REDUCING THE TIME REQUIREMENT IN DIRECT-READING NOISE MEASUREMENTS

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1. Introduction

Nowadays noise measurements attain an ever increasing importance in the widening field of application for electronic measurements. As the treatment of noise in passive linear networks today appears already as being classical, the attention of noise measuring techniques is at present focused primarily on active devices. Noise measurements play an extremely significant, and from the theoretical viewpoint, essential role in semiconductors, first of all in investigating flicker noise, not completely accounted for so far, but a more or less strict interdependence exists in semiconductors also between the stability of macroscopic parameters and noise characteristics. Noise measurements can prove a valuable factor in qualifying semiconductor products, as they occupy a special place in investigating the emission of electron tubes.

As to the further applications of noise in measurements, the transfer characteristics of complicated systems are frequently determined by means of noise response. Some acoustic and mechanical measurements (e.g. investigation of microphony in electron tubes), as well as the stochastic excitation of automatic control systems, may as well be classified as noise measurements.

Due to the random character of noise, the time of noise measurement cannot be reduced without limit; the narrower the frequency band the measurement is undertaken in, the longer the time necessary to achieve a specified accuracy of measurement. The measurements are particularly lengthy in the AF-range, owing to the inherent small absolute bandwidth.

The influence, exerted by the time of measurement on the accuracy, is easy to show. Let us assume a band-limited white noise, i.e. a noise with constant spectral density within the bandwidth B, cut out from the infinite spectrum. It follows from the sampling theory, valid for every band-limited signal, that not more than $2B$ independent measuring results can be obtained per unit of time this making possible the exact reproduction of the original function of the time. If the measurement is carried out as long as T, we obtain $2BT$ measuring points (Fig. 1). The average value plotted with a dotted line will be the more accurately obtained the higher the quantity $2BT$.

For averaging purposes an integrating network must be applied. In simple instruments ordinary RC-networks are used. If only a small variation in the output voltage of the integrating network is allowed, a high time constant must be chosen. In this case, however, a lengthy time is required to reach the steady state response (Fig. 1). This fact is extremely disturbing, if the noise voltage of several noise sources (e.g. transistors) is to be measured one after another with the same instrument. It will be shown that the problem

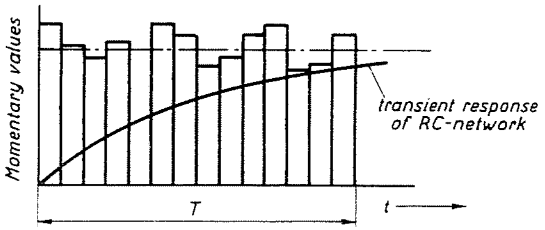

Fig. 1. Application of sampling theory to the measurement of band-limited noise

can easily be solved by making use of time-variable RC-networks: the time constant of the integrating network should gradually be increased from a small initial value at the beginning of the measurement up to the stationary amount. During this time the integrating network will be practically charged.

2. Uncertainty of noise power measurements of limited duration

As it is more easy to mathematically compute the noise power than the noise voltage, mostly the uncertainty of noise power measurements is analysed in the literature. The problem is approached by more, on the whole interdependent, but by different sequences of ideas. According to the sampling theory in a bandwidth B not more than

$$k = 2BT \tag{1}$$

independent random variables can be obtained during a time interval T [1]. The quantity k can be regarded as the number of statistically independent random variables. The result of the measurement — the mean value of the random variables — is the more accurate, the higher k is. Let us assume that the amplitude of the noise voltage to be measured is characterized by a normal probability density function centered on zero. Since, as already stated, not the noise voltage, but the squared noise voltage proportional to the noise power will be measured, the mean value of the statistically independent random variables — called in the following the result of measurement —

follows the so called χ^2 (chi-squared) density function. If $k \to \infty$, the probability density function becomes extremely narrow, and its centrum equals the mean square. But if k is not a large number, the results of measurement show considerable spread, and at $k < 20$ the value obtained will show a not negligible deviation from that for $k \to \infty$. The deviation is negative, which can be attributed to the fact that if but few measurements are performed the probability of the (not squared) noise voltage to deviate from zero drops.

The chi-squared density function is described in numerous ways in the literature.

Let $\xi_1, \xi_2, \ldots \xi_n$ denote independent random variables of the same normal distribution with the common probability density function

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

The distribution

$$\frac{\chi_n^2}{\sigma^2} = \left(\frac{\xi_1}{\sigma}\right)^2 + \left(\frac{\xi_2}{\sigma}\right)^2 + \ldots + \left(\frac{\xi_n}{\sigma}\right)^2$$

has a density function

$$q_n(x) = \frac{x^{\frac{n}{2}-1} e^{-\frac{x}{2}}}{2^{\frac{n}{2}} \Gamma\left(\frac{n}{2}\right)} \qquad \text{for } x > 0$$

and a distribution function

$$Q_n(x) = \frac{1}{2^{\frac{n}{2}} \Gamma\left(\frac{n}{2}\right)} \int_0^x t^{\frac{n}{2}-1} e^{-\frac{t}{2}} dt$$

These functions are not easy to treat, that is the reason why they are mostly plotted [2] or tabulated [1, 3, 4].

