THE TOLERANCES OF LINEAR NETWORKS AND SYSTEMS IN THE TIME, FREQUENCY AND COMPLEX FREQUENCY DOMAINS*

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From among the far-flung problems arising in connection with sensitivity and tolerance analysis the following questions will be dealt with here only:

1. For a given network, how the sensitivities and tolerances in the time, frequency and complex frequency domains can be determined?

2. What is the effect of the input waveform tolerance on the network response?

3. What connection exists between the various tolerances, e.g. between the tolerances of the real and imaginary parts or in case of frequency and time domains?

1. The computation of tolerances

Fig. 1 shows the symbols used to characterize the linear networks in the time, frequency and complex frequency domains. The system may be described, for instance, in the time domain with the weighting function $k(t)$, in the frequency domain with the amplitude characteristic $A(\omega)$ and the phase characteristic $b(\omega)$. In many cases it is more convenient to characterize the system in the time domain with the transit function $h(t)$, in the frequency domain with the logarithmic amplitude characteristic $a(\omega) = \ln A(\omega)$ and the group delay time characteristic $\tau = \frac{\mathrm{d}b}{\mathrm{d}\omega}$. Further, the following symbols are used: $p = \sigma + j\omega$ for the complex frequency, $K(p)$ for the transfer function, p_i' for the zeros and p_i'' for the poles. If no distinctions between poles and zeros should be made, the symbol p_i is used. In writing the second form of $K(p)$ it was assumed that $p_i = 0$. If $p_i = 0$, also a multiplication factor of the form p^l occurs and, consequently, m or n will be diminished by l. The network functions $k(t)$, $K(j\omega)$ and $K(p)$ will in common be symbolized by y.

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Time domain	Frequency domain	Complex frequency domain
t	ω	$p = \sigma + j\omega$
$k(t)$	$K(j\omega) = A(\omega)e^{-jb(\omega)}$ $a = \ln A(\omega)$ $\tau = \frac{db}{d\omega}$	$K(p) = \frac{A(p)}{B(p)} = k_1 \frac{\prod_1^m (p - pi')}{\prod_1^n (p - pi'')} =$ $= k_2 \frac{\prod_1^m \left(1 - \frac{p}{pi'}\right)}{\prod_1^n \left(1 - \frac{p}{pi''}\right)}$

Fig. 1. Characterization of linear networks

Fig. 2 shows the mutual connection of the tolerances and sets the tasks of the investigation. $Z_i = Z_{i0} + a_i$ is the true value of the i-th quantity determining the network, e.g. that of an impedance, where Z_{i0} is the nominal value of the i-th quantity and a_i is a random variable with zero mean. Usually,

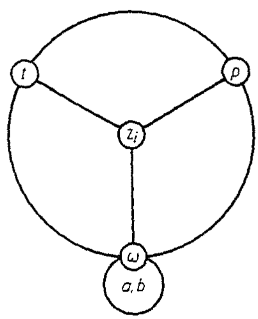

Fig. 2. Mutual connections between the tolerances

a_i is a complex value. One group of problems is to compute from the probability distribution of the network elements, assumed to be known, the tolerances of the networks characteristics in the time, frequency and complex frequency domains. Another and much more difficult group of problems is to determine the tolerances of the network elements from the tolerances set in the time, frequency or complex frequency domains. The third group of problems is the mutual connection of the tolerances, e.g. the conversion of time domain and frequency domain tolerances. Within the frequency domain in case of minimum phase networks there also appears the mutual connection between the phase characteristic tolerance and that of the logarithmic amplitude characteristic.

The sensitivity of the $y = f(x)$ connection is $S = \frac{dy}{dx}$ and its tolerance is $\Delta y = S\,\Delta x$. The connection $y = f(x)$ has the relative sensitivity $S_r = \frac{d \ln y}{d \ln x} = \frac{dy/y}{dx/x} = \frac{x}{y}\frac{dy}{dx}$ and the relative tolerance $\frac{\Delta y}{y} = S_r \frac{\Delta x}{x}$. The relative sensitivity will not be used below and it is to be mentioned that the definition of sensitivity used here differs from the original definition of Bode and can be used advantageously in all three domains.

