RELIABILITY INVESTIGATIONS OF ELECTRIC DISTRIBUTION NETWORKS

by

Z. REGULY

Department of Electric Power Plants, Polytechnical University, Budapest

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Presented by Prof. Dr. O. P. GESZTI

An important factor among the requirements to be satisfied in the distribution of electric power is that of service reliability. Although the degree of service reliability of a given network can be judged empirically by qualitative estimation, a numerical investigation seems a better approach to the problem. Methods of probability calculus and mathematical statistics offer the possibility for a correct evaluation of service reliability. Therefore, it is considered appropriate to give a short survey of mathematical concepts and operations applied for this purpose.

1. Mathematical survey

1.1 *Applied basic concepts of set theory*

Set : as a mathematical conception, is a collection of elements having some kind of property in common (e.g. pages of a book, persons forming the audience of a lecture, etc.).

Symbol: A, B, C, etc.

Elements of a set : a, b, h, etc.

a is an element of set A: $a \in A$,

a is not an element of set A: $a \notin A$.

Subset : is a part of the basic set (e.g. the set A of books dealing with mathematics is a subset of the set H of all books). Symbol: $A \subset H$. In an extreme case a subset may be equal to the basic set.

A set is called *empty set* or *zero set*, when having no elements. Symbol:

$$0.$$

Let H be a basic set with subsets A, $B \ldots K$. Applying the *operations of set algebra*, new sets may be obtained:

Union of subsets A and B: is the set of those elements of set H, which are elements of A or B or both. Symbol:

$$A \cup B.$$

Intersection of subsets A and B:
is the set of elements of H, which belong to both A and B. Symbol:

$$A \cap B.$$

Difference set of A and B:
is the set of those elements of A, which do not belong to B. Symbol:

$$A - B.$$

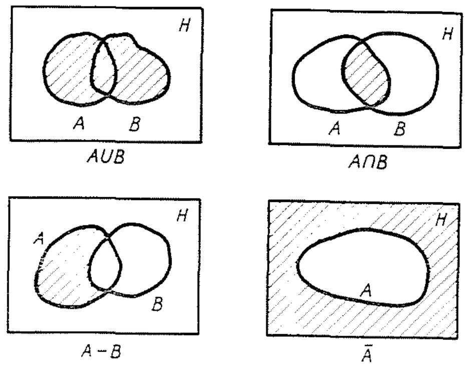

Fig. 1.1

Complement of A:
is the difference set of basic set H, and subset A. Symbol: CA or $\bar{A}$

$$CA = \bar{A}, = H - A.$$

Set of sets T is a sum of subsets belonging to the basic set H for which

$$\begin{array}{lcl} A \subset T & \text{and} & A \cup B \subset T \\ B \subset T & & A \cap B \subset T \\ & & A \cap B \subset T \\ & & 0 \subset T \end{array}$$

i.e., also the results of operations performed with the subsets are elements of T (thus, T is in fact the set of subsets).

These operations are clearly represented by the so-called Venn diagrams (see Fig. 1.1). The results are shown by the shaded portions of the figures.

Application of set theory for the description of network reliability

Let H be the set of service conditions of a network
A be the set of available service conditions of element a

B be the set of available service conditions of element b
then, the meaning of $A \cup B$: either a or b or both are available
the meaning of $A \cap B$: both a and b are available

$\bar{A}$: a is unavailable
$A \cap B$: a is unavailable and
b is available.

First, an element is considered as available, if free of any defect and is considered as being unavailable, if defective. Later on, the condition "unavailable" will be extended to the condition in which the element is taken out of service for maintenance (Section 4). Element "a" will be understood to mean a facility installed at point "a" of the network. Hence, $\bar{A}$ means that no element is available at point "a", or in other words, element "a" has become defective and is being repaired or under replacement. After replacement the new facility will be the one called element "a".

1.2 *Fundamental concepts of probability theory*

The service reliability of networks can be expressed by probability figures. As an introduction it seems appropriate to give a brief account of the axioms of probability calculus. Since our investigations will be based on the concepts of set theory, in the following sections a short survey of the set-theoretical axioms of Kolmogorov will be given.

I. Let the set of elementary events of a phenomenon (e.g. the set of services conditions of a network) be H.

II. Let T be a (set of sets), which is a collection (set) of subsets (A, B) of H, where the sets

$$A \cup B$$
$$A \cap B$$
$$A \cap \bar{B}$$
$$0$$

also belong to T.

Each one of sets A, B represents one portion of elementary events of the phenomenon, e.g. A represents the available condition of circuit breakers, and B the available condition of transformers, whereas $\bar{A}$ and $\bar{B}$ represent the unavailable condition of circuit breakers and transformers, respectively.

III. Let H also be a member of set of sets T (H is the basic set, i.e. the collection of all possible service conditions), while the set of sets T is the collection of groups of elements selected according to II, i.e. the subsets (A, B, ...) of set H, with set H itself included.

IV. To each element of T, (where each element is a set in itself) a number $P(A)$ can be attached, which is called probability. Its value is

$$0 \leq P(A) \leq 1.$$

V. To the basic set H the value 1 is attached (expressing the fact that one out of all possible service conditions will surely occur). To set 0 the value 0 is attached (it is impossible that none of the service conditions will occur).

