THE DETERMINATION OF SIMPLE QUADRATIC INTEGRALS BY ROUTH-COEFFICIENTS

by

J. LEHOCZKY

Department of Automation, Polytechnical University, Budapest

(Received January 20, 1966)

Presented by Prof. Dr. F. CSÁKI

A dynamic control process is called optimum, i.e. the individual characteristics of the system are optimum, when a certain predetermined integral reaches its minimum value:

$$I = \int_0^\infty f\left(x(t), t\right) \; dt = \min.$$

Signal $x(t)$ might be the weight-function of the system $[w(t)]$, the transient component of the step response $[v(t)]$, the error-signal of the control process $[x_e(t)]$, or when investigating the effects of the disturbance, the controlled characteristic $[x_c(t)]$. From among the different integral-criterions most often the quadratic criterions are used, which can be applied for the investigations into swinging processes too, as the compution of these is the simplest.

The simple quadratic integral criterion often also appears in the time weighted, or quadratic-time weighted form. So the most often used integral evaluations are:

$$I = \int_0^\infty x^2(t) \; dt$$

$$J = \int_0^\infty tx^2(t) \; dt$$

$$K = \int_0^\infty t^2 x^2(t) \; dt$$

Literary texts deal amply with the integral-criterions of linear, one-variable control systems with concentrated parameters. On the basis of the published, precalculated relations the integral values, — when the Laplace transformation form of the signal in question, resp. their constant coefficients are known, — can be computed.

However, the simple quadratic integral criterions can be conceived and computed also by the coefficients of the Routh-scheme, used for stability investigations. The presentation of this is the paper's purpose.

1. Basic concepts

If the function $x(t)$ is the solution of the differential equation

$$a_0 \overset{(n)}{x} + a_1 \overset{(n-1)}{x} + \ldots + a_{n-1} \dot{x} + a_n x = 0 \tag{1—1}$$

with reference to the initial conditions

$$\overset{(n-1)}{x}(0), \quad \overset{(n-2)}{x}(0), \ldots, \dot{x}(0), \quad x(0)$$

when the Laplace transformation form of function $x(t)$ is:

$$X(s) = \frac{a_0 x(0) s^{n-1} + [a_0 \dot{x}(0) + a_1 x(0)] s^{n-2} + \ldots + [a_0 \overset{(n-1)}{x}(0) + \ldots + a_{n-1} x(0)]}{a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n} \tag{1—2}$$

It can easily be seen, that if the initial conditions of the differential equation are:

$$x(0) = 1 \text{ and } \overset{(n-1)}{x}(0) = \ldots = \dot{x}(0) = 0 \tag{1—3}$$

then

$$X(s) = X_1(s) = \frac{a_0 s^{n-1} + a_1 s^{n-2} + \ldots + a_{n-2} s + a_{n-1}}{a_0 s^n + a_1 s^{n-1} + \ldots a_{n-1} s + a_n} \tag{1—4}$$

Further if the initial conditions are:

$$\overset{(n-1)}{x}(0) = 1 \text{ and } \overset{(n-2)}{x}(0) = \ldots = x(0) = 0 \tag{1—5}$$

then

$$X(s) = X_2(s) = \frac{a_0}{a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n} \tag{1—6}$$

Let the transfer function of the control system be

$$W(s) = \frac{a_n}{a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n} \tag{1—7}$$

If a unity step signal acts upon the system input

$$x_v(t) = 1(t); \quad X_v(s) = \frac{1}{s} \tag{1—8}$$

then the transform of the controlled characteristic is

$$X_c(s) = W(s)\, X_v(s) \tag{1—9}$$

and the transformation form of the control error signal is

$$X_e(s) = X_v(s) - X_c(s) = X_v(s) - W(s) \cdot X_v(s) = X_v(s)\,[1 - W(s)] \tag{1—10}$$

Substituting relations (1—7), resp. (1—8) we obtain as a result:

$$X_e(s) = \frac{1}{s}\left[1 - \frac{a_n}{a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n}\right] =$$

$$= \frac{a_0 s^{n-1} + a_1 s^{n-2} + \ldots + a_{n-2} s + a_{n-1}}{a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n} \tag{1—11}$$

which evidently agrees with relation (1—4). On the basis of the above argument we can state:

If the transfer function of the system is (1—7) and we examine the error-signal effected by the unity-step, then the transformed function of the signal agrees with relation (1—4) and the signal itself is the solution — involving the initial conditions according to (1—3) — of a differential equation similar to the general equation (1—1), (which can be conceived on the basis of relation $x_e(s)$).