The tolerance of the network function $y = y_0 + \Delta y$ is

$$\Delta y = \sum_{i=1}^{N} S_i\, a_i \tag{1}$$

where y_0 is the nominal value of the network function $[k_0(t), K_0(j\omega), K_0(p)]$, S_i is the sensitivity relating to the i-th element, and N is the number of the quantities determining the network. The sensitivities and the tolerances for the various domains are shown in Fig. 3. The formulae giving the tolerances are obtained in the following way.

Time domain	Frequency domain	Complex frequency domain
t	ω	$p = \sigma + j\omega$
$S_i(t) = \frac{\partial k_0}{\partial Z_{i0}}$	$S_i(j\omega) = \frac{\partial K_0(j\omega)}{\partial Z_{i0}}$	$S_i(p) = \frac{\partial K_0(p)}{\partial Z_{i0}}$
$\Delta k(t) \approx \sum_{pi''} \sum_{pi'} c_i' e^{p_{0i}'t} \Delta p_i' +$ $+ \sum_{pi''} \sum_{pi''} c_i'' e^{p_{0i}''t} \Delta p_i'' +$ $+ \sum_{pi''} \sum_{pi''} c_i'' t^p e^{p_{0i}''t} \Delta p_i''$	$\Delta a = \ln \frac{A(\omega)}{A_0(\omega)} \approx \sum_i \mathcal{R}e \frac{S_i a_i}{K_0}$ $\Delta b = b - b_0 \approx \sum_i \mathcal{I}m \frac{S_i a_i}{k_0}$ $\Delta a \leq \frac{1}{A_0} \sum_i \lvert S_i \rvert \lvert a_i \rvert$	$\Delta p_i = p_i - p_{i0} =$ $\approx \Delta K_a \frac{1}{\left.\frac{\partial K_0}{\partial p_{i0}}\right\rvert_{p = p_{i0}}}$

Fig. 3. Sensitivities and tolerances

In the frequency domain ω it is very important that from the sensitivity $S_i(j\omega) = \frac{\partial K_0(j\omega)}{\partial Z_{i0}}$ and from the tolerances a_i besides the tolerance $\Delta K(j\omega)$ also the tolerances of $A(\omega)$, $b(\omega)$ and $a(\omega)$ could be computed. Since

$$K(j\omega) = K_0(j\omega) + \sum_{i=1}^{N} S_i\, a_i =$$

$$K_0(j\omega)\left[1 + \sum_{i=1}^{N} \frac{S_i\, a_i}{K_0}\right] = K_0(j\omega)\left[1 + \sum_{i=1}^{N} \mathcal{R}e \frac{S_i\, a_i}{K_0} + j \sum_{i=1}^{N} \mathcal{I}m \frac{S_i\, a_i}{K_0}\right],$$

using Taylor series expansion we obtain

$$|K(j\omega)| = A(\omega) = A_0 \sqrt{\left(1+\sum_{i=1}^{N} \mathcal{Re}\,\frac{S_i\,a_i}{K_0}\right)^2 + \left(\sum_{i=1}^{N} \mathcal{Im}\,\frac{S_i\,a_i}{K_0}\right)^2} \approx$$

$$\approx A_0 \left(1+\sum_{i=1}^{N} \mathcal{Re}\,\frac{S_i\,a_i}{K_0}\right), \tag{2}$$

i.e.

$$b - b_0 = -\operatorname{arc\,tg} \frac{\sum_{i=1}^{N} \mathcal{Im}\,\frac{S_i\,a_i}{K_0}}{1+\sum_{i=1}^{N} \mathcal{Re}\,\frac{S_i\,a_i}{K_0}} \approx -\sum_{i=1}^{N} \mathcal{Im}\,\frac{S_i\,a_i}{K_0}. \tag{3}$$

In terms of equation (2)

$$\Delta a = \ln \frac{A(\omega)}{A_0(\omega)} \approx \ln \left(1+\sum_{i=1}^{N} \mathcal{Re}\,\frac{S_i\,a_i}{K_0}\right)$$

$$\approx \sum_{i=1}^{N} \mathcal{Re}\,\frac{S_i\,a_i}{K_0}. \tag{4}$$

The latter equation takes in the worst case the form

$$\Delta a \leq \sum_{i=1}^{N} \left|\frac{S_i\,a_i}{K_0}\right| = \frac{1}{A_0} \sum_{i=1}^{N} |S_i|\,|a_i|. \tag{5}$$

The equations (3), (4), (5) in the practical computations proved to be simple, good approximations.

