OVER-GROUND POWER TRANSMISSION LINE SYSTEMS

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1. Introduction

In most cases, power transmission lines and systems are horizontally conducted over the dissipative ground.

The accepted calculation method for over-ground three-phase power transmission line systems is based upon the resolution to reduce it to its symmetrical components [1]. The method of symmetrical components may be applied, in addition to three-phase lines, to systems of any optional number of lines which, however, are considered theoretically accurate only within such symmetrical layout types — as demonstrated underneath, — where the ground effects are neglected.

Fundamental work with respect to the theory of aerial power line systems has been conducted by HAYASHI [2]. In his calculations, however, ground effects are incorrectly taken into account, thus necessitating the further development of this theory. This method applies, nevertheless, well enough to power transmission line systems consisting of lead pairs with the presence of the ground neglected. The theory could be developed, however, for application to power transmission line systems other than those consisting of conductor pairs, such as three-phase power transmission lines. The results permit the calculation of reflection and input impedance as well.

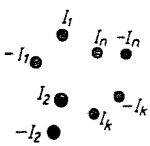
The solution thus obtained appears to complete satisfactorily the approximate results rendered by the electromagnetic theory of the power transmission line [3] as the latter may be successfully employed with information concerning the electromagnetic field available failing, however, to permit reflection and input impedance calculations.

The present paper deals with the ground effects by making use of the well-known theory of ground return power transmission lines [4], [5], [6].

First the electromagnetic field of aerial power transmission line systems will be determined and then, by combining the two theories referred to above, the projected problem will be solved.

The generalized Kelvin equations of coupled Lecher wires

Let us investigate a power transmission line system consisting of parallel located round wire pairs of $-n$ number (Fig. 1), with the ground effects neglected. The current flowing in one of the leads of a conductor pair returns in the other one.



F . 1

The magnetic flux, Φ_k, enclosed by the unit stretch of the k-th wire pair in a given system may be expressed by the i_j current of each wire pair, and by means of the external induction coefficients per unit length.

$$\Phi_k = \sum_{j=1}^{n} L_{kj} i_j \quad k = 1, 2, \ldots, n \tag{1}$$

where

$$L_{kj} = L_{jk} = \frac{\mu}{\pi} \ln \frac{r_{jk}}{a_{jk}} . \tag{2}$$

L_{kj} is a mutual induction coefficient, if $j \neq k$, and then

$$r_{jk} = \sqrt{d_1 d_2} \qquad a_{jk} = \sqrt{b_1 b_2} . \tag{3}$$

If $j = k$, L_{kj} represents the coefficient of self-induction.

$$L_{kk} = \frac{\mu}{\pi} \ln \frac{r_{kk}}{a_{kk}} . \tag{4}$$

The meaning of the symbols b_1, b_2, d_1, d_2, a_{kk}, and r_{kk} involved in (3) and (4) is explained by Fig. 2.

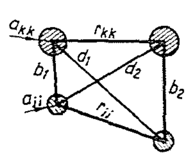


Fig. 2

Expressing (1) for each conductor pair, the equations obtained may be summarized within the following matrix equation:

$$\Phi = \frac{\mu}{\pi} \boldsymbol{M} \cdot \mathbf{i} \tag{5}$$

where $\boldsymbol{\Phi}$ and $\mathbf{i}$ are column vectors consisting of Φ_k and i_k elements, respectively, and $\boldsymbol{M}$ is a quadratic symmetrical matrix as follows:

$$\boldsymbol{M} = \begin{bmatrix} \ln \frac{r_{11}}{a_{11}} & \ln \frac{r_{12}}{a_{12}} \ldots \ln \frac{r_{1n}}{a_{1n}} \\ \ln \frac{r_{21}}{a_{21}} & \ln \frac{r_{22}}{a_{22}} \ldots \ln \frac{r_{2n}}{a_{2n}} \\ \cdot & \cdot \quad \cdots \quad \cdot \\ \ln \frac{r_{n1}}{a_{n1}} & \ln \frac{r_{n2}}{a_{n2}} \ldots \ln \frac{r_{nn}}{a_{nn}} \end{bmatrix} \tag{6}$$

Let us apply the induction law to the surface extended by the unit element of dz-length pertaining to the k-th conductor pair (Fig. 3). The cal-

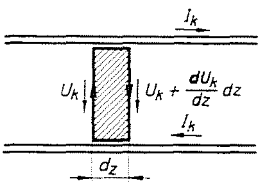


Fig. 3

culations are restricted to phenomena sinusoidally varying in time. The usual complex method of expression is employed:

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{s} = - u_k + \left(u_k + \frac{\mathrm{d}u_k}{\mathrm{d}z} \mathrm{d}z \right) +$$

$$+ 2\, i_k\, Z_{bk}\, \mathrm{d}z = - j\omega \Phi_k\, \mathrm{d}z. \tag{7}$$

Z_{bk} is the internal impedance of one of the leads of the k-th wire pair per unit length as determined by the skin effect. In determining Z_{bk}, it may be assumed that its value is affected by the electromagnetic fields of the adjacent leads only to a negligible degree.

Formulae (1) and (7) reveal that

$$-\frac{\mathrm{d}u_k}{\mathrm{d}z} = \sum_{j=1}^{n} Z_{jk}\, i_j \qquad k = 1, 2, \ldots, n \tag{8}$$

where

$$Z_{jk} = \begin{cases} j\omega\, L_{jk} & j \neq k \\ j\omega\, L_{kk} + 2Z_{bk} & j = k \end{cases} \tag{9}$$

The matrix equation obtained through (8) is:

$$-\frac{\mathrm{d}}{\mathrm{d}z}\mathbf{u} = \boldsymbol{Z}\,\mathbf{i} \tag{10}$$

where $\mathbf{u}$ is a column vector composed of u_k potentials.

The impedance matrix per unit length $\boldsymbol{Z}$ involved by (10) may be expressed by the sum of two matrices:

$$\boldsymbol{Z} = \frac{j\omega\mu}{\pi}\boldsymbol{M} + \boldsymbol{Z}_b \tag{11}$$

where the internal impedance matrix $\boldsymbol{Z}_b$ is a diagonal matrix.

$$\boldsymbol{Z}_b = 2\begin{bmatrix} Z_{b1} & 0 & \ldots & 0 \\ 0 & Z_{b2} & \ldots & 0 \\ \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & \ldots & Z_{bn} \end{bmatrix}. \tag{12}$$

Expression (10) represents a differential equation system involving the components of $\mathbf{u}(z)$ and $\mathbf{i}(z)$ However, to define the solution, a further relation between $\mathbf{u}$ and $\mathbf{i}$ is needed. For this purpose, the potential of the k-th conductor pair should be expressed by means of the charge q_j per unit length of the conductor pairs

$$u_k = \sum_{j=1}^{n} p_{jk}\, q_j \tag{13}$$

where

$$p_{jk} = \frac{1}{\pi\varepsilon_k}\ln\frac{r_{jk}}{a_{jk}}. \tag{14}$$

ε_k is the complex permittivity of the dielectric between the leads:

$$\varepsilon_k = \varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right) \tag{15}$$

where ε is the permittivity, and σ the specific conductivity of the dielectric.

Using (13), and taking (14) as well as (6) into consideration, the following matrix equation may be written:

$$\mathbf{u} = \boldsymbol{P} \cdot \mathbf{q} = \frac{1}{\pi\varepsilon_k} \boldsymbol{M}\mathbf{q}. \tag{16}$$

The elements of matrix $\boldsymbol{P}$ are the p_{jk} figures involved by (14), and $\mathbf{q}$ is a column vector composed of the q_k figures.

On grounds of (16)

$$\mathbf{q} = \pi\varepsilon_k \boldsymbol{M}^{-1} \mathbf{u}. \tag{17}$$

The relation between $\mathbf{q}$ and $\mathbf{i}$ as based upon the equation of continuity is

$$j\omega\mathbf{q} + \frac{\mathrm{d}}{\mathrm{d}z}\mathbf{i} = 0. \tag{18}$$

Substituting $\mathbf{q}$ from (17) to (18) reveals that

$$-\frac{\mathrm{d}}{\mathrm{d}z}\mathbf{i} = j\omega\varepsilon_k \pi\boldsymbol{M}^{-1}\mathbf{u} = \boldsymbol{Y}\mathbf{u} \tag{19}$$

where $\boldsymbol{Y}$ is the admittance matrix per unit length

$$\boldsymbol{Y} = j\omega\varepsilon_k \pi\boldsymbol{M}^{-1}. \tag{20}$$

Equation (19) and (10), together, represent a differential equation system with $\mathbf{u}(z)$ and $\mathbf{i}(z)$ being readily determined thereof. Using the derivative of the equations corresponding to z, a differential equation for $\mathbf{u}$ and $\mathbf{i}$, each, can be obtained.

$$\begin{aligned} \frac{\mathrm{d}^2}{\mathrm{d}z^2}\mathbf{u} &= \boldsymbol{Z}\boldsymbol{Y}\mathbf{u} = \boldsymbol{\Gamma}^2\mathbf{u} \\ \frac{\mathrm{d}^2}{\mathrm{d}z^2}\mathbf{i} &= \boldsymbol{Y}\boldsymbol{Z}\mathbf{i} = \boldsymbol{\Gamma}^2\mathbf{i}. \end{aligned} \tag{21}$$

These equations represent the Kelvin equations generalized to the given homogeneous power transmission line system [7], [8], [9]. The relations concerning the transmission matrix $\boldsymbol{\Gamma}$ involved by the equations are:

$$\begin{aligned} \boldsymbol{\Gamma}^2 &= \boldsymbol{Z}\boldsymbol{Y} \qquad & \boldsymbol{\Gamma}^{*2} &= \boldsymbol{Y}\boldsymbol{Z} \\ \boldsymbol{\Gamma} &= \sqrt{\boldsymbol{Z}\boldsymbol{Y}} \qquad & \boldsymbol{\Gamma}^{*} &= \sqrt{\boldsymbol{Y}\boldsymbol{Z}}. \end{aligned} \tag{22}$$

(The asterisk indicates the transform of the matrix.)

