# DESIGN OF COUNTING-TYPE DIGITAL CONTROLLERS

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## 1. Introduction

Sampled-data control systems are usually compensated by pulse operation compensating elements. They can be built by analogue and digital technique. Two ways of the realization by analogue elements are known:

*a)* Direct processing of pulse amplitude modulated signals by delayline networks. Storage and selection of pulses can be carried out by delaying the pulses.

*b)* Pulsed passive networks. The sampled or digital signals are converted analogue and the required shaping is carried out by passive networks.

Methods of digital realization are:

*a)* Using universal digital computers. Storage of the digital input is carried out by the computer itself, according to a preliminary program.

*b)* Using special-purpose machine. If no computer is in the production process the few, special operations required can easily be implemented by counting-type elements.

Great advantages of digitally implemented controllers are that they may be directly connected with other digital systems, multichannel compensating organs can easily be made with them. Digital controllers are favourable if great accuracy and time-constants varying in a wide range are required. Compared with analogue controllers considerably greater time-constants can be reliably implemented by simple means.

## 2. Pulse transfer function of PID-type digital controller

### *2.1. Pulse transfer function of proportional, integrating, and differentiating branches*

Many control problems can be solved with the use of PI, PID-type controllers. The input of the controller is sampled or digital, its output actuates a data hold. The quantization errors are disregarded, the coded signals are considered to be equal with the sample amplitudes.

Usually zero-order data holds are used in practice, which have the transfer function:

$$G_T(s) = \frac{1 - e^{-Ts}}{s} \tag{1}$$

where $T = 2\pi/\omega_s$, the sampling period.

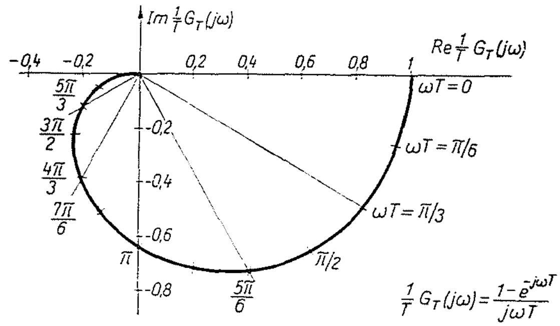


*Fig. 1.* Transfer locus of zero-order data hold

The transfer locus is shown in *Fig. 1.* The continuous output made from the sampled input has considerable amplitude and phase error, so its effect should be taken into consideration in case of high accuracy in the proportional branch, and in the differentiating and integrating branches on any way.

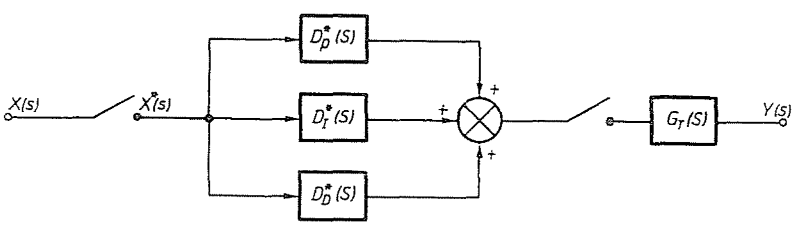


*Fig. 2.* Schematic diagram of PID controller

A PID-type controller can be built based on *Fig. 2.* Laplace-transform of the output — using the symbols of Fig. 2 — is

$$Y(s) = X^*(s)\, D^*(s)\, G_T(s)\,,$$

where

$$D^*(s) = D_P^*(s) + D_D^*(s) + D_I^*(s)$$

$X^*(s)$ is the Laplace-transform of the sampled input, which — with the help of the Laplace-transform of the continuous signal — is:

$$X^*(s) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(s + jn\,\omega_s). \tag{2}$$

If the amplitude density spectrum of the input has no higher components than $\frac{\omega_s}{2}$, half of the sampling frequency, and the output is considered only in the $0 \leq \omega \leq \frac{\omega_s}{2}$ range, then:

$$Y(s) = \frac{1}{T} X(s)\, D^*(s)\, G_T(s). \tag{3}$$

An over-all transfer function may be introduced relating to the input and the output:

$$W(s) = \frac{Y(s)}{X(s)} = \frac{1}{T} D^*(s)\, G_T(s) \tag{4}$$

$W(s)$ transfer function is a function of $T$. $D^*(s)$, the transfer function of the digital controller may be derived so that $W(s)$ should best approximate $G(s)$, the transfer function of a continuous ideal PID element:

$$G(s) = K_p \left(1 + T_d\, s + \frac{1}{T_i\, s}\right) \tag{5}$$

where $K_p$, $T_d$, $T_i$ are the coefficient of proportionality, differentiating and integrating time-constants of continuous ideal controller, respectively.

