# CALCULATION OF THE ZERO-SEQUENCE CURRENT DISTRIBUTION ALONG TRANSMISSION LINES

By

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(Received August 11, 1965)

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## 1. Description of the problem, definition of concepts

When in a network with effectively grounded neutral a ground fault (e.g. flashover of an insulator string) occurs, a zero-sequence current will flow through the fault, the circuit being completed by the ground return path and, if there are such present, also by the ground wires. The *distribution* of this zero-sequence current among ground wires and ground return path shows a *changing pattern* in the spans of the line sections in the vicinity of the fault and the feeding point (Fig. 1).

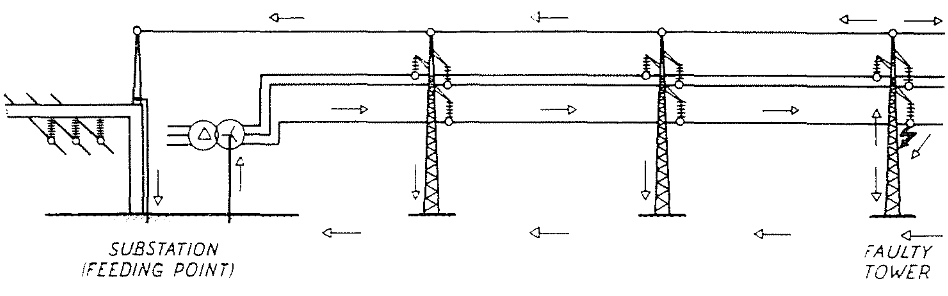



*Fig. 1*

In the 120—220 kV national grid the most frequently occuring faults are the phase-to-ground ones. Depending on the actual conditions of supply, a ground fault may be *fed either from one side or from both sides*. In any case, the fault current will enter the tower, where the fault occured, and one part of this current will flow through the tower into the ground and the other part into the ground wires, but in such a way that the ground wires will carry a portion of that latter part *in either direction*. In case of insulator-string flashovers the resistance between phase conductor and ground wires is equal to the impedance of the arc, while between phase conductor and ground there is a considerably higher impedance, viz. that of the tower footing. Hence, the zero-sequence current distribution in the ground wires shows a pattern in the surroundings of the fault according to which the current carried by the ground wires varies in each subsequent span, and is higher than that flowing in the spans lying sufficiently far away from the fault, where the value of current ceases to change from span to span. This final, unchanging value is called *steady-state value of current along the line.* Receding from the fault the current flowing in the ground wires is gradually reduced to this steady-state value.

The current flowing into the ground through any tower of the faulty line is equal to the difference of currents flowing in the ground wires of adjacent spans and is called *tower current.* The line sections in the vicinity of the fault and feeding point, respectively, in which the magnitude of current flowing in the ground wires varies from span to span, is called *the section with end-effect.* Accordingly, two kinds of end-effect may be distinguished, *viz. that in the surroundings of the fault and that near the feeding point.* The current distribution developing along the section with end-effect is termed *end-effect current distribution.*

The steady-state current along the line develops – *per definitionem* – in that section of an overhead line where the value of tower currents decays to zero. In connection with this steady-state current it should be noted that if a phase-to-ground fault occurs on an overhead line disconnected from one end, i.e. the fault is supplied from one side only, the magnitude of the steady state current flowing in the ground wire in the line section extending beyond the fault (in the direction opposite the supply point) is *zero.* This is not the case with faults supplied from both sides. The Bauch phenomenon brings about the condition equivalent to a supply from both ends.

## 2. Necessity of solving the questions of end-effect current distribution; problems encountered in the calculations

The knowledge of the principles to be followed in finding the end-effect current distribution is a necessity for the following practical reasons:

*a)* Due to the ever increasing short-circuit ratings, the phase-to-ground fault currents of networks with effectively grounded neutrals will further increase in the future. With respect to the electromagnetic interference affecting telecommunication lines, the end-effect is a favourable phenomenon. The knowledge of end-effect current distribution is indispensable in interference calculations.

*b) The potential rise* of substation grounding grids and tower groundings can only be determined if the end-effect current distribution is known.

*c)* The *thermal stresses* imposed on ground wires can be computed on the basis of end-effect current distribution.

The *factors influencing* the end-effect current distribution are as follows:

*a)* number, material, dimensions and arrangement of ground wires,

*b)* saturation, in the case of steel ground wires,

*c)* value of soil resistivity,

*d)* value of tower footing resistances,

*e)* distance between feeding point and fault,

*f)* circuit arrangement of the zero-sequence network, conditions of supply (feeding from one side or both sides).

Considering the factors enumerated above, the end-effect current distribution may be followed in *several ways:*

*a)* by performing field measurements,

*b)* carrying out investigations on an a.c. network analyser,

*c)* by means of calculations.

Based on the rather extensive related literature, to find by means of calculations the zero-sequence current distribution the main task to be solved is to develop a *calculation method suitable for generalization*. The method must take into consideration the resistive and inductive components of the self- and mutual impedances, the factors characterizing the fault and supply conditions, the variation of distance between feeding point and fault, the configuration of the network, etc.

In the present paper a *new calculation method* based on *matrix calculus* is described. The method fully satisfies the above outlined requirements. The method lends itself to digital computer processing.

As regards the other means of determining the zero-sequence current distribution, the performance of field measurements is always a suitable means of completing the picture furnished by the calculations. Nevertheless, due to the varying local conditions, it could be found difficult to generalize the results obtained from singular measurements. A further drawback of measurements lies in the time-consuming and rather costly field preparations, and the need of disconnecting the line to be investigated. Thus, it is clear that a *general investigation* of end-effect current distribution *cannot be performed only by means of measurements made on overhead transmission lines*. A description of field measurements performed by the author is to be found in the literature [23].