In the complex frequency domain $p = \sigma + j\,\omega$ the effect of the network element's tolerances on the poles and zeros can be expressed in the following form:

$$\Delta p_i = p_i - p_{i0} = \Delta K_a \frac{1}{\left.\frac{\partial K}{\partial p_i}\right|_{p_{i0}}}. \tag{6}$$

Formula (6) follows from the two ways of writing the transfer function tolerance:

$$\sum_{p_i} \frac{\partial K_0}{\partial p_i}(p_i - p_{i0}) = \sum_{i=1}^{N} \frac{\partial K_0}{\partial Z_{i0}}\,a_i = \Delta K_a. \tag{7}$$

Consequently, ΔK_a is the transfer function tolerance resulting from the tolerances of all network elements at the point $p = p_{i0}$. Since $K_0(p)$ is a rational fractional function, the expression $\left.\frac{\partial K_0}{\partial p_i}\right|_{p_{i0}}$ can be relatively easily computed. Special forms of the relation (6) can be found in works of PAPOULIS and HOROWITZ.

In the time domain t the character of the tolerance of the weighting function $k(t)$ can be judged in the knowledge of the tolerances of poles and zeros. If the transfer function is a proper fraction and the poles are single, then in terms of the expansion theorem

$$k_0(t) = \sum_{i=1}^{n} \frac{A(p''_i)}{B'(p''_i)} e^{p''_i t} =$$

$$= k_1 \sum_{i=1}^{n} \frac{(p''_i - p'_1)(p''_i - p'_2)\dots(p''_i - p'_m)}{(p''_i - p''_1)\dots 1 \dots (p''_i - p''_m)} e^{p''_i t}.$$

This expression must be differentiated with respect to the zeros and poles for determining the tolerance:

$$\Delta k(t) \approx \sum_{p''_i} \sum_{p'_i} c'_i e^{p''_i t} \Delta p'_i +$$

$$+ \sum_{p''_i} \sum_{p''_i} c''_i e^{p''_i t} \Delta p''_i +$$

$$+ \sum_{p''_i} \sum_{p''_i} t c''_i e^{p''_i t} \Delta p''_i . \qquad (8)$$

It can be seen that besides the original time functions $e^{p''_i t}$ there also occur the time functions $te^{p''_i t}$.

The logarithmic amplitude and phase plots, the so called Bode plots are very useful in the analysis and synthesis of linear systems. In the knowledge of pole and zero tolerances the tolerances of logarithmic amplitude and phase characteristics can be expressed. The results are shown in Figs 4, 5 and 6.

