STATISTICAL SYNTHESIS OF SAMPLED-DATA CONTROL SYSTEMS

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1. Conditions of optimization

A generally accepted principle of the synthesis of linear control systems is the following. The given data are: (1) The correlation functions $\varphi_{ss}(\tau)$, $\varphi_{sn}(\tau)$ and $\varphi_{nn}(\tau)$ of the input $r = s + n$ of the system, which is regarded as being stationary and ergodic, or the Laplace transforms of these, the power spectra $\Phi_{ss}(s)$, $\Phi_{sn}(s)$, and $\Phi_{nn}(s)$, respectively, (2) the ideal weighting function $y(t)$, which orders the ideal output signal $i(t)$ to the input signal $s(t)$. This physically realizable and stable system is regarded as the *optimum* which ensures a minimum of the mean square error. The problem is the determination of the weighting function $w(t)$ of this system.

As was shown in a paper published earlier [22], in the case of a linear sampled-data system two sorts of mean square errors may be defined: the discrete mean square error ζ^2 and the continuous mean square error ξ^2:

$$\zeta^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} e^2(nT)\,, \tag{1}$$

$$\xi^2 = \lim_{T'\to\infty} \frac{1}{2T'} \int_{-T'}^{T'} e^2(t)\, dt\,, \tag{2}$$

where $e(t) = c(t) - i(t)$ is the error signal, $c(t)$ the actual output signal, and T the sampling period. The ideal output signal can be ordered in three different ways to the input signal: (Problem I:) to the sampled control input $s^*(t)$ by a continuously operating ideal system, (Problem II:) to the continuous input signal $s(t)$ by a continuously operating ideal system, (Problem III:) to the continuous input signal $s(t)$ by a discretely operating ideal system. The expressions of ζ^2 and ξ^2 are given for each of the three problems.

It is obvious that the system ensuring the minimum of the discrete mean square error can be more simply calculated, and in general it results in compensators which are more simple to realize. The system ensuring the minimum of the continuous mean square error, in turn, is regarded as having a better operation.

Instead of demanding the physical realizability, in practice it is generally sufficient to demand the causality of the system, the condition of which is known to be

$$w(t) = 0, \ t < 0. \tag{3}$$

Both the causality and the stability of the sampled-data system are ensured, if all the poles of the discrete transfer function of $W(Z)$ are *outside the unit circle* $|Z| = |e^{-sT}| = 1$, (i. e. inside the unit circle $|z| = |e^{sT}| = 1$). If $W(Z)$ is rational this condition can be written in the form

$$\frac{1}{W(Z_i)} = 0 \, , \quad \frac{1}{W(Z_i, m)} = 0 \, , \quad |Z_i| > 1 \, , \tag{4}$$

since the poles of $W(Z, m)$ and $W(Z)$ are identical.

2. Minimization of the discrete mean square error

The expression for the discrete mean square error, according to [22 (64), (66)] is

$$\zeta^2 = \frac{1}{2\pi \mathrm{j}} \oint Z^{-1} \Psi_{ee}(Z) \, \mathrm{d}Z, \qquad \Gamma: |Z| = 1 \, , \tag{5}$$

$$\begin{aligned} \Psi_{ee}(Z) = \Psi_{ii}(Z) - W(Z)\,\Psi_{ir}(Z) - W(Z^{-1})\,\Psi_{ri}(Z) + \\ + W(Z^{-1})\,W(Z)\,\Psi_{rr}(Z) \, , \end{aligned} \tag{6}$$

where $\Psi(Z)$ represents the discrete transform of the simple correlation sequences. Let us determine the discrete transfer function $W_{\mathrm{opt}}(Z)$ which satisfies (4) and ensures the minimum of ζ^2 in the structure shown in Fig. 1, where $G(Z)$ and $G(Z, m)$ are the given transfer functions of the controlled plant, while $D(Z)$ and $B(Z)$ are the transfer functions of the impulse compensators which may be chosen.