The necessary condition is that

$$W(s) = W(s)|_{T=0} + \frac{\partial W(s)}{\partial T}\bigg|_{T=0} T + \frac{\partial^2 W(s)}{\partial T^2}\bigg|_{T=0} \frac{T^2}{2!} + \ldots \tag{6a}$$

the Taylor series at $T = 0$ should best approximate $G(s)$. So

$$W(s)|_{T=0} = G(s) \quad \text{and} \quad \frac{\partial^i W(s)}{\partial T^i}\bigg|_{T=0} = 0 \tag{6b}$$
$$(i = 1, 2, 3, \ldots)$$

If

$$D^*(s) = \frac{a_0 + a_1 e^{-Ts} + a_2 e^{-2Ts} + a_3 e^{-3Ts} + \ldots}{1 + b_1 e^{-Ts} + b_2 e^{-2Ts} + b_3 e^{-3Ts} + \ldots} \tag{7}$$

then it can be shown, that depending on the degree of accuracy the coefficients of pulse transfer function of the digital controller in the proportional, integrating, and differentiating branches should be chosen according to Tables 1, 2, and 3, respectively, to fulfill the conditions of (6a) and (6b).

The amplitude and phase characteristics of the proportional branch supplemented with the compensating element is shown in *Fig. 3.* The deviation

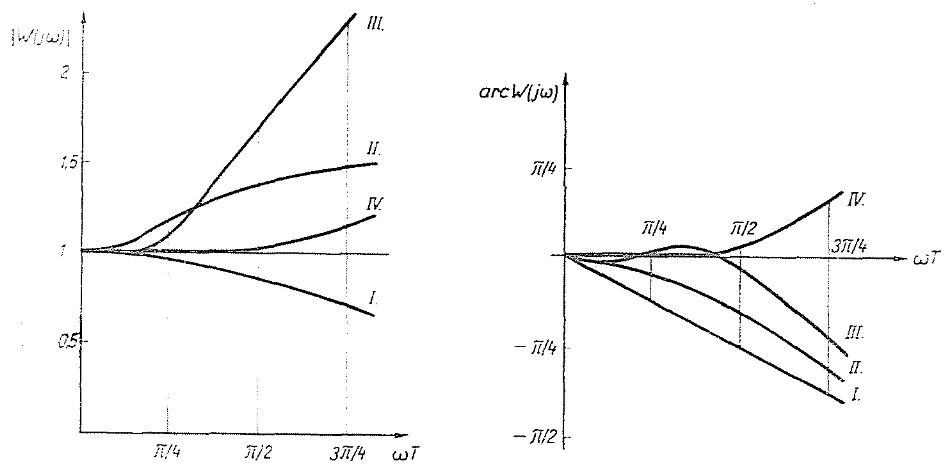


*Fig. 3.* Amplitude and phase characteristics of the proportional branch supplemented with digital controller

percentage of the amplitude characteristics of the integrating organ in case of various integrating methods from the ideal is shown in *Fig. 4.*

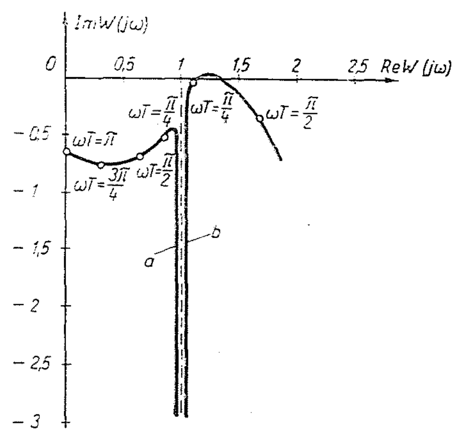


*Fig. 4.* W($j\omega$) transfer locus of PI-type digital controllers

E. g.: The Fourier transform of the integrating organ in case of trapezoidal integration is:

$$D^*(j\omega) = \frac{K_p T}{2T_i} \frac{1 + e^{-j\omega T}}{1 - e^{-j\omega T}} = \frac{1}{j} \frac{K_p T}{2T_i} \operatorname{ctg} \pi \cdot \frac{\omega}{\omega_s}$$

The frequency response of the ideal integrating organ is given by

$$G(j\omega) = \frac{1}{j} \frac{K_p}{T_i \omega} .$$

The integration error is:

$$h = 1 - \pi \frac{\omega}{\omega_s} \operatorname{ctg} \pi \frac{\omega}{\omega_s}$$

The method proposed can also be used for designing arbitrary pulse transfer functions. In case of an integrating organ the various known numerical integration methods (square, trapezoidal, Simpson rule) can be employed directly.