VI. If $A \cap B = 0$ (A and B mutually exclude each other, i.e. their simultaneous occurrence is impossible):

$$P(A \cup B) = P(A) + P(B).$$

VII. Generalization of Axiom VI: If $A_1, A_2, \ldots A_n$ elements of T and $A_i \cap A_k = 0$ then the probability is:

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_{0=1}^{n} P(A_j).$$

From the theorems that can be deduced from the above axioms only those are summarized, which will be utilized in the following discussion:

a) Let us assume that the sets of events — briefly referred to as events, in the followings — $A_1, A_2, A_3 \ldots A_n$ are independent of each other. This means that none of them is the cause of another (e.g. while a breakage of an insulator string and a line fault are interdependent events: a busbar fault in the distribution network and a transmission line fault in the 120 kV grid are independent events). Thus, if events $A_1, A_2, \ldots A_n$ are independent, then the probability of their simultaneous occurrence is

$$P(A_1 \cap A_2 \ldots \cap A_n) = P(A_1) \cdot P(A_2) \ldots P(A_n) = \prod_{k=1}^{n} P(A_k).$$

b) If the condition set up by Axiom VI fails to apply

$$A \cap B \neq 0\,,$$

then, according to a), if A and B are independent of each other:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

In the case of three service conditions (A, B, C), if

$$A \cap B \neq 0, \quad A \cap C \neq 0, \quad B \cap C \neq 0\,,$$

then, if A, B and C are independent of each other:

$$\begin{aligned} P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A) \cdot P(B) - \\ - P(A) \cdot P(C) - P(B) \cdot P(C) + \\ + P(A) \cdot P(B) \cdot P(C). \end{aligned}$$

2. Investigation of networks

The dependability of a network can be determined by means of two characteristic properties, these being the reliability rate and the unavailability rate.

The *reliability rate* of a network, or network element "A" is expressed by the number

$$P(A) = R_A$$

determining the probability of the occurrence of service conditions (A) under which the network is capable of supplying the load.

The *unavailabilty rate* of a network or network element "A" is expressed by the number

$$P(\bar{A}) = Q_A$$

expressing the probability of service conditions ($\bar{A}$) under which the network is incapable of supplying the load.

Since a network is either capable or incapable of supplying the load, we may write

$$P(A \cup \bar{A}) = P(A) + P(\bar{A}) = \underline{R_A + Q_A = 1}\,.$$

2.1 *Series arrangement of network elements*

Let "a" and "b" network elements be connected in series (Fig. 2.1). Further, let A and B be the sets of service conditions of network elements a and b respectively, with said elements available.

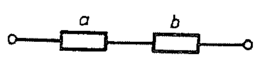

Fig. 2.1

The resultant network is available, if both elements are available (Fig. 2.2):

$$A \cap B;$$

the probability of this condition is

$$P(A \cap B) = \underline{R_{\text{series}} = R_A \cdot R_B}\,.$$

The resultant network is unavailable, if either "a" or "b", or both, are unavailable (Fig. 2.2):

$$\bar{A} \cup \bar{B}$$

the probability of this condition is

$$P(\bar{A} \cup \bar{B}) = Q_{\text{series}} = Q_A + Q_B - Q_A \cdot Q_B.$$

Since, generally $Q_i \ll 1$, with good approximation:

$$Q_{\text{series}} = Q_A + Q_B.$$

In the following, by expending the concept of series network elements, the method of the so-called "complete system of events" will be described, which has been developed for calculating the resultant reliability rate or unavailability rate of a network.[1]

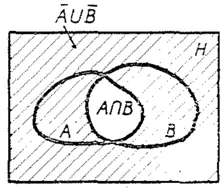

Fig. 2.2

Let us write all combinations that can be obtained from the sets of service conditions of network elements "a" and "b", which can occur simultaneously.

$A \cap B$ both components available
$\bar{A} \cap B$ a out, b available
$A \cap \bar{B}$ a available, b out
$\bar{A} \cap \bar{B}$ both out.

Any pair of the above cases can be excluded, since no element can be available and unavailable simultaneously. Thus, their probabilities can be added (Axiom VI).

Since all possible service conditions (subsets) have been written, the union of subsets means the basic set (H) of all possible service conditions. Since, however, one of these service conditions will certainly occur, the probability of the basic set is 1 (as stated by Kolmogorov's Axiom V).

Hence, the probability of possible service conditions is as follows:

$$\begin{array}{ll} P(A \cap B) = R_A R_B & \text{available} \\ P(\bar{A} \cap B) = Q_A R_B & \text{unavailable} \\ P(A \cap \bar{B}) = R_A R_B & \text{unavailable} \\ \underline{P(\bar{A} \cap \bar{B}) = Q_A Q_B} & \text{unavailable} \\ P(H) \quad\quad = 1. & \end{array}$$

[1] This method was first used in this country by the Budapest Electricity Works.

The sum of probabilities is 1. From the table the probability of availability

$$R_A R_B = R_{\text{series}}$$

and that of unavailability

$$Q_A R_B + R_A Q_B + Q_A Q_B = Q_{\text{series}}$$

can immediately be seen.

Two different results are obtained for the value of Q_{series}. These two results, however, are equal:

$$Q_A R_B + R_A Q_B + Q_A Q_B = Q_A (1 - Q_B) + (1 - Q_A) Q_B + Q_A Q_B =$$
$$= Q_A - Q_A Q_B + Q_B - Q_A Q_B + Q_A Q_B = Q_A + Q_B - Q_A Q_B.$$

Although a rather simple example has been chosen to demonstrate the application of the method, the principles used are of general validity and, therefore, there is no need to verify its applicability to more involved cases.

2.2 *Parallel arrangement of network elements*

Let circuit elements a and b be connected in parallel. Let A and B represent the availabilities of circuit elements a and b, respectively.

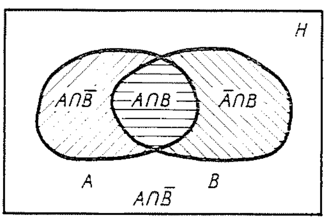

Fig. 2.3

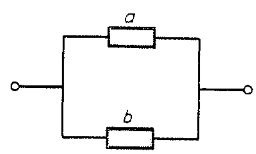

Fig. 2.4

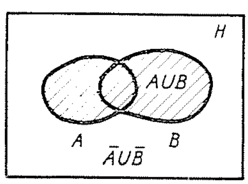

Fig. 2.5

The resultant network is available, if either "a", or "b", or both, are available (Fig. 2.5)

$$A \cup B,$$

the probability of which is

$$P(A \cup B) = \underline{R_{\text{parallel}} = R_A + R_B - R_A R_B}$$

(the product term cannot be neglegted, since $R \approx 1$!). The resultant network is unavailable, if "a" and "b" are simultaneously unavailable (Fig. 2.5):

$$\bar{A} \cap \bar{B},$$

the probability of which is

$$P(\bar{A} \cap \bar{B}) = \underline{Q_{\text{parallel}} = Q_A Q_B} .$$

Using the method of "complete system of events":