If numerous measurements are carried out, the chi-squared density function can be approximated by a normal distribution with the variance

$$\sigma^2 = \frac{2}{k} \tag{2}$$

It follows from the properties of the normal distribution that 68.3 per cent of the results of measurements fall in the band of width $\pm\sigma$, while the probability of falling in the interval $\pm 2\,\sigma$ amounts to 95 per cent. Let us assume a permissible error area of $\pm h$ with respect to the accurate mean, in which p per cent of all measurements must be located. If, e.g. $h = \sigma$, 68.3 per cent of

$$k = \frac{2}{\sigma^2} = \frac{2}{h^2} \tag{2a}$$

measurements fall in the band of width $\pm h$. If higher confidence is required,

$$k = \frac{2f(p)}{\sigma^2} = \frac{2f(p)}{h^2} \tag{3}$$

and

$$T = \frac{k}{2B} = \frac{f(p)}{h^2} \frac{1}{B} \tag{4}$$

where $f(p)$ depends on the confidence level (see Table I) and h stands for the relative error. The values of $f(p)$ have been determined on grounds of the properties of the normal distribution.

Table I

p (per cent)	68.3	90	95	98	99.7
$f(p)$	1	2.7	4	5.4	9

Thus, if a noise power of the bandwidth B is repeatedly measured with an integrating network operating as long as T, the results obtained will fall with a probability of p into an interval, having a relative width of $\pm h$.

3. Measurement of noise power by a direct reading instrument

The usual arrangement for noise measurement is shown in Fig. 2. The wide-band (for simplicity's sake white) noise of the noise source F followed by a band-pass filter, is connected to a square-law detector. The output signal of the square-law detector controls through an RC-network a *dc* vacuum tube voltmeter. Obviously, the greater the time constant of the RC-network, the smaller the fluctuation of the indication with respect to the mean square of the noise voltage is. The effect exerted by the bandwidth of the noise input is not too easy to see.

According to DAVENPORT and ROOT [5] the frequency spectrum of the signals appearing in various points of the circuit are shown in Fig. 3, under the assumption that the signal before the bandpass has a continuous and constant spectrum (white noise), and the transfer characteristic of the band-pass is approximately rectangular. To avoid difficulties in the origin of the spectrum we define the frequency range between $-\infty$ and $+\infty$ and convert the spectrum function, defined originally, but for positive frequencies, into an even function.

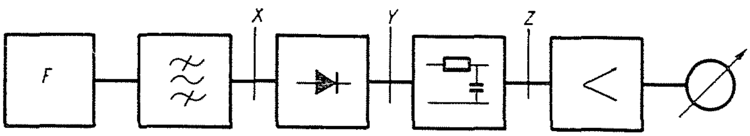

Fig. 2. Block diagram of a direct-reading noise measuring instrument

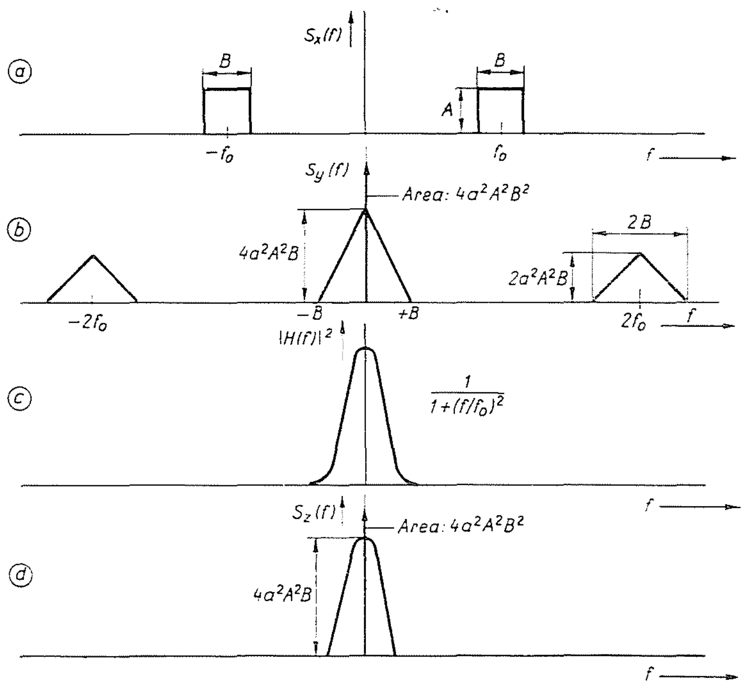

Fig. 3. Power spectrum of the signals, generated in the nodes X, Y and Z of Fig. 2

The energy spectrum function at the output of the band-pass

$$S_x(f) = \begin{cases} A, & \text{if } |f_0 - B/2| < |f| < |f_0 + B/2| \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

After the square-law detector having the amplitude characteristic $y = ax^2$ the spectrum will consist of two components; i.e. of the spectrum line

$$\overline{S}_y = 4a^2 A^2 B^2 \delta(f) \tag{6}$$

located in the origin and marking the mean square, and of the *ac* components

$$\tilde{S}_y = 4a^2A^2(B - |f|) \quad \text{for } |f| < B \tag{7}$$

characteristic of the fluctuation (if we drop the terms symmetric to $\pm 2f_0$, which are of no interest from the point of view of the following).