### 2.2. *Pulse transfer function of PI-type controller*

The pulse transfer function of a PI organ can be determined by using proportional and integrating components of various accuracy given in Tables 1 and 2.

**Table 1**

Coefficients of digital controller in the proportional branch at various accuracy approximation of the ideal $G(s) = K_p$ transfer coefficient

| | $a_0$ | $a_1$ | $a_2$ | $b_1$ | Note |
|---|---|---|---|---|---|
| I | $K_p$ | — | — | — | |
| II | $\frac{3K_p}{2}$ | $-\frac{K_p}{2}$ | — | — | $D^*(s)$ rational integral function |
| III | $\frac{11}{6} K_p$ | $-\frac{7}{6} K_p$ | $\frac{2}{6} K_p$ | — | |
| IV | $\frac{5}{2} K_p$ | $\frac{1}{2} K_p$ | — | 2 | $D^*(s)$ rational fraction function |

*a)* Using row I. of both tables:

$$D^*(s) = K_p \left(1 + \frac{T}{T_i} \frac{1}{1 - e^{-Ts}}\right) = K_p \frac{\left(1 + \frac{T}{T_i}\right) - e^{-Ts}}{1 - e^{-Ts}} \tag{8}$$

**Table 2**

Coefficients of digital controller in the integrating branch at various accuracy approximation of the ideal $K_p/T_is$ continuous transfer function

| | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $b_1$ | $b_2$ | $b_3$ | Integration method |
|---|---|---|---|---|---|---|---|---|
| I | $\frac{T}{T_i}K_p$ | — | — | — | $-1$ | — | — | Square law |
| II | $\frac{T}{2T_i}K_p$ | $\frac{T}{2T_i}K_p$ | — | — | $-1$ | — | — | Trapezoidal law |
| III | $\frac{T}{3T_i}K_p$ | $\frac{4T}{3T_i}K_p$ | $\frac{T}{3T_i}K_p$ | — | 0 | $-1$ | — | 1/3 Simpson rule |
| IV | $\frac{3T}{8T_i}K_p$ | $\frac{9T}{8T_i}K_p$ | $\frac{9T}{8T_i}K_p$ | $\frac{3T}{8T_i}K_p$ | 0 | 0 | $-1$ | 3/8 Simpson rule |

and

$$W(s) = \frac{1}{T} D^*(s)\, G_T(s) = K_p \left( \frac{1 - e^{-Ts}}{T_s} + \frac{1}{T_i s} \right) \tag{9}$$

*b)* The row II in Table 1 may be used in case the $P$ branch is to be implemented with higher accuracy. The pulse transfer function of the digital controller using square law integration is again:

$$D^*(s) = K_p \frac{\left(1{,}5 + \frac{T}{T_i}\right) - 2e^{-Ts} + 0{,}5e^{-2Ts}}{1 - e^{-Ts}} \tag{10}$$

The transfer locus of PI-type digital controller for the cases *a)* and *b)* is given in Fig. 4, provided $T_i = 10\ T$.

**Table 3**

Coefficients of digital controller in the differentiating branch at various accuracy approximation of the ideal $K_dT_ds$ continuous transfer function

| | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $b_1$ | $b_2$ | $b_3$ |
|---|---|---|---|---|---|---|---|
| I | $\frac{T_d}{T}K_p$ | $-\frac{T_d}{T}K_p$ | — | — | — | — | — |
| II | $\frac{2T_d}{T}K_p$ | $-\frac{2T_d}{T}K_p$ | — | — | 1 | — | — |
| III | $\frac{3T_d}{T}K_d$ | 0 | $-\frac{3T_d}{T}K_p$ | — | 4 | 1 | — |
| IV | $\frac{8T_d}{3T}K_p$ | 0 | 0 | $-\frac{8T_d}{3T}K_p$ | 3 | 3 | 1 |

**Table 4**

The deviation percentage of the amplitude characteristics of the integrating branch from the ideal, the data hold is not considered

| | I Square | II Trapezoid | III 1/3 Simpson | IV 3/8 Simpson |
|---|---|---|---|---|
| | rule | | | |
| $\omega = \frac{\omega_S}{6}$ | +4.7 | − 9.3 | +0.78 | +1.12 |
| $\omega = \frac{\omega_S}{4}$ | +9.5 | −21.4 | +1.06 | +1.19 |

## 3. Open-cycle PI digital controller

PI-type organ is often used for a single and multichannel controller because of its simple structure. Stepping motor, electric servomotor or electronic integrating organ may be used as the integrating element. The design method is shown for PI controller because of its simple structure.