Service conditions	Probabilities	Resultant network
$A \cap B$	$R_A R_B$	available
$\bar{A} \cap B$	$Q_A R_B$	available
$A \cap \bar{B}$	$R_A Q_B$	available
$\bar{A} \cap \bar{B}$	$Q_A R_B$	unavailable

$$R_A R_B + Q_A R_B + R_A Q_B = R_{\text{resultant}}$$
$$Q_A Q_B = Q_{\text{resultant}}$$

2.3 *Combination of series and parallel elements*

Let network elements a, b, c be connected as shown in Fig. 2.6. The network is built up of series and parallel elements. The resultant network is available, if "a" and "b" or "c" (or "a" and "b" and "c") are simultaneously available (Fig. 2.7):

$A \cap (B \cup C)$, the probability of which is

$$P[A \cap (B \cap C)] = R_A (R_B + R_C - R_B R_C) = R_A (1 - Q_B Q_C) .$$

The resultant network is unavailable (Fig. 2.7), if

$$\bar{A} \cap (\bar{B} \cap \bar{C}),$$

the probability of which is

$$P[\bar{A} \cup (\bar{B} \cap \bar{C})] = Q_A + Q_B Q_C - Q_A Q_B Q_C .$$

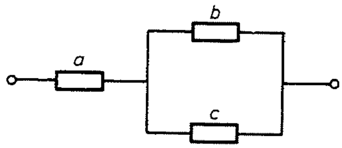

Fig. 2.6

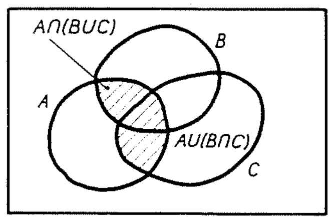

Fig. 2.7

Now applying the method of "complete system of events" (Fig. 2.8):

Service conditions	Probabilities	Resultant network
$A \cap B \cap C$	$R_A R_B R_C$	available
$\bar{A} \cap B \cap C$	$Q_A R_B R_C$	unavailable
$A \cap \bar{B} \cap C$	$R_A Q_B R_C$	available
$A \cap B \cap \bar{C}$	$R_A R_B Q_C$	available
$\bar{A} \cap \bar{B} \cap C$	$Q_A Q_B R_C$	unavailable
$\bar{A} \cap B \cap \bar{C}$	$Q_A R_B Q_C$	unavailable
$A \cap \bar{B} \cap \bar{C}$	$R_A Q_B Q_C$	unavailable
$\bar{A} \cap \bar{B} \cap \bar{C}$	$Q_A Q_B Q_C$	unavailable

$$R_{\text{resultant}} = R_A (R_B R_C + Q_B R_C + R_B Q_C) .$$

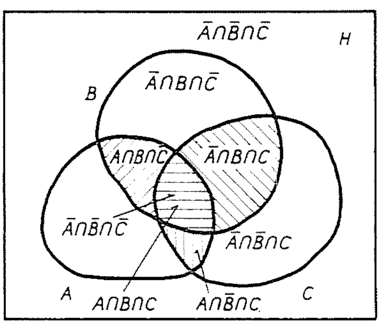

Fig. 2.8

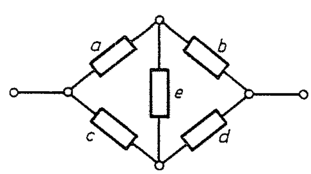

Fig. 2.9

2.4 *Mesh-connected elements*

Let the connection of elements be as shown in Fig. 2.9: The network is available, if between the terminals at least one path exists along which all elements are available. Thus it is assumed that in the case if any one possible path is available, the network is also available.

The possible paths in the network:

$$A\cap B$$
$$A\cap E\cap D$$
$$C\cap E\cap B$$
$$C\cap D\,.$$

The network is available, if one or more of the above paths are available:

$$(A\cap B)\cup(A\cap E\cap D)\cup(C\cap E\cap B)\cup(C\cap D).$$

The probability of this condition is:

$$\begin{aligned}R_e = {} & P(A\cap B)+P(A\cap E\cap D)+P(C\cap E\cap B)+P(C\cap D)-\\ & -P(A\cap B\cap A\cap E\cap D)-P(A\cap B\cap C\cap E\cap B)-\\ & -P(A\cap B\cap C\cap D)-P(A\cap E\cap D\cap C\cap E\cap B)-\\ & -P(A\cap E\cap D\cap C\cap D)-P(C\cap E\cap B\cap C\cap D)+\\ & +P(A\cap B\cap A\cap E\cap D\cap C\cap E\cap B)+\\ & +P(A\cap B\cap A\cap E\cap D\cap C\cap D)+\\ & +P(A\cap B\cap C\cap E\cap B\cap C\cap D)+\\ & +P(A\cap E\cap D\cap C\cap E\cap B\cap C\cap D)-\\ & -P(A\cap B\cap A\cap E\cap D\cap C\cap E\cap B\cap C\cap D)\,,\end{aligned}$$

since $A\cap A = A$; $B\cap B = B$; etc.

$$\begin{aligned}R_e = {} & P(A)\,P(B)+P(A)\,P(D)\,P(E)+P(B)\,P(C)\,P(E)+\\ & +P(C)\,P(D)-P(A)\,P(B)\,P(D)\,P(E)-P(A)\,P(B)\,P(C)\,P(E)-\\ & -P(A)\,P(B)\,P(C)\,P(D)-P(A)\,P(C)\,P(D)\,P(E)-\\ & -P(B)\,P(C)\,P(D)\,P(E)+2\,P(A)\,P(B)\,P(C)\,P(D)\,P(E)\,.\end{aligned}$$

The same result may be obtained by using the method of "complete system of events", although in that case the latter procedure is somewhat more lengthy.

