THE GRAPH THEORETICAL BASIS OF TRANSIENT STABILITY STUDIES IN POWER SYSTEMS

by

L. RÁCZ

Department for Electric Power Plants, Polytechnical University, Budapest

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Presented by Prof. Dr. O. P. GESZTI

To introduce theoretical considerations let us inspect the single two-machine system (with one degree of freedom) shown in Fig. 1. Each of the machines have their neutral point grounded, the machine parameters are the same; machine 1 generates P_v electrical power, which is consumed totally by machine 2 operating as a motor. The three-phase connection plan corresponding to the system of Fig. 1 is shown in Fig. 2. According to graph theory the synchronous machine is a multiterminal electromechanical component [8] and this way the graph of the system on the basis of Fig. 2 is shown in Fig. 3. The vertex r is regarded as reference vector; one must refer to the shaft torsion angle of machines 1 and 2 — caused by the mechanical power delivered (branches $b-r$ and $r-k$). Let us now inspect the star-like subgraph with six elements and four vertices, containing the topological informations of the interconnected three-phase stators. It can be seen that the subgraphs of stators 1 and 2 are both representing a tree; if we regard subgraph 1 as a tree of the six elemented graphs studied, then subgraph 2 is forming the chord system belonging to it and vica versa.

Let us now select subgraph 1 as a tree; in this case the fundamental circuit equations for the stator phase voltages in matrix notation are as follows:

$$\mathbf{B} \cdot U = 0, \tag{1a}$$

where **B** is the fundamental circuit matrix belonging to the selected chord system.

Writing (1a) in detail:

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} U_{1a} \\ U_{1b} \\ U_{1c} \\ U_{2a} \\ U_{2b} \\ U_{2c} \end{bmatrix} = 0. \tag{1b}$$

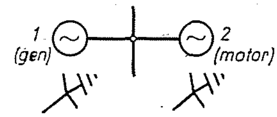


Fig. 1

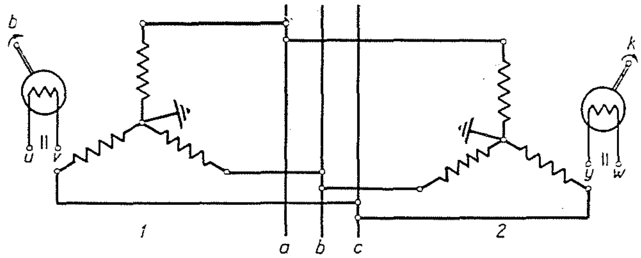


Fig. 2

After introducing the column vector of the stator phase voltages: $U_s = \begin{bmatrix} U_a \\ U_b \\ U_c \end{bmatrix}$ we can simplify (1b) as follows:

$$[\mathbf{E}\ \ \mathbf{E}] \cdot \begin{bmatrix} U_{1s} \\ U_{2s} \end{bmatrix} = 0. \tag{1}$$

On the basis of quite similar considerations we can write the cut-set equation for the stator phase currents; regarding the selected tree it has the form:

$$\mathbf{A} \cdot \mathbf{I} = [\mathbf{E} - \mathbf{E}] \cdot \begin{bmatrix} I_{1s} \\ I_{2s} \end{bmatrix} = 0, \tag{2}$$

where $\mathbf{A}$ is the cut-set matrix, while I_s signifies the column vector of stator phase currents. Equations (1) and (2) are completely describing the topology of the two-machine system shown in Fig. 1. However, for the determination of phase voltages, phase currents and the relative angles of the rotors whether

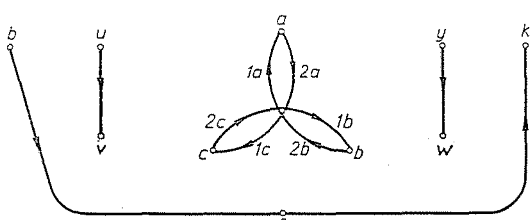


Fig. 3

in static or in transient state it is necessary to write down the terminal equations of the synchronous machines as multiterminal graph elements. The transient voltage equations of the three-phase circuits and the rotor field circuit

—neglecting the effect of the damping circuits and in view of the symmetrical structure of the machine—are as follows:

$$\begin{bmatrix} U_a \\ U_b \\ U_c \\ U_g \end{bmatrix} = \begin{bmatrix} R_G + pL_{aa} & pL_{ab} & pL_{ab} & pL_{ga} \\ pL_{ab} & R_G + pL_{aa} & pL_{ab} & pL_{ab} \\ pL_{ab} & pL_{ab} & R_G + pL_{aa} & pL_{ab} \\ pL_{ga} & pL_{ga} & pL_{ga} & R_g + pL_{gg} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_y \\ I_g \end{bmatrix} \qquad (3)$$

where all of the induction coefficients are trigonometric functions of the 2θ angles except L_{gg}; that is to say equation (3) is a system of differential equations with time varying coefficients. Expounding equation (3) according to the indicated partitioning (along the dashed lines) and using the introduced notation we obtain:

$$U_s = (\mathbf{R}_{ss} + \mathbf{pL}_{ss}) \cdot I_s + pL_{sg} \cdot I_g \qquad (3a)$$

$$U = {}_g p\,(L_{sg})_t \cdot I_s + (R_g + pL_{gg}) \cdot I_g. \qquad (3b)$$

(All the notations are clear on the basis of (3).)

One can now apply the Park transformation, i.e.:

$$\begin{bmatrix} U_d \\ U_q \\ U_0 \end{bmatrix} = U_P = \mathbf{T}_\mathbf{P} \cdot U_s \text{ and } \begin{bmatrix} I_d \\ I_q \\ I_0 \end{bmatrix} = I_P = \mathbf{T}_\mathbf{P} \cdot I_s, \qquad (4)$$

where the transformation matrix:

$$\mathbf{T}_\mathbf{P} = \frac{2}{3} \cdot \begin{bmatrix} \cos\theta & \cos\left(\theta + \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) \\ -\sin\theta & -\sin\left(\theta + \frac{2\pi}{3}\right) & -\sin\left(\theta - \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \qquad (4a)$$

while the inverse matrix:

$$\mathbf{T}_\mathbf{T}^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta - \frac{2\pi}{3}\right) & 1 \end{bmatrix}, \qquad (4b)$$

here: $\Theta = \Theta_0 + \omega t + \delta$.

Substituting (4) into (3a) and (3b) the result is:

$$U_P = \left[\mathbf{T}_\mathbf{P} \cdot (\mathbf{R}_{ss} + \mathbf{pL}_{ss}) \cdot \mathbf{T}_\mathbf{P}^{-1}\right] I_P + \left[\mathbf{T}_\mathbf{P} \cdot pL_{sg}\right] I_g \qquad (5a)$$

and

$$U_g = \left[p(L_{gs})_t \cdot \mathbf{T}_\mathbf{P}^{-1}\right] I_P + (R_{gg} + pL_{gg}) \cdot I_g. \qquad (5b)$$

In equs. (5a) and (5b) the products in brackets give the impedance matrix of equ. (3) — the latter being written in the original three-phase system —transformed to Park's reference frame. After completing the indicated operation we obtain the transformed voltage equations; in matrix notation:

$$\begin{bmatrix} U_d \\ U_q \\ U_0 \\ \cdot\ \cdot \\ U_g \end{bmatrix} =$$

$$= \begin{bmatrix} R_s + pL_{dd} & -(\omega + \Delta\omega) L_{qq} & 0 & . & pM_{gd} \\ (\omega + \Delta\omega) L_{dd} & R_s + pL_{qq} & 0 & . & (\omega + \Delta\omega) M_{gd} \\ 0 & 0 & R_0 + pL_0 & . & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{3}{2} pM_{gd} & 0 & 0 & . & R_g + pL_{gg} \end{bmatrix} \cdot \begin{bmatrix} I_d \\ I_q \\ I_0 \\ \cdot\ \cdot \\ I_g \end{bmatrix} \quad (6)$$

where $\Delta\omega = p\delta$, while the elements L_{dd}, L_{qq} and M_{gd} of the new impedance matrix can be computed from the mean and absolut values of the L_{aa}, L_{ab} and L_{ag} elements (being functions of 2θ).