Solution of the equations and its interpretation

The solution of the first equation under (22) is:

$$\mathbf{u}(z) = e^{-\boldsymbol{\Gamma} z}\,\mathbf{U}^{+} + e^{\boldsymbol{\Gamma} z}\,\mathbf{U}^{-} \tag{23}$$

The explanation of the matrix functions $e^{\pm \boldsymbol{\Gamma}(z)}$ in (23) will be dealt with later. The figures $\mathbf{U}^{+}$ and $\mathbf{U}^{-}$ are constants and, as it will be seen later, column vectors composed of potential values transmitted to directions $+z$ and $-z$.

By substituting (23) into (10), then differentiating in correspondence with z, and by multiplying the equation from the left side on by $\boldsymbol{Z}^{-1}$, the solution for $\mathbf{i}$ is obtained.

$$\mathbf{i}(z) = \boldsymbol{Z}_0^{-1}\,(e^{-\boldsymbol{\Gamma} z}\,\mathbf{U}^{+} - e^{+\boldsymbol{\Gamma} z}\,\mathbf{U}^{-})\,. \tag{24}$$

$\boldsymbol{Z}_0$ is the natural impedance matrix

$$\boldsymbol{Z}_0 = (\boldsymbol{Z}^{-1}\boldsymbol{\Gamma})^{-1} = \boldsymbol{\Gamma}^{-1}\boldsymbol{Z}\,. \tag{25}$$

The expression $e^{\pm \boldsymbol{\Gamma} z}$ in (23) and (24) may be determined on the basis of the general relations concerning matrix functions. First the characteristic values of the matrix $\boldsymbol{\Gamma}^2$ should be calculated. Let us indicate these by γ^2. The equation relating to γ^2 is:

$$|\boldsymbol{\Gamma}^2 - \gamma^2 \boldsymbol{E}| = 0 \tag{26}$$

where $\boldsymbol{E}$ is the unit matrix. Subsequently, the investigations will be restricted to that case where the minimum equation of $\boldsymbol{\Gamma}^2$ has only simplex root values. Generally, (26) has n root values. One mode pertains to each of the different roots. The modes should be indicated by Greek key-letters in the suffices (α, β, . . . ν).

Expression (26) may be re-written, by using (11), (20), and (22), to the below form:

$$|\boldsymbol{\Gamma}^2 - \gamma^2 \boldsymbol{E}| = |\boldsymbol{G}^2 - g^2 \boldsymbol{E}| = 0 \tag{27}$$

where

$$\boldsymbol{G}^2 = j\omega\varepsilon_k \pi Z_b \boldsymbol{M}^{-1} \tag{28}$$

and

$$\boldsymbol{\Gamma}^2 = \boldsymbol{G}^2 + k^2 \boldsymbol{E} \tag{29}$$

furthermore,

$$\gamma^2 = g^2 + k^2 \tag{30}$$

where k is the propagation coefficient of the plane wave travelling in the given dielectric. The g^2 values are the characteristic values of matrix $\boldsymbol{G}^2$. Generally, the expression of $\boldsymbol{G}^2$ is much simpler than that of $\boldsymbol{\Gamma}^2$, its characteristic values are, therefore, more readily determined.

Knowing the γ^2 values, the Lagrangian polynomes of n-th order [9] can be obtained these being the functions of γ^2 and of the following characteristics:

$$L_\varkappa(\gamma^2) = \begin{cases} 1 & \gamma^2 = \gamma_\varkappa^2 \\ 0 & \gamma^2 = \gamma_\eta^2 \end{cases} \tag{31}$$

$$\eta = \alpha, \beta, \ldots, \nu \qquad \eta \neq \varkappa .$$

The Lagrangian polynomes are expressed as follows:

$$L_\varkappa(\gamma^2) = \prod_{\substack{\eta=\alpha \\ \eta\neq\varkappa}}^{\nu} \frac{\gamma^2 - \gamma_\eta^2}{\gamma_\varkappa^2 - \gamma_\eta^2} . \tag{32}$$

In (32), let us now substitute γ^2 with the $\boldsymbol{\Gamma}^2$ matrix

$$\boldsymbol{L}_\varkappa(\boldsymbol{\Gamma}^2) = \prod_{\substack{\eta=\alpha \\ \eta\neq\varkappa}}^{\nu} \frac{\boldsymbol{\Gamma}^2 - \gamma_\eta^2 \boldsymbol{E}}{\gamma_\varkappa^2 - \gamma_\eta^2} . \tag{33}$$

Matrix $\boldsymbol{\Gamma}^2$ may be broken down in accordance with the Lagrangian polynomes defined by (33):

$$\boldsymbol{\Gamma}^2 = \sum_{\varkappa=\alpha}^{\nu} \gamma_\varkappa^2 \boldsymbol{L}_\varkappa(\boldsymbol{\Gamma}^2) . \tag{34}$$

Similarly to (34), an f($\boldsymbol{\Gamma}^2$) function of matrix $\boldsymbol{\Gamma}^2$ may be obtained as well [9]:

$$f(\boldsymbol{\Gamma}^2) = \sum_{\varkappa=\alpha}^{\nu} f(\gamma_\varkappa^2) \boldsymbol{L}_\varkappa(\boldsymbol{\Gamma}^2) . \tag{35}$$

On the basis of (35), the propagation matrix $\boldsymbol{\Gamma}$ can be determined:

$$\boldsymbol{\Gamma} = \sum_{\varkappa=\alpha}^{\nu} \gamma_\varkappa \boldsymbol{L}_\varkappa(\boldsymbol{\Gamma}^2). \tag{36}$$

By making use of (35), with (25) as a basis, the natural impedance matrix expression $\boldsymbol{Z}_0$ is obtained.

Furthermore, (35) reveals that

$$e^{\mp \boldsymbol{\Gamma} z} = \sum_{\varkappa=\alpha}^{\nu} e^{\mp \gamma_\varkappa z} \boldsymbol{L}_\varkappa(\boldsymbol{\Gamma}^2). \tag{37}$$

Substituting (37) into (23), the function of u(z) and i(z), respectively, may be determined:

$$\mathbf{u}(\mathbf{z}) = \sum_{\varkappa=\alpha}^{\nu} \boldsymbol{L}_\varkappa(\boldsymbol{\Gamma}^2) [e^{-\gamma_\varkappa z} \mathbf{U}^+ + e^{\gamma_\varkappa z} \mathbf{U}^-] \tag{38}$$

and

$$\mathbf{i}(z) = \boldsymbol{Z}_0^{-1} \sum_{\varkappa=\alpha}^{\nu} \boldsymbol{L}_\varkappa(\boldsymbol{\Gamma}^2) [e^{-\gamma_\varkappa z} \mathbf{U}^+ - e^{\gamma_\varkappa z} \mathbf{U}^-]. \tag{39}$$

On grounds of (38) and (39), the solution of the generalized Kelvin equations may be interpreted. The solution concerning either potential or current consists of two parts. One is composed of generally damped waves travelling to direction $+z$, and the other of those to direction $-z$: the terms of the sum correspond to different modes. Each mode has one pertaining γ^2 and two γ propagation coefficients different only by sign according to the wave travelling to direction $+z$ or $-z$. Generally, the number of modes is in agreement with that of the conductor pairs. If the characteristic equation of $\boldsymbol{\Gamma}^2$ (26) has coincident root figures, the number of modes will be lower than that of the wire pairs. Both $\mathbf{U}^+$ and $\mathbf{U}^-$ progressing to direction $+z$ and $-z$, respectively, can be broken down to the sum of individual modes. Actually, (38) contains this resolution. The $\mathbf{U}_\varkappa(z)$ potential column vector pertaining to the $\varkappa$-th mode is the $\varkappa$-th term of equation (38).

$$\mathbf{u}_\varkappa(z) = \boldsymbol{L}_\varkappa(\boldsymbol{\Gamma}^2)\,\mathbf{U}[e^{-\gamma_\varkappa z}{}^+ + e^{\gamma_\varkappa z}\,\mathbf{U}^-] = e^{-\gamma_\varkappa z}\,\mathbf{U}_\varkappa^+ + e^{\gamma_\varkappa z}\mathbf{U}_\varkappa^- \qquad (40)$$

Potential column vectors $\mathbf{U}_\varkappa^+$ and $\mathbf{U}_\varkappa^-$ are the eigenvectors of matrix $\boldsymbol{\Gamma}^2$. In other words, equation (39) presents the resolution of $\mathbf{u}(z)$ by eigenvectors mathematically, and by modes physically.

If the number of modes is in agreement with that of the conductor pairs then, according to (40), the relation of the potential of individual wire pairs is of a given value for a given mode, that is, by assuming the potential of one conductor pair the potential of the rest of the wire pairs is determined. If a mode has coincident root figures pertaining, then the given mode can exist for optionally assumed values of as many potentials, as manifold the root is.