If we examine, however, the weight-function of the system from comparing relations (1—6) and (1—7), it is given, that the transform of the weight function, i.e. the transfer function is:

$$W(s) = \frac{a_n}{a_0} X_2(s) \tag{1—12}$$

where $X_2(s)$ is the form in accordance with the initial conditions (1—5) of the transformed general equation (1—2). From relation (1—12) it can be seen, that transfer function $W(s)$ differs only by a constant transfer factor from relation (1—6).

The weight function of the system therefore is proportional to the solution, involving the initial conditions (1—5), of a differential equation conceived on the basis of $w(s)$ and similar to general equation (1—1).

On the basis of the coefficients of differential equation (1—1) we can write the Routh-scheme, used for stability investigations of linear systems, as follows:

$$
\begin{array}{llll}
R_{00} = a_0 & R_{01} = a_2 & R_{02} = a_4 & \dots \\
R_{10} = a_1 & R_{11} = a_3 & R_{12} = a_5 & \dots \\
R_{20} = \dfrac{a_1 a_2 - a_0 a_3}{a_1} & R_{21} = \dfrac{a_1 a_4 - a_0 a_5}{a_1} & . & \dots \\
R_{30} = \dfrac{R_{20} R_{11} - R_{10} R_{21}}{R_{20}} & . & . & \dots \\
\dots\dots & \dots\dots & \dots\dots & \dots \\
R_{(n-1)0} & & & \\
R_{n0} = a_n & & &
\end{array}
$$

Let us further introduce the following notations, which contain a certain set of Routh-coefficients weighted by the initial conditions of the differential equation:

$$
\begin{aligned}
R_0 &= 0 \\
R_1 &= R_{00}\ x(0) \\
R_2 &= R_{00}\ \dot{x}(0) + R_{10}\, x(0) \\
R_3 &= R_{00}\ \ddot{x}(0) + R_{10}\, \dot{x}(0) + R_{01}\, x(0) \\
R_4 &= R_{00}\ \dddot{x}(0) + R_{10}\, \ddot{x}(0) + R_{01}\, \dot{x}(0) + R_{11}\, x(0) \\
R_5 &= R_{00}\ \overset{(\mathrm{IV})}{x}(0) + R_{10}\, \dddot{x}(0) + R_{01}\, \ddot{x}(0) + R_{11}\, \dot{x}(0) + R_{02}\, x(0) \\
&\dots\dots\dots\dots
\end{aligned}
$$

2. A simple quadratic integral criterion

The double of the integral value

$$I = \int_0^\infty x^2(t)\, dt$$

can be expressed according to EFFERTZ and CREMER, in the following form:

$n = 1$

$$2 \int_0^\infty x^2(t)\, dt = \frac{R_{00}}{R_{10}}\, x^2(0) \tag{2—1}$$

$n = 2$

$$2\int_0^\infty x^2(t)\,dt = \frac{R_{00}}{R_{10}}x^2(0) + \frac{R_{10}}{R_{20}}\left[\frac{R_{00}}{R_{10}}\dot{x}(0) + x(0)\right]^2 \tag{2—2}$$

$n = 3$

$$2\int_0^\infty x^2(t)\,dt = \frac{R_{00}}{R_{10}}x^2(0) + \frac{R_{10}}{R_{20}}\left[\frac{R_{00}}{R_{10}}\dot{x}(0) + x(0)\right]^2 +$$
$$+\frac{R_{20}}{R_{30}}\left[\frac{R_{00}}{R_{20}}\ddot{x}(0) + \frac{R_{10}}{R_{20}}\dot{x}(0) + x(0)\right]^2 \tag{2—3}$$