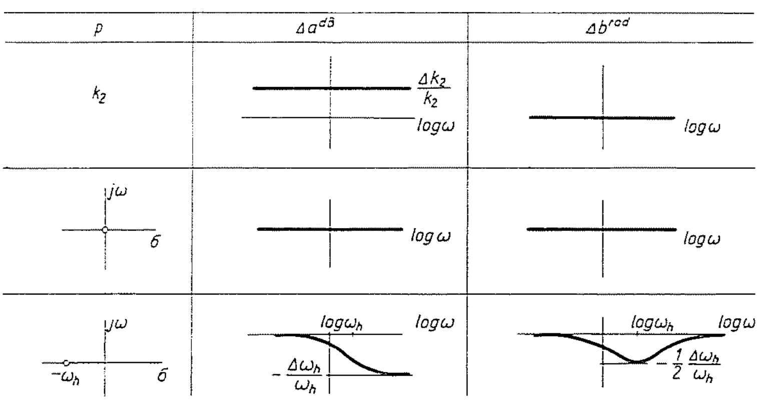

Fig. 4. Tolerances in logarithmic amplitude and phase plots I

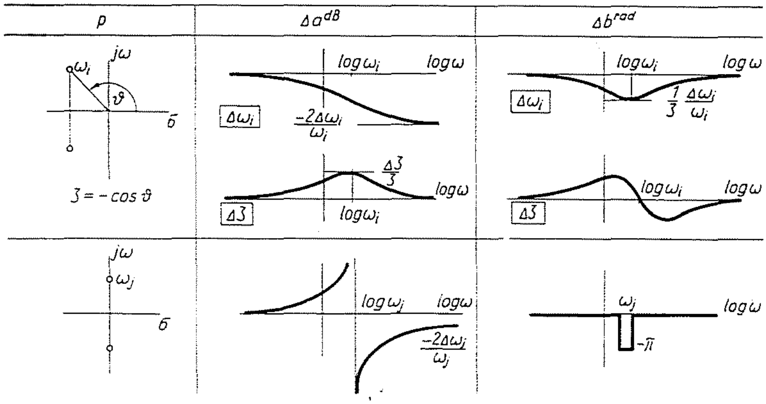

Fig. 5. Tolerances in logarithmic amplitude and phase plots II.

Root factors	Δ_a^{NP}	Δb^{rad}
k_2	$\frac{\Delta k_2}{k_2}$	0
$1 + \frac{p}{\omega_h}$	$-\frac{\left(\frac{\omega}{\omega_h}\right)^2}{1+\left(\frac{\omega}{\omega_h}\right)^2}\frac{\Delta\omega_h}{\omega_h}$	$-\frac{\frac{\omega}{\omega_h}}{1+\left(\frac{\omega}{\omega_h}\right)^2}\frac{\Delta\omega_h}{\omega_h}$
$1 + 2\zeta\frac{p}{\omega_i} + \left(\frac{p}{\omega_i}\right)^2$ $D = 1 + (4\zeta^2 - 2)\left(\frac{\omega}{\omega_i}\right)^2 + \left(\frac{\omega}{\omega_i}\right)^4$	$\Delta a_{\omega i} = -\frac{-2\left(\frac{\omega}{\omega_i}\right)^4 - (4\zeta^2-2)\left(\frac{\omega}{\omega_i}\right)^2}{D}\frac{\Delta\omega_i}{\omega_i}$ $\Delta a_\zeta = \frac{4\zeta\left(\frac{\omega}{\omega_i}\right)^2}{D}\Delta\zeta$	$\Delta b_{\omega i} = \frac{-2\zeta\frac{\omega}{\omega_i} - 2\zeta\frac{\omega^3}{\omega_i^3}}{D}\frac{\Delta\omega_i}{\omega_i}$ $\Delta b_\zeta = \frac{2\zeta\frac{\omega}{\omega_i}\left[1-\left(\frac{\omega}{\omega_i}\right)^2\right]}{D}\Delta\zeta$
$1 + \left(\frac{p}{\omega_j}\right)^2$	$2\frac{\left(\frac{\omega}{\omega_j}\right)^2}{1-\left(\frac{\omega}{\omega_j}\right)^2}\frac{\Delta\omega_j}{\omega_j}$	0 $\quad \omega < \omega_{jo}$ 0 $\quad \omega > \omega_j$ $-\pi \quad \omega_{jo} > \omega < \omega_j$

Fig. 6. Tolerances in logarithmic amplitude and phase plots III.

In the analysis it was assumed that the character of the factors remains invariable, which means that the sign of the constant k_2 as well as the root at the origin do not vary, the root lying on the real axis varies only on the real axis and does not become a pair of conjugate complex zeros etc. In Fig. 4 the first row shows the tolerance of the constant k_2 in the transfer function.

The second row is the case of a zero at the origin. The third row indicates the effect of the variation of a zero being on the negative real axis. The variation of the zero has a considerable influence on the amplitude and phase characteristics.