The signal $y(t)$ with a frequency spectrum $S_y = \bar{S}_y + \tilde{S}_y$ is applied to the *RC*-network, having a transfer function

$$H(f) = \frac{1}{1 + jf/f_a} \tag{8}$$

Since $S_y(f)$ denotes an energy spectrum, the magnitude of the transfer function must be squared:

$$|H(f)|^2 = \frac{1}{1 + (f/f_a)^2} \tag{9}$$

The spectral density of the output of the integrating network is

$$S_z(f) = \bar{S}_z(f) + \tilde{S}_z(f)$$

with a *dc* component of

$$\overline{S_z}(f) = 4a^2A^2B^2\delta(f) \text{ since } |H(0)|^2 = 1 \tag{10}$$

and an *ac* component of

$$\tilde{S}_z(f) = 4a^2 A^2 (B - |f|) \frac{1}{1 + (f/f_a)^2} \quad \text{for } |f| < B \tag{11}$$

We obtain the *dc* mean square and the mean square of the fluctuation by integration:

$$\overline{z^2} = \int_{-\infty}^{\infty} \overline{S_z}(f)\, df = 4a^2 A^2 B^2 \tag{12}$$

$$\sigma_z^2 = \int_{-\infty}^{\infty} \tilde{S}_z(f)\, df = 4a^2 A^2 \left[\int_{-B}^{0} \frac{B + f}{1 + (f/f_a)^2} df + \int_{0}^{B} \frac{B - f}{1 + (f/f_a)^2} df \right] \tag{13}$$

Some computation yields

$$\frac{\sigma_z^2}{\overline{z^2}} \cong \frac{f_a}{B} \left[\pi - \frac{f_a}{B} 2 \ln \frac{B}{f_a} \right] \tag{14}$$

The second term has the form $\frac{\ln x}{x}$ and vanishes for $x \to \infty$. Thus, in case of sufficiently low f_a/B we obtain

$$\frac{\sigma_z}{\sqrt{\overline{z^2}}} = \sqrt{\pi \frac{f_a}{B}} \tag{15}$$

Setting the time constant $\tau = \frac{1}{2\pi f_a}$ results

$$\frac{\sigma_z}{\sqrt{\overline{z^2}}} = \frac{1}{\sqrt{2B\tau}} \tag{16}$$

From equation (16) we may conclude: the quotient of fluctuation and root mean square depends on the ratio between the bandwidths of integrator and band-pass. If f_a/B is adequately small, the fluctuation has a normal distribution, thus the pointer of the *DC* instrument stays in the range $\sqrt{\overline{z^2}} \pm \sigma_z$ during 68.3 per cent of the time of measurement. Increasing this domain to $\pm 2\sigma_z$, the probability rises to 95 per cent.

4. Time requirement of direct-reading measurement

In chapter 2 it has been shown that measuring noise power with adequate accuracy requires an adequate amount of information. If the bandwidth B and the relative error h are specified, the minimum time requirement T of the measurement is determined by them. Undertaking a number of measurements with an ideal integrator operating as long as T, the results will with the probability p fall into the interval having the relative width $\pm h$.

In chapter 3 the fluctuation exhibited by the pointer of a direct-reading instrument with the time constant τ has been computed. Let us now investigate the relation between the time of measurement T and the time constant τ, if the errors due to the finite duration of measurement and the fluctuation of the pointer are assumed to be equal:

$$\frac{2}{2BT} = \frac{1}{2B\tau} \tag{17}$$

i.e.

$$\tau = \frac{T}{2} \tag{18}$$

Let us assume e.g. that the noise of individual transistors is measured consecutively. According to Fig. 3 the noise power can be split into two components: a *dc* component and an *ac* one of continuous spectrum. As it is the *ac* component which gives rise to fluctuation, only the *dc* component can be regarded as a useful signal.

According to equation (12), at the moment when a transistor to be measured is switched on, a unit step of

$$\overline{z^2} = 4a^2A^2B^2 \tag{12}$$

appears at the output of the square-law detector. The measuring circuit having a time constant τ will be charged at $t = \tau$ to 63 per cent of the amplitude of the unit step, while it will reach 86.5 per cent at $t = 2\tau$. Thus, if the measurement were finished at the moment T, the measuring circuit could not reach its stationary state. But waiting for a complete charging up ($t \gg \tau$, or at least $t > 2\tau$) would unnecessarily raise the amount of information gained of a random variable (noise voltage or noise power). The accuracy of the measurement will not be considerably higher, since the pointer fluctuates just the same in the vicinity of the mean, affected by the time constant.