$n = 4$

$$2\int_0^\infty x^2(t)\,dt = \frac{R_{00}}{R_{10}}x^2(0) + \frac{R_{10}}{R_{20}}\left[\frac{R_{00}}{R_{10}}\dot{x}(0) + x(0)\right]^2 +$$
$$+\frac{R_{20}}{R_{30}}\left[\frac{R_{00}}{R_{20}}\ddot{x}(0) + \frac{R_{10}}{R_{20}}\dot{x}(0) + x(0)\right]^2 +$$
$$+\frac{R_{30}}{R_{40}}\left[\frac{R_{00}}{R_{30}}\dddot{x}(0) + \frac{R_{10}}{R_{30}}\ddot{x}(0) + \left(\frac{R_{00}}{R_{10}} + \frac{R_{20}}{R_{30}}\right)\dot{x}(0) + x(0)\right]^2 \tag{2—4}$$

The evaluation becomes especially simple, if the initial conditions are

$$\overset{(n-1)}{x}(0) = \ldots = \dot{x}(0) = 0 \text{ and } x(0) = 1 \tag{2—5}$$

because then:

$$2\int_0^\infty x^2(t)\,dt = \frac{R_{00}}{R_{10}} + \frac{R_{10}}{R_{20}} + \frac{R_{20}}{R_{30}} + \ldots + \frac{R_{(n-1)0}}{R_{n0}} \tag{2—6}$$

It is to be noted, that when $x(0) \neq 1$, then the right side has to be multiplied by the value of $x^2(0)$. Substituting the initial conditions (2—5) into equation (1—2), we obtain relation (1—4), which implies, that with such initial conditions the transform of function $x(t)$ will be equal to the transformed function (1—4). If the transfer function of the systems equals relation (1—7), then relation (1—4) is the transformed function of the error effected by the unity step, therefore, the quadratic integral criterion conceived with reference to the error effected by the unity step, will just equal relation (2—6), so

$$\int_0^\infty x_e^2(t)\,dt = \frac{1}{2}\left(\frac{R_{00}}{R_{10}} + \frac{R_{10}}{R_{20}} + \ldots + \frac{R_{(n-1)0}}{R_{n0}}\right)$$

On the other hand, it is

$$\overset{(n-1)}{x}(0) = 1 \text{ and } \overset{(n-2)}{x}(0) = \ldots = x(0) = 0 \tag{2—7}$$

then

$$\int_0^\infty x^2(t)\,dt = \frac{R_{00}^2}{2R_{(n-1)0}\,R_{n0}} \tag{2—8}$$

Thus this method of computation can be used for the evaluation of the transformed function signal in the form of

$$X(s) = \frac{a_0}{a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n} \tag{2—9}$$

As the above relation — but for a constant transfer factor — equals the transformation form of the weight function

$$W(s) = \frac{R_{n0}}{R_{00}} X(s) \tag{2—10}$$

therefore

$$\int_0^\infty w^2(t)\,dt = \frac{R_{00}}{2R_{(n-1)0}}$$

Example 2.1. Let the position control system according to Fig. 1 be given. The transfer functions of the opened and closed system are:

$$Y(s) = \frac{A}{s(1 + sTg)(1 + sTg)}$$

$$W(s) = \frac{A}{TgT_m s^3 + (Tg + T_m)s^2 + s + A}$$

The transform of the control error is:

$$X_e(s) = \frac{T_g T_m s^2 + (T_g + T_m)s + 1}{T_g T_m s^3 + (T_g + T_m)s^2 + s + A}$$

On the basis of the typical equation the coefficients of the Routh scheme are:

$$R_{00} = T_g T_m \qquad\qquad R_{01} = 1$$
$$R_{10} = T_g + T_m \qquad\qquad R_{11} = A$$
$$R_{20} = \frac{T_g + T_m - A T_g T_m}{T_g + T_m}$$
$$R_{30} = A$$