We have to add the mechanical hunting equation of the synchronous machine to equ. (6); the mechanical power transmitted by the shaft of the synchronous machine is as follows:

$$P_m = U_d \cdot I_d + U_g \cdot I_q + (D + pT_\omega) . p\delta. \quad (7)$$

Equations (6) and (7) are the terminal equations of the synchronous machine regarded as a multiterminal graph element. The transformation equation (4) can be written for both of machines 1 and 2:

$$U_{P1} = \mathbf{T}_P \cdot U_{s1} \quad U_{P2} = \mathbf{T}_P \cdot U_{s2}.$$

Premultiplying the fundamental circuit equ. (1) with $\mathbf{T}_P$ the result is:

$$\mathbf{T}_P [\mathbf{E}\,\mathbf{E}] \begin{bmatrix} U_{1s} \\ U_{2s} \end{bmatrix} = \mathbf{T}_P \mathbf{E} \cdot U_{1s} + \mathbf{T}_P \cdot \mathbf{E} \cdot U_{2s} = \mathbf{E} \cdot \mathbf{T}_P \cdot U_{1s} + \mathbf{E} \cdot \mathbf{T}_P \cdot U_{2s} =$$

$$= [\mathbf{E}\,\mathbf{E}] \cdot \begin{bmatrix} U_{1P} \\ U_{2P} \end{bmatrix} = 0 \quad (8)$$

and similarly in the case of equ. (2)

$$\mathbf{T}_P \cdot [\mathbf{E}\ -\mathbf{E}] \begin{bmatrix} I_{1s} \\ I_{2s} \end{bmatrix} = [\mathbf{E}\ -\mathbf{E}] \begin{bmatrix} I_{1P} \\ I_{2P} \end{bmatrix} = 0 . \quad (9)$$

That is to say, we can apply the fundamental circuit matrix and the cut-set matrix as well for the voltage and current vectors transformed into Park's reference frame, on the basis of equs. (8) and (9). But since equs. (6) and (7) can be written for both of machines 1 and 2, the transient stability of the two-machine system shown in Fig. 1 can be studied by substituting these equations into equs. (8) and (9). In relations (8)—(9) the Park transformation could be directly applied to the original three-phase graph-equations because the six-branched (three-phase) subgraph showing the stator connections (Fig. 3) had its fundamental circuit i.e. cut-set matrix composed of two unit matrices. However, one cannot transform the graph itself, since the submatrix of $d-q-o$ quantities in equ. (6) is not diagonal — and is not even symmetric— i.e. there is an interaction between the d and q quantities at all the synchronous machines (and at the passive elements, too). Thereby one should always have to determine the **A** and **B** matrices from the three-phase graph, which is very complicated by more composed systems. To avoid these difficulties it is practical to apply a further variable-transformation, according to the following equations (the general variable is v, which can designate voltage and current as well):

$$v_\varrho = \frac{1}{\sqrt{2}}(v_d + jv_q)$$

$$\hat{v}_\varrho = \frac{1}{\sqrt{2}}(v_d - jv_q) \tag{10}$$

$$v_0 = v_0 .$$

In accordance with that, the transformation matrix is:

$$\mathbf{T}_\varrho = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & j & 0 \\ 1 & -j & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}. \tag{10a}$$

After which we can write (9) in matrix notation:

$$v_\varrho = \mathbf{T}_\varrho \cdot v_P . \tag{10b}$$

One can also easily realize the following:

$$\mathbf{T}_\varrho^{-1} = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 & 1 & 0 \\ -j & j & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}. \tag{10c}$$

Let us further assume that the synchronous saliency is negligible (which will not cause great error, the biggest part of the machines in the system being turbogenerators with cylindrical rotors); in this case in equ. (6): $L_{dd} =$

$=L_{qq}$. It is now practical to expound equ. (6) according to the marked partitioning:

$$U_P = \mathbf{Z}_P \cdot I_P + Z_{Pg} \cdot I_g$$
$$U_g = Z_{gP} \cdot I_P + Z_g \cdot I_g . \tag{10a}$$

(The notation is evident after comparing with equ. (6).)

We can now apply the transformation (10b) for the voltages and currents:

$$U = [\mathbf{T}_\varrho^{-1} \cdot \mathbf{Z}_P \cdot \mathbf{T}_\varrho] \cdot I_P + [\mathbf{T}_\varrho^{-1} \cdot Z_{Pg}] \cdot I_g \tag{11a}$$

$$U_g = [Z_{gP} \cdot \mathbf{T}_\varrho] \, I_P + Z_g \cdot I_g . \tag{11b}$$

Computing the products in brackets we obtain:

$$[\mathbf{T}_\varrho^{-1} \cdot \mathbf{Z}_P \cdot T_\varrho] = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 & j & 0 \\ 1 & -j & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix} \begin{bmatrix} R_s + pL_{dd} & -(\omega + \Delta\omega) L_{dd} & 0 \\ (\omega + \Delta\omega) L_{dd} & R_s + pL_{dd} & 0 \\ 0 & 0 & R_0 + pL_0 \end{bmatrix} \cdot$$

$$\cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 \\ -j & j & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix} = \begin{bmatrix} Z_{dd} + pL_{dd} & 0 & 0 \\ 0 & Z_{dd} + pL_{dd} & 0 \\ 0 & 0 & R_0 + pL_0 \end{bmatrix} . \tag{12a}$$

Which is already a diagonal matrix.