On grounds of equation (39), $\mathbf{i}(z)$ can be broken down by modes identical to $\mathbf{u}(z)$, and this might be commented in exactly the same way as the resolution of $\mathbf{u}(z)$ had been explained.

Power transmission line system termination. Reflection calculations

The values of $\mathbf{U}^+$ and $\mathbf{U}^-$ can be determined from the limit conditions specified by the termination of the power transmission line system.

Let us define the relations in case of the limit conditions referred to underneath. The wire pairs are of equal length. At the initial section of the power transmission line system, given $\mathbf{U}_0$ tension is switched onto each power transmission line pair and, at the end of the power transmission line system, each conductor pair is terminated by a given impedance Z_t (Fig. 4).

First the limit conditions specified by the terminations should be investigated. For simplicity's sake, the initial point of co-ordinate z should be assumed at the point of the terminations (Fig. 4). By means of the termination impedance values, let us construct a diagonal matrix Z_t having, along its main diagonal, the individual termination impedance values, Z_{tk} located.

The Ohm law as expressed in a matrix equation form applying to the individual leads at the point of termination is:

$$\mathbf{U} = \mathbf{u}(0) = \boldsymbol{Z}_t\, \mathbf{i}(0) = \boldsymbol{Z}_t\, \mathbf{I} \tag{41}$$

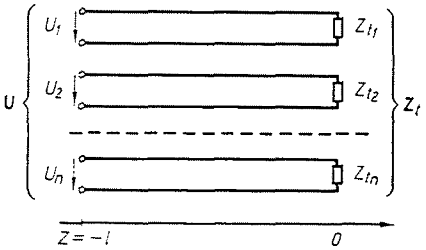


Fig. 4

It should be taken into consideration that the sum of Lagrangian polynomes in the matrix defined by (33) represents the unit matrix $\boldsymbol{E}$.

$$\sum_{\varkappa=\alpha}^{\nu} \boldsymbol{L}_\varkappa(\boldsymbol{\Gamma}^2) = \boldsymbol{E}. \tag{42}$$

From (42) and (38), at the point of $z = 0$, it appears that

$$\mathbf{U} = \mathbf{U}^+ + \mathbf{U}^- . \tag{43}$$

Similarly to (43), on the basis of (39) and (41) it is seen that

$$\mathbf{I} = \boldsymbol{Z}_0^{-1} (\mathbf{U}^+ - \mathbf{U}^-) . \tag{44}$$

Substituting (43) and (44) into (41) reveals that

$$\mathbf{U}^+ + \mathbf{U}^- = \boldsymbol{Z}_t\, \boldsymbol{Z}_0\, (\mathbf{U}^+ - \mathbf{U}^-) . \tag{45}$$

The relation between $\mathbf{U}^+$ and $\mathbf{U}^-$ is offered, in a definition-like manner, by scattering matrix $\boldsymbol{S}$ applying to the tensions:

$$\mathbf{U}^- = \boldsymbol{S}\mathbf{U}^+ . \tag{46}$$

By transposition, (45) can be written in a form similar to (46) resulting in the value of $\boldsymbol{S}$:

$$\boldsymbol{S} = [\boldsymbol{Z}_t\, \boldsymbol{Z}_0^{-1} + \boldsymbol{E}]^{-1}\, [\boldsymbol{Z}_t\, \boldsymbol{Z}_0 - \boldsymbol{E}] . \tag{47}$$

The scattering matrix relating to currents can be determined by using (24) as a basis. Current column vectors $\mathbf{I}^+$ and $\mathbf{I}^-$ are defined for the $z = 0$ point:

$$\mathbf{I}^+ = \boldsymbol{Z}_0^{-1}\, \mathbf{U}^+$$

$$\mathbf{I}^- = -\boldsymbol{Z}_0^{-1}\, \mathbf{U}^- . \tag{48}$$

By utilizing (46) and (48), a relation between $\mathbf{I}^+$ and $\mathbf{I}^-$ may be written:

$$\mathbf{I}^- = \boldsymbol{Z}_0^{-1}\,\mathbf{U}^- = -\,\boldsymbol{Z}_0\,\mathbf{S}\mathbf{U}^+ = -\,\boldsymbol{Z}_0^{-1}\,\mathbf{S}\boldsymbol{Z}_0\,\mathbf{I}^+. \tag{49}$$

The scattering matrix $\boldsymbol{S}_i$ relating to current is expressed, with (47) being made use of, thus:

$$\boldsymbol{S}_i = -\,\boldsymbol{Z}_0\,\boldsymbol{S}\boldsymbol{Z}_0 = -\,(\boldsymbol{Z}_t + \boldsymbol{Z}_0)^{-1}\,(\boldsymbol{Z}_t - \boldsymbol{Z}_0). \tag{50}$$

Knowing the scattering matrices, the reflected column vectors $\mathbf{U}^-$ and $\mathbf{I}^-$ existing at the termination of the conductor systems can be determined by using equations (46) and (48).

Now the conditions existing at the initial part of the power transmission line system will be investigated. Assume the initial section of the power transmission line as the supply point at the location $l = -z$ (Fig. 4). With respect to (23) and (46), the value of column vector $\mathbf{U}_0$ created by the supply tensions is:

$$\mathbf{U}_0 = \mathbf{U}\,(-\,l) = (e^{\Gamma l} + e^{\Gamma l}\,\mathbf{S})\,\mathbf{U}^+. \tag{51}$$

According to the previous assumption, $\mathbf{U}_0$ is a given value, thus (51) may be used to determine $\mathbf{U}^+$:

$$\mathbf{U}^+ = (e^{\Gamma l} + e^{\Gamma l}\,\mathbf{S})^{-1}\,\mathbf{U}_0\,. \tag{52}$$

Knowing $\mathbf{U}^+$, (46) permits the determination of $\mathbf{U}^-$. By now, the value of each quantity included by the expressions of potential and current column vectors described by equations (23), (24), (38), and (39) could be determined.

In addition, the possible relationship between the potential column vector $\mathbf{U}_0$ and current column vector $\mathbf{I}_0$ existing at the supply point should be examined. The expression of $\mathbf{I}_0$ is obtained from equation (24):

$$\mathbf{I}_0 = \mathbf{i}\,(-l) = \boldsymbol{Z}_0^{-1}\,(e^{\Gamma l} - e^{-\Gamma l}\,\mathbf{S})\,\mathbf{U}^+\,. \tag{53}$$

Substituting (53) into (52), it will be seen that

$$\mathbf{I}_0 = \boldsymbol{Z}_0^{-1}\,(e^{\Gamma l} - e^{\Gamma l}\,\mathbf{S})\,(e^{\Gamma l} + e^{\Gamma l}\,\mathbf{S})\,\mathbf{U}_0 = \boldsymbol{Y}_{be}\,\mathbf{U} = \boldsymbol{Z}_{be}^{-1}\,\mathbf{U}_0\,. \tag{54}$$

The input admittance matrix $\boldsymbol{Y}_{be}$, and the input impedance matrix $\boldsymbol{Z}_{be}$ are expressed from (54) thus:

$$\boldsymbol{Y}_{be} = \boldsymbol{Z}_0^{-1}\,(e^{\Gamma l} - e^{\Gamma l}\,\mathbf{S})\,(e^{\Gamma l} + e^{\Gamma l}\,\mathbf{S})^{-1}$$

$$\boldsymbol{Z}_{be} = (e^{\Gamma l} + e^{-\Gamma l}\,\mathbf{S})\,(e^{\Gamma l} - e^{\Gamma l}\,\mathbf{S})^{-1}\,\boldsymbol{Z}_0\,. \tag{55}$$

In case of a given $\mathbf{U}_0$, and knowing the input impedance matrix, $\mathbf{I}_0$ can be determined.

The ideal power transmission line system

Now the system consisting of ideal power transmission lines will be studied. The internal impedance Z_b of all conductor pairs amounting to zero, the internal impedance matrix $\boldsymbol{Z}_b$ as defined by equation (12) is similarly equal to zero. With respect to equation (28), $|\boldsymbol{G}^2| = 0$ and, therefore, all characteristic values of $\boldsymbol{G}^2$ amount to zero: $g_z^2 = 0$. Thus, from (29) and (30), the values of $\boldsymbol{\Gamma}^2$ and γ^2 are:

$$\gamma^2 = k^2 \tag{56}$$

$$\boldsymbol{\Gamma}^2 = k^2\boldsymbol{E}$$

that is, $\boldsymbol{\Gamma}^2$ is proportional to **E** having only one characteristic value: k^2. All U-vectors are eigenvectors. This means that only one mode will exist, and the pertinent propagation coefficient will agree with the propagation coefficient existing within the given medium in case of plane waves. In the line system, the ratio of tensions propagating to the same direction is independent of coordinate z.

It follows from relation $\boldsymbol{Z}_b = \boldsymbol{0}$, on the basis of equations (11) and (20), that

$$\boldsymbol{Z} = \boldsymbol{Y}^{-1}\,. \tag{57}$$

The characteristic impedance matrix of the power transmission line is expressed, with respect to (11), (25), and (56), thus:

$$\boldsymbol{Z}_0 = \boldsymbol{\Gamma}^{-1}\boldsymbol{Z} = \frac{1}{k}\boldsymbol{Z} = \frac{1}{\pi}\sqrt{\frac{\mu}{\varepsilon_k}}\,\boldsymbol{M}\,. \tag{58}$$

In case of a perfect dielectric, there is a real permittivity obtained: $\varepsilon_k = \varepsilon_r \cdot \varepsilon_0$ and, in this instance, $\boldsymbol{Z}_0$ is similarly a real value:

$$\boldsymbol{Z}_0 = \frac{120}{\sqrt{\varepsilon_r}}\,\boldsymbol{M}\,. \tag{59}$$

The scattering matrix $\boldsymbol{S}$ of the ideal power transmission line may also be calculated by utilizing (47). The expression of $\boldsymbol{Z}_0$ included by the formula can be obtained from (58) and (59), respectively.