### *3.1. Digital controller using stepping motor*

The advantage of stepping motor is that besides integration it implements the digital-analogue converter and zero — order hold also.

Disadvantage of stepping motor is that the output of controller is an angular displacement. Because of this no direct feedback from the output of controller to its input is possible.

In a number of problems it is possible to make a profit out of this disadvantage and the stepping motor may act as an actuator organ of the control circuit.

#### *3.1.1. Block diagram of PI controller*

Inserting a reversible counter into the proportional branch that realizes differentiation in the time-domain *(Fig. 5)* the adjustment of the proportionality factor is carried out by a readout unit ($K_1$) connected to the reversible counter. The readout unit may contain e.g. two variable frequency ($f_A$, $f_B$) pulse generators. The reversible counter is reset by the $f_A$ frequency pulse sequence, during reset signals of the $f_B$ frequency pulse generator are put to the stepping motor through a gate circuit. Then

$$K_1 = \frac{f_B}{f_A}$$

Readout of the reversible counter may also be done by counting-type multiplyer unit implementing coded-signal — pulse number conversion.

The counting-type multiplyer unit adjusting $K_2$ coefficient is inserted into the integrating branch. The pulse sequences from the two branches

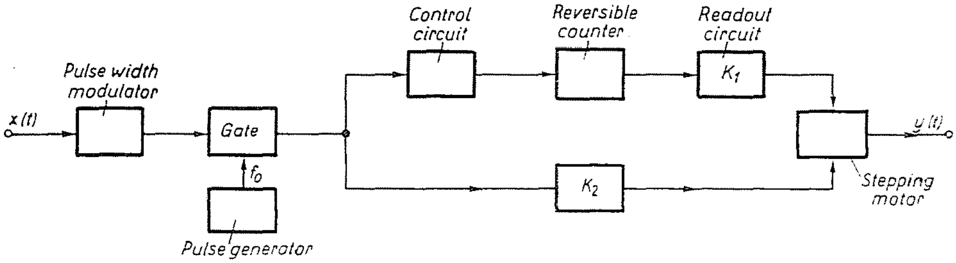


*Fig. 5.* Block diagram of PI controller

arriving one after the other succesively are put on the stepping motor. A pulse number proportional to the sampled signal should be put on the reversible counter and the ($K_2$) counting-type multiplyer unit. In case of continuous input this can be done by converting the input to pulse width modulated signal and gating an $f_0$ frequency reference oscillator signals with it.

### *3.1.2. Determination of pulse transfer function of PI controller*

The $z^{-1}$ term necessary for differentiation is implemented by the storage of pulse number corresponding to the value of sampled signal at the instant $t = (n-1)T$. The amplitudes of sampled signal at $t = (n-1)T$ and $t = nT$ instants be $x(nT - T)$ and $x(nT)$; the pulse numbers corresponding to the sampled values $m_{n-1}$ and $m_n$, respectively. The output of the reversible counter after $t = nT$ instant will be:

$$y_1(nT) = x(nT) - x(nT - T) \qquad \text{and} \qquad \Delta m_n = m_n - m_{n-1}$$

Let the initial angular displacement of the stepping motor be $y_2(nT-T)$ At $t = nT$ the input of the motor is $K_2 m_n$ pulses. Having turned by an angle corresponding to these pulses the input of the motor is $K_1 \Delta m_n$ pulses. The motor sums up the number of pulses. Let us assume, for the sake of simplicity, that the motor takes the total $M(nT)$ input at $t = nT$ instant, then

$$y_2(nT) = y_2(nT - T) + K_3 M(nT) \tag{11}$$

where $K_3$ is the transfer coefficient of the motor and

$$M(nT) = K_1 \Delta m_n + K_2 m_n, \qquad \text{and} \qquad M(z) = K[K_1(1 - z^{-1}) + K_2]X(z)$$

The pulse transfer function of the motor is:

$$G_m(z) = \frac{Y_2(z)}{M(z)} = \frac{K_3}{1 - z^{-1}} \tag{12}$$

Considering the fact that the motor implements zero-order hold also, the over-all pulse transfer function will be:

$$D(z) = \frac{Y_2(z)}{X(z)} = K_4\left[1 + \frac{K_2}{K_1}\frac{1}{1 - z^{-1}}\right]$$

where $K_4 = K\,K_1K_3$.