It is known [1, 10] that the stability of the system can be ensured in a practically satisfactory manner only if $W(Z)$ contains all the zeros of $G(Z)$ which are inside the unit circle. Let

$$G(Z) = P(Z)\, G_0(Z), \tag{7}$$

where the polynomial $P(Z)$ contains all the zeros of $G(Z)$ which are inside the unit circle, thus all the zeros of $G_0(Z)$ are already outside the unit circle. A typical case of this kind is when $G(s) = e^{-snT} G_0(s)$, thus $P(Z) = Z^n$ (delayed controlled plant).

Accordingly the transfer function of the closed system should have the form

$$W(Z) = P(Z)\, V(Z), \tag{8}$$

where $V(Z)$ is the auxiliary function which is to be determined. This may have, by force of (4), only poles outside the unit circle.

Let us employ on determining $V(Z)$ the simplified method of CSÁKI [14, 20]. Let us introduce the auxiliary functions

$$\Psi'_{ir}(Z) = P(Z)\,\Psi_{ir}(Z)\,, \Psi'_{ri}(Z) = P(Z^{-1})\,\Psi_{ri}(Z)\,,$$

$$\Psi'_{rr}(Z) = P(Z)\,P(Z^{-1})\,\Psi_{rr}(Z)\,; \tag{9}$$

$$A(Z) \equiv \frac{\Psi'_{ri}(Z)}{\Psi_{rr}(Z)}\,,\ A(Z^{-1}) \equiv \frac{\Psi'_{ri}(Z^{-1})}{\Psi'_{rr}(Z^{-1})} = \frac{\Psi'_{ir}(Z)}{\Psi'_{rr}(Z)} \tag{10}$$

determined by the data. By eliminating the functions $\Psi_{ri}(Z)$ and $\Psi_{ir}(Z)$, relationship (6) can be written in the following form:

$$\begin{aligned}\Psi_{ee}(Z) = \Psi_{ii}(Z) - A(Z)\,A(Z^{-1})\,\Psi'_{rr}(Z) + \\ + \left[A(Z) - V(Z)\right]\left[A(Z^{-1}) - V(Z^{-1})\right]\Psi'_{rr}(Z).\end{aligned} \tag{11}$$

The first two terms cannot be influenced by the choice of $V(Z)$. The last term is not negative on the unit circle ($Z = e^{j\varphi}$), thus we obtain the minimum value of ζ^2 if the last term is zero, i. e. if

$$V_0(Z) = A(Z) \equiv \frac{\Psi'_{ri}(Z)}{\Psi'_{rr}(Z)}\,. \tag{12}$$

This $V_0(Z)$, however, also contains poles which are inside the unit circle. Write (12) in the form

$$\Psi'_{rr}(Z)\, V_0(Z) - \Psi'_{ri}(Z) = 0. \tag{13}$$

If we write in place of $V_0(Z)$ the function $V_{\text{opt}}(Z)$ which has no poles inside the unit circle, then zero cannot figure at the right side. CSÁKI implicitly assumes that by substituting $V_{\text{opt}}(Z)$ we should write at the right side a function $F_-(Z)$ having poles *only* inside the unit circle.

$$\Psi'_{rr}(Z)\, V_{\text{opt}}(z) - \Psi'_{ri}(Z) = F_-(Z). \tag{14}$$

Let us factorize the function $\Phi(Z)$ to the product of two functions

$$\Phi(Z) = \Phi^+(Z) \cdot \Phi^-(Z) \tag{15}$$

where all the poles and zeros of $\Phi^+ (Z)$ are outside the unit circle, consequently all the poles and zeros of $\Phi^- (Z)$ are inside the unit circle. Thus (14) may be written in the form

$$[\Psi'_{rr}(Z)]^+ V_{\text{opt}}(Z) = \frac{\Psi'_{ri}(Z)}{[\Psi'_{rr}(Z)]^-} + \frac{F_-(Z)}{[\Psi'_{rr}(Z)]^-} . \tag{16}$$

The left side has poles only outside the unit circle, while the second term of the right side only inside it. Decompose an arbitrary function $\Phi(Z)$ to a sum:

$$\Phi(Z) = [\Phi(Z)]_+ + [\Phi(Z)]_- \equiv \Phi_+(Z) + \Phi_-(Z), \tag{17}$$

where all the poles of part $[\quad]_+$ are outside the unit circle, and those of part $[\quad]_-$ inside the unit circle (there is no restriction for the zeros). Accordingly the left side of (16) is equal to part $[\quad]_+$ of the first term at the right side.