The expression of the input admittance calculated by means of (55) is somewhat simplified if (54) is made use of:

$$\boldsymbol{Y}_{be} = \boldsymbol{Z}_0^{-1}\,(e^{kl}\,\boldsymbol{E} - e^{kl}\,\boldsymbol{S})\,(e^{-kl}\,\boldsymbol{E} + e^{-kl}\,\boldsymbol{S})^{-1}. \tag{60}$$

In connection with the calculation of the ideal power transmission line system, it might be noted furthermore that certain authors failed to observe the fact according to which all roots of equation (26) coincide in this case and, therefore, only one γ^2 will exist [2], [7].

Three-phase power transmission line without neutral wire

The three-phase power transmission line does not consist of wire pairs may be, however, retraced to a system composed of such. Systems with or without a neutral wire demand separate discussions. First the system without neutral wire will be studied (Fig. 5).

Corresponding to practice, assume for simplicity's sake identical radii for all three conductors (calculations for different radii may also be made). Consider the wires as a system consisting of two wire pairs where one of the

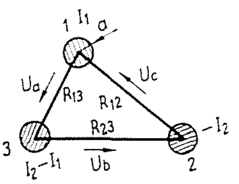


Fig. 5

wires (No. 3 in this example) is common, that is, wires 1—3 and 3—2, respectively, form two wire pairs. Tension $u_a(z)$ is created between leads 1—3, and that of $u_b(z)$ between wires 3—2. The propagation of only these two phase voltages must be studied, as the third one $u_c(z)$ may be determined with them.

$$u_a(z) + u_b(z) + u_c(z) = 0. \tag{61}$$

In wires 1 and 2, currents I_1 and I_2 are, respectively, flowing whereas lead 3 has current $I_2 - I_1$ flowing within.

The value of the mutual inductance between the two wire pairs is:

$$L_{12} = \frac{\mu}{\pi} \ln \frac{\sqrt{R_{12}\, a}}{\sqrt{R_{13}\, R_{23}}} \tag{62}$$

With (6) and (62) taken into consideration, matrix $\boldsymbol{M}$ may be expressed thus

$$\boldsymbol{M} = \begin{bmatrix} \ln \dfrac{R_{13}}{a} & \ln \dfrac{\sqrt{R_{12}\, a}}{\sqrt{R_{13}\, R_{23}}} \\ \ln \dfrac{\sqrt{R_{12}\, a}}{\sqrt{R_{13}\, R_{23}}} & \ln \dfrac{R_{23}}{a} \end{bmatrix}. \tag{63}$$

When determining $\boldsymbol{Z}_b$, it must be taken into account that currents I_1 and I_2 are flowing through wire 3 causing there a voltage drop. Correspondingly, equation (8) may be written in the following form:

$$-\frac{\mathrm{d}\,u_a}{\mathrm{d}z} = i_1\,(j\omega\,L_{11} + 2\,Z_b) + i_2\,(j\omega\,L_{12} + Z_b)$$

$$-\frac{\mathrm{d}\,u_b}{\mathrm{d}z} = i_1\,(j\omega\,L_{12} + Z_b) + i_2\,(j\omega\,L_{22} + 2\,Z_b). \tag{64}$$

Based upon equation (64), the expression of matrix $\boldsymbol{Z}_b$ is:

$$\boldsymbol{Z}_b = Z_b \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}. \tag{65}$$

Knowing matrices $\boldsymbol{M}$ and $\boldsymbol{Z}_b$ as given by equations (63) and (65), respectively, the characteristics of the power transmission line system ($\boldsymbol{G}^2$, $\boldsymbol{\Gamma}^2$, $\boldsymbol{Z}_0$, $g_\varkappa^2$, and $\gamma_\varkappa^2$) can be determined by using equations (25), (27), (28), (29), and (30).

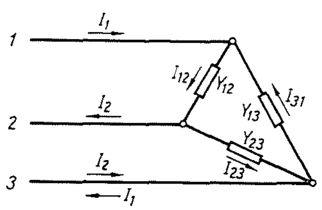


Fig. 6

The termination of three-phase lines without neutral may be represented by either star or delta connections. As the neutral points of the star connection have no separate terminals, it may be converted to a delta connection. Thus, an investigation concerning only the delta connection appears to be quite sufficient (Fig. 6).

In order to simplify calculations, terminations will be taken into consideration through admittance. The node equation for each branch point may be written as follows:

$$I_1 = I_{12} - I_{13} = U_1\,(Y_{12} + Y_{13}) + U_2\,Y_{12}$$

$$I_2 = I_{12} - I_{23} = U_1\,Y_{12} + U_2\,(Y_{12} + Y_{23}). \tag{66}$$

Summarized in a matrix equation:

$$\mathbf{I} = \boldsymbol{Y}_t\,\mathbf{U} \tag{67}$$

where

$$Y_t = \begin{bmatrix} Y_{12} + Y_{13} & Y_{12} \\ Y_{12} & Y_{12} + Y_{23} \end{bmatrix} = \boldsymbol{Z}_t^{-1}. \tag{68}$$

Knowing matrix $\boldsymbol{Zf}$, scattering matrix $\boldsymbol{S}$ can be obtained from (47), and thus all characteristic terms required for the calculation of a three-phase power transmission line without neutral have been determined.

With respect to the above statements, now the conditions existing with the three leads symmetrically arranged will be studied. The leads are equally spaced (Fig. 7).

$$R_{12} = R_{13} = R_{23} = r.$$

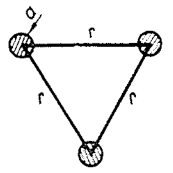


Fig. 7

The expression of matrix $\boldsymbol{M}$, as based on (63), is

$$\boldsymbol{M} = \begin{bmatrix} \ln \frac{r}{a} & \ln \sqrt{\frac{a}{r}} \\ \ln \sqrt{\frac{a}{r}} & \ln \frac{r}{a} \end{bmatrix} = \ln \frac{r}{a} \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix} \tag{69}$$

and the inverse of matrix $\boldsymbol{M}$ is:

$$\boldsymbol{M}^{-1} = \frac{4}{3} \frac{1}{\ln \frac{r}{a}} \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}. \tag{70}$$

As for matrix $\boldsymbol{G}^2$, with (28), (6), (65), and (70) taken into consideration, it will be seen that

$$\boldsymbol{G}^2 = \frac{2Z_b}{\ln \frac{r}{a}} j\omega\varepsilon_k \pi \boldsymbol{E}. \tag{71}$$

The two eigenvalues of $\boldsymbol{G}^2$ coincide:

$$g^2 = \frac{2Z_b j\omega\varepsilon_k \pi}{\ln \frac{r}{a}}. \tag{72}$$

With respect to (30), likewise one value for γ^2 is obtained:

$$\gamma^2 = \frac{2Z_h j\omega\varepsilon_k \pi}{\ln \frac{r}{a}} + j\omega\varepsilon_k j\omega\mu . \tag{73}$$

This means that only one mode will exist, and the amplitude ratio of the surge voltages travelling in the lead to the same direction will not depend on the z co-ordinate. Voltages may be arbitrarily broken down to the sum of two different components as these travel with identical propagation coefficients.

The usual calculation method of three-phase power transmission lines is represented by the resolution to symmetrical components [1]. Mains without neutral (without ground) have only positive and negative sequence symmetrical components but none of zero sequence. Consequently, if wave phenomena should also be taken into consideration, calculations with symmetrical components render, theoretically, correct results only in case of a symmetrical layout. With the presence of the ground taken also into account, however, the symmetrical component method does not ensure correct results, even in case of symmetrical arrangements.

Three-phase power transmission line with neutral

Now the three-phase power transmission line system with neutral will be discussed (Fig. 8). For simplicity's sake assume that the three outers have identical radii (a). The radius of the neutral is a_0. The currents of the three outers

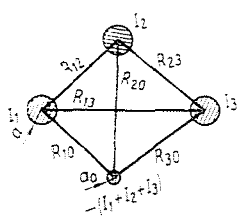


Fig. 8

(I_1, I_2, and I_3) return in the neutral wire, that is, all three outers constitute wire pairs with the neutral, the latter being the common lead of the three wire pairs.