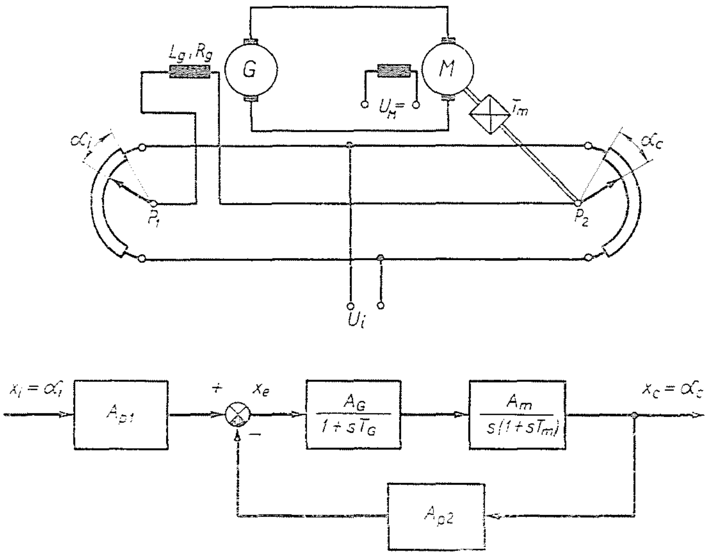

Fig. 1

The quadratic integral value of the system weight function is:

$$\int_0^\infty w^2(t)\, dt = \frac{R_{00}}{2R_{20}} = \frac{(T_g + T_m)\, T_g\, T_m}{2(T_g + T_m - A T_g T_m)}$$

In the case of a unity step input signal the quadratic integral of the system control error is:

$$\int_0^\infty x_h^2(t)\, dt = \frac{1}{2}\left(\frac{R_{00}}{R_{10}} + \frac{R_{10}}{R_{20}} + \frac{R_{20}}{R_{30}}\right) =$$

$$= \frac{1}{2}\left(\frac{T_m T_g}{T_g + T_m} + \frac{(T_g + T_m)^2}{T_g + T_m - A T_g T_m} + \frac{T_g + T_m - A T_g T_m}{A(T_g + T_m)}\right)$$

It is to be noted, that if the input signal is not an $1(t)$ unity step function, i.e. $x(0) \neq 1$, but some other step function, then the right side must be multiplied by the square of the input step function, i.e. by $x^2(0)$.

3. Time weighted quadratic integral criterion

The computation of the time weighted quadratic integral value of signal $x(t)$

$$J = \int_0^\infty t x^2(t)\, dt \tag{3—1}$$

can be deduced from the computation of the simple quadratic integral value.

In accordance with the well-known theses of the Laplace transformation and utilizing the complex convolution the time-weighted quadratic integral — when the transformed function $X(s)$ of signal $x(t)$ is known — can be computed. The utilized relations are:

$$\mathscr{L}[x^2(t)] = \int_0^\infty x^2(t)\, e^{-st}\, dt$$

$$\mathscr{L}[tx^2(t)] = \int_0^\infty tx^2(t)\, dt = -\lim_{s\to 0} \frac{\partial}{\partial s} \int_0^\infty x^2(t)\, e^{-st}\, dt \tag{3—2}$$

where according to the Parseval-thesis

$$\int_0^\infty x^2(t) e^{-st}\, dt = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} X(s)\, X(-s)\, ds$$

The time weighted quadratic integral values of signals possessing a transformed function like (1—2), plotted against the constant coefficients of the transformed function are published in references [2, 10]. Computing the integral values with the aid of the Routh coefficients, we obtained the following results:

$n = 1$

$$\int_0^\infty tx^2(t)\, dt = \frac{1}{4} \frac{R_1^2}{R_{10}^2}$$

$n = 2$

$$\int_0^\infty tx^2(t)\,dt = \frac{1}{4}\frac{R_2^2}{R_{20}^2} - \frac{R_1 R_2}{2R_{10} R_{20}} + \frac{1}{2R_{10}^2}\left(R_1^2 + \frac{R_{00}}{R_{20}} R_2^2\right)$$

$n = 3$

$$\int_0^\infty tx^2(t)\,dt = \frac{1}{4}\frac{R_3^2}{R_{30}^2} - \frac{1}{2R_{20}}\left[\frac{R_1 R_2}{R_{10}} + \frac{R_2 R_3}{R_{30}}\right] +$$
$$+ \frac{1}{2R_{20}^2}\left[R_1^2 \frac{R_{01} + R_{10} R_{30}}{R_{10}^2} + (R_2^2 - 2R_1 R_3)\left(\frac{R_{00} R_{01}}{R_{10}^2} + 1\right) +\right.$$
$$\left. + R_3\left(\frac{R_{00}^2}{R_{10}^2} + \frac{R_{10}}{R_{30}}\right)\right] \tag{3—3}$$