The case of conjugate complex zeros is shown in Fig. 5. The conjugate complex root is described by the absolute value of ω_i and by the phase ϑ. For the latter the parameter $\zeta = -\cos\vartheta$ is introduced. In case of a conjugate complex root the effects of the absolute value and the phase angle tolerances must be analysed separately as shown in Fig. 5. In the last row of Fig. 5 the effect of the tolerance of the pure imaginary zero is indicated.

Fig. 6 summarizes the formulas of the root factors and those of the tolerances of the logarithmic amplitude and phase plots. In the terms of the foregoing, the following cases are to be found here: $k_2 =$ const., zero on the real axis, conjugate complex root, pure imaginary root. As had been assumed the zero located at the origin remains invariably in the origin, thus is not shown in the table.

In the case of frequency transformations which are usually made in the design of filters, since the main geometric structure of the filter remains unchanged, the expression of the sensitivities is the same for low-pass and transformed impedances:

$$S_i = \frac{\partial K_0^L(p, Z_{i0}^L)}{\partial Z_{i0}^L} = \frac{\partial K_0^T(p, Z_{i0}^T)}{\partial Z_{i0}^T}. \tag{9}$$

Here L relates to the low-pass filter and T to the transformed one. However, the tolerance of the low-pass filter and that of the transformed filter are different, because the variations of the impedances are different: $\Delta a_i^L \neq \Delta a_i^T$.

2. The effect of the input waveform tolerance

If the Dirac delta pulse or the unit step function is considered as input signal, then their tolerance appears only with a constant multiplication factor in the weighting function as well as in the transit function. The analysis of the systems in the time domain is made in connection with the measurement of the response to the periodic signal. The period, the rise time and the overshoot of these test signals are established by international recommendations. The computation of the system response to periodic signal even in case of applying the very well useable Laplace transformation is by order of magnitude more complicated than the computation of the response to unit step function. An acceptable compromise between complexity of computation and practically utilizable result is the computation of the response to the ramp step. The ramp step is shown in Fig. 7. With $t \leqslant 0$ its value is zero, from $t = 0$ to $t = t_r$ it varies linearly and is constant for $t \geqslant t_r$. The two interesting intervals are marked with I and II.

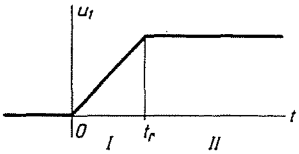

Fig. 7

Let us analyse the effect of the rise time t_r of the ramp step $u_1 = u_1(t,t_r)$ on the response $u_2(t)$ of the system. The response of the system is

$$u_2(t, t_r) = \int_0^t k(t-\tau)\, u_1(\tau, t_r)\, d\tau\,. \tag{10}$$

Since

$$\frac{\partial u_2}{\partial t_r} = \int_0^t k(t-\tau) \frac{\partial u_1(\tau, t_r)}{\partial t_r}\, \mathrm{d}\tau\,,$$

the tolerance of the response is

$$\Delta u_2 = \frac{\partial u_2}{\partial t_r} \Delta t_r = \int_0^t k(t-\tau) \frac{\partial u_1(\tau, t_r)}{\partial t_r} \Delta t_r\, \mathrm{d}\tau =$$

$$= \int_0^t k(t-\tau)\, \Delta u_1\, \mathrm{d}\tau \tag{11}$$

where

$$\Delta u_1 = \frac{\partial u_1(\tau, t_r)}{t_r} \Delta t_r\,. \tag{12}$$

In domain I

$$u_1^I = \frac{\tau}{t_r}\quad \Delta u_1^I = -\frac{\tau}{t_r^2} \Delta t_r = -u_1^I \frac{\Delta t_r}{t_r}\,.$$

Substituting this in (11) and using (10), the result

$$\Delta u_2^I = -\frac{\Delta t_r}{t_r} \int_0^t k(t-\tau)\, u_1^I\, \mathrm{d}\tau = -u_2^I \frac{\Delta t_r}{t_r} \tag{13}$$

will be obtained.