Values r and a in matrix $\boldsymbol{M}$ are transformed as follows:

$$\begin{array}{llll} r_{11} = R_{10} & r_{11} = \sqrt{R_{10} R_{20}} & a_{12} = \sqrt{R_{12}\, a_0} & a_{11} = \sqrt{a\, a_0} \\ r_{22} = R_{20} & r_{13} = \sqrt{R_{10} R_{30}} & a_{13} = \sqrt{R_{13}\, a_0} & a_{22} = \sqrt{a.\, a_0} \\ r_{33} = R_{30} & r_{23} = \sqrt{R_{20} R_{30}} & a_{23} = \sqrt{R_{23}\, a_0} & a_{33} = \sqrt{a.\, a_0}. \end{array} \tag{74}$$

The meaning of the symbols $R_{12}, R_{13}, R_{23}, R_{10}, R_{20}$ and R_{30} are understood through Fig. 8. Substituting (74) into (6), it will be seen that

$$\boldsymbol{M} = \begin{bmatrix} \ln \dfrac{R_{10}}{\sqrt{a.a_0}} & \ln \dfrac{\sqrt{R_{10}\,R_{20}}}{\sqrt{R_{12}\,a_0}} & \ln \dfrac{\sqrt{R_{10}R_{30}}}{\sqrt{R_{13}\,a_0}} \\ \ln \dfrac{\sqrt{R_{10}\,R_{20}}}{\sqrt{R_{12}\,a_0}} & \ln \dfrac{R_{20}}{\sqrt{a\,a_0}} & \ln \dfrac{\sqrt{R_{20}\,R_{30}}}{\sqrt{R_{23}\,a_0}} \\ \ln \dfrac{\sqrt{R_{10}\,R_{30}}}{\sqrt{R_{13}\,a_0}} & \ln \dfrac{\sqrt{R_{20}\,R_{30}}}{\sqrt{R_{23}\,a_0}} & \ln \dfrac{R_{30}}{\sqrt{a\,a_0}} \end{bmatrix}. \tag{75}$$

When expressing matrix $\boldsymbol{Z}_b$, it must be remembered that all phase currents flow through the neutral and cause there a voltage drop.

$$-\frac{du_1}{dz} = i_1\,(j\omega\,L_{11} + Z_b + Z_{b0}) + i_2\,(j\omega\,L_{12} + Z_{b0}) + i_3\,(j\omega\,L_{13} + Z_{b0})$$

$$-\frac{du_2}{dz} = i_1\,(j\omega\,L_{12} + Z_{b0}) + i_2\,(j\omega\,L_{22} + Z_b + Z_{b0}) + i_3\,(j\omega\,L_{23} + Z_{b0})$$

$$-\frac{du_3}{dz} = i_1\,(j\omega\,L_{13} + Z_{b0}) + i_2\,(j\omega\,L_{23} + Z_{b0}) + i_3\,(j\omega\,L_{33} + Z_b + Z_{b0}). \tag{76}$$

With respect to (76), matrix $\boldsymbol{Z}_b$ may be expressed as follows:

$$\boldsymbol{Z}_b = \begin{bmatrix} Z_b + Z_{b0} & Z_{b0} & Z_{b0} \\ Z_{b0} & Z_b + Z_{b0} & Z_{b0} \\ Z_{b0} & Z_{b0} & Z_b + Z_{b0} \end{bmatrix}. \tag{77}$$

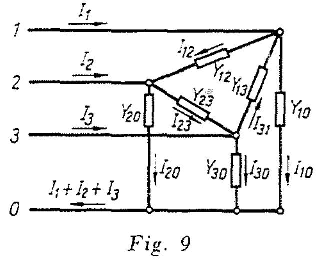


Fig. 9

Knowing $\boldsymbol{M}$ and $\boldsymbol{Z}_b$ as introduced by (75) and (77) respectively, the characteristics of the power transmission line system ($\boldsymbol{G}^2$, $\boldsymbol{\Gamma}^2$, $\boldsymbol{Z}_0$, g^2, and γ^2) can be now determined by using equations (25), (27), (28), (29), and (30) as bases.

When calculating loading impedance, it must not be forgotten that the termination may be of star or delta connection as well. Hence, the general example will be studied when consumers of star as well as of delta connection jointly load the power transmission line system (Fig. 9).

The admittances of star connection will be indicated by Y_{10}, Y_{20}, and Y_{30} while those of delta connection by Y_{13}, Y_{23}, and Y_{33}, respectively.

On grounds of Fig. 9, the node equations may be written as follows:

$$\begin{aligned} I_1 &= I_{10} + I_{12} - I_{13} = U_1 (Y_{10} + Y_{12} + Y_{13}) - U_2 Y_{12} - U_3 Y_{13} \\ I_2 &= I_{20} + I_{23} - I_{12} = - U_1 Y_{12} + U_2 (Y_{20} + Y_{12} + Y_{23}) - U_3 Y_{23} \\ I_3 &= I_{30} + I_{31} - I_{23} = - U_1 Y_{13} - U_2 Y_{23} + U_3 (Y_{30} + Y_{13} + Y_{23}). \end{aligned} \tag{78}$$

Equation (78) presents the expression of loading admittance matrix $\mathbf{Y}_t$:

$$\mathbf{Y}_t = \begin{bmatrix} Y_{10}+Y_{12}+Y_{13} & -Y_{12} & -Y_{13} \\ -Y_{12} & Y_{20}+Y_{12}+Y_{23} & -Y_{23} \\ -Y_{13} & -Y_{23} & Y_{30}+Y_{13}+Y_{23} \end{bmatrix} = \mathbf{Z}_t^{-1}. \tag{79}$$

In case of star connection it reads:

$$\mathbf{Y}_t = \begin{bmatrix} Y_{10} & 0 & 0 \\ 0 & Y_{20} & 0 \\ 0 & 0 & Y_{30} \end{bmatrix} \tag{80}$$

whereas in case of delta connection:

$$Y_t = \begin{bmatrix} Y_{12}+Y_{13} & -Y_{12} & -Y_{13} \\ -Y_{12} & Y_{12}+Y_{23} & -Y_{23} \\ -Y_{13} & Y_{23} & Y_{13}+Y_{23} \end{bmatrix}. \tag{81}$$

By this method, the calculation of the three-phase power transmission line with neutral has been traced back to the calculation of a power transmission line system composed of three transmission line pairs.

Now let us discuss the layout of a three-phase power trasnmission line with a symmetric neutral (Fig. 10). The phase leads are equally spaced.

$$R_{12} = R_{13} = R_{23} = R$$

$$\sqrt{R_{10} R_{20}} = \sqrt{R_{10} R_{30}} = \sqrt{R_{20} R_{30}} = r = \frac{R}{\sqrt{3}}.$$

The outers are at equal distances also to the neutral:

$$r_{11} = r_{22} = r_{33} = R_{10} = R_{20} = R_{30} = r = \frac{R}{\sqrt{3}}$$

The other quantities included by matrix $\boldsymbol{M}$ are:

$$a_{12} = a_{13} = a_{23} = b = \sqrt{R\, a_0}$$

$$a_{11} = a_{22} = a_{33} = \sqrt{a_v\, a_0} = a$$

where a_v represents the radius of the outers, and a_0 that of the neutral.

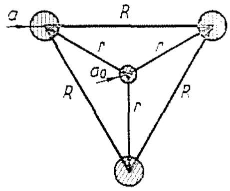


Fig. 10

Substituting the aforeasaid figures into (6), the expression of matrix $\boldsymbol{M}$ is obtained:

$$\boldsymbol{M} = \begin{bmatrix} \ln \frac{r}{a} & \ln \frac{r}{b} & \ln \frac{r}{b} \\ \ln \frac{r}{b} & \ln \frac{r}{a} & \ln \frac{r}{b} \\ \ln \frac{r}{b} & \ln \frac{r}{b} & \ln \frac{r}{a} \end{bmatrix}. \tag{82}$$

From equation (82)

$$\boldsymbol{M}^{-1} = \frac{1}{L} \begin{bmatrix} \ln \frac{r^2}{a \cdot b} & -\ln \frac{r}{b} & -\ln \frac{r}{b} \\ -\ln \frac{r}{b} & \ln \frac{r^2}{a \cdot b} & -\ln \frac{r}{a} \\ -\ln \frac{r}{b} & -\ln \frac{r}{b} & \ln \frac{r^2}{a \cdot b} \end{bmatrix} \tag{83}$$

where

$$L = \ln \frac{r}{a} \ln \frac{r^2}{a \cdot b} - 2 \ln \frac{2r}{b} \tag{84}$$

and

$$\boldsymbol{P} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \tag{85}$$

Matrix $\boldsymbol{Z}_b$ may be expressed, on the basis of (70), by means of matrices $\boldsymbol{E}$ and $\boldsymbol{P}$:

$$\boldsymbol{Z}_b = Z_{b0}\, 3\, \boldsymbol{P} + Z_b\, \boldsymbol{E}\,. \tag{86}$$

By substituting (83) and (86) into (28), the expression of $\boldsymbol{G}^2$ is arrived at:

$$\boldsymbol{G}^2 = \frac{j\omega\varepsilon_k \pi}{L}\left[\left(Z_{b0} \ln \frac{a^2}{b^2} - Z_b \ln \frac{r}{a}\right) 3\, \boldsymbol{P} + Z_b \ln \frac{r^2}{ab^2}\, \boldsymbol{E}\right]. \tag{87}$$