$n = 4$

$$\int_0^\infty tx^2(t)\,dt = \frac{1}{4}\frac{R_4^2}{R_{40}^2} - \frac{1}{2R_{30}}\left[2R_1 \frac{R_{40}}{R_{10} R_{20}} +\right.$$
$$+ 2\frac{R_{00}}{R_{10} R_{20}}(R_3^2 - 2R_4 R_2) + \frac{R_{01}}{R_{10} R_{20}}(R_2^2 - 2R_3 R_1) +$$
$$+ \frac{1}{R_{20}}(R_2 R_3 - 3R_4 R_1) + R_1 R_2 \frac{R_{11}}{R_{10} R_{20}} + \frac{R_3 R_4}{R_{40}} +$$
$$\left. + R_4^2 \frac{R_{01} R_{00} + R_{10}^2}{R_{40} R_{10} R_{20}}\right] + \frac{1}{2R_{30}^2}\left[\frac{R_{00}^2 R_{11}^2}{R_{20}^2 R_{10}^2} +\right.$$
$$\left. + \frac{R_3^2 - 2R_2 R_4}{R_{20}^2} + \frac{R_{11}}{R_{10} R_{20}^2}(R_2^2 - 2R_1 R_3) + R_4^2 \frac{1}{R_{40} R_{20}}\right] \cdot$$
$$\cdot (R_{11} R_{10} - 4R_{00} R_{40} + R_{01}^2)$$

The evaluation becomes more simple, if we write the time weighted quadratic integral with reference to the error effected by the unity input signal. In this case in sense of the transform function described in (1—4):

$$R_1 = R_{00},\ R_2 = R_{10},\ R_3 = R_{01}, \ldots \tag{3—4}$$

as

$$\overset{(n-1)}{x}(0) = \ldots \dot{x}(0) = 0 \text{ and } x(0) = 1$$

On the other hand, if

$$\overset{(n-1)}{x}(0) = 1 \text{ and } \overset{(n-2)}{x}(0) = \ldots = x(0) = 0$$

then the integral involving the weight function can be computed in a more simple way, because

$$R_n = R_{00} \text{ and } R_{n-1} = \ldots = R_1 = 0 \tag{3—5}$$

If we substitute the (3—5) relations into (3—3) equations, further taking relation (2—10) into consideration, we obtain the following simple results, which can be utilized for the weight function evaluation:

$n = 1$

$$\int_0^\infty t w^2(t)\, dt = \frac{1}{4} \frac{R_{00}}{R_{10}}$$

$n = 2$

$$\int_0^\infty t w^2(t)\, dt = \frac{1}{4} \frac{R_{00}}{R_{20}} + \frac{R_{00}}{2R_{10}^2}$$

$n = 3$

$$\int_0^\infty t w^2(t)\, dt = \frac{1}{4} \frac{R_{00}}{R_{30}} + R_{00} \left(\frac{R_{00}^2 R_{30}}{R_{10}^2} + R_{10} \right)$$

$n = 4$

$$\int_0^\infty t w^2(t)\, dt = \frac{1}{4} \frac{R_{00}}{R_{40}} - \frac{R_{00}}{2R_{30}} \cdot \frac{R_{01} R_{00} + R_{10}^2}{R_{10} R_{20}} +$$

$$+ (R_{11} R_{10} - 4R_{40} R_{00} + R_{01}^2) \left(\frac{R_{00} R_{11}^2 R_{40}}{2R_{30}^2 R_{20}^2 R_{10}^2} + \frac{R_{00}}{R_{20}} \right) \tag{3—6}$$

