matrix $\mathbf{A}$ to be written and inverted. The surplus work involved in sensitivity determination consists in writing and multiplication of matrices $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$.

However, in special cases, simplifications can be made.

Employ a voltage source for excitation of capacitances and a current source for inductances. It is evident from Fig. 4 that under such conditions the pertaining outputs will be just the state variables (capacitive voltage and inductive current).

Of those excitations, the ones resulting in other than improper systems are marked w_s (located outside of a capacitive loop or inductive cut set). All the rest of excitations are marked w_r, including the excitations of controled sources, resistances, capacitive loops and inductive cut sets.

This distinction will split the state variables as well. The excitations pertaining to those marked x_s will not result in improper systems, whereas those pertaining to x_i will result in improper systems.

Now let us write state-variable equations as

$$\dot{\mathbf{x}} = [\mathbf{A}_s \; \mathbf{A}_i] \begin{bmatrix} \mathbf{x}_s \\ \mathbf{x}_i \end{bmatrix} + [\mathbf{B}_s \; \mathbf{B}_r] \begin{bmatrix} \mathbf{w}_s \\ \mathbf{w}_r \end{bmatrix} \tag{43}$$

$$\begin{bmatrix} \mathbf{y}_s \\ \mathbf{y}_r \end{bmatrix} = \begin{bmatrix} \mathbf{C}_s \\ \mathbf{C}_r \end{bmatrix} \begin{bmatrix} \mathbf{x}_s \\ \mathbf{x}_i \end{bmatrix} + \begin{bmatrix} \mathbf{D}_s \\ \mathbf{D}_r \end{bmatrix} \begin{bmatrix} \mathbf{w}_s \\ \mathbf{w}_r \end{bmatrix} \tag{44}$$

As has been mentioned above, the outputs $\mathbf{y}_s$ are just the state variables $\mathbf{x}_s$; hence

$$\mathbf{C}_s = \mathbf{I}$$

and

$$\mathbf{D}_s = \mathbf{0} \tag{45}$$

will be obtained.

Furthermore, it can be proved (see Appendix) that

$$\mathbf{B}_s = -\mathbf{A}_s . \tag{46}$$

Using (43) and (44) and carrying out the multiplication of (42), the transfer matrix will take the form of

$$\mathbf{T} = \begin{bmatrix} -\mathbf{\Phi A}_s & \mathbf{\Phi B}_r \\ -\mathbf{C}_r \mathbf{\Phi A}_s & \mathbf{C}_r \mathbf{\Phi B}_r \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{D}_r \end{bmatrix} \tag{47}$$

where $\mathbf{\Phi}$ is used for denoting the relationship

$$\mathbf{\Phi} = (p\mathbf{I} - \mathbf{A})^{-1} . \tag{48}$$

It follows from relationship (47) that no outputs and inputs have to be defined specially for the state variables $\mathbf{x}_s$.

Relative to a state-variable analysis with the assumption of a single input and output, the sensitivity calculations involve the only complication by the introduction of new inputs $\mathbf{w}_r$ and outputs $\mathbf{y}_r$.

Accordingly, the method is applied in the following manner.

1. Find the particular state variables belonging to $\mathbf{x}_s$ (at which the voltage source connected in series with the capacitance and the current source connected in parallel with the inductance will not produce an improper system).

2. Select suitable excitations for all the rest of components by consulting Fig. 4.

3. Afterwards, take into account only the excitations defined in the preceding clause (together with the pertaining outputs). Thus coefficient matrixes $\mathbf{B}_r$, $\mathbf{C}_r$, $\mathbf{D}_r$ have to be written.

4. Matrix $\mathbf{A}$ is independent of the numbers of outputs and inputs. The respective line and column of matrix $\mathbf{T}$ can be calculated with reference to realtionship (47).

7. Appendix

Theorem: if a linear system can be written as

$$\dot{\mathbf{x}} = [\mathbf{A}_s \vdots \mathbf{A}_i] \begin{bmatrix} \mathbf{x}_s \\ \cdots \\ \mathbf{x}_i \end{bmatrix} + [\mathbf{B}_s \vdots \mathbf{B}_r] \begin{bmatrix} \mathbf{w}_s \\ \cdots \\ \mathbf{w}_r \end{bmatrix} + \mathbf{B}_r^{(1)} \dot{\mathbf{w}}_r + \dots \tag{49}$$

where $\mathbf{w}_s$ are voltage sources (connected in series with the capacitances) or current sources connected in parallel with the inductances, then

$$\mathbf{B}_s = \mathbf{A}_s \, . \tag{50}$$

Proof

The unique solution gives evidence of the fact that a given arrangement may be associated with a single matrix $\mathbf{B}$ only. The investigation is carried out for capacitances only and, on account of the superposition principle, for the case of $\mathbf{w}_r = 0$. For inductances, the theorem can be proved in a similar manner.