Let the rise time of the response be designed by t_f and let it be assumed that $t_f \leqslant t_r$. In terms of Fig. 8

$$\frac{\Delta u_2^I}{t_{f2} - t_{f1}} = \frac{\Delta u_2^I}{\Delta t_f} = \frac{\partial u_2^I}{\partial t},$$

of which, using equation (13), we obtain

$$\Delta t_f = t_{f2} - t_{f1} = \frac{\Delta u_2^I}{\frac{\partial u_2^I}{\partial t}} = \frac{u_2^I}{\frac{\partial u_2^I}{\partial t}} \frac{\Delta t_r}{t_r}. \tag{14}$$

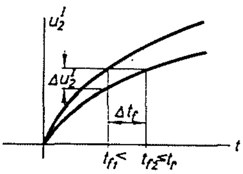

Fig. 8

Equations (13) and (14) can be used advantageously in the evaluation of measurement results if the network elements have nominal values and only the rise time of the input signal varies.

3. The mutual connections between the tolerances

3.1. *The connection of the tolerances of the real and imaginary parts*

Let the real part of the transfer characteristic be $K(j\,\omega)$ of the four-terminal network shown in Fig. 9. $A(\omega)$ and its imaginary part $B(\omega)$. In the knowledge of the network elements Z_i the real functions $A(\omega)$ and $B(\omega)$ can be determined.

Fig. 9

Since the values of the network elements depend on chance, $A(\omega)$ and $B(\omega)$ can be considered as stochastical processes. Further on, the connection between the characteristics (mean value, variance, correlation function, spectral density) of the stochastical processes $A(\omega)$ and $B(\omega)$ shall be dealt with.

It is known that the real and imaginary parts of the complex functions regular in the right half-plane and on the j axis are connected by the Hilbert transformation. In this case

$$B(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{A(\tau)}{\omega - \tau} \, d\tau , \tag{15a}$$

$$A(\omega) - A(\infty) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{B(\tau)}{\omega - \tau} \, d\tau . \tag{15b}$$

Equation (15) can be considered as a convolution with the weighting function $k_H = \frac{1}{\pi t}$. Consequently, the Hilbert transformation can be substit-

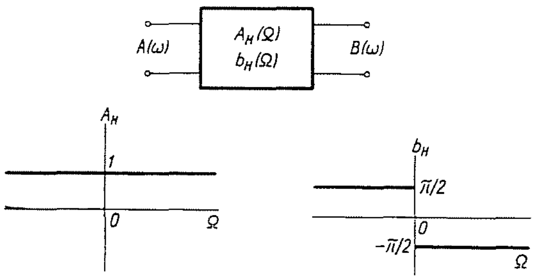

Fig. 10

uted in the frequency domain Ω by the transfer characteristic $K_H(j\Omega)$ or by an equivalent amplitude characteristic $A_H(\Omega)$ and phase characteristic $b_H(\Omega)$. Performing the Fourier transformation of the weighting function k_H we obtain

$$K_H(j\Omega) = \mathcal{F} k_H(t) = \begin{cases} -j & \Omega > 0 \\ j & \Omega < 0 \end{cases} \tag{16a}$$

respectively,

$$A_H(\Omega) = 1 \tag{16b}$$

$$b_H(\Omega) = \begin{cases} -\frac{\pi}{2} & \Omega > 1 \\ \frac{\pi}{2} & \Omega < 1 \end{cases} \tag{16c}$$

The four-terminal network given by equations (16) by realizing the Hilbert transformation is shown in Fig. 10. Consequently, the Hilbert transformation is equivalent to the passage through a linear system having the input $A(\omega)$ and the output $B(\omega)$.

The application of the relations concerning the linear transformation of the stochastical processes is very simple, since $A_H(\Omega) = 1$, and gives the following results for the mean value, the variance, the correlation function and spectral density of the tolerances ΔA and ΔB:

$$M\ [\Delta A] = M\ [\Delta B] = 0 \tag{17a}$$

$$D^2\ [\Delta A] = D^2\ [\Delta B] \tag{17b}$$

$$R\ [\Delta A] = R\ [\Delta B] \tag{17c}$$

$$G\ [\Delta A] = G\ [\Delta B]\,. \tag{17d}$$

Therefore, the statistical characteristics of the tolerances are the same for the real and imaginary parts of the transfer characteristic, if the tolerances do not alter the character of the network, that is, if the Hilbert transformation can be applied.