Similarly:

$$[Z_{gP} \cdot \mathbf{T}_\varrho] = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{3}{2} \cdot pM_{gd} & \frac{3}{2} \cdot pM_{gd} & 0 \end{bmatrix}$$

and

$$[\mathbf{T}_\varrho^{-1} \cdot Z_{Pg}] = \frac{1}{\sqrt{2}} \begin{bmatrix} pM_{gd} + jX_{gd} \\ pM_{gd} - jX_{gd} \\ 0 \end{bmatrix} . \tag{12b}$$

where:

$$Z_{dd} = R_s + j \, \omega \, L_{dd} + j \, \Delta\omega \cdot L_{dd'} \text{ and } X_{gd} = \omega \, M_{gd} + \Delta\omega \, M_{gd} . \tag{12c}$$

After substituting equ. (12) into equ. (10) the result is:

$$\begin{bmatrix} U_\varrho \\ \hat{U}_\varrho \\ U_0 \\ \cdot\cdot \\ Ug \end{bmatrix} = \begin{bmatrix} Z_{dd} + p \, L_{dd} & 0 & 0 & \frac{1}{\sqrt{2}} (p \, M_{gd} + j \, X_{gd}) \\ 0 & \hat{Z}_{dd} + p \, L_{dd} & 0 & \frac{1}{\sqrt{2}} (p \, M_{gd} - j \, X_{gd}) \\ 0 & 0 & \mathrm{R}_0 + p \, L_0 & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \frac{3}{\sqrt{6}} \cdot p \, M_{gd} & \frac{3}{\sqrt{6}} \cdot p \, M_{gd} & 0 & \mathrm{R}_g + p \, L_{gg} \end{bmatrix} \cdot \begin{bmatrix} I_\varrho \\ \hat{I}_\varrho \\ I_0 \\ \cdot\cdot \\ I_g \end{bmatrix} . \tag{13}$$

By using equs. (7) and (10) and completing the prescribed operations we obtain the following expression for the mechanical power:

$$P_m = (U_\varrho \cdot \hat{I}_\varrho + \hat{U}_\varrho \cdot I_\varrho + (D + p\,T_\omega)) \cdot p\delta = U_\varrho \hat{I}_\varrho + \hat{U}_\varrho I_\varrho + (D + p\,T_\omega) \cdot \Delta\omega\,. \tag{14}$$

Equs. (13) and (14) are the terminal equations of the synchronous machine in the new reference frame. Premultiplying now the fundamental circuits (8) and (9), respectively, cut-set equations by $\mathbf{T}_\varrho$ the result is — on the analogy of (8):

$$[\mathbf{E}\,\mathbf{E}] \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = 0 \tag{15}$$

and

$$[\mathbf{E} - \mathbf{E}] \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = 0. \tag{16}$$

Equs. (15) and (16) are the fundamental circuit and cut-set equations of the system shown in Fig. 1. after accomplishing the new ("ϱ") transformation. It is also possible here to directly apply the fundamental circuit (**B**) and cut-set matrix (**A**) to the voltage and current vectors transformed into the new "ϱ" system, on the analogy of the considerations made in the case of equs. (8)—(9). However, we are now in a much more advantageous position, since according to (13) the submatrix of stator quantities (in the left upper corner) is a diagonal matrix — that is to say, the ϱ, $\hat{\varrho}$ and 0 quantities are independent of each other. It is possible now to draw the subgraph belonging to (15) and (16); we accomplish in this case the inverse of the usual procedure: we have to seek the graph and its formulation tree suitable for the fundamental circuit and cut-set equations — these latter containing the same topological informations as the graph. This graph is shown in Fig. 4. In the studied case we were able to transform the graph itself. The subgraph could be divided into three separate parts, since the ϱ, $\hat{\varrho}$ and 0 quantities are independent of each other. If the neutral point of the generator is not earthed (corresponding to the general practice, then the subgraph of the zero sequence variables and with that the proper row and column of the impedance matrix must be cancelled.

In writing equ. (13) as a hypermatrix-equation for both of the synchronous machines, using the relations (15) and (1) and adding to that

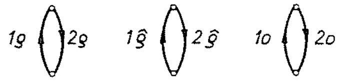
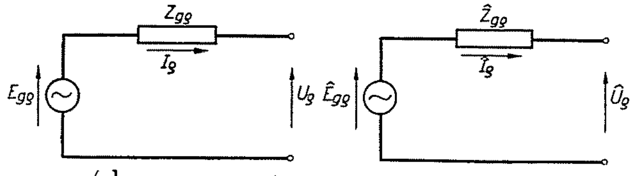


Fig. 4

equ. (14), their result is a six-order system of differential equations (with the following variables: $U_{1\varrho}$, $\hat{U}_{1\varrho}$, $I_{1\varrho}$, $\hat{I}_{1\varrho}$, I_g, $\Delta\omega_1 - \Delta\omega_2$; there are no zero-sequence quantities because of the symmetry of the system) which can be solved in the knowledge of the initial values. These latter can be determined for U_ϱ,



$$E_{g\varrho} = \left(\frac{1}{\sqrt{2}} \cdot pMgd + jx_{gd}\right) I_g \qquad \hat{Z}_{g\varrho} = \hat{Z}_{du} + pL_{du}$$

$$Z_{g\varrho} = Z_{dd} + pL_{dd}$$

Fig. 5

$\hat{U}_\varrho$, I_ϱ and $\hat{I}_\varrho$, respectively, in transforming the phase voltage and current values of the steady state; the initial value of the field current is: $I_{go} = \dfrac{U_g}{R_g}$, while $\Delta\omega_1 = 0$ and $\Delta\omega_2 = 0$, since in the steady state the machines are rotating with synchronous speed. The $\delta_1(t)$ and $\delta_2(t)$ time-functions can be computed by integrating the $\Delta\omega_1$ and $\Delta\omega_2$ relative angle velocity values for the studied time interval, beginning at the moment: $t = 0$.

The lower limit of the integral is the initial value of δ, which can be taken as zero.

The following procedure is the same as in the case of more complicated power systems, i.e. we have to draw the (equal) ϱ and $\hat{\varrho}$ subgraphs — on the basis of the normal single-line connection scheme of the system. According to equ. (13) there is a voltage source in the connection scheme of both the ϱ and $\hat{\varrho}$ components; the equivalent scheme of the generator according to (13) is to be seen in Fig. 5. We have now to inspect the transformed equations of other elements which the power systems are composed of.

1. Transformers

The T equivalent scheme of single-phase transformers — regarded as four-poles — is widely known; this has to be repeated three times by three-phase transformers ensuring the topologically correct interconnections. Since the open circuit impedance of transformers is by 2—3 orders greater than other (series) impedances occurring in transient stability studies, the neglection of those — i.e. the omission of the cross branches in the T equivalent scheme — causes, but very little, error. Further, if the computations are made in per unit system, then the transformer ratios will not figure either in the equivalent scheme, which is simplified thereby to symmetrical three-phase series impedances. (One can easily realize this latter statement by star/star connected transformers; however, if one of the windings is delta-connected it is always possible to determine the equivalent star winding.) On this basis the terminal matrix-equation of three-phase transformers (taking into account the phase-symmetry) is:

$$\begin{bmatrix} \Delta u_a \\ \Delta u_b \\ \Delta u_c \end{bmatrix} = (r + \omega l) \cdot \mathbf{E} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \mathbf{Z} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}, \tag{17}$$

where Δu is the longitudinal voltage-drop, while r and ωl are the resistance and inductive reactances, respectively, (all of the quantities are expressed in a p.u. system, signified by minuscule notation); $\mathbf{Z}$ is the diagonal impedance matrix of the transformer.