Example 3.1. The position control system shown in FIG. 1 is given. The transfer functions and the Routh scheme coefficients are identical with those in example 2.1. In the case of a unity step input signal the time weighted quadratic integral of the system control error, considering that

$$R_1 = T_g R_m, \qquad R_2 = T_g + T_m, \qquad R_3 = 1$$

gives the following result:

$$\int_0^\infty t x_e^2(t)\, dt = \frac{1}{4A^2} - \frac{T_m + T_g}{2(T_g + T_m - A T_g T_m)} \left(\frac{T_g + T_m}{A} + T_g T_m \right) +$$

$$+ \frac{(T_m + T_g)^2}{2(T_m + T_g - A T_m T_g)} \left\{ T_g^2 T_m^2 \frac{1 + A(T_g + T_m)}{(T_g + T_m)^2} + \right.$$

$$+ \left[(T_g + T_m)^2 - 2T_g T_m \right] \left[\frac{T_g T_m}{(T_g + T_m)^2} + 1 \right] +$$

$$\left. + \frac{T_g^2 T_m^2}{(T_g + T_m)^2} + \frac{T_g + T_m}{A} \right\}$$

and the time weighted quadratic integral of the system weight function on the basis of formula (3—6) is:

$$\int_0^\infty t w^2(t)\,dt = \frac{1}{4}\,\frac{T_g T_m}{A} + T_g T_m \left[\frac{T_g^2 T_m^2}{(T_g + T_m)^2} + T_g + T_m\right]$$

4. Square-time weighted integral criterion

The computation of the square-time weighted integral value

$$K = \int_0^\infty t^2 x^2(t)\,dt$$

can equally be deduced from the computation of the simple quadratic integral.
The utilized relations are:

$$\mathscr{L}[tx(t)] = -\frac{\partial}{\partial s} X(s)$$

$$\int_0^\infty t^2 x^2(t)\,dt = \int_0^\infty (tx(t))^2\,dt \tag{4—1}$$

$$\int_0^\infty x^2(t)\,dt = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} X(-s)\,X(s)\,ds\,.$$

The order of equation of the transformed function derived denominator will be doubled in relation to the original one, due to the differentiation, and therefore, the computation of the square-time weighted integral criterions, even for systems with very small orders of equation, is very complex and difficult to handle. However, for the sake of completeness we can put down the values of the first two integrals, expressed with the aid of the Routh coefficients:

$n = 1$
$$\int_0^\infty t^2 x^2(t)\,dt = \frac{R_{00}}{4R_{10}^3}\,R_1^2$$

$n = 2$
$$\int_0^\infty t^2 x^2(t)\,dt = \frac{1}{4}\,\frac{R_{00}}{R_{10}^3} + R_1^2\left(\frac{R_{00}\,R_{01}^2}{2R_{10}^3} + \frac{R_{00}^2}{4R_{10}\,R_{01}^3} + \frac{R_{00}^3}{4R_{10}^3\,R_{01}^2}\right) + R_2^2\left(\frac{R_{00}^2\,R_{01}}{R_{10}^3} + \frac{R_{00}}{4R_{10}\,R_{01}^2} + \frac{R_{10}}{4R_{01}^3}\right) - \frac{R_1 R_2}{2}\left(\frac{R_{00}\,R_{01}}{R_{10}^2} + \frac{R_{00}}{R_{01}^3} + \frac{R_{00}^2}{R_{10}^2\,R_{01}^3}\right) \tag{4—2}$$

Comment: In the case of systems with higher orders of equation the computation by Routh coefficients of the square-time weighted integrals is very intricate. In the sense of the (4—1) relations they can be traced back to the relations in Part 1, only it must be remembered, that e.g. the computation of the square-time weighted integral criterion of a third-degree system can be deduced from a sixth-degree common quadratic integral expression, where the involved Routh coefficients will, of course, have a different meaning than the Routh coefficients of the original third-degree system. The evaluation is simpler, if

$$x(0) = 1 \quad \text{and} \quad \overset{(n-1)}{x}(0) = \ldots = \dot{x}(0) = 0$$

In this case:

$$R_1 = R_{00},\ R_2 = R_{10},\ R_3 = R_{01},\ \ldots$$

On the other hand, if

$$\overset{(n-1)}{x}(0) = 1 \quad \text{and} \quad \overset{(n-2)}{x}(0) = \ldots = x(0) = 0$$

then

$$R_n = R_{00} \quad \text{and} \quad R_{n-1} = \ldots = R_1 = 0$$

Therefore, the square-time weighted integrals involving the weight function are:

$$n = 1 \qquad \int_0^\infty t^2 w^2(t)\, dt = \frac{1}{4} \frac{R_{00}^2}{R_{10}^2}$$

$$n = 2 \qquad \int_0^\infty t^2 w^2(t)\, dt = \frac{1}{4} \frac{R_{20}}{R_{10}^3} + R_{00} \left(\frac{R_{00}^2 R_{01}^2}{R_{10}^3} + \frac{R_{00}}{4R_{10} R_{01}} + \frac{R_{10}}{4R_{01}^2} \right) \tag{4—3}$$

as with a second order system $R_{01} = R_{20}$.