Comparing this with equation (8), the proportionality factor, and the time-constant of integration will be:

$$K_p = K_4 = K\,K_1\,K_3\,, \quad \text{and} \quad T_i = T\frac{K_1}{K_2} \tag{13}$$

respectively.

### *3.1.3. Structure of PI controller*

Simplified block diagram of the PI controller and its time diagram are shown in *Figs. 6* and *7*, respectively.

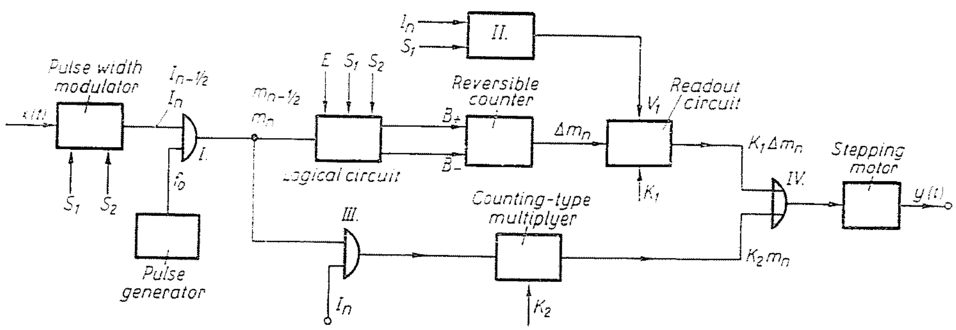


*Fig. 6.* Simplified block diagram of PI controller

For the implementation of the $z^{-1}$ term double the sampling frequency in the differentiating branch. The sampling instants will be $t = nT - T/2$ and $t = nT$.

Thus the Laplace-transform of the differentiating element is:

$$Y(s) = K_4\left[\frac{1 - e^{-\frac{T}{2}s}}{T_s} + \frac{1}{T_i\,s}\right]X(s) \tag{14}$$

Because of this the accuracy of proportional component deteriorates. With the same accuracy at the same sampling frequency the allowable maximum frequency component of the input is halved.

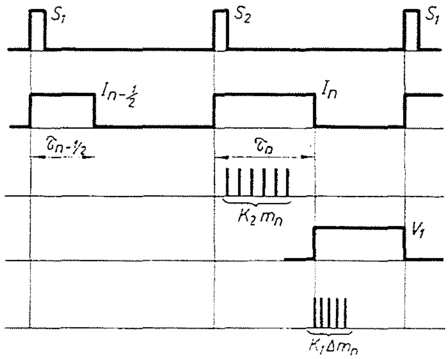


*Fig. 7.* Time diagram of PI controller

If a positive sign is represented by logical YES level at the $E$ sign bit of the pulse sequence, at the instants of $S_1$ and $S_2$ the logical variables are $E_1$ and $E_2$, respectively, then the logical equations driving the summing ($B_+$) and subtracting ($B_-$) inputs of the reversible counter are

$$B_+ = \bar{E}_1 S_1 \vee E_2 S_2, \quad \text{and} \quad B_- = E_1 S_1 \vee \bar{E}_2 S_2$$

respectively, where $\vee$ assigns inclusive *OR* (logical sum).

At the time of $S_1$ a pulse assigned $I_{n-1/2}$, whose width is $\tau_{n-1/2}$ proportional to the amplitude $x(nT - T/2)$ lets through Gate I to the reversible counter $m_{n-1/2}$ pulses. At the time $S_2$ a pulse assigned $I_n$, whose width is $\tau_n$ proportional to the amplitude $x(nT)$ makes the motor step by $K_2 m_n$ pulses through the counting-type multiplying unit. At the instant $I_n$ extincts at the output of Gate II $V_1$ control signal appears which is ended by the next $S_1$ sampling signal appearing at the instant $t = nT + T/2$. (This can be so done for example, that the trailing edge of $I_n$ sets and the leading edge of $S_1$ resets a flip-flop.)