Transforming equ. (17) directly into the ϱ system on the basis of App. 1 we obtain:

$$\begin{bmatrix} \Delta u_\varrho \\ \Delta \hat{u}_\varrho \\ \Delta u_0 \end{bmatrix} = [\mathbf{T}_{\varrho P}^{-1} \cdot \mathbf{Z} \cdot \mathbf{T}_{\varrho P}] \cdot \begin{bmatrix} i_\varrho \\ \hat{i}_\varrho \\ i_0 \end{bmatrix}, \tag{18a}$$

where the triple product in brackets can be computed using equ. (17):

$$\mathbf{T}_{\varrho P}^{-1} \cdot \mathbf{Z} \cdot \mathbf{T}_{\varrho P} = (r + \omega\, 1) \cdot \mathbf{T}_{\varrho P}^{-1} \cdot \mathbf{E} \cdot \mathbf{T}_{\varrho P} = (r + \omega\, 1) \cdot \mathbf{E} = \mathbf{Z}, \tag{18b}$$

i.e. the impedance matrix is invariant to the $\mathbf{T}_{\varrho P}$ transformation, and therefore:

$$\begin{bmatrix} \Delta u_\varrho \\ \Delta \hat{u}_\varrho \\ \Delta u_0 \end{bmatrix} = \mathbf{Z} \cdot \begin{bmatrix} i_\varrho \\ \hat{i}_\varrho \\ i_0 \end{bmatrix}. \tag{18}$$

Relation (18) is the terminal equation of the transformer in the ϱ system; the suitable graph of the transformer is shown in Fig. 6.

The graph is composed of a forest containing three separate trees, similarly to the subgraph of the synchronous machine phase quantities. One can ascertain from equ. (18) that the transformer is represented in the ϱ system by identical impedances in all the three component networks in the same way as in the case of the ordinary symmetrical components. On this basis the zero sequence equivalent scheme (the third equ. of (18)) can be determined in accordance with the connection group from the theory of symmetrical components.

2. Transmission lines

Neglecting the shunt admittances (which cause, but very little, fault in the case of not too long transmission lines in transient stability studies) transmission lines can also be represented in every phase by series impedances. The terminal equations of transmission lines symmetrized by phase-change can be written on the analogy of equ. (17); after transforming with the matrix $\mathbf{T}_{\varrho P}$ we obtain equations corresponding to (18) in the ϱ system; the graph is the same as that of Fig. 6. An essential difference compared with the trans-

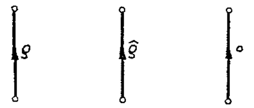


Fig. 6

formers is that in the zero sequence equation we obtain impedances differing from the other two (because of the earth return circuit); this Z_0 can be computed from the Carson—Pollaczek relations.

According to the above considerations the transformed equations and the graph of the other three-phase network elements (choke, consumer etc.) can be determined in exactly the same way as in Chapters 1 and 2.

3. The algorithm to follow in the case of complicated systems

We have seen in the former paragraphs that the advantage of transforming the transient voltage equations of whatsoever complicated symmetrical, three-phase power systems containing synchronous machines with cylindrical rotors is, that the resulting equations for the $\varrho, \hat{\varrho}$, and zero-sequence components are completely independent of each other. The zero-sequence component appears only if there is an asymmetrical shunt or series fault in the system, that is to say, in case of symmetrical (whether steady state or transient) relations there exist only ϱ and $\hat{\varrho}$ voltage and current components, resp. However, these two component systems being independent of each other and their graphs identical (Figs 4 and 6) one can draw two separate identical graphs for the power system studied, the structure of which is equivalent with the usual single line connection scheme of the same system.

Let us now inspect, for instance, the four machine power system shown in Fig. 7 (which has 3 degrees of freedom). Let us assume that in steady state operating conditions of the system at one of the busses there occurs an abrupt

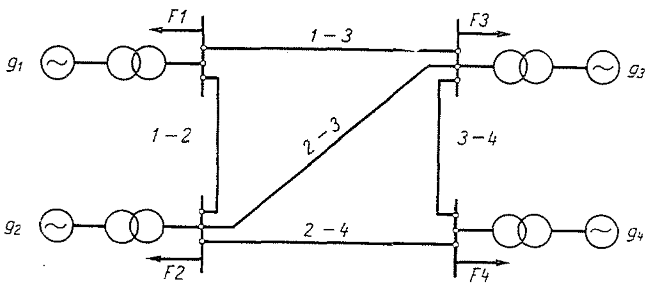


Fig. 7

consumption-change and by that electromechanical transients arise. To determine this we have to construct the graph of the system in accordance with the above principles; only one of them is shown here, the subgraphs of three phase quantities transferred into the $\underset{\sim}{\varrho}$ and $\underset{\sim}{\hat{\varrho}}$ systems being identical.

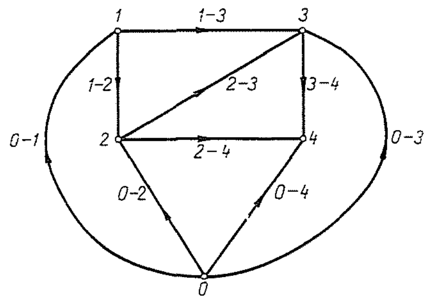


Fig. 8

For the sake of simplicity the subgraphs representing the field circuit and the mechanical connections, resp., of the synchronous machines (in this case forming a forest composed of 5 trees — on the analogy of Fig. 3) were not indicated on Fig. 8. The number of vertices in the graph of Fig. 8 is five, while that of the elements is nine (the graph vertex designated by 0 is the neutral-bar of the $\underset{\sim}{\varrho}$ and $\underset{\sim}{\hat{\varrho}}$ networks, resp.), according to which the rank of the graph is four and its nullity five. Since the rank of a graph is equal to that of its cut-set matrix, while its nullity gives the rank of the fundamental circuit matrix, here it is more practical to use the cut-set equations for the computation as this way the inversion of a smaller matrix will be needed. We shall regard the busbar-consumers as those having constant current (I_F) demand, that is to say, they are represented as current sources. Consequently, the generator and the consumer of whichever busbar can be regarded as concentrated graph elements, the terminal equation of which on the basis of Kirchhoff's laws written for Fig. 9 is as follows:

$$I_H + I_F = Y_g(\Delta U_g + E_g). \tag{19}$$

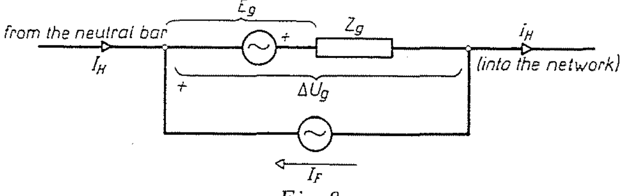


Fig. 9

where:

$$Y_g = 1/Z_g.$$

(That is valid both in the system of the ϱ and $\hat{\varrho}$ components, resp.)