Example 4.1. Compute the square-time weighted integral of the error function involving a unity step input signal of the tracking control system shown in Fig. 2, when the transfer functions are:

$$Y(s) = \frac{A}{(1 + sT_m)\, s}$$

$$W(s) = \frac{A}{T_m s^2 + s + A}$$

$$X_e(s) = \frac{T_m s + 1}{T_m s^2 + s + A}$$

The Routh-coefficients of the system are:

$$R_{00} = R_m \qquad R_{01} = A \qquad R_1 = T_m$$
$$R_{10} = 1 \qquad R_2 = 1$$
$$R_{20} = A$$

The required relation on the basis of expression (4—2) is:

$$\int_0^\infty t^2 x_e^2(t)\,dt = \frac{1}{4A^3} + \frac{T_m}{4}\left(1 + \frac{1}{A^2}\right) + \frac{T_m^2}{2}\left(A + A^2 - \frac{1}{A^3}\right) +$$
$$+ \frac{T_m^3}{2}\left(\frac{1}{2A^3} - \frac{1}{A^2}\right) + \frac{T_m^4}{4A^2}$$

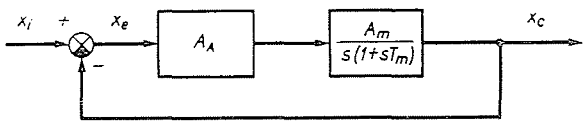

Fig. 2

Example 4.2. Compute the square-time weighted integral of the weight function of the system shown in Fig. 2.

$$\int_0^\infty t^2 w^2(t)\,dt$$

The required integral on the basis of relation (4—4) is:

$$\int_0^\infty t^2 w^2(t)\,dt = \frac{1}{4} A + T_m\left(T_m^2 A^2 + \frac{T_m}{4A} + \frac{1}{4A^2}\right)$$

Summary

The paper presents the computation of simple quadratic integrals, often used for the optimation of systems, with the aid of Routh coefficients. It introduces particularly the computations of the common, the time-weighted and the square-time weighted simple quadratic integrals. At the end of the sections applications are shown by simple examples.

References

1. WIENER, N.: Cybernetics. 1948.
2. NEWTON, G. C., GOULD, L. A. JR., KAISER, J. F.: Analytical Design of Linear Feedback Controls. John Wiley and Sons, 1957.
3. Фельдбаум, А. А.: Интегральные критерии качества регулирования. Автоматика и Телемеханика, 1948, 9. № 1.
4. Пугачев, В. С.: Теория случайных функций и ее применение к задачам автоматического управления. Москва, Физматгиз, изд. 1960.
5. WIENER N.: Extrapolation, Intrapolation and Smoothing of Stationary Time Series. New York, Mc Graw-Hill Book Co., 1961.
6. Красовский, А. А.: Интегральные оценки качества процесса регулирования. Москва, Машгиз, 1949.
7. KIPINIAK, W.: Dynamic Optimization and Control. New York 1961.
8. TSIEN H. S.: Technische Kybernetik. Stuttgart 1957.
9. EFFERTZ, F. H., KOLBERG, F.: Einführung in die Dynamik selbsttätiger Regelungssysteme. VDI-Verlag Düsseldorf, 1963.
10. CREMER, H., EFFERTZ, F. H.: Über die algebraischen Kriterien für die Stabilität von Regelungssystemen. Math. Annalen 137 (1959).
11. CREMER, L.: Ein neues Verfahren zur Beurteilung der Stabilität linearer Regelungssysteme. ZAMM **25**,27(1947).
12. EFFERTZ, F. H.: Darstellung der quadratischen Regelfläche als Funktion der Routhschen Probefunktionen und der Anfangswerte der Regelungstechnik. Moderne Theorien und ihre Verwendbarkeit 298. München, 1957.

János LEHOCZKY, Budapest XI., Egry József u. 18—20. Hungary