There is, however, a method to eliminate this paradox: the measuring circuit must be transformed into a time-variable arrangement of increasing time constant. It will be shown that as a function of the time-dependence of the time constant the charging time may considerably be reduced, enabling the optimum utilization of the time duration by all means necessary for the measurement.

5. Time-variable indicator

According to Fig. 4 let the indicator, connected to the square-law rectifier, to consist of a time-variable resistor $R(t)$, a condenser of constant capacitance, and a pointer instrument without inertia and consumption. The time constant $R(0)C = \tau_0$ at the start must be considerably smaller than the time constant $R(T)C = \tau_\infty$. There is no general method to design such circuits containing passive, linear and time-variable components [6, 7].

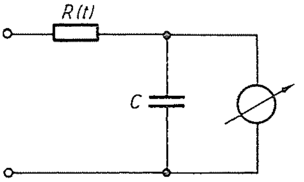

Fig. 4. Time-variable RC-integrator

Let

$$C(t) = C = \text{const},$$
$$R(t) = R_0 + \frac{t}{T} R_1 \quad \text{for} \quad 0 < t < T \tag{19}$$

and apply the unit step voltage of the form

$$u = U \cdot 1(t) \tag{20}$$

to the input.

The integro-differential equation satisfied by the current of the circuit is given by

$$i(t)\, R(t) + \frac{1}{C} \int i(t)\, dt = U \cdot 1(t) \tag{21}$$

Differentiating both sides of equation (21) in the range $t > 0$ we obtain

$$\frac{di}{dt} R(t) + i \frac{dR(t)}{dt} + \frac{i}{C} = 0 \tag{22}$$

Substitution of $R(t)$ from equation (19) yields

$$\frac{di}{dt} \left(R_0 + \frac{t}{T} R_1 \right) + i \frac{R_1}{T} + \frac{i}{C} = 0 \tag{23}$$

For simplicity, let us introduce the notations

$$\frac{R_1}{R_0} = a$$
$$R_0 C = \tau_0 \tag{24}$$
$$R_1 C = \tau_1 = a\tau_0$$

With notations (24) equation (23) may be written in the following form:

$$\frac{di}{dt} \tau_0 \left(1 + a \frac{t}{T} \right) + i \left(1 + \frac{a\tau_0}{T} \right) = 0 \tag{25}$$

Differential equation (25) can be separated:

$$\frac{di}{i} = - \frac{1 + a\tau_0/T}{1 + at/T} \, \frac{dt}{\tau_0} \tag{26}$$

The differential equation has the general solution

$$i = K\left(1 + a\frac{t}{T}\right)^{-\left(1+\frac{T}{a\tau_0}\right)} \tag{27}$$

The initial condition of the circuit is

$$i(0) = \frac{U}{R_0} = K \tag{28}$$

so that we get

$$i = \frac{U}{R_0}\left[1 + a\frac{t}{T}\right]^{-\left(1+\frac{T}{a\tau_0}\right)} \tag{29}$$

The *RC*-networks having time-variable and invariant components can be compared by determining the current through resistor R at $t = T$. In case of the time-variable network the condenser is charged to a higher extent, so that we obtain a smaller $i(T)$ than is usual in *RC*-networks. For $t \geqq T$ the circuits are equivalent.

Let the current flowing at the time T be denoted by $i(T) = I_1$ for the time-variable circuit and by $i'(T) = I_1'$ for the circuit with constant components. With these notations we obtain

$$I_1 = \frac{U}{R_0}(1 + a)^{-\left(1+\frac{T}{a\tau_0}\right)} \tag{30}$$

and

$$I_1' = \frac{U}{R_0 + R_1}e^{-\frac{T}{\tau_0+\tau_1}} \tag{31}$$

In equation (31) $R_0 + R_1$ and $\tau_0 + \tau_1$ have been substituted, because the two networks are equivalent at $t > T$ only in this case. The ratio of the currents amounts to

$$\frac{I_1}{I_1'} = (1 + a)^{-\frac{T}{a\tau_0}}e^{\frac{T}{(1+a)\tau_0}} \tag{32}$$

In order to obtain the extreme of I_1/I_1' it is necessary to determine that of the expression

$$\varphi(a) = -\frac{T}{a\tau_0}\ln(1 + a) + \frac{T}{(1 + a)\tau_0} \tag{33}$$

The extreme exists, since for $a \to 0$ the first term tends to $-\dfrac{aT}{a\tau_0}$, the second approaches T/τ_0, while for $a \to \infty$, $\varphi(a)$ vanishes. The locus of the extreme is given by the graphic solution of equation

$$\frac{\tau_0}{T}\frac{d\varphi(a)}{da} = \frac{1}{a^2}\ln(1+a) - \frac{1}{a(1+a)} - \frac{1}{(1+a)^2} = 0 \tag{34}$$

and is found to appear at $a = 2$. Inserting into equation (32) gives

$$\frac{I_1}{I_1'} = \exp\frac{T}{\tau_0}\left[\frac{1}{1+a} - \frac{\ln(1+a)}{a}\right] = \exp\left(-0.22\frac{T}{\tau_0}\right) \tag{35}$$