2.5 *System provided with a reserve element*

Let us consider a bus of high degree of service reliability. With good approximation

$$R_S = 1\,.$$

Two substations (I and II) are fed from this bus through elements "a" and "b", further, a reserve supply is available through element "t". (Fig. 2.10)

The power demands of substations I and II are equal and correspond to the transfer capacity of "a" and "b" respectively, further, the transfer capacities of "*a*", "*b*" and "*t*" are also equal. Therefore, the network is considered available, if at least two transmission branches are available. The network is capable of supplying one half of the load with only one transmission branch available.

The reliability of the network can be studied by means of the method of "complete system events".

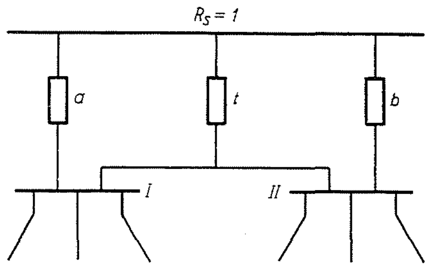

Fig. 2.10

Service conditions:	Probabilities:	Resultant network:
$A \cap B \cap T$	$R_A R_B R_T$	available
$A \cap B \cap T$	$Q_A R_B R_T$	available
$A \cap B \cap T$	$R_A Q_B R_T$	available
$A \cap B \cap T$	$R_A R_B Q_T$	available
$A \cap B \cap T$	$Q_A Q_B R_T$	available for half load
$A \cap B \cap T$	$Q_A R_B Q_T$	available for half load
$A \cap B \cap T$	$R_A Q_B Q_T$	available for half load
$A \cap B \cap T$	$Q_A Q_B Q_T$	unavailable.

Summing up the rows of acceptable service conditions, the following resultants are obtained:

$$R_e = R_A R_B R_T + Q_A R_B R_T + R_A Q_B R_T + R_A R_B Q_T .$$

$$Q_e = Q_A Q_B Q_T .$$

The probability of half-load network availability is:

$$R_{e\,\mathrm{half}} = Q_A Q_B R_T + Q_A R_B Q_T + R_A Q_B Q_T .$$

3. Selection of probability variables

With respect to service reliability of a system two questions may arise:

a) how many unavailabilities occur within a given period,
b) what is the duration of these unavailabilities.

Correspondingly, two probability variables should be introduced:

a) frequency of unavailabilities,
b) duration of unavailabilities.

It seems convenient to jointly investigate the two variables. This can be done in two different ways.

I. The durations of unavailabilities may be handled as discrete variables, i.e. after selecting characteristic outage durations, such as $T_0 = 0.1$, 0.5, 2, 5 hours, etc. The number of unavailabilities outlasting the selected periods T_0 should be investigated:

$$\bar{A}(t \geqq T_0),$$

where: t [hours] duration of one outage,
T_0 [hours] selected discrete periods.

Value of the unavailability rate (probability of failure) is:

$$Q_{AT_0} = P[\bar{A}(t \geqq T_0)],$$

$$Q_{AT_0} = \frac{N[\bar{A}(t \geqq T_0)]}{N_r},$$

where: $N\,[\bar{A}(t \geqq T_0)$ [days]: number of days, with element $\bar{A}$ unavailable for a period of $t \geqq T_0$,
N_r [days]: number of days of the period of investigation.

The calculations described above should be performed for each period of T_0, considering the unavailability rates Q and availability rates R of each circuit element referring to each period T_0 concerned.

Example. Let us determine from statistical data the unavailability rate Q_A of circuit breaker type PTK 601—20/1000. Statistical data are at disposal for a period of 5 years, thus the period of investigation is $N_v - 5.365 -$ $- 1825$ days. Number of circuit breakers involved: 50. (The more units investigated, the more comprehensive conclusions can be obtained.) According to the statistical data, the number of days with any one of the 50 circuit breakers out of service (except for maintenance reasons):

for a period of $t = 0.1$ hours; 50 days: per breaker: $N = 1$ day
for a period of $t = 0.5$ hours, 10 days: per breaker: $N = 0.2$ day
for a period of $t = 5$ hours, 3 days: per breaker: $N = 0.06$ day

Hence, the unavailability rate of circuit breaker: for outage durations of

$$t = 0.1 \qquad\qquad t = 0.5$$

$$Q_{A0.1} = \frac{1}{1825} = 5.8 \cdot 10^{-4}, \qquad Q_{A0.5} = \frac{0.2}{1825} = 1.1 \cdot 10^{-4},$$

$$t = 5$$

$$Q_{A5} = \frac{0.06}{1825} = 0.34 \cdot 10^{-4}.$$

II. The second method of joint investigations of the two probability variables:

The frequency of unavailability can be dealt with as a discrete variable, i.e. a few frequency values are to be selected, such as $K_0 = 10^{-2}$, 10^{-1}, 1, etc. representing the number of unavailabilities occurring within the investigated period, and, again over the same period, the overall duration of unavailabilities whose occurrence outnumber K_0 should be investigated:

$$\bar{A}(k \geqq K_0),$$

where k [occurrence/T_v]: occurrence of one outage within the investigated period (T_v),

K_0 [occurrence/T_v]: discrete values selected.

Unavailability rates:

$$Q_{AK_0} = P\,[\bar{A}(k \geqq K_0)],$$

$$Q_{AK_0} = \frac{T[\bar{A}(k \geqq K_0)]}{T_v}.$$

where $T\ [\bar{A}(k \geqq K_0)]$ [hours]: overall duration of unavailabilities whose occurrence outnumbers K_0,

$T_v\ [\bar{A}(k \geqq K_0)]$ [hours]: period of investigation (usually one year: 8760 hours).

The calculations described under II should be performed for all selected values of K_0, substituting the corresponding unavailability rates Q and availability rates R. Obviously, the values Q and R of the various network elements should always be those belonging to the same value of K_0.

Example : Let us determine, by means of this second method, the unavailability rate (Q_A) of circuit-breaker type PTK 601—20/1000 of the previous example. Statistical data as before refer to 50 breakers and to a period of 5 years.