$$[\Psi'_{rr}(Z)]^+ V_{\text{opt}}(Z) = \left[\frac{\Psi'_{ri}(Z)}{[\Psi'_{rr}(Z)]^-}\right]_+ . \tag{18}$$

Let p designate the degree of the polynomial $P(Z)$ having poles only inside the unit circle, then evidently

$$P(Z^{-1}) = Z^{-p} P_0(Z);\; P_0(Z) = Z^p P(Z^{-1}), \tag{19}$$

where all the zeros of the polynomial $P_0(Z)$ are outside the unit circle. Accordingly

$$[\Psi'_{rr}(Z)]^+ = [P(Z)\, Z^{-p} P_0(Z)\, \Psi_{rr}(Z)]^+ = P_0(Z)\, \Psi^+_{rr}(Z),$$

$$[\Psi'_{rr}(Z)]^- = Z^{-p} P(Z)\, \Psi^-_{rr}(Z). \tag{20}$$

As a final result, the expression for the physically realizable, stable and optimum transfer function is found to be

$$W_{\text{opt}}(Z) = P(Z)\, V_{\text{opt}}(Z) = \frac{P(Z)}{P_0(Z)\, \Psi^+_{rr}(Z)} \left[\frac{P_0(Z)\, \Psi_{ri}(Z)}{P(Z)\, \Psi^-_{rr}(Z)}\right]_+ . \tag{21}$$

The correlation sequences can be expressed by the correlation functions and the ideal transfer function [22].

$$\Psi_{rr}(Z) = \Phi_{rr}(Z); \tag{22}$$

$$\Psi_{ri}(Z) = Y(Z)\, \Phi_{rs}(Z), \text{ (Problem I, and III)}; \tag{23}$$

$$\Psi_{ri}(Z) = \Phi_{ri}(Z) = \mathfrak{z}'[Y(s)\, \Phi_{rs}(s)], \text{ (Problem II)}. \tag{24}$$

We wish to remark the following on these results. If the occurring functions are rational, as it is in most cases, then the operation $[\quad]_+$ simply denotes the formation of the sum of partial fractions pertaining to the poles outside the unit circle. This is the consequence of the application of the variable Z, since the situation is more complicated in the case when using an expression in terms of the variable $z = Z^{-1}$ [21]. It can further be shown, e. g. by the variation method as employed by Tou [6], that the assumption of Csáki is not restricting the function class, thus the solution (21) is really optimum. This verification is also necessary because without the introduction of the notations (9) we obtain such an expression in place of (21) which differs from relationships (21) only in the fact that 1 is to be written in place of both $P_0(Z)$ values. This $W(Z)$ is physically realizable but the system is not optimized thereby. If $P(Z) = 1$, then $P_0(Z) = 1$, thus there is no difference between the two results.

3. Minimization of the continuous mean square error

The expression for the continuous mean square error, in view of [22(70), (72)] is found to be

$$\xi^2 = \frac{1}{2\pi \mathrm{j}} \oint_{\Gamma} Z^{-1} \overline{\Psi}_{ee}(Z)\, \mathrm{d}Z\,, \tag{25}$$

$$\overline{\Psi}_{ee}(Z) \equiv \int_0^1 \Psi_{ee}(Z; m) dm = \int_0^1 [\Psi_{ii}(Z; m) - Z^{-1} W(Z, m)\, \Psi_{ir}(Z; m, 0) - \\ - ZW(Z^{-1}, m)\, \Psi_{ri}(Z; 0, m) + W(Z^{-1}, m)\, W(Z, m)\, \Psi_{rr}(Z)]\, \mathrm{d}Z\,. \tag{26}$$