Evidently, a capacitance charged to a voltage U_0 is equivalent to an uncharged capacitance connected in series with a voltage source of $E_0 = U_0$.

Be initial value of $\mathbf{u}_0$ assigned to the capacitors involved in $\mathbf{x}_s$; determine the output at an excitation of $\mathbf{w} = 0$.

The state variables will be

$$\mathbf{x}_s(t) = e^{\mathbf{A}t} \, \mathbf{u}_0 \, . \tag{51}$$

Now establish the same conditions but with uncharged capacitors connected in series with voltage sources $\mathbf{w}_s = \mathbf{u}_0$.

Of course, the potential difference developing across the poles of the initial capacitor will be (Fig. 12)

$$\mathbf{x}_s(t) = \mathbf{x}_1(t) + \mathbf{w}_s \tag{52}$$

where

$$\mathbf{x}_1(t) = \int_0^t e^{\mathbf{A}(t-\tau)}\,\mathbf{B}_s\,\mathbf{w}_s\,d\tau\,. \tag{53}$$

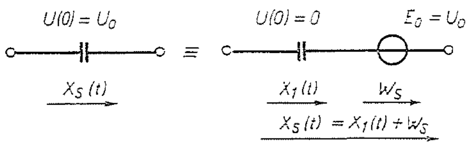

Fig. 12

The value of $\mathbf{B}_s$ can be determined by equalizing equations (51) and (52). Namely:

$$e^{\mathbf{A}t}\,\mathbf{u}_0 = \mathbf{w}_s + \int_0^t e^{\mathbf{A}(t-\tau)}\cdot\mathbf{B}_s\,\mathbf{w}_s\,d\tau\,. \tag{54}$$

After integration ($\mathbf{B}_s$ and $\mathbf{w}_s = \mathbf{u}_0$ are constant),

$$e^{\mathbf{A}t}\,\mathbf{u}_0 = \mathbf{u}_0 + e^{\mathbf{A}t}\cdot(-\mathbf{A}^{-1}\cdot e^{\mathbf{A}t} + \mathbf{A}^{-1})\,\mathbf{B}_s\,\mathbf{u}_0\,. \tag{55}$$

Rearranging,

$$(e^{\mathbf{A}t} - \mathbf{I})\,\mathbf{u}_0 = e^{\mathbf{A}t}\,[-\mathbf{A}^{-1}\cdot e^{\mathbf{A}t} + \mathbf{A}^{-1}]\,\mathbf{B}_s\,\mathbf{u}_0\,. \tag{56}$$

On account of the definition of matrix functions in terms of infinite series, the product in brackets on the right side can be interchanged. Accordingly

$$(e^{\mathbf{A}t} - \mathbf{I}) = \mathbf{u}_0 = (e^{\mathbf{A}t} - \mathbf{I})\,\mathbf{A}^{-1}\cdot\mathbf{B}_s\,\mathbf{u}_0\,. \tag{57}$$

The term (57) will only be valid if

$$[\mathbf{A}_s\;\mathbf{A}_i]^{-1}\cdot[\mathbf{B}_s\;\mathbf{B}_i] = [\underbrace{\mathbf{I}_s}_{s}\;\underbrace{\mathbf{M}_i}_{i}] \tag{58}$$

where $\mathbf{I}_s$ is a unit matrix, and $\mathbf{M}_i$, $\mathbf{B}_i$ are omitted. Hence

$$\mathbf{B}_s = \mathbf{A}_s\,. \tag{59}$$

*

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Summary

The method here described can be applied for the determination of circuit component sensitivities without the need of derivation. Bychovsky's theorem has been generalized to active networks, parasitic components and any kind of transfer function.

It has been pointed out that—with the excitations and outputs suitably selected — the sensitivity of the network with respect to any component can be determined from a row and a column of the transfer matrix.

Adopting this method to state-variable analysis, it has been pointed out that — with some limitations — it is not necessary to define special inputs and outputs for the state-variable components. Thus the calculation of coefficients of matrices **B**, **C** and **D** as well as transfer matrix **T** has been largely simplified. Accordingly, the sensitivities can be determined in a single step of analysis, involving a slight surplus work.

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