On grounds of chapter 4 we chose

$$\frac{\tau}{T} = \frac{\tau_0 + \tau_1}{T} = \frac{1}{2} \tag{36}$$

and since

$$\tau_1 = a\tau_0 = 2\tau_0 \tag{37}$$

we may write

$$\frac{I_1}{I_1'} = \exp(-0.22.6) = \frac{1}{3.65} \tag{38}$$

It is worth recalling that in the time invariant network a current of $I_1' = i'(0)e^{-2} = 0.135\ i(0)$ is flowing at the moment $t = 2(\tau_0 + \tau_1)$, so that the voltage across the condenser can reach but 86.5 per cent of the steady-state value. As a contrast, the time-variable network allows

$$I_1 = \frac{I_1'}{3.65} = \frac{0.135}{3.65}\, i'(0) = 0.037\, i'(0) \tag{39}$$

i.e. the steady-state value will be approached by 3.7 per cent.

The result obtained can also be put in another way: in the circuit of constant components a current of

$$I_1' = \frac{U}{R_0(1+a)} e^{-\frac{T}{(1+a)\tau_0}} = \frac{U}{3R_0} e^{-0.33\frac{T}{\tau_0}} \tag{40}$$

is flowing at the time T. The time-variable network "hurries" by $0.22\ T/\tau_0$ compared with the circuit with constant components, so that the time necessary to charge the condensers to the same extent will be shorter by a factor

$$\frac{0.33}{0.33 + 0.22} = 0.6 \tag{41}$$

Fig. 5 demonstrates both current curves. The vertical axis is normalized to the initial current $i'(0)$ of the time invariant network, while the horizontal one to the steady-state time constant $\tau_0 + \tau_1$.

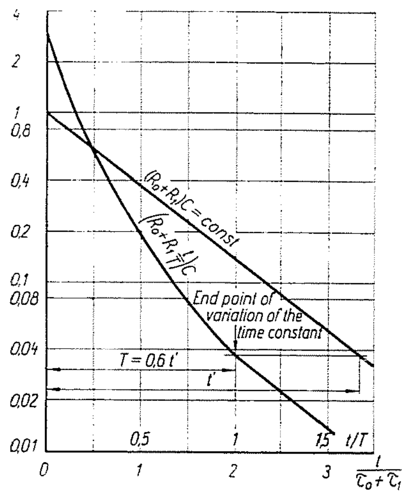

Fig. 5. Input current of integrators with invariant and time-varying parameters, respectively, as a function of time

So far t/T has been handled as a parameter which can be arbitrarily chosen. If we stipulate in advance

$$\frac{\tau}{T} = \frac{\tau_0 + \tau_1}{T} = \frac{1}{2} \tag{42}$$

the substitution $\tau_1 = a\tau_0$ yields

$$\frac{T}{\tau_0} = 2(1 + a) \tag{43}$$

Introducing equation (43) into (33) we obtain

$$\varphi(a) = -\frac{2(1 + a)}{a} \ln(1 + a) + 2 , \tag{44}$$

a function with no extreme.

Fig. 6 is a family of output voltage curves plotted with a as a parameter, computed by means of the equation

$$U_c(t) = U1(t) - i(t)\,R(t) =$$
$$= U - U\left(1 + a\frac{t}{T}\right)^{-\frac{T}{a\tau_0}} = U\left[1 - \left(1 + a\frac{t}{T}\right)^{-\frac{2(1+a)}{a}}\right] \tag{45}$$

It can be seen that no considerable improvement results at $a > 2$.

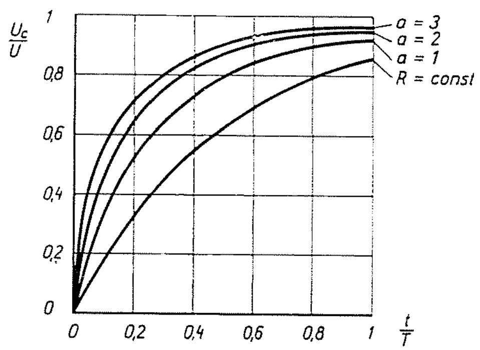

Fig. 6. Output voltage of an integrator with time-variable parameters as a function of time

Similar calculations can be carried out for time dependences other than linear [8]. If e.g.

$$R = R_0 + R_1(1 - e^{-t/T}), \tag{46}$$

the charging current amounts to

$$i = \frac{U}{R_0}\,\frac{e^{-\frac{t}{\tau_0}\frac{1}{1+a}}}{\left[1 + a\,(1 - e^{-t/T})\right]^{\left(\frac{T}{\tau_0+\tau_1}+1\right)}} \tag{47}$$

while a square-law time function

$$R = R_0 + R_1\left(\frac{t}{T}\right)^2 \tag{48}$$

gives

$$i = \frac{U}{R_0}\,\frac{\exp\left[-\frac{T}{\tau_0\sqrt{a}}\,\mathrm{arc\,tg}\left(\sqrt{a}\,\frac{t}{T}\right)\right]}{1 + a\left(\frac{t}{T}\right)^2} \tag{49}$$