Now $\boldsymbol{\Gamma}^2$ can be termed with (87) and (29) being made use of:

$$\boldsymbol{\Gamma}^2 = \boldsymbol{G}^2 + k^2 \boldsymbol{E} \begin{bmatrix} A & B & B \\ B & A & B \\ B & B & A \end{bmatrix} \tag{88}$$

where

$$A = 2\left(Z_{b0} \ln \frac{a}{b} + Z_b \ln \frac{r}{b}\right) + k^2$$

$$B = Z_{b0} \ln \frac{a^2}{b^2} - Z_b \ln \frac{r}{a}\,. \tag{89}$$

The equation suitable to determine the eigenvalue of $\boldsymbol{\Gamma}^2$, with (81) and (26) taken into consideration, is:

$$|\boldsymbol{\Gamma}^2 - \gamma^2 \boldsymbol{E}| = \begin{vmatrix} A-\gamma^2 & B & B \\ B & A-\gamma^2 & B \\ B & B & A-\gamma^2 \end{vmatrix} =$$

$$= (A - B - \gamma^2)^2\, (A + 2\, B - \gamma^2) = 0. \tag{90}$$

Equation (90) reveals that two of the three eigenvalues coincide, that is, only two different eigenvalues are obtained:

$$\gamma_\alpha^2 = A - B \tag{91}$$

$$\gamma_\beta^2 = A + 2\, B.$$

Using (33), (88), and (91), the matrix Lagrangian polynomes are determined thus:

$$\boldsymbol{L}_\alpha(\boldsymbol{\Gamma}^2) = \frac{\boldsymbol{\Gamma}^2 - \gamma_\beta^2 \boldsymbol{E}}{\gamma_\alpha^2 - \gamma_\beta^2} = -\frac{1}{3}\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\boldsymbol{L}_\beta(\boldsymbol{\Gamma}^2) = \frac{\boldsymbol{\Gamma}^2 - \gamma_\alpha^2 \boldsymbol{E}}{\gamma_\beta^2 - \gamma_\alpha^2} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \tag{92}$$

Now the surge voltage $\mathbf{u}(z)$ travelling to the $+z$ direction should be broken down to the two modes with (92) and (40) being made use of for this purpose:

$$\mathbf{u}_\alpha(z) = \boldsymbol{L}_\alpha(\boldsymbol{\Gamma}^2)\, e^{-\gamma_\alpha z}\, \mathbf{U}^+ = \frac{-e^{-\gamma_\alpha z}}{3}\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} U_1^+ \\ U_2^+ \\ U_3^+ \end{bmatrix} =$$

$$= \frac{-e^{-\gamma_\alpha z}}{3}\begin{bmatrix} -2U_1^+ + U_2^+ + U_3^+ \\ U_1^+ - 2U_2^+ + U_3^+ \\ U_1^+ + U_2^+ - 2\,U_3^+ \end{bmatrix} \tag{93}$$

and

$$\mathbf{U}_\beta(z) = \boldsymbol{L}_\beta(\boldsymbol{\Gamma}^2)\, e^{-\gamma_\beta z}\, \mathbf{U}^+ = \frac{e^{-\gamma_\beta z}}{3}\,(U_1^+ + U_2^+ + U_3^+)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{94}$$

Studying the resolution of $u(z)$ to $u_\alpha(z)$ and $u_\beta(z)$, respectively, it may be stated that $u_\beta(z)$ corresponds to the symmetrical component of zero sequence, whereas $u_\alpha(z)$ represents the sum of the components of positive and negative sequence types. In other words, the calculation using symmetrical components will result, in case of the symmetrical layout as illustrated by Fig. 10, in such a resolution where there is only one propagation coefficient pertaining to the components of positive and negative sequence, respectively. In addition, the calculation shows that the sum of the positive and negative sequence components may be resolved in some other way as well (such as to components α and β). If the wave phenomena are also to be taken into account, that is, when the power transmission line is studied as a network of divided parameters, then the resolution to symmetrical components will, theoretically, not give correct results with an asymmetrical layout employed. The calculation correct also theoretically is performed by resolution to modes as referred to above.

With the ground taken into consideration, asymmetrical components do not ensure a correct result for a symmetrical arrangement, either.

Ground-return type power transmission line

The scheme of a ground-return type power transmission line is illustrated by Fig. 11. The loading impedance is cut in between a lead parallel to the ground and the ground proper. Thus, the conductor and the ground jointly represent the power transmission line. In calculations, ground is assumed as limited by a horizontal plane and being homogeneous. Its characteristic parameters are: specific conductivity σ_f, permittivity ε_f, and permeability μ_0.

Electric and magnetic fields must meet different limit conditions on the surface of the ground. More exactly, E_x, $(\sigma + j\omega\varepsilon)\, E_y$, E_z, H_x, H_y, and H_z

must be continuous values. The electromagnetic field may be defined as the sum of a field pertaining to a conductor located in an open space and of an additional field existing due to the presence of ground. The conductor field is represented by a Sommerfeld surface wave [10]. Limit conditions can be satisfied by expressing the additional field existing, due to the presence of ground, in the form of the Fourier integral [4], [5], [6].

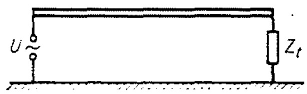


Fig. 11

With the limit conditions satisfied, for the component of the electric field conforming to the direction of propagation, at a point characterized by given co-ordinates x and y, the following expression as written in the Fourier integral form is obtained:

$$E_z = I \frac{g}{j\omega\varepsilon_k} \frac{1}{2\pi a \, \mathrm{H}_1^{(1)}(ga)} \Bigg| \mathrm{H}_1^{(1)}\left(g\sqrt{x^2+(l-y)^2}\right) - $$

$$- \frac{2}{\pi j} \int_0^{\infty} V(a) \frac{e^{-(l-y)\sqrt{a^2-g^2}}}{\sqrt{a^2-g^2}} \cos ax \, \mathrm{d}a \Bigg| \tag{95}$$

where I is the current flowing in the lead, while $H_0^{(1)}$ and $H_1^{(1)}$ are the Hankel functions of zero and first order, respectively, and of the first kind, and

$$V(a) = \frac{\begin{vmatrix} g^2 k^2 & f^2 k_0^2 & 0 \\ a & a & f^2\sqrt{a^2-g^2} + g^2\sqrt{f^2-g^2} \\ \sqrt{a^2-g^2} & \sqrt{a^2-f^2} & a(k^2-k_0^2) \end{vmatrix}}{\begin{vmatrix} g^2 k^2 & f^2 k_0^2 & 0 \\ a & a & f^2\sqrt{a^2-g^2} + g^2\sqrt{f^2-g^2} \\ -\sqrt{a^2-g^2} & \sqrt{a^2-f^2} & a(k^2-k_0^2) \end{vmatrix}} \tag{96}$$

where

$$k_f^2 = (\sigma_f + j\omega\varepsilon_f) j\omega \mu_0$$

$$f^2 = \gamma^2 - k_f^2 . \tag{97}$$

It should be noted here that the results obtained by Carson (43) may be regarded as approximating (95) and (96).

Retraction of the calculation of an aerial lead system to that of wire pairs

Let us study the aerial network consisting of n conductors as illustrated by Fig. 12. One end of the conductors is supplied with a given voltage as compared to the ground. At the other end of the wire system the lines are terminated by given impedances. The termination impedances may be cut in between either two conductors or one of the conductors and the ground. In order to solve the problem, the E_z value produced on the surface of the individual conductors must be determined. First the E_{zkj} value produced on the surface of

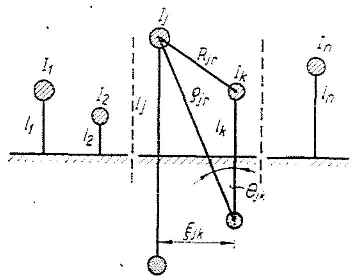


Fig. 12

the k-th lead by the I_j current flowing in the j-th lead and by the current produced in the ground upon the effect of I_j should be determined. Using the symbols of Fig. 12, ($x = \xi_{jk}$, $l = l_j$, and $y = l_k$), its value is

$$\sqrt{x^2 + (l - y)^2} = R_{jk}\,.$$

According to equation (95) and the literature

$$E_z = I_j\,(Z_{jk} - Z_{fjk}) \tag{98}$$

where

$$Z_{jk} = \frac{1}{2\pi a_j}\,\frac{g}{2\pi\varepsilon_0}\,\frac{1}{\mathrm{H}_1^{(1)}(g a_j)}\left[\mathrm{H}_0^{(1)}(g R_{jk}) - \mathrm{H}_0^{(1)}(g \varrho_{jk})\right] \tag{99}$$

and

$$Z_{fjk} = \frac{1}{2\pi a_j}\,\frac{g}{j\omega\varepsilon_0}\,\frac{1}{\mathrm{H}_1^{(1)}(g a_j)}\,\frac{2}{\pi j}\int_0^\infty (V(a) - 1)\,\cdot$$

$$\cdot\,\frac{e^{-(l_k + l_j)\sqrt{a^2 - g^2}}}{\sqrt{a^2 - g^2}}\,\cos a\,\xi_{jk}\,da. \tag{100}$$

As a definition, Z_{jk} is the mutual impedance per unit length of the j-th and k-th conductors with an ideal ground assumed, that is, its value can be determined by image formation [3] whereas Z_{fjk} is the correction due to the dissipative ground. The numerical determination of Z_{fjk} will be dealt with in the next paragraph. The approximate value of Z_{jk} can be determined by substituting into (99) the low argument approximate expression of the Hankel-functions.

$$Z_{jk} = \frac{g^2}{2\pi j \omega \varepsilon_0} \ln \frac{\varrho_{jk}}{R_{jk}}. \tag{101}$$

On the basis of (98) and making use of the superposition theorem, the E_z value produced on the surface of the k-th lead may be expressed as the sum of E_{zjk} field intensities generated by the currents flowing in the individual conductors and in the earth. By using (101), it will be seen that

$$E_{zk} = \sum_{j=1}^{n} I_j (Z_{jk} - Z_{fjk}) =$$

$$= \sum_{j=1}^{n} I_j \left(\frac{g_2}{2\pi j \omega \varepsilon_0} \ln \frac{\varrho_{jk}}{R_{jk}} - Z_{fjk} \right) \qquad k = 1, 2, \cdots, n\,. \tag{102}$$