Let us select the (Lagrangian) tree seen in Fig. 10 of the graph of Fig. 8

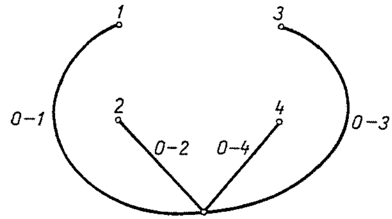


Fig. 10

for writing the equations. The cut-set matrix is the following:

$$\mathbf{A} = \begin{array}{c} \\ 0-1 \\ 0-2 \\ 0-3 \\ 0-4 \end{array} \begin{array}{c} \begin{array}{ccccccccc} 0-1 & 0-2 & 0-3 & 0-4 & 1-2 & 1-3 & 2-3 & 2-4 & 3-4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} . \tag{20}$$

The E_g electromotive force and the Y_g admittance appearing in equ. (19) are, for example, on the basis of the ϱ component network in Fig. 5 as follows:

$$E_g = \left(\frac{1}{\sqrt{2}} p\, M_{gd} + j\, X_{gd} \right) \cdot I_g$$

and (21)

$$Z_g = Z_{dd} + p\, L_{dd} .$$

The branch equations corresponding to equ. (19) can be written for every branch of the network with the only restriction that in the equations of the complement (the chords) of the chosen trees E_g and I_F are equal to zero. In accordance with that the branchmatrix equation of the network has been given by equation (22a) (omitting the ϱ indices for simpler notation)

$$\begin{bmatrix} I_{H0-1} \\ I_{H0-2} \\ I_{H0-3} \\ I_{H0-4} \\ I_{H1-2} \\ I_{H1-3} \\ I_{H2-3} \\ I_{H2-4} \\ I_{H3-4} \end{bmatrix} + \begin{bmatrix} I_{F1} \\ I_{F2} \\ I_{F3} \\ I_{F4} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} =$$

$$
= \begin{bmatrix}
Y_{g1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & Y_{g2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Y_{g3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & Y_{g4} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Y_{1-2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & Y_{1-3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Y_{2-3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Y_{2-4} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Y_{3-4}
\end{bmatrix} \cdot \begin{bmatrix}
U_{g1} + E_{g1} \\
U_{g2} + E_{g2} \\
U_{g3} + E_{g3} \\
U_{g4} + E_{g4} \\
U_{1-2} + 0 \\
U_{1-3} + 0 \\
U_{2-3} + 0 \\
U_{2-4} + 0 \\
U_{3-4} + 0
\end{bmatrix} \tag{22a}
$$

or more shortly:

$$I_H + I_F = \mathbf{Y}_g \cdot (U + E_g). \tag{22b}$$

After premultiplying this with the cut-set matrix and rearranging, we obtain:

$$\mathbf{A} \cdot I = \mathbf{A}(\mathbf{Y}_g \cdot U + \mathbf{Y}_g E_g - I_F). \tag{23}$$

But according to the cut-set equations:

$$\mathbf{A} \cdot I_H = 0. \tag{24}$$

In this way:

$$\mathbf{A} \cdot \mathbf{Y}_g \cdot U = \mathbf{A}(I_P - \mathbf{Y}_g E_g). \tag{25}$$

Introducing now the column vector of the cut-set (or with another terminology: node-pair) voltages (U_v), where:

$$U = \mathbf{A}_t \cdot U_v,$$

and in using this latter in equ. (25) there results the node-pair system of equations of the network for the chosen tree:

$$(\mathbf{A} \cdot \mathbf{Y}_g \cdot \mathbf{A}_t) \cdot U_v = \mathbf{A}(I_F - \mathbf{Y}_g \cdot E_g). \tag{26}$$

The triple product in brackets on the left side of the equation is the node-pair admittance matrix of the network for the chosen tree:

$$\mathbf{Y}_v = \mathbf{A} \cdot \mathbf{Y}_g \cdot \mathbf{A}_t, \tag{26a}$$

at the same time the quantity in brackets on the right side is the column vector of the resulting cut-set (node-pair) currents:

$$I_v = A(I_F - \mathbf{Y}_g \cdot E_g). \tag{26b}$$

The $\mathbf{Y}_v$ matrix is in our case:

$$Y_v = \begin{bmatrix} Y_{g1}+Y_{1-2}+Y_{1-3} & -Y_{1-2} & -Y_{1-3} & 0 \\ -Y_{1-2} & Y_{g2}+Y_{1-2}+Y_{2-3}+Y_{2-4} & -Y_{2-3} & Y_{2-4} \\ -Y_{1-3} & -Y_{2-3} & Y_{g3}+Y_{1-3}+Y_{2-3}+Y_{3-4} & -Y_{3-4} \\ 0 & -Y_{2-4} & -Y_{3-4} & Y_{g4}+Y_{2-4}+Y_{3-4} \end{bmatrix}.$$

That is to say, a matrix of the fourth order, which is nonsingular and, therefore, in solving equ. (26) we obtain the cut-set (node-pair) voltages:

$$U_v = \mathbf{Y}_v^{-1} \cdot I_v \,. \tag{27}$$

(One has to count the $\mathbf{Y}_v^{-1}$ inverse matrix at the beginning of the transient stability studies.) The elements of the column vector U_v are equivalent with the tree branch voltages, i.e.:

$$U_{vi} = U_{gi}, \text{ where: } i = 1, 2, 3, 4 \,.$$

But since the 0 vertex of the tree in Fig. 10 is the neutral point of the component network studied (ϱ or $\hat{\varrho}$), U_{gi} is the voltage of the i^{th} busbar. The ϱ and $\hat{\varrho}$ components of the generator currents can now be computed from equ. (19) (the value $I_H + I_F$ being the stator current of the synchronous machine). The following has still to be mentioned:

The terminal equation of the synchronous machine (13) transformed into the ϱ reference system was deduced from those written in Park's reference system, i.e. in a reference $d-q$ frame fixed firmly to the rotor of the synchronous machine. It is well known that there is an angle difference among the $d-q$ frames of the single generators of the system in steady state, too (the so-called load-angles); in transient state these angle differences vary periodically in time. The effect of these phenomena on the equations transformed into the ϱ system is that the Θ_0 relative angles occurring in the expression of the Θ arguments which appear in the elements of the $\mathbf{T}_{\varrho P}$ transformation matrix [see Appendix 1, equ. (f2) — the variable being: $\Theta_0 + \omega t + \delta$ according to equ. (4c)], are different for the single synchronous machines. One has therefore to carry out the computation in determining the pretransient steady state still in the system of the symmetrical phase quantities. For symmetry reasons the system can be represented by its well-known single phase equivalent scheme; this way (on the analogy of the concrete example of Figs 7—10) the node-pair admittance matrix will evidently be the same as matrix $\mathbf{Y}_v$ in equ. (16c). It became evident, namely, in paragraphs 1 and 2 that the impedance of passive network elements is invariant to the $\mathbf{T}_{\varrho P}$ transformation; on the other hand, it is to be seen from equs. (13)—(12c) and from Fig. 5, resp., that one can represent the synchronous machine in steady state by its open circuit terminal voltage and synchronous reactance after the $\mathbf{T}_{\varrho P}$ transformation, too. We have to form the inverse of the $\mathbf{Y}_v$ matrix already for the steady state computation — and by storing this we shall be able to use it for the transient state computation supposed the members like pL and $\Delta\omega L$ (Fig. 5 and equ. (12c)) are negligible. In the knowledge of the steady state (and of the single Θ_0 angles) one can determine the initial ϱ and $\hat{\varrho}$ values of the parameters by applying the $\mathbf{T}_{\varrho P}$ transformation. On the basis of App. 1 and equ. (4c) the U_ϱ and $\hat{U}_\varrho$ components of whichever synchronous machine are as follows (it was mentioned above that zero-sequence voltage and current will not arrive to the generator):

$$U = \frac{\sqrt{2}}{3}\left[U_a \cdot e^{-j\Theta} + U_b\, e^{-j\Theta} \cdot e^{-j\frac{2\pi}{3}} + U_c \cdot e^{-j\Theta} \cdot e^{j\frac{2\pi}{3}}\right] =$$

$$= e^{-j\delta} \cdot \left[U_a \cdot e^{-j(\Theta_0 + \omega t)} + U_b \cdot e^{-j(\Theta_0 + \omega t)} \cdot e^{-j\frac{2\pi}{3}} + \right.$$

$$\left. + U_c\, e^{-j(\Theta_0 + \omega t)}\, e^{j\frac{2\pi}{3}}\right] = e^{-j\delta} \cdot U_{\varrho 0}, \qquad (28)$$

where $U_{\varrho 0}$ is the initial value of U_ϱ; quite similarly: $\hat{U}_\varrho = e^{j\delta} \cdot \hat{U}_{\varrho 0}$.