### 3.1.4. *Design of PI controller*

The allowable maximum input frequency — ($\omega_M$) — is determined by the accuracy of the proportional and integrating components, respectively. The maximum frequency can be evaluated graphically using the amplitude and phase characteristics already given.

A possible solution of pulse width modulation is the time-transformation method. If the slope of the linear ramp is $\alpha$, the input is $x(t) = A \sin \omega t$ sinusoidal signal, then the width of the pulse appearing at the $nT$ instant is

$$\tau_n = \frac{\tau_0 |\sin \omega n T|}{1 - \omega \tau_0 |\cos \omega n T|} \cong \tau_0 |\sin \omega n T| \quad \text{and} \quad \tau_0 = \frac{A}{\alpha} \tag{15}$$

Since the relative error of pulse width is:

$$\delta = \omega \tau_0 |\cos n \omega T|$$

the necessary condition is:

$$\omega_M \tau_0 = \omega_M \frac{A}{\alpha} \leq \delta_{\max}$$

The amplitude of input varies between $A_m$ and $A_M$. Altering the amplitude the width of the corresponding pulses also varies

$$\tau_{0\,\min} = \frac{A_m}{\alpha} \quad \text{and} \quad \tau_{0\,\max} \leq \frac{A_M}{\alpha}\,. \tag{16}$$

For the slope of the linear ramp the following condition results:

$$\alpha \geq \frac{\omega_M A_M}{\delta_M}$$

The signals of the $f_0$ frequency generator are gated by the $\tau_n$ width pulse, the minimum and maximum number of pulses will be:

$$m_{\min} \leq f_0 \tau_{0\,\min}\,; \quad m_{\max} \leq f_0 \tau_{0\,\max} \tag{17}$$

respectively.

Differentiation occurs by producing the difference of $m_n$ and $m_{n-1/2}$. The difference at the sinusoidal input is $\Delta x \leq A\omega T$, and the number of pulses corresponding to it is:

$$\Delta m \leq f_0 \tau_0 \omega T \tag{18}$$

If $(\Delta m)_{\min}$ minimum pulse number is required in the reversible counter then it determines the minimum frequency of the input in case of a given minimum input amplitude.

Capacity of the reversible counter is determined by $\tau_{0\,\max}$ corresponding to the maximum input and the maximum input frequency. So

$$(\Delta m)_{\max} \leq f_0 \tau_{0\,\max} \omega_M T$$

The minimum sampling period is determined by the maximum width of $\tau_{n-1/2}$, and $\tau_n$ and the maximum tracking frequency of the motor. It was assumed that the motor could follow $K_2 m_n$ pulses. Since their frequency is max. $f_0$, and if $f_{mot}$ is the tracking frequency of the motor, then

$$f_{mot} \geq f_0$$

The $K_1 \Delta m_n$ pulses are sensed by the motor after signal $I_n$ ended. So the minimum of sampling period is:

$$T_{min} = 2\tau_{0\,max} + \frac{(K_1 \Delta m_n)\,max}{f_{mot}} \tag{19}$$

The transfer coefficient of the proportional branch depends on the number of steps belonging to unit input. The number of pulses corresponding to unit amplitude input is

$$K_1 \tau_{01} f_0 = K_1 \frac{\tau_{0\,max}}{A_M} f_0$$

and so

$$K_p = K_1 K_3 \frac{\tau_{0\,max}}{A_M} f_0 \tag{20}$$

### *3.2. Digital controller using electric servomotor*

The summing and zero-order hold is implemented by a constant velocity servomotor. The motor may be driven, for example, by a constant amplitude signal whose width corresponds to the number of pulses.

The theoretical structure of the PI controller differs from that given in *Fig. 6* that the output of OR Gate IV drives the motor through a pulse number — pulse width converter. It is advisable to make the units in such a way that the periodicity of output pulses should be constant.

$K_K$, the transfer coefficient of the pulse number — pulse width converter is determined by the output pulse width corresponding to unit input (single pulse).

The controller's proportionality factor is the output angular displacement in case of unit input. The number of pulses given for unit input is $\tau_{01} f_0$, so if the motor is running with $\omega_0$ constant angular velocity

$$K_p = K_k \tau_{01} f_0 \omega_0 = K_k \frac{\tau_{0M}}{A_M} f_0 \omega_0 \tag{21}$$

The time-constant of integration is given by

$$T_i = T\frac{K_1}{K_2}$$

Other characteristics of the controller can be evaluated simiarly to that given in point 3.1.