The selected values for K_0 [occurrences/year] are 0.01, 0.1 and 0.5 and refer to one breaker. With the 50 circuit breakers the following unavailabilities occurred (except for maintenance reasons):

$k' \geqq$ 2.5 outages with overall duration of $T' = 1000$ hours,
$k' \geqq$ 25 outages with overall duration of $T' =$ 250 hours,
$k' \geqq$ 125 outages with overall duration of $T' =$ 50 hours.

The above values reduced to one year and one breaker:
$k \geqq 0.01$ [occurrence/year]: $T - 4$ hours,
$k \geqq 0.1$ [occurrence/year]: $T - 1$ hour,
$k \geqq 0.5$ [occurrence/year]: $T - 0.2$ hours.

($k \geqq 0.01$ may cover e.g. all unavailabilities that occurred,
$k \geqq 0.1$ may cover e.g. unavailabilities due to leakage and valve-box failures,
$k \geqq 0.5$ may cover e.g. unavailabilities due to valve-box failures).

Hence, the unavailability rates of the circuit breaker:

with $k \geqq 0.01$ $\qquad$ $k \geqq 0.1$

$$Q_{A1} = \frac{4}{8760} = 4.6 \cdot 10^{-4}, \qquad Q_{A5} = \frac{1}{8760} = 1.14 \cdot 10^{-4},$$

$k \geqq 0.5$

$$Q_{A20} = \frac{0.2}{8760} = 0.23 \cdot 10^{-4}.$$

4. Consideration of maintenance outages

In the above investigations any element in troublefree condition has been considered available, and the elements affected by a failure were only regarded as unavailable. However, there is a service condition of network elements left out of consideration so far, namely the condition of a piece of equipment while it is taken out of service for maintenance. Extending our investigations to include this condition, the unavailabilities may be attributed to two reasons:

a) failure,

b) maintenance.

The difference between the two groups lies in the character of their occurrence, the former being a random phenomenon, while the latter can be made to occur at a predetermined time.

A failure can either directly bring about a disturbance, or may only cause unavailability (e.g. in the case of excessive temperature rise of a transformer terminal connection the trouble can be eliminated by switching over to a standby unit), reducing though the resultant service reliability of the network, but causing no disturbance.

In evaluating the reliability of the various network elements there is no difference between the two conditions of unavailability. The element investigated by itself is in both cases unavailable, and whether or not such an unavailability will lead to a disturbance will depend on the availability of a path parallel to the unavailable one and capable of taking over the load. The unavailability rate is, thus, a figure, which is characteristic of the network and not of the element.

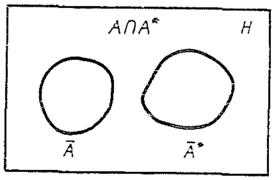

Fig. 4.1

Maintenance only causes unavailability, weakening thereby the resultant reliability of the network.

According to that stated above, two new sets of service conditions are to be introduced:

element "*a*" is under maintenance $\bar{A}^*$,
element "*a*" is not under maintenance A^*.

The interdependence of sets A and A^* must first be cleared. The possibility of a failure of a network element while it is under maintenance can be excluded, this being considered as an impossible event. On the other hand, — mostly in the case of apparatus — while repairing a defect which has caused a disturbance, normal maintenance of the piece of equipment will at the same time be performed (such occurrences can be evaluated by introducing conditional probabilities). However, it should be noted, that in case of minor defects, in order to minimize the duration of the disturbance, usually no maintenance is permitted to take place. Thus, the coincidence of a defect and a maintenance period can only occur in case of serious damages, although in such cases a total replacement of the defective element (e.g. of an apparatus) will often take place. The number of such occurrences is further reduced by the possibility of defects closely preceded by a maintenance period.

Justified by the considerations outlined above, the coincidence of a maintenance period and a defect will be neglected:

$$\bar{A} \cap \bar{A}^* = 0 \tag{4.1}$$

hence (Fig. 4.1):

$$\bar{A} \cap A^* = \bar{A} \tag{4.2}$$

$$\bar{A}^* \cap A = \bar{A}^* \tag{4.3}$$

and

$$A \cap A^* = (\overline{\bar{A} \cup \bar{A}^*}). \tag{4.4}$$

Due to relation (4.1), the events

$$\bar{A} \text{ and } A^*$$

as well as

$$A \text{ and } A^*$$

$$A \text{ and } \bar{A}^*$$

are not independent of each other. (Defect $\bar{A}$ can only occur, if there is no maintenance A^*, and maintenance $\bar{A}^*$ can only take place, if no defect to the element occurred.)

Hence, theorem *b)* of Section 1.2 cannot be applied here:

$$P(\bar{A} \cap A^*) \neq P(\bar{A}) \cdot P(A^*),$$

and

$$P(A \cap \bar{A}^*) \neq P(A) \cdot P(\bar{A}^*),$$

but, according to relations (4.2) and (4.3)

$$P(\bar{A} \cap A^*) = P(\bar{A}) = Q_A,$$

and

$$P(A \cap \bar{A}^*) = P(\bar{A}^*) = Q_A^*,$$

$$P(A \cap A^*) = P(\overline{\bar{A} \cup \bar{A}^*}) - 1 - [P(\bar{A}) + P(\bar{A}^*)] = R_A - Q_A.$$

If simultaneous occurrence of several service conditions is investigated, the service conditions not independent on each other should be substituted by their resultant

e.g.:

$$A \cap \bar{A}^* \cap B,$$

where B is independent of A and $\bar{A}^*$, while A and $\bar{A}^*$ are not independent of each other. Substituting the dependent elements with their resultant:

$$A \cap \bar{A}^* \cap B = \bar{A}^* \cap B.$$

So, among the conditions investigated there will no elements dependent on each other be left and, thus, their probabilities can be multiplied together.