3.2. *The connection between the tolerance of the time and frequency domains*

To begin with, let the effect of small sinusoidal variations of the phase characteristic in the time domain be investigated. Let the frequency of the sinusoidal variation Ω_0 and its amplitude be Δb (Fig. 11):

$$b = \Delta b \sin\ \Omega_0\, \omega\,. \tag{18}$$

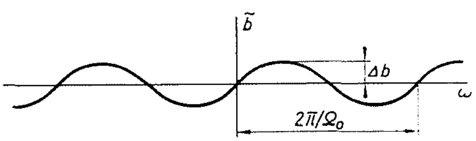

Fig. 11

Then in the output signal beside the original signal u_{20} we get two echos

$$u_{2\tilde{b}} = u_{20} + \frac{\Delta b}{2} u_{20}(t + \Omega_0) - \frac{\Delta b}{2} u_{20}(t - \Omega_0)$$

that is, for the tolerance in the time domain we obtain the inequalities

$$u_{2\tilde{b}} = \max |u_{2\tilde{b}} - u_{20}| \leqslant \Delta b\ \max |u_{20}| \tag{19a}$$

$$\frac{\Delta u_{2b}}{\max |u_{20}|} < \Delta b\,. \tag{19b}$$

A similar result is obtained if with a linear phase variation a cosinoidal amplitude variation is assumed. Fig. 12 shows the waveform distorsion appearing in various situations, for the case of square wave.

In a general case the uppermost limit of the tolerance characterizing the resulting waveform distorsion is obtained by adding the small sinusoidal and cosinoidal effects

$$\frac{\Delta u_2}{\max |u_{20}|} \leqslant \int_{-\infty}^{\infty} [\,|S_{\Delta\tilde{A}}(\Omega)| + |S_{\Delta\tilde{b}}(\Omega)|\,]\,\mathrm{d}\Omega\,. \tag{20}$$

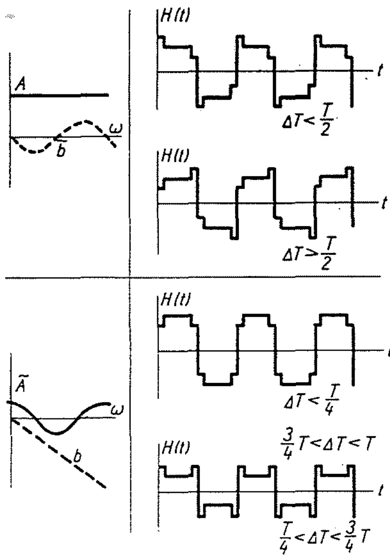

Fig. 12. Frequency characteristics and waveform distorsion

Here $S_{\Delta\tilde{A}}(\Omega)$ and $S_{\Delta\tilde{b}}(\Omega)$ are the tolerance spectra of the amplitude and of the phase characteristic, respectively.

The tolerance frequency Ω and the tolerance spectrum $S(\Omega)$ introduced to describe the deviation of the real and imaginary parts as well as of the amplitude and phase characteristics from the nominal are conducive to the better understanding of the connection of tolerances.

Summary

The tolerances of the logarithmic amplitude characteristic and those of the phase characteristic can be expressed as a sum of real random variables. Knowing the tolerances of poles and zeros simple formulae for the tolerances of the weighting function as well as for those of the logarithmic amplitude and phase characteristics (Bode diagrams) can be deduced.

In the case of frequency transformation the sensitivities computed for the low-pass filter can be used for the computation of tolerances.

After defining the response to the ramp step, a simple formula for the tolerance of the rise time is given. The connection between the real and imaginary parts of the network function in terms of the Hilbert transformation is analysed. The mutual connections between the tolerances of the frequency and time domains are treated by introducing the notion of the tolerance spectrum.

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