Since the closed system (Fig. 1.) contains only impulse compensators, thus as is known,

$$W(Z, m) = \frac{G(Z, m)}{G(Z)} W(Z)\,. \tag{27}$$

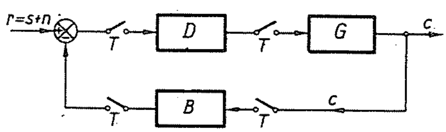

Fig. 1

By substituting this into (26), the expression can be written as follows

$$\overline{\Psi}_{ec}(Z) = \overline{\Psi}_{ii}(Z) - W(Z)\, \overline{\Psi}_{ir}(Z) - W(Z^{-1})\, \overline{\Psi}_{ri}(Z) + \\ + W(Z^{-1})\, W(Z)\, \overline{\Psi}_{rr}(Z)\,, \tag{28}$$

where the functions with a dash above, depend only on the given quantities:

$$\overline{\Psi}_{ii}(Z) \equiv \int_0^1 \Psi_{ii}(Z; m)\, dm\,, \tag{29}$$

$$\overline{\Psi}_{ir}(Z) \equiv \frac{Z^{-1}}{G(Z)} \int_0^1 G(Z, m)\, \Psi_{ir}(Z; m, 0)\, dm\,, \tag{30}$$

$$\overline{\Psi}_{ri}(Z) \equiv \frac{Z}{G(Z^{-1})} \int_0^1 G(Z^{-1}, m) \Psi_{ri}(Z; 0, m)\, dm, \tag{31}$$

$$\overline{\Psi}_{rr}(Z) \equiv \frac{\Psi_{rr}(Z)}{G(Z)\, G(Z^{-1})} \int_0^1 G(Z, m)\, G(Z^{-1}, m)\, dm. \tag{32}$$

We note that $\overline{\Psi}_{ir}(Z) = \overline{\Psi}_{ri}(Z^{-1})$.

By comparing the relations (25) and (5), and (28) and (6), respectively, it is evident that the expressions for ξ^2 and ζ^2 are completely identical, only in place of the functions $\Psi(Z)$ we should write $\overline{\Psi}(Z)$ with identical indices. Thus we may take over also the final result (21).

$$W_{\mathrm{opt}}(Z) = \frac{P(Z)}{P_0(Z)\overline{\Psi}_{rr}^{+}(Z)} \left[\frac{P_0(Z)\, \overline{\Psi}_{ri}(Z)}{P(Z)\, \overline{\Psi}_{rr}^{-}(Z)} \right]_+ . \tag{33}$$

Of this $W_{\mathrm{opt}}(Z, m)$ may be expressed, on the basis of (27).

For purposes of practical calculations the correlation sequences may be expressed by the correlation functions [22]:

$$\overline{\Psi}_{rr}(Z) = \frac{\Phi_{rr}(Z)}{G(Z)\, G(Z^{-1})} \int_0^1 G(Z, m)\, G(Z^{-1}, m)\, dm\,, \tag{34}$$

$$\overline{\Psi}_{ri}(Z) = \frac{\Phi_{rs}(Z)}{G(Z^{-1})} \int_0^1 G(Z^{-1}, m)\, Y(Z, m)\, dm\,, \tag{35}$$

(Problem I)

$$\overline{\Psi}_{ri}(Z) = \frac{1}{G(Z^{-1})} \int_0^1 G(Z^{-1}, m)\, \mathfrak{z}'_m \left[Y(s)\, \Phi_{rs}(s) \right] dm, \tag{36}$$

(Problem II)

$$\overline{\Psi}_{ri}(Z) = \frac{Y(Z)}{G(Z^{-1})} \int_0^1 G(Z^{-1}, m)\, \Phi_{rs}(Z, m)\, \mathrm{d}m\,, \tag{37}$$

(Problem III).

In this way the optimum transfer function for all the six problems is expressed with the aid of the correlation functions, of the transfer function of the controlled plant, and of the ideal transfer function, i.e. of the given quantities.

4. Calculation of the optimum compensators

With the knowledge of the optimum transfer function we can determine the transfer function of the impulse compensator(s) of the given structure.