4.1 *Evaluation of the maintenance of individual elements*

According to the foregoing statements, the following service conditions of element *"a"* can occur.

element "a" is in working order: A
element "a" is unavailable due to a defect: $\bar{A}$
element "a" is not under maintenance: A^*
element "a" is unavailable due to maintenance: $\bar{A}^*$
element "a" is, thus, available, if in working order and not under maintenance:

$$A \cap A^*,$$

and unavailable, if

$$\bar{A} \cap A^*.$$

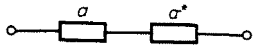

Fig. 4.2

Collating this result with those of Section 2.1, the maintenance of element "a" can be regarded as being equivalent to the connecting of an element a^* in series with the original element "a" (Fig. 4.2). Hence, the probability of availability of element "a" is

$$P(A \cap A^*) .$$

To calculate this value, a further probability variable is to be introduced, the *probability of maintenance :*

$$P(\bar{A}^*) = Q_A^* .$$

This relation can be interpreted in two different ways, as described in Section 3:

I. $$Q_A^* = \frac{N(t \geq T_0)}{N_v} ,$$

where t [hours] = duration of maintenance,
N [days] = number of days of maintenance, or:

II. $$Q_A^* = \frac{T^-(k \geq K_0)}{T_v} ,$$

where k [occurrences/T_v] = frequency of maintenance,
T [hours] = duration of maintenance.

With these values, the probability of availability of element "a" is:

$$P(A \cap A^*) = \underline{R_A - Q_A^* = R_e} .$$

The probability of unavailability of element "a" is:

$$P(\bar{A} \cup \bar{A}^*) = \underline{Q_A + Q_A^*} ,$$

because $\bar{A} \cap \bar{A}^* = 0$ (Axiom VI, Section 1.2).

With the method of "complete system of events":

Service conditions	Probabilities	Resultant
$A \cap A^*$	$R_A - Q_A^*$	available
$A \cap A^* = \bar{A}$	Q_A	unavailable
$A \cap \bar{A}^* = \bar{A}^*$	Q_A^*	unavailable.

4.2 *Evaluation of maintenance of series elements*

The maintenance of a system consisting of series elements can always be arranged in such a way that all its elements are to be maintained simul-

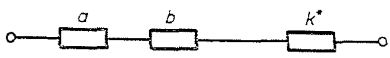

Fig. 4.3

taneously. In the calculations it will be assumed that the frequency of maintenance is equal for all series elements (e.g. each element is maintained once a year). In such a case, the outage duration of the series path due to maintenance will be determined by the element requiring the longest duration of maintenance. Let this element be "k". The equivalent circuit of the series path is shown in Fig. 4.3.

The series system is available, if

$$A \cap B \cap K^* .$$

Its probability is

$$P(A \cap B \cap K^*) = \underline{R_A R_B - Q_K^* = R_e} .$$

The series system is unavailable, if "a", or "b", or both, are defective, or maintenance is taking place:

$$\bar{A} \cup \bar{B} \cup \bar{K}^* = (\bar{A} \cup \bar{B}) \cup \bar{K}^* .$$

However, according to the former condition:

$$(\bar{A} \cap \bar{B}) \cap \bar{K}^* = 0 ,$$

thus

$$P[(\bar{A} \cup B) \cup K^* = P(\bar{A} \cup \bar{B}) + P(K^*)] ,$$

$$\underline{Q_e = Q_A + Q_B - Q_A Q_B + Q_K^*} .$$

When using the method of "complete system of events" it should be taken into account that the event of maintenance ($\overline{K}^*$) excludes the possibility of a defect according to Section 4.

Service conditions:	Probabilities:	Resultant network:
$A \cap B \cap K^*$	$R_A R_B - Q_K^*)$	available
$(\bar{A} \cap B) \cap K^* = \bar{A} \cap B$	$Q_A R_B$	unavailable
$(A \cap \bar{B}) \cap K^* = A \cap \bar{B}$	$R_A Q_B$	unavailable
$(\bar{A} \cap \bar{B}) \cap K^* = \bar{A} \cap \bar{B}$	$Q_A Q_B$	unavailable
$(A \cap B) \cap K^* = \overline{K}^*$	Q_K^*	unavailable

where, in case of $(\bar{A} \cap B) \cap K^*$, the first term indicates the disturbance of

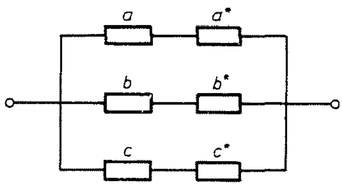

Fig. 4.4

the series system, hence

$$(A \cap B) \cap K^* = \bar{A} \cap B \, .$$

Similarly:

$$(\bar{A} \cap B) \cap \overline{K}^* = 0 \, ,$$

these rows need not even be indicated.

The above example refers to two series elements only, but the method can be extended to cover any number of series elements.

4.3 *Evaluation of maintenance of parallel elements*

A parallel system is shown in Fig. 4.4. Each branch of the above parallel system can be a resultant of series elements. In this case, A represents the resultant available service conditions of the series elements, and $\bar{A}^*$ the service conditions of the series system under maintenance periods, as defined in Section 4.2.

Maintenance of parallel elements should be planned so as to avoid simultaneous maintenance outage of two parallel elements.