### 3.3. *Electronic digital controller*

#### 3.3.1. *Structure of PI controller*

Open-cycle PI controller can be implemented based on the shown schematic in *Fig. 8*. Its time diagram is given in *Fig. 9*. The counting-type multiplyer inserted into the proportional branch transmits $K_1 m_n$ pulses to the pulse

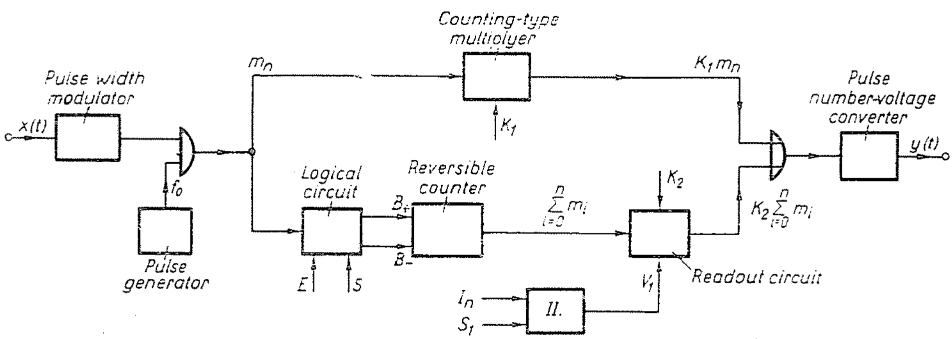


*Fig. 8.* Block diagram of electronic PI controller

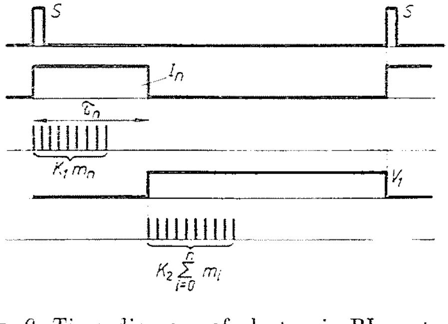


*Fig. 9.* Time diagram of electronic PI controller

number — voltage ($D/A$) converter placed at the output. In the integrating branch $K_2 m_n$ pulses are given to or subtracted from the preceding output of integrating branch depending upon the sign of $x(nT)$.

Having completed the proportional component (at the end of $I_n$ signal) use the stored

$$K_2 m_n + K_2 \sum_{i=0}^{n-1} m_i = K_2 \sum_{i=0}^{n} m_i$$

pulse number. The integrated component is put to the input of the $D/A$ converter using some kind of readout from the reversible counter.

### 3.3.2. *Transfer function of PI controller*

There is a unit in the proportional branch of the controller multiplying by $K_1$ factor, the integrating branch performs the summation by square law. The pulse transfer function of the controller is as follows,

$$D(z) = K\left[K_1 + \frac{K_2}{1 - z^{-1}}\right] = KK_1\left[1 + \frac{K_2}{K_1}\frac{1}{1 - z^{-1}}\right]$$

where $K$ is the over-all transfer coefficient of the pulse width modulator and the $D/A$ converter.

Using Eq. (8) and taking into consideration the effect of the zero-order hold placed at the output yields

$$K_p = KK_1; \qquad T_i = T\frac{K_1}{K_2}$$

The controller can be designed in the way shown previously.

Block diagram of open-cycle PID controller is shown in Fig. 10. The system can be built up already by joining the appropriate parts of Figs. 6 and 8.

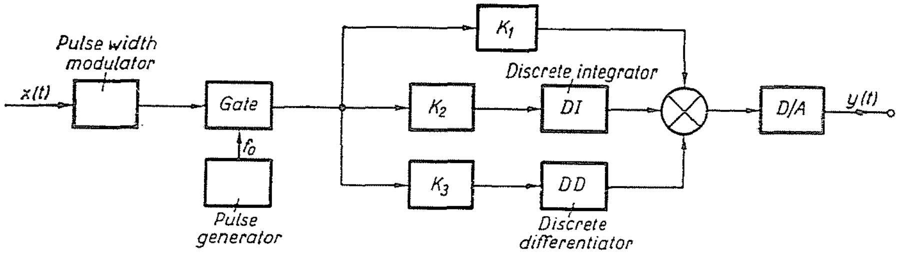


*Fig. 10.* Schematic diagram of open-cycle PID controller

## 4. Realization of general digital controller

Designing sampled data systems the optimal solution in many cases requires a digital controller having a higher power of $z^{-1}$ than that given in (8) and (10). Because of this more complex controllers are necessary than the PID controller. If the pulse transfer function to be implemented is