6. Experimental results

The most simple integrator with time-variable parameters is shown in Fig. 7. The network consists of an indirectly heated thermistor. Before the measurement the switch is in the illustrated position, so that a small resis-

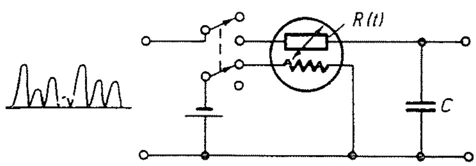

Fig. 7. Integrator with time-variable parameters, comprising an indirectly heated thermistor

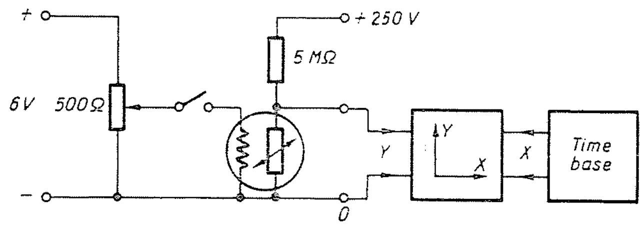

Fig. 8. Circuit for investigating the time dependence of thermistor resistance

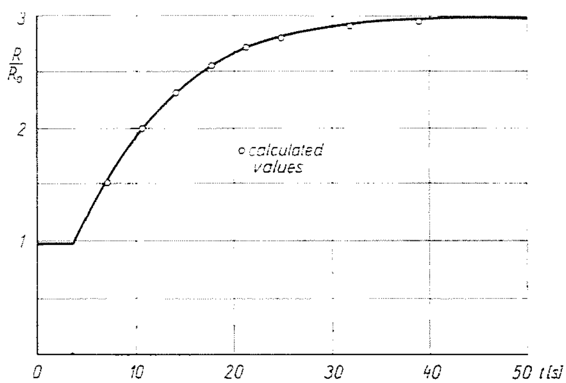

Fig. 9. The resistance of the thermistor as a function of time after switching off the heating

tance is realized by warming up the thermistor. Switching off the heating at $t = 0$ the noise voltage, rectified linearly or according to square-law, is applied to the input of the fourpole. The resistance of the cooling thermistor, as well as the time constant of the integrator, is now gradually increasing.

First the function $R(t)$ has been determined by means of the circuit shown in Fig. 8.

It was established during the preliminary investigations that the thermistor in the evacuated bulb does not meet our demands in its original state, as it has a very long time constant (of some minutes). Therefore, the tip of the bulb was broken and we allowed air to leak into the system, considerably reducing the time constant. To protect the thermistor against uncontrollable air flows, the bulb was not removed.

The curves obtained experimentally are shown in Fig. 9. The abscissas denote the time in seconds, while the resistance values are normalized to R_0 (i.e. the resistance to be measured at $t = 0$). This plot is, of course, not exponential, because notwithstanding the exponential drop in the thermistor temperature subsequent to switching off its heating, the resistance is not a linear function of temperature:

$$R = R' e^{B_t/\Theta} \tag{50}$$

where B_t denotes a constant depending on the material of the thermistor, and Θ designates the temperature. Let Θ_1 stand for the ambient temperature and $\Delta\Theta$ the temperature rise due to heating. Thus

$$R(t) = R' \exp \frac{B_t}{\Theta_1 + \Delta\Theta e^{-t/T}} \approx R' \exp \frac{B_t}{\Theta_1} \left[1 - \frac{\Delta\Theta}{\Theta_1} e^{-t/T}\right] \tag{51}$$

If we consider that at $t = 0$ the resistance of the thermistor amounts to

$$R_0 = R' \exp \frac{B_t}{\Theta_1 + \Delta\Theta_1} \approx R' \exp \frac{B_t}{\Theta_1} \left[1 - \frac{\Delta\Theta}{\Theta_1}\right] \tag{52}$$

and the resistance of the cold thermistor is given by

$$R_\infty = R' \exp \left[\frac{B_t}{\Theta_1}\right] \tag{53}$$

it follows that

$$\frac{R_\infty}{R_0} \cong \exp \left[\frac{B_t}{\Theta_1} \frac{\Delta\Theta}{\Theta_1}\right] \tag{54}$$

Introduction of equation (54) in (51) yields

$$\frac{R(t)}{R_0} = \left(\frac{R_\infty}{R_0}\right)^{(1-e^{-t/T})} \tag{55}$$

The values computed from equation (55) are also illustrated in Fig. 9.

The next experiment was carried out with the circuit of Fig. 7. Applying the unit step voltage to the input of the RC-network we simultaneously switched off the heating of the thermistor. Fig. 10 shows the obtained output voltages with the aid of a line recorder. To facilitate comparison the curves for integrators, comprising the constant resistors R_0 and R_∞ respectively, are also plotted. It can be seen that the output voltage of the integrator with time-varying parameters increases initially with a slope, corresponding to R_0.