These conditions can be expressed as follows:

$$\bar{A}^* \cap \bar{B}^* = \bar{A}^* \cap \bar{C}^* = \bar{B}^* \cap \bar{C}^* = A^* \cap \bar{B}^* \cap \bar{C}^* = 0 \, .$$

From that it follows:

$$\bar{A}^* \cap B^* \cap C^* = \bar{A}^* \cap B^* = \bar{A}^* \cap C^* = \bar{A}^* ,$$

$$P(A^* \cap B^* \cap C^*) = 1 - P(\bar{A}^* \cup \bar{B}^* \cup \bar{C}^*) =$$

$$= 1 - (Q_A + Q_B + Q_C) = 1 - \sum_{i=1}^{n} Q_i$$

$$P(A^* \cap B^* \cap C^* \cap A \cap B \cap C) = 1 - P(\bar{A}^* \cup \bar{B}^* \cup \bar{C}^* \cup \bar{A} \cup \bar{B} \cup \bar{C} =$$

$$= 1 - [Q_A^* + Q_B^* + Q_C^* + (1 - R_A R_B R_C)] =$$

$$= -(Q_A^* + Q_B^* + Q_C^*) + R_A R_B R_C .$$

In addition to the above, Eqs. 4.2 and 4.3 are also to be applied. The system can be investigated with the method of the "complete system of events". Six elements are contained in the system, thus one of the combinations of the six elements would appear in each row of the table. Since, however, only the probabilities of independent events can be multiplied together as stated in *b)* of Section 1.2, the combinations with elements not independent of each other are to be transformed by means of the relations shown above, for the interdependence of elements are defined by these relations. E.g.:

$$A^* \cap B^* \cap C^* \cap A \cap B \cap C = A^* \cap B^* \cap C^* \cap B \cap C = \text{(because } A \cap \bar{A}^* = \bar{A}^*)$$

and, further, $= \bar{A}^* \cap B \cap C$

(because $A^* \cap B^* \cap C^* = A^*$).

Similarly:

$$A^* \cap B^* \cap C^* \cap \bar{A} \cap B \cap C = B^* \cap C^* \cap \bar{A} \cap B \cap C .$$

In setting up the table of the "complete system of events", the first group of service conditions will consist of the possible variations with element "*a*" under maintenance. In this case it is assumed (Eq. 4.2) that during this period "*a*" cannot become defective and the possible variations are combinations of "*b*" and "*c*".

The second and third service conditions are similar to the first. The fourth group is constituted by the variations of service conditions in which no element is taken out for maintenance. Since probabilities of independent events only can be multiplied together (*b)* of Section 4.2), the transformations mentioned above should be performed.

The service condition of the resultant network is shown in the third/fourth columns. If one of the three parallel branches is capable alone of transmitting the full load, then the third column applies, whereas if two branches are needed to carry the full load, then the fourth column is valid.

Adding the probabilities of the corresponding rows the probabilities of available, unavailable and half-load service conditions can be computed. The example shown refers to the case of only three parallel branches, but the method can be applied to any number of branches.

Service conditions	Probabilities:	Resultant network, with 1 branch required for carrying	2 branches the full load
$\overline{A}^* \cap B \quad C$	$Q_A^* R_B R$	available	available
$\overline{A}^* \cap \overline{B} \cap C$	$Q_A^* Q_B R$	available	half-load
$\overline{A}^* \cap B \cap \overline{C}$	$Q_A^* R_B Q_C$	available	half-load
$\overline{A}^* \cap \overline{B} \cap \overline{C}$	$A_A^* Q_B Q_C$	unavailable	unavailable
$\overline{B}^* \cap A \cap C$	$Q_B^* R_A R_B$	available	available
$\overline{B}^* \cap \overline{A} \cap C$	$Q_B^* Q_A R_C$	available	half-load
$\overline{B}^* \cap A \cap \overline{C}$	$Q_B^* R_A Q_C$	available	half-load
$\overline{B}^* \cap \overline{A} \cap \overline{C}$	$Q_B^* Q_A Q_C$	unavailable	unavailable
$\overline{C}^* \cap A \cap B$	$Q_C^* R_A R_B$	available	available
$\overline{C}^* \cap \overline{A} \cap B$	$Q_C^* Q_A R_B$	available	half-load
$\overline{C}^* \cap A \cap \overline{B}$	$Q_C^* R_A Q_B$	available	half-load
$\overline{C}^* \cap \overline{A} \cap \overline{B}$	$Q_C^* Q_A Q_B$	unavailable	unavailable
$A^* \cap B^* \cap C^* \cap A \cap B \cap C$	$-(Q_A^* + Q_B^* + Q_C^*) \quad + R_A R_B R_C$	available	available
$B^* \cap C^* \cap \overline{A} \cap B \cap C$	$-(Q_B^* + Q_C^*) \quad + Q_A R_B R_C$	available	available
$A^* \cap C^* \cap A \cap \overline{B} \cap C$	$-(Q_A^* + Q_C^*) \quad + R_A Q_B R_C$	available	available
$A^* \cap B^* \cap A \cap B \cap \overline{C}$	$-(Q_A^* + Q_B^*) \quad + R_A R_B Q_C$	available	available
$C^* \cap \overline{A} \cap \overline{B} \cap C$	$-Q_C^* \quad + Q_A Q_B R_C$	available	half-load
$A^* \cap A \cap \overline{B} \cap \overline{C}$	$-Q_A^* \quad + R_A Q_B Q_C$	available	half-load
$B^* \cap \overline{A} \cap B \cap \overline{C}$	$-Q_B^* \quad + Q_A R_B Q_C$	available	half-load
$\overline{A} \cap \overline{B} \cap \overline{C}$	$+ Q_A Q_B Q_C$	unavailable	unavailable

5. Evaluation of statistical data

The availability calculations, as outlined above, should be performed by utilizing the probability variables (Q, R, Q) of a network or networks. The theoretical values of these variables have been defined in Sections 3 and 4. A further task is to compute the practical, numerical values of these variables, which can be performed on the basis of statistical data.

In Section 3 the probability variables Q have been determined in two different ways (I/Q_{AT_0}; and II/Q_{AK_0}).

The statistical data have to be evaluated correspondingly, in the following way:

a) Let us examine network element "*a*" (e.g. a given type of circuit breaker or a 10-kilometre section of a given type of transmission line, etc.). From these network elements — of which a great number is present in a network — a sample containing n elements should be selected. The probability figures of the network element concerned shall then be determined on the basis of this sample.

b) The period of investigation should be fixed according to Method I: N_v days, according to Method II: T_v hours.

c) From statistical data the unavailability rate shall be determined for each element of the sample consisting of n elements. According to Method I:

$$q_i = \frac{N_i\,[\bar{A}(t \geqq T_0)]}{N_v} .$$

Values of T_0 shall be selected (0.1, 0.5, 2, 10 hours), and those of q_i be calculated for each value of T_0. According to Method II:

$$q_i = \frac{T_i\,[A(k - K_0)]}{T_v} .$$

Values of K_0 shall be selected (0.01, 0.1, etc. per year) and those of q_i be calculated for each value of K_0.

d) The expectation value of q is considered as the unavailability rate (uncertainty) of the network under investigation. Best approximation is obtained from the arithmetic mean:

$$q_A = Q_A = \frac{q_1 + q_i + \ldots + q_n}{n} .$$

This value is to be determined for all values of T_0 and K_0, respectively.