Which means that the U_ϱ and $\hat{U}_\varrho$ values of any of the synchronous machines can be computed during the transient state from the initial values of the ϱ components by applying the rotation according to equ. (28) with the prevailing $\delta(t)$ angle.

The terminal equations (13) of the synchronous machines are differential equations; but since the coefficient matrix in the ϱ system contains complex numbers it is impossible to solve them on an analog computer. System transients can be studied, therefore, only on a digital computer with the aid of the Runge—Kutta integration procedure. It is practical to accomplish the computations in one time step according to the following program:

1. One has to solve the node-pair matrix equation of phase quantities in the pretransient symmetrical steady state — on the analogy of equs. (22)—(27) — i.e. there is to be formed the inverse of the $\mathbf{Y}_v$ matrix. It is the same task as that by the usual load-flow study of the network; we can do this by an iterative procedure after having the $\mathbf{Y}_v^{-1}$ matrix.

2. In the knowledge of the initial values of node voltages and generator currents from step 1 one has to determine the initial values of

a) the generator open circuit voltages and the field currents,

b) the quantities transformed into the ϱ system on the basis of App. 1 — i.e. in the knowledge of the generator phase currents and node phase voltages.

3. We have to determine the new ϱ and $\hat{\varrho}$ graph of the network modified by the cause producing transients (network-modification, short circuit etc.),

and after this we have to form the $\mathbf{Y}_v$ and the $\mathbf{Y}_v^{-1}$ matrix, respectively, both in the ϱ and $\hat{\varrho}$ systems. (App. 2 describes the manner of connecting the ϱ, $\hat{\varrho}$, and 0 component networks in case of asymmetrical shunt faults.)

4. The ϱ and $\hat{\varrho}$ components of the busbar voltages have to be determined and after that the $(I_H + I_F)$ column vector (the generator currents) from equ. (22b).

5. One has to compute the change in

a) the current I_g and then the $E_{g\varrho}$ and $\hat{E}_{g\varrho}$ voltages, resp.

b) the angle δ

from the differential equations (13) and (14), resp. during the time step studied at each of the synchronous machines.

(We have to mention that if the effect of the damping coils is to be accounted for the change in the $E_{g\varrho}$ and $\hat{E}_{g\varrho}$ voltages must be determined from the (f7) differential equation of App. 3.)

6. The modified values of $E_{\varrho g}$ and $\hat{E}_{g\varrho}$ (caused by the change of angle) can be computed from the new value of the δ angle — which is determined for the end of the time step studied — by using equ. (28).

7. Finally one has to count the I_v node-pair current vector; for the next time step the computation is repeated from point 4.

The above computation program consisting of 7 points can be regarded as a part of the block scheme of the digital algorithm.

It is still to be mentioned that the graph of multinode, multiloop power systems formed in accordance with Fig. 8 is generally nonplanar, and has therefore the disadvantage of possessing no dual graph; however, the cut-set and fundamental circuit equations can be written and the computations completed in the same way as above.

4. The manner of considering the effect of voltage and turbine governors

According to literature in many cases it is expedient to take the effect of voltage and turbine governors into consideration by transient stability studies. As a starting point of solving the problem on the basis of graph theory we have to regard Fig. 3 again, which contains the complete graph of the two-machine system shown in detail on Fig. 2. The six-branched subgraph in the middle is here characteristic for the stator connections; subgraphs $u-v$ and $y-w$, resp., give information on the topology of the field-circuits, while subgraph $b-r-k$ on that of the mechanical relations. However, the subgraph of stator quantities was influenced by the transformation into the ϱ system, while that of the latter quantities remained unaltered. Let us now inspect one by one the possibilities of taking into account the voltage and turbine governor, respectively.

a) As long as the excitation voltage was assumed to be constant the subgraphs of the excitation circuits had no part when writing down the equations. Our task has, however, increased now as we have to complete that part of Fig. 3 which is related to the excitation circuits. The excitation voltage of the synchronous machine is supplied by a direct current generator, while the excitation circuit of this latter is supplied by a separate auxiliary generator in the case of bigger synchronous machines. Further we have to take into account that the voltage-regulator — the input quantity of which is the terminal voltage of the synchronous machine (or in the network of Fig. 7 the voltage of busbar 1 characterized by the graph-element 0—1 on Fig. 8) — is acting on the field-

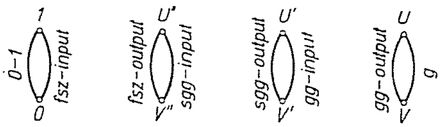


Fig. 11

g: field circuit of the synchronous machine
gg: main exciter d. c. generator
sgg: auxiliary exciter d. c. generator
fsz: voltage regulator

circuit of the auxiliary exciter; on this basis the field-circuit subgraph (u—v) of e.g. synchronous machine 1 (in the system of Fig. 7) must be completed according to Fig. 11.

In Fig. 11 the main and auxiliary exciter and the voltage regulator equipment, respectively, are regarded as two-port networks — having input and output terminals; according to which their graph consists of two separate (input and output) elements.

Two-port networks — as is well known — are characterized by their transfer functions, which give the response of the system for a unit-step stimulus, and which can be expressed in terms of the Laplace operator, or — after inverse transformation — as a time-function. The graph of the synchronous machine given in Fig. 3 is now modified in the way that the number of vertices is increased by four, the number of non-connected subgraphs by two, and accordingly the rank of the graph has grown by two while its nullity by four. The rank of the graph being, however, equal to that of the matrix A, the rank of the problem (its degree of freedom) is augmented by two at each of the synchronous machines owing to the consideration of voltage regulator, — if using the method of investigation proposed in connection with Fig. 8. In this case the steps written in the 3rd paragraph have to be evidently completed by the step-by-step solution of the differential equations describing the two-port elements of Fig. 11.

b) Similarly, in so far as the mechanical power, i.e. the driving moment — transmitted through the shafts of the synchronous machines — is assumed to be constant, the $b-r-k$ subgraph in Fig. 3 — in which synchronous machine 1 is characterized by the $b-r$ element — can be omitted. However, if we wish to take into account the effect of the turbine governor the subgraph describing the topology of mechanical relations has to be completed. The mechanical power received by the generator is delivered by the turbine. This latter can also be regarded as a two-port transfer-element, characterized by its specified transfer function.

The power delivered by the steem streaming in, can be regarded as the input quantity of the turbine, while its output quantity is the mechanical power given to the synchronous machine. On the other hand, the steem streaming into the turbine can be regarded as the output parameter of the governor,

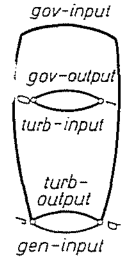


Fig. 12
gen: generator
turb: turbine
gov: turbine governor

the input of which is the $\Delta\omega$ angle velocity deviation. (The turbine governor itself is also a composed transfer element, consisting of a centrifugal measurer, hydraulic amplifier and operating element — this latter is the steem inlet valve of the turbine.) On the basis of the above considerations the subgraph of mechanical relations of machine 1 (in the system of Fig. 7) must be completed in accordance with Fig. 12.