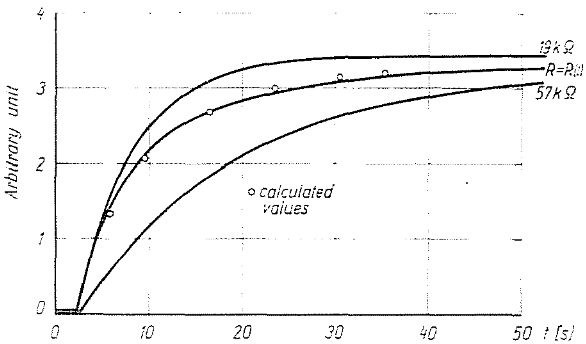

Fig. 10. Output voltage of the circuit shown in Fig. 7, stimulated by a unit step

The curve can also be checked by means of computation. Substituting relation (55) into the differential equation (22) we obtain — omitting the details —

$$i = \frac{U}{R_0}\,\varrho^{(e^{-x}-1)}\,\varrho^{\left[\frac{\overline{Ei}(\lambda e^{-x})-Ei(\lambda)}{x_0\varrho}\right]} \tag{56}$$

where

$$\varrho = \frac{R_\infty}{R_0}, \qquad \lambda = \ln \varrho\,, \qquad x = \frac{t}{T}$$

and

$$\overline{Ei}\,(\xi) = \int_{-\infty}^{\xi} \frac{e^{\eta}}{\eta}\,d\eta \qquad \text{for} \qquad \xi > 0 \tag{57}$$

the range of the integral exponential function, defined for positive independent variables. The values obtained from equation (56) are given in Fig. 10.

Figs. 11 and 12 show the cathode-ray oscilloscope photographs of the response functions, produced by a noise input. For control purposes we plotted the charge process of a time invariant integrator at the same time.

The two curves were shown on the screen of a double-beam oscilloscope with a scanning cycle of some 50 seconds.

On the photographic record it can readily be observed that both integrators are equivalent from the time the steady state prevails; they respond in the same manner to noise stimuli. The output voltage of the time-variable

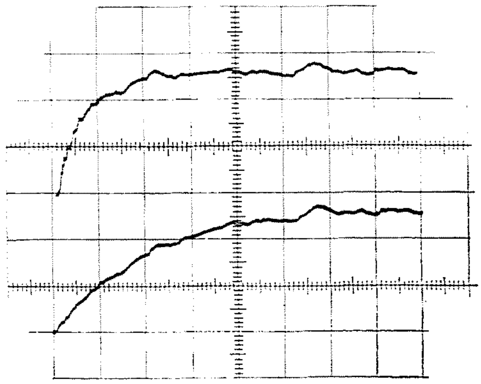

Fig. 11. Output voltage of an integrator with time-variable parameters (above) and steady state equivalent integrator with constant parameters (below), in case of noise input

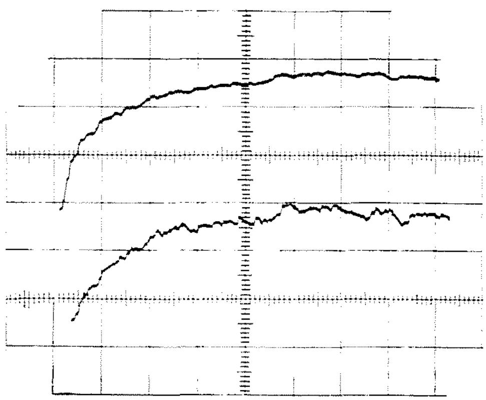

Fig. 12. Output voltage of an integrator with time-variable parameters (above) and an initially equivalent integrator with invariant parameters (below), in case of noise input

integrator (above) rises at the beginning more rapidly than that of the circuit with constant parameters (below). Fig. 12 serves as an illustration for the other comparison: the two circuits are initially equivalent, but reaching the steady-state the ratio of their time constant equals 3 : 1. This can easily be seen from the extent of the fluctuations.

The curves now and then also show a falling character, which of course results from the random nature of the input voltage.

7. Suggested circuit

The two poles of time-varying resistance based on thermal effect (e.g. thermistor, incandescent lamp) are not favourable for two reasons. If their heating-up is performed by the noise to be measured, the time constant of the integrator will depend on the level of the input. To heat the two-pole with an auxiliary current would, however, cause considerable difficulties, as it is very complicated to separate heater current and noise signal. On the other hand, the realizable time constant depends on the thermal time constant, so that it cannot be chosen freely. In all circuits described previously the circuit parameters are determined by the features of the thermistor.