The value of Q_A will be the more accurate, the higher is the value of n. In order to simplify the calculation, Sections *c)* and *d)* may be combined:

$$Q_A = \frac{\dfrac{N_1}{N_v} + \dfrac{N_2}{N_v} + \ldots + \dfrac{N_n}{N_v}}{n} = \frac{N_1 + N_2 + \ldots + N_n}{nN_v} ,$$

or

$$Q_A = \frac{\dfrac{T_1}{T_v} + \dfrac{T_2}{T_v} + \ldots + \dfrac{T_n}{T_v}}{n} = \frac{T_1 + T_2 + \ldots + T_n}{nT_v} ,$$

i.e.

$$Q_A = \frac{1}{nN_v} \sum_1^n N_i\,[\bar{A}(t \geqq T_0)] . \tag{5.1}$$

or

$$Q_A = \frac{1}{nT_v} \sum_1^n T_i\,[\bar{A}(k \geqq K_0)] . \tag{5.2}$$

e) The maintenance outage rate of the network element concerned is now also to be determined. This can be done by means of formulae (5.1) and (5.2). By evaluating the statistical values all kinds of unavailability of the investigated network element should be taken into account, i.e. not only those due to a disturbance but any event that caused unavailability. If, namely, one of several parallel elements in a system became unavailable, no outage occurred. This condition shall also be considered in the statistical valuation (e.g. if a terminal of a transformer exhibited an excessive temperature rise, and a standby unit was put into service, and the transformer originally in service was taken out for repair: no outage was caused, yet an unavailability occurred). During the unavailability of such elements the reliability of the resultant network is reduced!

Attention should be drawn to the requirement that statistical surveys must be extended to cover all unavailability conditions!

The following statistical data referring to unavailability conditions should be included:

Network element (e.g. 100 km transmission line);
Date of unavailabilities;
Duration (t) of unavailabilities;
Cause of unavailabilities (insulator, conductor, etc.);
Loss of kilowatthours, if any.

From the above figures the following data should be determined:

$$N\,[\bar{A}(t \geqq T_0)]\,,$$

$$\text{and } T\,[\bar{A}(k \geqq K_0)]\,,$$

where k indicates the rate of occurrence for each cause.

The data should preferably be processed by means of punched-card type computers.

6. Evaluation of the results of reliability investigations

Performing the operations shown in Sections 2 and 4 with the probability variables described in Sections 3, 4, and 5, a resultant unavailability rate is obtained. The result of these operations is as follows:

With Method I:

$$Q_e = \frac{N[(t \geqq T_0)]}{N_v} \quad \text{frequency [occurrence/year] for various periods } T_0\,.$$

With Method II:

$$Q_e = \frac{T[(k \geqq K_0)]}{T_v} \quad \text{duration [hour/year] for various frequencies } K_0\,.$$

The result in both cases is a two-variable probability function. The valuation of this function can be performed considering two different aspects:

6.1 *Technical aspect*

The calculations give information on the reliability of the investigated system. Comparison between several different systems can also be made on this basis.

6.2 *Economic aspect*

No firmly established methods exist for the economic valuation of reliability or for assessing the expectable value of energy not supplied to the consumers in consequence of a disturbance.

a) In principle, the expectable value of energy loss due to a disturbance lasting a given period of T_v can be evaluated by means of the following formula:

$$\tilde{W} = \tilde{T} \cdot P_{\text{average}} \text{ [kWh]},$$

where $\tilde{T}$ [hours] is the expectable duration of system unavailability (outage) within the investigated period T_v:

$$\tilde{T} = T/k \leqq K_{0\,\min}) \,.$$

P (kW) is the average load of the system.

Now, the problem is to find the economic value of lost energy W, i.e. the economic damage caused by this outage. The determination of this damage is extremely difficult, because it cannot be taken as equal to the value of products that could have been manufactured by means of this electrical energy, since the loss of production due to an outage is one that can subsequently be made good. An irreparable loss is represented by the idle time of workers and employees, or the damage resulting from a manufacturing process that cannot be carried on after the recovery of supply voltage, etc. The assessment of damage due to a disturbance imposed on public consumers is a further problem. According to results of a few attempts made in this country to find numerical values for this damage, the figures are in the range of 10 to 50 Forints/kWh.

These, however, can by no means be regarded as well-established figures, and many authors admit that this problem has not been solved anywhere yet [7]. Some attemps were made in France by EDF, but according to recent information, also these failed. The general opinion is that such specific values may be found for individual industrial plants, but values of universal validity could not be found so far.

b) The valuation of a given unavailability rate may perhaps be possible in the case of industrial plants where the economic damage dependent on the duration of outage is such a way that no damage is caused by a short-time disturbance, minor damage is caused by a longer disturbance and severe harm is done by a disturbance of longer duration (e.g. in metallurgical industry and mines, where life hazards may also be involved).

7. Summary

Using set-theory and probability arithmetic a general method can be developed for numerical evaluation of resultant reliability of widely varying network configurations. Similarly, the maintenance outage rates can be correctly evaluated.

Reliability or expected outage rates of network elements can be evaluated by means of mathematical statistics. Existing statistical records on system disturbances must be completed with data required for reliability calculations. Evaluation of data is preferably performed by means of punch-card computers.

Based on these theoretical considerations detailed computations will be performed to evaluate the reliability of actual networks. The results will be published at a later date.

*

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Zoltán Reguly, Budapest XI., Egry József u. 18—20. Hungary