The value of E_{zk} may be expressed also by the internal field of the k-th lead. From this:

$$E_{zk} = I_k Z_{bk} \tag{103}$$

where Z_{bk} is the internal impedance of the k-th lead as related to the lead unit length with the skin effect taken into account. By combining (102) and (103), the following matrix equation is obtained:

$$\left[\frac{g_2}{\pi j \omega \varepsilon_0} \boldsymbol{M} - (\boldsymbol{Z}_j + \boldsymbol{Z}_{br}) \right] \mathbf{I} = 0. \tag{104}$$

The $\boldsymbol{M}$ of (104) is identical to the half matrix $\boldsymbol{M}$ defined by (6), if each lead forms a wire pair with its image.

$$\boldsymbol{M} = \frac{1}{2} \begin{bmatrix} \ln \frac{\varrho_{11}}{R_{11}} & \ln \frac{\varrho_{12}}{R_{12}} & \cdots & \ln \frac{\varrho_{1n}}{R_{1n}} \\ \ln \frac{\varrho_{21}}{R_{21}} & \ln \frac{\varrho_{22}}{R_{22}} & \cdots & \ln \frac{\varrho_{2n}}{R_{2n}} \\ \cdot & \cdot & \cdots & \cdot \\ \ln \frac{\varrho_{n1}}{R_{n1}} & \ln \frac{\varrho_{n2}}{R_{n2}} & \cdots & \ln \frac{\varrho_{nn}}{R_{nn}} \end{bmatrix}. \tag{105}$$

$\boldsymbol{Z}_{bv}$ included by (104) is a diagonal matrix

$$\boldsymbol{Z}_{bv} = \begin{bmatrix} Z_{b1} & 0 & \dots 0 \\ 0 & Z_{b2} & \dots 0 \\ \cdot & \cdot & \dots \cdot \\ 0 & 0 & \dots Z_{bn} \end{bmatrix}. \tag{106}$$

The Z_{bjk} elements of matrix Z_{bv} included by (104) are defined by equation (100). The approximate value of the integral seen there will be determined in the next paragraph. Now matrices $\boldsymbol{Z}_f$ and $\boldsymbol{Z}_{bv}$ will be combined into a single matrix, $\boldsymbol{Z}_b$

$$\boldsymbol{Z}_b = \boldsymbol{Z}_{bv} + \boldsymbol{Z}_f\,. \tag{107}$$

Substituting this into (104), and multiplying the equation from the right-hand side on, by $j\omega\varepsilon_k \pi \boldsymbol{M}^{-1}$:

$$(j\omega\varepsilon_k \boldsymbol{M}^{-1} \boldsymbol{Z}_b - g^2 \boldsymbol{E})\,\mathbf{I} = \mathbf{0}\,. \tag{108}$$

Equation (108) represents a homogeneous linear equation system for the $\mathbf{I}$ currents flowing in the individual wire pairs. This has a solution other than trivial, only in that case when the value of the determinant obtained from the coefficients of the equations is equal to zero. Thus with equation (28) taken into consideration, it will be obtained that

$$(\boldsymbol{G}^{*2} - g^2 \boldsymbol{E}) = 0. \tag{109}$$

The g^2 figures rendered by equation (109) represent the eigenvalues of $\boldsymbol{G}^{*2}$. Since the eigenvalues of matrices $\boldsymbol{G}^2$ and $\boldsymbol{G}^{*2}$ are identical, equation (109) corresponds to (27).

The calculations resulted in matrix $\boldsymbol{G}^*$ instead of $\boldsymbol{G}$ as equation (108) has been relating here to currents and not to voltages.

The results obtained so far has led back the calculation of over-ground line systems to the discussion of power transmission line systems. Accordingly, the calculation of over ground line systems necessitates the formation of lead images. The following calculations assume all conductors to form a wire pair with their own images. Thus matrix $\boldsymbol{M}$ of (185) may be calculated. Taking the dissipative ground into consideration is performed by using the impedance matrix $\boldsymbol{Z}_b$ which, due to $\boldsymbol{Z}_f$, will not represent a diagonal matrix now as in (12). With the matrices $\boldsymbol{Z}_b$ and $\boldsymbol{M}$ determined, matrices $\boldsymbol{\Gamma}^2$, $\boldsymbol{G}^2$, and $\boldsymbol{Z}_0$ as well as the values g^2 and γ^2 can be calculated with the method used for wire pairs. Consequently, the number of modes will generally equal that of the conductors.

Determination of the ground impedance matrix

The elements of matrix $\boldsymbol{Z}_f$ are determined by equation (100). Substituting the low argument approximate term of the Hankel function in equation (100) will show

$$Z_{fjk} = \frac{g^2}{2\pi j\omega\varepsilon_0}\int\limits_0^\infty [V(\alpha)+1]\,\frac{e^{-(l_k+l_j)\sqrt{\alpha^2-g^2}}}{\sqrt{\alpha^2-g^2}}\cos\alpha\xi_{jk}\,d\alpha. \tag{110}$$

With the expression of $V(\alpha)$ as given by (96) taken into consideration, integral (110) may be approximated giving the below result:

$$Z_{fjk} = \frac{g^2}{2\pi j\omega\varepsilon_0}\,\frac{2k_0^2}{k_f^2-k_0^2}\left\{\frac{1}{g^2}\,\frac{j\pi f}{4\varrho_{jk}}\left[e^{-j\Theta_{jk}}\mathbf{H}_1(\varrho jfe^{j\Theta_{jk}}) + e^{j\Theta_{jk}}\,\mathbf{H}_1(\varrho_{jk}jfe^{-j\Theta_{jk}}) - e^{-j\Theta_{jk}}N_1(\varrho_{jk}jfe^{j\Theta_{jk}}) - e^{j\Theta_{jk}}N_1(\varrho_{jk}jfe^{-j\Theta_{jk}})\right] + \frac{\pi}{2j}\left[\sin\Theta_{jk}\mathrm{H}_0^{(1)}(g\varrho_{jk}) + \frac{1}{g\varrho_{jk}}\cos 2\Theta_{jk}\mathrm{H}_1^{(1)}(g\varrho_{jk})\right]\right\} + \frac{\pi}{2j}\left[\frac{k^2}{k_f^2+k_0^2}\left(1+\frac{k_0^2}{g^2}\right)\cdot \mathrm{H}_0^{(1)}\left(\sqrt{g^2+\frac{k_0^2}{k_0^2+k_0^2}}\,\varrho_{jk}\right) - \mathrm{H}_1^{(1)}(g\varrho_{jk})\right] \tag{111}$$

where

$$l_k+l_j+j\xi_{jk} = \varrho_{jk}\,e^{j\Theta_{jk}} \quad l_k+l_j-j\xi_{ik} = \varrho_{jk}\,e^{-j\Theta_{jk}} \tag{112}$$

The meaning of ϱ_{jk} and Θ_{jk} can be seen in Fig. 12. $N_1(z)$ represents the Neumann function of first order and $\mathbf{H}_1(z)$ the Struve function of first order defined by the following sequence:

$$\mathbf{H}_1(z) = \sum_{m=0}^{\infty}\frac{(-1)^m\left(\frac{z}{2}\right)^{2(m+1)}}{\Gamma\left(m+\frac{3}{2}\right)\Gamma\left(m+\frac{5}{2}\right)} = \frac{2}{\pi 3}\left(z^2-\frac{z^4}{15}+\frac{z^6}{525}-+\ldots\right). \tag{113}$$

As shown by equation (111), Z_{fjk} is the transcendental function of g and f. Since (g) and (f) are unknown and their determination is feasible only with Z_{fjk} known, the determination of the numerical values appears, for a given task rather complex. Fortunately, however, with the displacement current in the ground ($\omega\varepsilon_f \ll \sigma_f$) being negligible, expression (109) will be much simplified. For the ground parameters encountered in practice, this assumption is permissible up to about 1 megacycle per second. In case of the above assumption, the last (third) term of (111) will seem negligible as compared to the rest of the terms. The low argument Hankel functions may be substituted with approximate terms. The term containing function $H_0^{(1)}$ will change negligibly as compared to the term containing function $H_1^{(1)}$. Now introducing the new variable r_{jk}:

$$r_{jk}^2 = j(\varrho_{jk} f)^2 = \omega\mu_0\, \sigma_f\, \varrho_{jk}^2. \tag{114}$$

With the above statements in mind, the approximate value of Z_{fjk} will appear as

$$Z_{fjk} = \frac{\omega\mu_0}{\pi}\left[\frac{\pi}{4}\frac{\sqrt{j}}{r_{jk}}\left(e^{-j\Theta_{jk}}\, \mathbf{H}_1(r_{jk}\, e^{j\Theta_{jk}}) + e^{j\Theta_{jk}}\, \mathbf{H}_1(r_{jk}\, e^{-j\Theta_{jk}}) - \right.\right.$$

$$\left.\left. - e^{-j\Theta_{jk}}\, N_1(r_{jk}\, e^{j\Theta_{jk}}) - e^{j\Theta_{jk}}\, N_1(r_{jk}\, e^{-j\Theta_{jk}})\right) - \frac{\cos 2\Theta_{jk}}{r_{jk}}\right]. \tag{115}$$