The rank of the graph is increased by one, its nullity by three. In using the mathematical model based on the cut-set equations the rank of the problem (the degree of freedom) has grown by one at each of the machines, and the steps given in paragraph 3 must be completed with the step-by-step integration of the differential equations describing the transfer elements of Fig. 12.

Appendix 1

The combination of the "ϱ" and the Park transformation.

In three-phase systems we can change over from phase quantities to the "ϱ" reference frame quantities, by using equs. (4) and (10) followingly:

$$v_\varrho = \mathbf{T}_\varrho \cdot v_P = \mathbf{T}_\varrho \cdot \mathbf{T}_P \cdot v_s = \mathbf{T}_{\varrho P} \cdot v_s \,. \tag{f1}$$

Which can be expounded on the basis of equs. (4a) and (10a):

$$\mathbf{T}_{\varrho}\cdot\mathbf{T}_P=\mathbf{T}_{\varrho P}=\frac{1}{\sqrt{2}}\,\frac{2}{3}\cdot\begin{bmatrix}1 & j & 0\\ 1 & -j & 0\\ 0 & 0 & \sqrt{2}\end{bmatrix}\begin{bmatrix}\cos\theta & \cos\left(\theta+\frac{2\pi}{2}\right) & \cos\left(\theta-\frac{2\pi}{3}\right)\\ -\sin\theta & -\sin\left(\theta+\frac{2\pi}{3}\right) & -\sin\left(\theta-\frac{2\pi}{3}\right)\\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{bmatrix}=$$

$$=\frac{\sqrt{2}}{3}\cdot\begin{bmatrix}e^{-j\Theta} & e^{-j\left(\Theta+\frac{2\pi}{3}\right)} & e^{-j\left(\Theta-\frac{2\pi}{3}\right)}\\ e^{j\Theta} & e^{j\left(\Theta+\frac{2\pi}{3}\right)} & e^{j\left(\Theta-\frac{2\pi}{3}\right)}\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{bmatrix}. \tag{f2}$$

Similarly, according to equs. (4b) and (10b):

$$\mathbf{T}_P^{-1}\cdot\mathbf{T}_{\varrho}^{-1}=\mathbf{T}_{\varrho P}^{-1}=\frac{1}{\sqrt{2}}\begin{bmatrix}e^{j\theta} & e^{-j\theta} & \sqrt{2}\\ e^{j\left(\Theta+\frac{2\pi}{3}\right)} & e^{-j\left(\Theta+\frac{2\pi}{3}\right)} & \sqrt{2}\\ e^{j\left(\Theta-\frac{2\pi}{3}\right)} & e^{-j\left(\Theta-\frac{2\pi}{3}\right)} & \sqrt{2}\end{bmatrix}. \tag{f3}$$

And the inverse transformation: $v_s=\mathbf{T}_{\varrho P}^{-1}\cdot v_{\varrho}$. (f4)

Appendix 2

The representation of asymmetrical shunt faults in the "ϱ" reference system.

Asymmetrical shunt faults create connections among the ϱ, $\hat{\varrho}$ and 0 sequence networks at the point of fault; for their determination we have to take only two points into account — the point of fault and the neutral bus (the vertex 0 in the graph of Fig. 8) — of the ϱ, $\hat{\varrho}$ and 0 sequence substitution schemes of the network studied. Let us now inspect the main types of asymmetrical shunt faults:

a) Phase-to-ground short circuit (in phase a).

The column vectors of voltages and current at the fault location (index h) are:

$$U_h=\begin{bmatrix}0\\ U_{hb}\\ U_{hc}\end{bmatrix}\quad\text{and}\quad I_h=\begin{bmatrix}I_{ha}\\ 0\\ 0\end{bmatrix}. \tag{f5}$$

Applying (f4) to (f5) we obtain: $U_{hs}=\mathbf{T}_{\varrho P}^{-1}\cdot U_{h\varrho}$. (f6)

After expounding (f6) the first scalar equation is as follows:

$$U_{h\varrho}\cdot e^{j\theta}+\hat{U}_{h\varrho}\cdot e^{-j\theta}+U_{h0}=0\,. \tag{f7}$$

Applying (f1) to (f5) the result is: $I_h = \mathbf{T}_{\varrho P} \cdot I_{hs}$. (f8)

Expounding this latter:

$$I_h = I_{ha} \frac{\sqrt{2}}{3} \cdot e^{-j\theta}, \hat{I}_{h\varrho} = I_{ha} \cdot \frac{\sqrt{2}}{3} \cdot e^{j\theta}$$

and (f9)

$$I_{h0} = \frac{1}{3} I_a .$$

On the ground of equs. (f7) and (f9) we can sketch the substitution scheme of phase-to-ground short circuit in the ϱ ref. system; this is shown in Fig. 13. (In this figure there are "ideal induction motors" connected to the ϱ and $\hat{\varrho}$ networks, while an ideal transformer is connected to the 0 sequence network.)

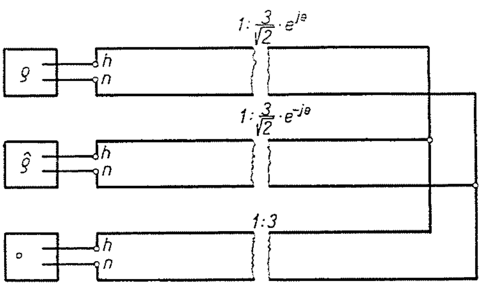


Fig. 13

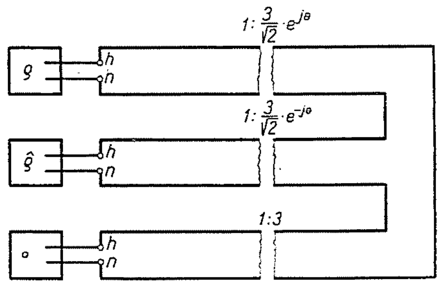


Fig. 14

b) Two-phase-to-ground short circuit (in the phases b—c)

After quite similar considerations we obtain:

$$I_{h\varrho} \cdot e^{j\theta} + \hat{I}_{h\varrho} \cdot e^{-j\theta} + I_{h0} = 0; \qquad \text{(f10)}$$

$$U_h = U_{ha} \cdot \frac{\sqrt{2}}{3} \cdot e^{-j\theta}, U_h = U_{ha} \cdot \frac{\sqrt{2}}{3} \cdot e^{j\theta}$$

and (f11)

$$U_{h0} = \frac{1}{3} U_{ha} .$$

The suitable substitution scheme in conformity with these latter equations is given in Fig. 13.

c) Phase-to-phase short circuit (in the phases b—c)

It is easy to realize analogously that one can now derive the substitution scheme from that shown in Fig. 14 by opening the zero sequence network between the points h and n (that is to say, in this case only the ϱ and $\hat{\varrho}$ networks are connected in parallel).