$$D(z) = \frac{a_0 + a_1 z^{-1} + \ldots + a_m z^{-m}}{1 + b_1 z^{-1} + \ldots + b_n z^{-n}} \tag{23}$$

and the poles in the $z$ plane are $p_1, p_2, \ldots p_n$, then by expanding it into partial fractions results in

$$D(z) = \sum_{i=1}^{n} \frac{K_i z^{-1}}{1 - p_i z^{-1}} \tag{24}$$

where $K_i$ is the coefficient of the partial fraction corresponding to $p_i$ pole.

The particular terms are realized separately and the outputs of the particular units are summed up. The relation between $Y_i(z)$ and $X(z)$, $z$ transform of the output of the $i$-th term and the input respectively is

$$Y_i(z) = K_i z^{-1} X(z) + p_i z^{-1} Y_i(z) \tag{25}$$

Eq. (25) may be modelled by a closed-loop logical block *(Fig. 11)*. For the effect of the sampling signal a number of pulses proportional to $K_i x(nT)$ gets onto the reversible counter. Sign of $x(nT)$ is considered by a logical

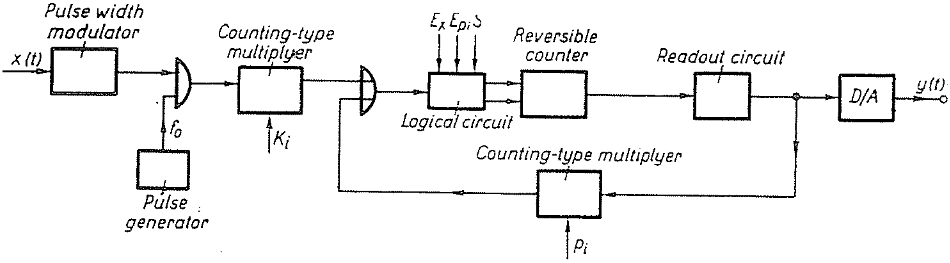


*Fig. 11.* Implementation of $D_i(z) = \frac{K_i z^{-1}}{1 - p_i z^{-1}}$ pulse transfer function

circuit. Then the content of the reversible counter will be read out, and the pulse number proportional to the value of $y(nT)$ will be put on the $D/A$ converter. At the same time a number of pulses corresponding to $p_i y(nT)$ — the sign of $p_i$ is also taken into account — gets on one input of the reversible counter. The counter produces the sum

$$K_i x(nT) + p_i y_i(nT) = y_i(nT + T)$$

but it is used only at the next sampling instant producing the storage.

If $p_i = 1$ the circuit becomes an integrating organ. By using it closed-loop PI controller can readily be implemented.

## Summary

In the case of given dynamic requirements, the coefficients of the digital controller are given in the paper, taking the hold circuit of the circuit into consideration. The design method proposed by the author can be employed for designing an arbitrary pulse transfer function. In the case of an integrating organ the various known numerical integration methods (square, trapezoidal, Simpson rule) can be employed directly.

The logical block diagram of a PI controller is given for the cases when the integrating element is a stepping motor, a constant velocity servomotor, or an electronic integrating organ. The transfer functions of the circuits are determined. A generalized design method is described, considering the lower and upper limit frequency of the input signal, and the dynamics of the input signal level.

For the electronic realization of a general pulse transfer function a method based on developing into partial functions has been elaborated. The individual partial fractions can be realized by a single-loop feedback network built up of units of the numerator of the system. The logical block diagram of the basic term of the general pulse transfer function is given.

## References

1. KESSLER, G.—ERLANGEN: Digitale Regelung der Relation zwei Drehzahlen. Elektrotechn. Z. **82**, 574—579. (1961).
2. KRUG, E. K.: Avtomatika i telemechanika XXIII 4 (1962)
3. ALEKSANDRIDI, T. M.—DILIGENSKY, S. N.: Digital Controllers. International Federation of Automatic Control Congress 1963. 519.
4. FRITZSCHE, W.: Vorteile und Grenzen digitaler Drehzahlregelungen. Regelungstechnik **12**, 104—112 (1964).
5. RECHBERGER, H.: The Application of Digital Differential Analysers in Control Loops, International Federation of Automatic Control Congress 1963. 065.

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