Examine the arrangement illustrated in Fig. 1, the transfer function of which is known to be

$$W(Z) = \frac{D(Z)\, G(Z)}{1 + B(Z)\, D(Z)\, G(Z)}\,. \tag{38}$$

If we choose one of the transfer functions $B(Z)$ or $D(Z)$ then the other is unequivocally determined.

$$D(Z) = \frac{W(Z)}{G(Z)} \frac{1}{1 - B(Z)\, W(Z)}\,, \tag{39}$$

$$B(Z) = \frac{1}{W(Z)} - \frac{1}{D(Z)\, G(Z)}\,. \tag{40}$$

If here $W(Z) = W_{\mathrm{opt}}(Z)$, then $D(Z)$ and $B(Z)$, respectively, are the transfer functions ensuring the optimum transfer function of the closed system.

The condition of the physical realizability of impulse compensators is that they should realize only delays. To this end $D(Z)$ and $B(Z)$ should be rational and bounded at the place $Z = 0$.

$$D(0) < \infty, \; B(0) < \infty. \tag{41}$$

Consider relation (7) and (8), where the conditions $G_0(0) \neq 0$ and $V(0) \neq 0$ are fulfilled:

$$D(Z) = \frac{V(Z)}{G_0(Z)} \frac{1}{1 - B(Z)\, P(Z)\, V(Z)}\,. \tag{42}$$

Take a physically realizable transfer function $B(Z)$ into consideration, then $B(0)$ is bounded and thus

$$D(0) = \frac{V(0)}{G_0(0)} \cdot \frac{1}{1 - B(0)\,P(0)\,V(0)} \tag{43}$$

is in any case bounded, if

$$B(0) \neq \frac{1}{P(0)\,V(0)} = \frac{1}{W(0)}\,. \tag{44}$$

Theoretically the stability of $D(Z)$ can easily be ensured, under the condition that it should have no poles inside the unit circle. Since $V(Z)$ has no such pole and $G_0\,(Z)$ has no such zero, the condition of this is found to be, in view of (42), that

$$B(Z_i)\;W(Z_i) = 1,\;\; |Z_i| > 1. \tag{45}$$

Frequently a rigid feedback, i. e. the application of $B(Z) = 1$, is sufficient. This may be allowed in case of

$$W(0) \neq 1;\;\; W(Z_i) = 1,\;\; |Z_i| > 1. \tag{46}$$

Then

$$D(Z) = \frac{W(Z)}{G(Z)}\,\frac{1}{1 - W(Z)} = \frac{V(Z)}{G_0\,(Z)}\,\frac{1}{1 - P(Z)\,V(Z)}\,,\, B(Z) = 1\,. \tag{47}$$

If (46) is not satisfied the choice of $B(Z) = B_0$ may still be suitable.

The reversed way, i. e. the choice of $D(Z)$ and the calculation of $B(Z)$ on the basis of (40) is less advisable since both the realization and the ensuring of stability are difficult.

5. Some critical remarks

In the course of the examination of the analysis problem [22] the concepts of results published in the literature were discussed. We have seen that different authors give the same result for the calculation of ζ^2 (at least for Problem I), while the expression for ξ^2 is only given correctly by Kuzin [8]. The result of Zypkin [15] is correct formally, though the theoretical foundations are controversial.

As regards the problem of synthesis, Tou [6, 9] determined for Problem I the transfer function ensuring the optimum of ζ^2. His starting point is the relation $W(Z) = K(Z)\,G(Z)$ in which he determines the optimum of $K(Z)$. It can be proved that this final result is actually identical with (21), though formally more complicated. It deserves attention that $W(Z)$ does not actually contain all the zeros of $G(Z)$, only those inside the unit circle. Since the expression for ξ^2 as given by TOU is incorrect, the result for the optimization is also incorrect. Nevertheless his method was used here in Section 3.

ZYPKIN [15] and KUZIN [8] solved only the minimization of ζ^2, but did not consider restriction (8), consequently the stability of systems designed with the aid of their method is practically insufficient.