Appendix 3

The ϱ system terminal equations of synchronous machines with cylindrical rotor, taking the damping coils into account.

$$
\begin{bmatrix} U_d \\ U_q \\ U_0 \\ \cdots \\ U_g \\ 0 \\ 0 \end{bmatrix} =
\left[\begin{array}{ccc:ccc}
R_s + p\,L_{dd} & -(\omega + \Delta\omega)\cdot L_{qq} & 0 & p\,M_{gd} & p\,M_{d1d} & -(\omega+\Delta\omega)\,M_{q1q} \\
(\omega + \Delta\omega)\,L_{dd} & R_s + p\,L_{qq} & 0 & (\omega+\Delta\omega)\,M_{gd} & (\omega+\Delta\omega)\,p\,M_{d1d} & p\,M_{q1q} \\
0 & 0 & R_0 + p\,L_0 & 0 & 0 & 0 \\
\hdashline
\frac{3}{2}\cdot p\,M_{gq} & 0 & 0 & R_g + p\,L_{gg} & p\,M_{g1g} & 0 \\
\frac{3}{2}\cdot p\,M_{d1d} & 0 & 0 & p\,M_{g1d} & R_{1d} + p\,L_{1d} & 0 \\
0 & \frac{3}{2}\cdot p\,M_{q1q} & 0 & 0 & 0 & R_{1q} + p\,L_{1q}
\end{array}\right] \cdot
\begin{bmatrix} I_d \\ I_q \\ I_0 \\ \cdots \\ I_g \\ I_{1d} \\ I_{1q} \end{bmatrix}
\qquad \text{(f12)}
$$

Equ. (6) gives the terminal equations of synchronous machines transformed into Park's reference frame, neglecting the damping coils. If we wish to consider these, too, we can derive the well-known equation (f12), where index $1d$ means direct axis, while index $1q$ quadrature axis damping coil quantities.

Expounding (f12) according to the marked partitioning and using the notation presented above, we obtain:

$$\begin{aligned} U_P &= \mathbf{Z}_P \cdot I_P + \mathbf{Z}_{Pr} \cdot I_r \\ U_r &= \mathbf{Z}_{rP} \cdot I_r + \mathbf{Z}_{rr} \cdot I_r , \end{aligned} \tag{f13}$$

where index r means rotor quantities.

Applying now the transformation (10b) for voltages and currents we obtain:

$$\begin{aligned} U &= [\mathbf{T}_\varrho^{-1} \cdot \mathbf{Z}_P \cdot \mathbf{T}_\varrho] \cdot I + [\mathbf{T}_\varrho^{-1} \mathbf{Z}_{Pr}] \cdot I_r \\ U_r &= [\mathbf{Z}_{rP} \cdot \mathbf{T}_\varrho] \cdot I_P + \mathbf{Z}_{rr} \cdot I_r . \end{aligned} \tag{f14}$$

The value of the triple product $\mathbf{T}_\varrho^{-1} \cdot \mathbf{Z}_P \cdot \mathbf{T}_\varrho$ was given in equ. (12a). Building similarly the two other products in brackets — on the analogy of equs. (10a) and (10c) — we obtain:

$$\mathbf{T}_\varrho^{-1} \cdot \mathbf{Z}_{Pr} = \frac{1}{\sqrt{2}} \begin{bmatrix} pM_{gd} + jX_{gd}, & pM_{d1d} + jX_{d1d}, & -X_{q1q} + jpM_{q1q} \\ pM_{gd} - jX_{gd}, & pM_{d1d} - jX_{d1d}, & -X_{q1q} - jpM_{q1q} \\ 0 & 0 & 0 \end{bmatrix}, \tag{f15}$$

where

$$X_{d1d} = (\omega + \Delta\omega) \cdot M_{d1d} \text{ and } X_{q1q} = (\omega + \Delta\omega) \cdot M_{q1q} .$$

Further:

$$\mathbf{Z}_{rP} \cdot \mathbf{T}_\varrho = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{3}{2} pM_{gd} & \frac{3}{2} pM_{gd} & 0 \\ \frac{3}{2} pM_{d1d} & \frac{3}{2} pM_{d1d} & 0 \\ -j\frac{3}{2} pM_{q1q} & j\frac{3}{2} pM_{q1q} & 0 \end{bmatrix}. \tag{f16}$$

Using equs. (12a), (f15) and (f16), we can transform (f12) into the ϱ system followingly:

$$
\begin{bmatrix} U_{\varrho} \\ \hat{U}_{\varrho} \\ U_0 \\ \cdot\ \cdot \\ U_g \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix}
Z_{dd} + pL_{dd} & & 0 & \frac{1}{\sqrt{2}}(pM_{gd} + jX_{gd}) & \frac{1}{\sqrt{2}}(pM_{1d} - jX_{d1d}) - X_{q1q} - jpM_{q1q} & \\
0 & Z_{dd} + pL_{dd} & 0 & \frac{1}{\sqrt{2}}(pM_{gd} - jX_{gd}) & \frac{1}{\sqrt{2}}(pM_{1d} - jX_{d1d}) - X_{q1q} - jpM_{q1q} & \\
0 & & R_0 + pL_0 & 0 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\frac{3}{\sqrt{6}} pM_{gd} & \frac{3}{\sqrt{6}} pM_{gd} & 0 & R_g + pL_{gg} & pM_{g1d} & 0 \\
\frac{3}{\sqrt{6}} pM_{d1d} & \frac{3}{\sqrt{6}} pM_{d1d} & 0 & pM_{g1d} & R_{1d} + pL_{1d} & 0 \\
0 & \frac{3}{\sqrt{6}} pM_{q1q} & 0 & 0 & 0 & R_{1g} + pL_{1g}
\end{bmatrix}
\cdot
\begin{bmatrix} I_{\varrho} \\ \hat{I}_{\varrho} \\ I_0 \\ \cdot\ \cdot \\ I_g \\ I_{1d} \\ I_{1q} \end{bmatrix}
\tag{f17}
$$

Equation (f17) is the ϱ system terminal voltage equation in matrix notation of the synchronous machine if making allowance for the damping coils.

Symbols

U and I: voltage in volts and current in amperes, resp.
L: Self-induction coefficient (and also mutual induction coefficient in the system of phase quantities) in henries
M: Mutual induction coefficient in henries
R, X, Z and Y: Resistance, reactance, impedance and admittance, resp., in ohms
P: Power in watts
D: Damping factor in W(rad)sec.
T: Moment of momentum in Wsec.
p: differential operator $\left(\frac{d}{dt}\right)$
ω: synchronous angle-velocity
$\theta = \theta_0 + \omega t + \delta$: The angle between the d axis of the synchronous machine and a reference vector at standstill
E: unit matrix, **T**: transformation matrix
Subscripts:
a, b, c: phase quantities
s, r: stator and rotor quantities, resp.
g: field circuit quantity
d, q: direct and quadrature axis quantities, resp., (in Park's reference frame)
$1d$, $1q$: parameters of the direct and quadrature axis damping coils, resp.
o: initial value or zero sequence quantity.

Summary

In this paper the graph theoretical relations of transient stability studies are discussed and a new coordinate transformation method is expounded. This method joins in itself the advantages of Fortescue's symmetrical component system with those of Park's reference frame, if the need arises for the exact digital calculation of the electromechanical transients in three-phase power systems containing synchronous machines with cylindrical rotors.

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László RÁCZ, Budapest XI., Egry József u. 18. Hungary