The Struve (110) and Neumann functions of (115) can be approximated by means of their sequences. Through approximation the following formula will apply to Z_{fjk}:

$$Z_{fjk} = \frac{\omega\mu_0}{\pi}(P_{jk} + j\, Q_{jk}) \tag{116}$$

where

$$P_{jk} = \frac{\pi}{8} - \frac{r_{jk}}{3\sqrt{2}}\cos\Theta_{jk} - \frac{r_{jk}^2}{16}\left((\ln m r_{jk} - \frac{5}{4}\right)\cos 2\Theta_{jk} +$$

$$+ \frac{r_{jk}^2}{16}\Theta_{jk}\sin 2\Theta_{jk} + \frac{r_{jk}^3}{45\sqrt{2}}\cos 3\Theta_{jk} - \frac{r_{jk}^4\pi}{1536}\cos 4\Theta_{jk} - \frac{r_{jk}^4}{16}\Theta_{jk}\sin 4\Theta_{jk} +$$

$$\frac{r_{jk}^5}{1575\sqrt{2}}\cos 5\Theta_{jk} + \frac{r_{jk}^6}{18432}\left(\ln m r_{jk} - \frac{47}{28}\right)\cos 6\Theta_{jk} - \frac{r_{jk}^6}{18432}\Theta_{jk}\sin 6\Theta_{jk} + \ldots$$

$$Q_{jk} = \frac{1}{4} - \frac{1}{2}\ln m r_{jk} + \frac{r_{jk}}{3\sqrt{2}}\cos\Theta_{jk} - \frac{r_{jk}}{64}\cos 2\Theta_{jk} + \frac{r_{jk}^3}{45\sqrt{2}}\cos 3\Theta_{jk} +$$

$$\frac{r_{jk}^4}{384}\left(\ln m r_{jk} - \frac{5}{3}\right)\cos 4\Theta_{jk} - \frac{r_{jk}^4}{1575\sqrt{2}}\cos 5\Theta_{jk} + \frac{r_{jk}^6\pi}{73728}\cos 6\Theta_{jk} + \ldots$$

$$m = 0.890536\ . \tag{117}$$

As opposed to (112), equation (116) does not depend on (g) and, with (116) valid, f and Z_{fjk} will depend — at a given frequency, — only on the geometrical dimensions and material constants. Were the above approximation not applicable, the Z_f and g^2 values as calculated from (116) might be considered as first approximate values. Substituting the obtained g and f values into (112), the Z_f and g values can be corrected. This correction may be performed repeatedly if necessary.

Determination of the cut-off impedance matrix

In accordance with the aforesaid statements, the matrix form image parameters of the aerial power transmission line system can be properly determined. In order to promote the calculation of the produced reflections as well, the expression of the loading (cut-off) impedance must also be determined. In attempting to express the loading impedance matrix, such a situation should be investigated where, at a given point, the power transmission line will be

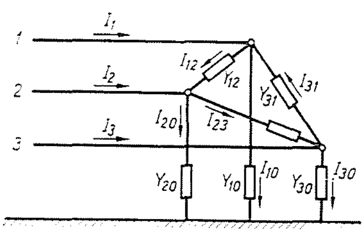


Fig. 13

terminated in such a manner as to have each conductor connected to all other conductors by means of the Y_{jk} admittance, and to the ground through the admittance Y_{k0} (Fig. 13).

(For easier understanding, the Figure has only three conductors entered.) The node law applying to the terminal of the k-th conductor reveals that

$$I_k = U_k(Y_{k0} + \sum_{\substack{j=1\\ j\neq k}}^{n} Y_{jk}) - \sum_{\substack{j=1\\ j\neq k}}^{n} U_j Y_{jk} \tag{118}$$

introducing the below notation:

$$Y_{kk} = Y_{k0} - \sum_{\substack{j=1\\ j\neq 1}}^{n} Y_{jk} \tag{119}$$

and substituting (119) into (118), it will be seen that

$$I_k = - \sum_{j=1}^{n} Y_{jk} U_k . \tag{120}$$

Expressing (120) for the node points of each lead terminal, the following matrix equation is obtained:

$$\mathbf{I} = \boldsymbol{Y}_t \, \mathbf{U} \tag{121}$$

where $\boldsymbol{Y}_t$ is the loading admittance matrix:

$$\boldsymbol{Y}_t = \boldsymbol{Z}_t^{-1} = -\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \tag{122}$$

With the wires loaded also at points other than their terminals or if the individual wires terminate at different points, each homogeneous section must be discussed separately.

Studying the layout illustrated by Fig. 14, it will be seen that a total of (n) aerial lines are run here in a length of l_1. At this l_1 length, similarly to the termination illustrated by Fig. 13, the wires are loaded with admittances characterized by matrix $\boldsymbol{Y}_{t1}$. Subsequently to length l_1, a number of k of the n

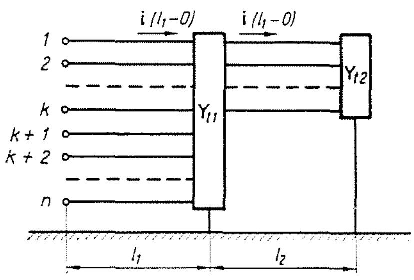


Fig. 14

wires proceed to a length of l_2 being, again similarly to the loading illustrated by Fig. 13, terminated at their ends with admittances characterized by matrix $\boldsymbol{Y}_{t2}$. First the homogeneous section l_2 will be studied. With $\boldsymbol{Y}_{t2}$ known, the $\boldsymbol{Y}_{be2}$ input admittance matrix expression of the section can be determined in accordance with formula (55). Knowing this, the below equation as related to point $z = l_1$ may be written:

$$\mathbf{i}^{[k]}(l_i + 0) = \boldsymbol{Y}_{be2}^{[k]} \, \mathbf{u}^{[k]}(l_i) \, . \tag{123}$$

Here $\mathbf{i}^{[k]}(l_1 + 0)$ is a column matrix composed of the values of currents flowing in the k proceeding wires as assumed at point $z = l_1$, whereas the elements of vector $\mathbf{u}^{[k]}$ (l_1) represent the voltages of these wires as compared to the ground. Matrix $\boldsymbol{Y}_{be_2}^{[k]}$ is of dimension k. Matrix equation (123) is equivalent to k scalar equations. Now let us extend the dimension of $\boldsymbol{Y}_{be_2}^{[k]}$ to n by making all elements in the rows under the k-th row and in the columns following the k-th column equal to zero.

$$Y_{be2}^{[n]} = \left[\begin{array}{ccc:ccc} & & & 0 & \dots & 0 \\ & \mathbf{Y}_{be2}^{[k]} & & \dots & & \dots \\ & & & 0 & \dots & 0 \\ \hdashline 0 & \dots & 0 & 0 & \dots & 0 \\ \dots & & \dots & \dots & & \dots \\ 0 & \dots & 0 & 0 & \dots & 0 \end{array}\right]. \tag{124}$$

With this being made use of, equation (123) may be re-written as an equation of n-dimension.

$$\mathbf{i}^{[n]}(l_i + 0) = \mathbf{Y}_{be2}^{[n]} \mathbf{u}^{[n]}(l_i). \tag{125}$$

where $\mathbf{u}^{[n]}$ (l_1) is a column vector composed of the voltages of n wires existing at point $z = l_1$ the first k elements of which are identical to those of $\mathbf{u}^{[k]}$ (l_1) while the rest of its elements equal to zero. Equation (125) consists of scalar equations of n number of which the first k equations are identical to those produced by (123) while the rest show that $\mathbf{i}_m$ $(l_1 + 0) = 0$, where $k < m < n$. Before point $z = l_1$, the currents in the wires may be combined into the column vector $\mathbf{i}(l_1 - 0)$. This consists of two parts. One is the column vector $\mathbf{i}^{[k]}$ $(l_1 + 0)$ symbolizing the proceeding currents as expressed by (125) whereas the other is represented by the column vector composed of the currents flowing off for loading purposes. This part can be calculated with matrix $\mathbf{Y}_{t1}$ being made use of:

$$\mathbf{i}^{[n]}(l_1 - 0) = (\mathbf{Y}_{be}^{[n]} + \mathbf{Y}_{t1})\, \mathbf{u}^{[n]}(l_1). \tag{126}$$

From (126), the admittance matrix $\mathbf{Y}_t$ loading the power transmission line system of l_1 length is obtained:

$$\mathbf{Y}_t = \mathbf{Y}_{be}^{[n]} + \mathbf{Y}_t\,. \tag{127}$$

The calculation presented above includes the following two special situations as well: *a)* the first k wires are not shunted at point $z = l_1$ and, *b)* the wires are of identical length but loading is not only at the terminals.

By now, matrix $\mathbf{M}$ (105), input impedance matrix $\mathbf{Z}_b$ (107) and (116), and loading admittance matrix $\mathbf{Y}_t$ (124) of the system have all been determined to promote the calculation of aerial line systems. Thus the calculation of aerial line systems has completely been led back to the calculation of systems composed of conductor sections. The method discussed above is suitable for power transmission lines either with or without neutral as well as for those of either single or double three-phase type. Since these do not require any special design, their separate discussion is not necessary.

Summary

Power transmission line systems are usually installed horizontally above ground level. The layout techniques of such systems reported on by the literature so far may be objected for various reasons. The present paper attempts to further develop the results published by the literature, and to render a more accurate theory as well as calculation method than those hitherto known.

In theory, first the generalized Kelvin equations concerning coupled power transmission lines and their solution are being dealt with. Then the reflection and input impedance calculations are discussed. Finally, by making use of the electromagnetic field theory of power transmission lines as well as of the theory of ground-return power transmission lines the results are generalized to promote the calculation of power transmission line systems installed above ground level.

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