6. Illustrative example

Let the problem be the designing of a follow-up system with minimum discrete mean square error, if the data are

$$G(s) = \frac{1 - e^{-sT}}{s} \, \frac{s - b}{(s+1)(s+2)} \; ; \; T = 0.2;$$

$$\Phi_{ss}(s) = \frac{2}{1 - s^2} \; ; \Phi_{nn}(s) = A; \Phi_{sn}(s) = \Phi_{ns}(s) = 0 .$$

By decomposing into partial fractions and employing the corresponding tables [19],

$$G(Z) = KZ \frac{1 - kZ}{(1 - pZ)(1 - p^2 Z)} \, , p \equiv e^{-0.2} = 0.81873 \, ,$$

$$K = p(1 - p) - 0.5(1 - p^2)b \, , \, k = p \, \frac{2 + (1 - p)\, b}{2p - (1 - p)\, b} \; .$$

The two cases should be examined separately.

$$\text{a)} \quad |k| < 1 \; ; \qquad b < 0 \, , \; b > \frac{4p}{(1 - p)^2} = 99.667 \, ,$$

$$\text{b)} \quad |k| > 1 \; ; \qquad 0 < b < \frac{4p}{(1 - p)^2} = 99.667 .$$

Accordingly

a) $P(Z) = Z, \; P_0(Z) = 1;$

b) $P(Z) = Z(1 - kZ), \; P_0(Z) = Z - k.$

The discrete transforms of the correlation functions are found to be

$$\Phi_{ss}(Z) = \frac{(1-p^2)Z}{(1-pZ)(Z-p)}, \quad \Phi_{nn}(Z) = A, \quad \Phi_{ns}(Z) = \Phi_{sn}(Z) = 0,$$

$$\Phi_{rr}(Z) = \Phi_{ss}(Z) + \Phi_{nn}(Z) = \frac{Ap}{q} \frac{(1-qZ)(Z-q)}{(1-pZ)(Z-p)}.$$

$$\Phi_{rr}^{+}(Z) = \frac{Ap}{q} \frac{1-qZ}{1-pt}, \qquad \Phi_{rr}^{-}(Z) = \frac{Z-q}{Z-p},$$

where q denotes the root, being inside the unit circle, of the following second degree equation

$$q^2 - \left(\frac{1+p^2}{p} + \frac{1-p^2}{Ap}\right) q + 1 = 0, \quad |q| < 1.$$

The optimum transfer function, on the basis of (21), in the case of $|k| < 1$, by substituting $P(Z) = Z$ and $P_0(Z) = 1$, is found to be

$$W_{opt}(Z) = \frac{Z}{\Phi_{rr}^{+}(Z)} \left[\frac{\Phi_{rs}(Z)}{Z\,\Phi_{rr}^{-}(Z)}\right]_{+} = \frac{q}{Ap} Z \frac{1-pZ}{1-qZ} \left[\frac{1-p^2}{(1-pZ)(Z-q)}\right]_{+}.$$

The operation $[\quad]_{+}$ denotes that we should take the partial fraction having the denumerator $(Z-q)$, thus

$$W_{\mathrm{opt}}(Z) = (p-q)\frac{Z}{1-qZ},$$

$$\zeta^2_{\min} = \frac{1-pq}{p-q}.$$

If, in turn, $|k| > 1$, then $P(Z) = Z(1-k)$ and $P_0(Z) = Z - k$, thus

$$W_{\mathrm{opt}}(Z) = \frac{(p-q)(1-kp)}{k(k-p)} \frac{Z(1-kZ)}{(1-qZ)(1-k^{-1}Z)},$$

$$\zeta^2_{\min} = \frac{1-p^2}{1-pq}\left[1 + p^2 - pq + \frac{2p^2(p-q)}{k-p} - \frac{p(1-p^2)(p-q)}{(k-p)^2}\right].$$

The discrete mean square error ζ^2 is a continuous function of the parameter b. It is interesting to note that $\zeta^2_{\min}$ has a maximum if $b = 2p(1+p^2) = 0.97971$ and then $\zeta^2_{\min} = 1$.

If $|k| - 1 \ll 1$, the transfer function $W_{\mathrm{opt}}(Z)$ cannot be accepted, since then $|k^{-1}| \approx 1$ and thus the stability margin is very small.