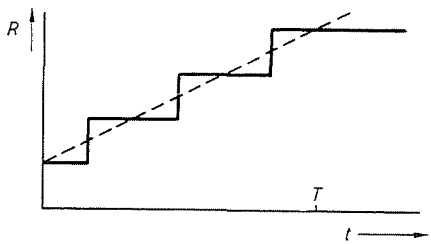

Fig. 13. Stepped charging resistor

In the following an integrating network easy to design and realize will be outlined. The variation of resistance is stepped according to Fig. 13. The specified (linear, exponential or arbitrary) variation in resistance can be approximated to any accuracy by increasing the number of steps. Luckily the accuracy of approximation is not of primary importance in this kind of application. However, it can be quantitatively seen that the higher the relative change in resistance, the more steps are needed. Remember that the time constant amounts initially to $\tau_0 = R_0 C$, resulting in a relative fluctuation

$$\frac{\sigma_z}{\sqrt{\overline{z^2}}} = \frac{1}{\sqrt{2B\tau_0}} \tag{16}$$

Let us now investigate the extreme case of one step. If immediately before switching from low time constant to high a considerable difference exists between instantaneous value and mean of the squared input noise voltage, the output voltage of the integrator with low time constant equally deviates to a great extent from the mean. The difference will be compensated but slowly after switching to a high time constant. In case of more steps, however, the fluctuations shrink continuously down to the value corresponding to the final time constant.

In order to check the integrator with stepped time constant the circuit presented in Fig. 14 has been developed.

At the beginning of the measurement switch K is opened. The condenser C_1 discharges through resistance R_7 (and the base circuit of transistor T_4) on the whole exponentially. The emitter of transistor T_4 becomes more and

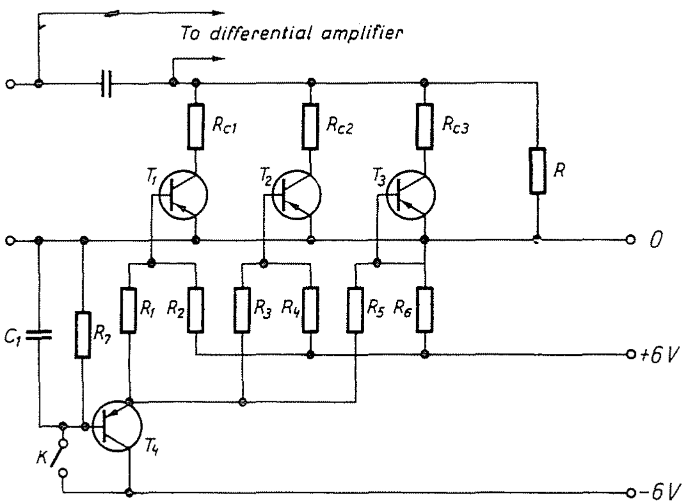

Fig. 14. Circuit to realize the resistance function shown in Fig. 13

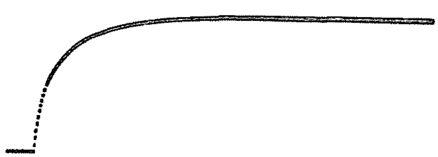

Fig. 15. Output voltage of the circuit given in Fig. 14 in case of unit step input

more positive, so that the originally saturated transistors gradually cut off and disconnect the resistors $R_{C1} \ldots R_{C3}$ in parallel to R.

Fig. 15 shows the cathode-ray oscilloscope photograph of the output voltage, generated by a unit step.

Summary

Narrow-band or low-frequency noise measurements are made difficult primarily by the length of time which is required. Owing to the random character of noise the time of measurement cannot be decreased arbitrarily. In this paper the author shows that the direct-reading noise measuring instruments, equipped with an integrating RC-network, utilize but poorly the time of measurement, causing an additional rise in time requirement. There is, however, a method for optimum efficiency: if an integrating RC-network with time-variable components

is applied, resulting in an increasing time constant, the transient response can be considerably shortened. The paper is intended to give a detailed picture of the integrating RC-network with linearly varying time constant and to derive formulas throwing some light on RC-networks with time constants other than linear time dependence. As illustration two practical circuits are presented.

References

1. Blackman, R. B.—Tukey, J. W.: The measurement of power spectra from the point of view of communication engineering. BSTJ **37**, 185—282, 485—570 (1958).
2. Slepian, D.: Fluctuations of random noise power. BSTJ. **37**, 163—184 (1958).
3. Crandall, S. H.: Random vibration. MIT, Chapman and Hall, John Wiley, 1958.
4. Schweizer, G.: Untersuchungen der auftretenden Fehler bei der Auswertung von statistischen Signalen. AEÜ **16**, 235—244 (1962).
5. Davenport, W. B.—Root, W. L.: An introduction to the theory of random signals and noise. McGraw—Hill, New York 1958.
6. Pipes, L. A.: Four methods for the analysis of time-variable circuits. IRE Trans. CT—**2**, 4—12 (1955).
7. Дорман, М. И.: Прохождение белого шума через некоторые цепи со скачкообразным изменением параметра. Радиотехника, **19**, 17—27 (1964).
8. Ambrózy, A.: Félvezető eszközök kisfrekvenciás zajmérésének néhány problémája. Kandidátusi disszertáció, Budapest, 1964.
9. Ambrózy A:. Reducing the time requirement in direct reading VLF noise measurement, Proc. IEEE **53**, 1161—1162 (1965).

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