Summary

Two kinds of the mean square error can be defined in the case of a stationary stochastic input signal in a sampled-data system, and the ideal output signal can be ordered in three ways to the input signal. For all the six problems a method is given for the calculation of such a system which is physically realizable, stable and ensures the minimum of the mean square error. For the calculation of the impulse compensators to be arranged in the forward and feedback path a method is given which ensures the possibility of their physical realization and stability, further the independence of the stability of the system of the accuracy of parameters.

References

1. Bertram, J. E.: Factors in the Design of Digital Controllers for Sampled-Data Feedback Systems. Trans. AIEE. pt. II. **75**, 151—159 (1956).
2. Chang, S. S. L.: Statistical Design Theory for Strictly Digital Sampled-Data Systems. Trans. AIEE. pt. I. **76**, 702—709 (1957).
3. Chang, S. S. L.: Statistical Design Theory of Digital Controlled Continuous Systems. Trans. AIEE. pt. II. **77**, 191—201 (1958).
4. Ragazzini, J. R.—Franklin, G. F.: Sampled-Data Control Systems, Chap. 10. McGraw-Hill Book. Comp. Inc. New York, 1958.
5. Цыпкин, Я. З.: Теория импульсных систем. Гл. III. 3, III. 9. Физматгиз, Москва, 1958.
6. Tou, J. T.: Statistical Design Theory of Digital Control Systems. IRE Trans. AC. **5**, 290—297 (1960).
7. Солодовников, В. В.: Статистическая динамика линейных систем автоматического управления. Гл. XII., XIII. Физматгиз, Москва, 1960.
8. Кузин, Л. Т.: Расчет и проектирование дискретных систем управления. Гл. VIII., IX. Машгиз, Москва, 1960.
9. Tou, J. T.: Statistical Design of Linear Discrete-Data Control Systems via the Modified z-transform Method. J. Franklin Inst. **271**, 249—262 (1961).
10. Chang, S. S. L.: Synthesis of Optimum Control Systems. Chap. 6. McGraw-Hill Book Comp. Inc. New York, 1961.
11. Jury, E. I.: Optimization Procedures for Sampled-Data and Digital Control Systems. Scientia Electrica **7**, 2—12 (1961).
12. Chang, S. S. L.: Optimum Transmission of Continuous Signal over a Sampled Data Link. Trans. AIEE. pt. II. **80**, 538—542 (1961).
13. Nishimura, T.: On the Modified z-Transform of Power Spectra Densities. IRE Trans. AC. **7**, 55—56 (1962).
14. Csáki, F.: Simplified Derivation of Optimum Transfer Functions in the Wiener—Newton Sense. Periodica Polytechnica, Electr. Eng. **6**, 237—245 (1962).
15. Цыпкин, Я. З.: Теория линейных импульсных систем. Гл. II. 8., III. 8., V. 10., V. 11. Физматгиз, Москва, 1963.
16. Jury, E. I.: Comments on the Statistical Design of Linear Sampled-Data Feedback Systems. IEEE Trans. AC. **10**, 215—216 (1965).
17. Steiglitz, K.—Franaszek, P. A.—Haddad, A. H.: IEEE Trans. AC. **10**, 216—217 (1965).
18. Галоускова, А.: Синтез многомерных линейных импульсных систем регулирования по квадратическим критериям. Труды Международной Конференции по Многомерным и Дискретным Системам Автоматического Управления. Секция Б (129—140), Прага, 1965.
19. Fodor, Gy.: Laplace-Transforms in Engineering. Chap. 38—41, 47. Akadémiai Kiadó, Budapest, 1965.
20. Csáki, F.: Optimum Pulse-Transfer Functions for Multivariable Digital Stochastic Processes. Periodica Polytechnica, Electr. Eng. **9**, 353—376 (1965).
21. Csáki, F.—Steiglitz, K.—Franaszek, P. A.—Haddad, A. H.: Discussion of "Comments on the Statistical Design of Linear Sampled-Data Feedback Systems" IEEE Trans. AC. **11**, 149—150 (1966).
22. Fodor, Gy.: Statistical Analysis of Sampled-Data Control Systems. Periodica Polytechnica, Electr. Eng. **10**, 251—263 (1966).

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