The use of a digital computer makes the application of an a.c. network analyser *dispensable*. Generally, the following aspects are against the use of an a.c. network analyser: inevitable inaccuracies in the settings and readings; time consumption of measurements, insufficiency of available elements in the network analyser and, in the case of provisional setups, the uncontrollable contact resistances.

## 3. Assumptions adopted for the calculations

Our investigations (calculation of basic data, setting up of equivalent circuits, development of calculation methods) have been based on the following assumptions:

*a)* the frequency of power transmission has been taken as 50 c/s,

*b)* no transient phenomena have been taken into consideration,

*c)* the calculation method applies to a single-circuit three-phase overhead transmission line,

*d)* between supply points and fault a symmetrically transposed line has been assumed,

*e)* line capacitances have been neglected,

*f)* series impedances have been considered as lumped elements,

*g)* zero-sequence self- and mutual impedances have been computed by means of the simplified Carson-Clem formulae,

*h)* the impedance of the arc has been brought about by a flashover of insulator, strings has been neglected,

*i)* dependence of steel-ground-wire impedance on the magnitude of current load has been left out of consideration, to avoid cumbersome successive approximation procedure,

*j)* homogenous soil resistivity and tower-footing resistance values and their independence of time and site have been assumed.

## 4. Setting up of the basic zero-sequence equivalent circuit

The phase-conductor/ground-wire system chosen as a setting-out arrangement and consisting of two parallel conductors with a ground return path is shown in Fig. 2, where the positive directions of current flow are also indicated. The following voltage equations may be written:

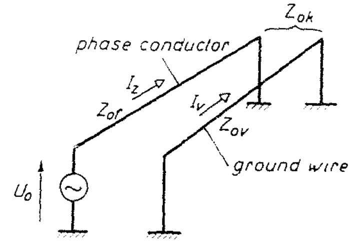



*Fig. 2*

for the loop of phase conductor/ground:

$$U_0 = I_z Z_{0f} + I_v Z_{0k} \qquad (1)$$

for the loop of ground wire/ground:

$$0 = I_z Z_{0k} + I_v Z_{0v} . \tag{2}$$

CLARKE [5] has given an equivalent circuit shown in Fig. 3, representing *one span* of a line, where the impedance $Z_{0g}$ of the ground-return path is taken into account, as an auxiliary quantity. The tower-footing resistance ($R_0$) also

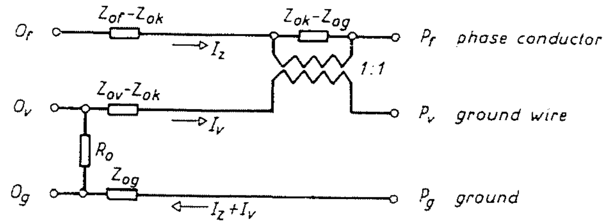



*Fig. 3*

appears in the equivalent network. The self-impedances given in the equivalent circuit represent the respective quantities of the ground-return path of one span. In the case of *zero-sequence* quantities, instead of $R_0$, the value $3\,R_0$ appears in the circuit diagram.

Considering the equivalent circuit of Fig. 3, and applying a short-circuit, first, across terminals $P_f$, $P_v$, $P_g$ and, then, across terminals $O_v$ and $O_g$, and inserting voltage $U_0$ across $O_f$ and $O_g$, and neglecting $R_0$, the following voltage equations can be written:

phase-conductor/ground:

$$U_0 = I_z (Z_{0f} - Z_{0k}) + (I_z + I_v)(Z_{0k} - Z_{0g}) + (I_z + I_v) Z_{0g} \tag{3}$$

ground-wire/ground:

$$0 = I_v (Z_{0v} - Z_{0k}) + (I_z + I_v)(Z_{0k} - Z_{0g}) + (I_z + I_v) Z_{0g} . \tag{4}$$

After performing the necessary operations and simplifications, Eqs. 3 and 4 are reduced to:

$$U_0 = I_z Z_{0f} + I_v Z_{0k} \tag{5}$$

$$0 = I_z Z_{0k} + I_v Z_{0v} \tag{6}$$

i. e. the relations thus obtained are identical to those given under 1 and 2.

For the sake of simplicity *the 1 : 1 ratio, ideal coupling transformer has been omitted* from the equivalent circuit of Fig. 3. Without presenting the detailed calculations, the equivalent circuit thus obtained, *consisting exclusively of selfimpedances*, is shown in Fig. 4 (simplifying the notations: $Z = Z_{0k} - Z_{0g}$).

The network short-circuited as detailed above must be simplified for the purpose of writing the voltage equations. The delta arrangement of impedances obtained by short-circuiting are, first, transformed into a star. Then, after lumping the impedances in each branch and omitting $R_0$, the voltage equations analogous to those obtained further above can be written, from which relations 1 and 2 can again be arrived at.

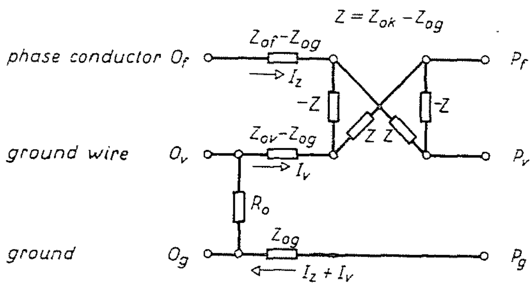



*Fig. 4*

After checking by means of the voltage equations, as well as by considering the identity of the driving point and transfer impedances, it can be stated that the equivalent networks of Figs. 3 and 4 are basically identical. In the following, the circuit diagram shown in Fig. 4 and containing self-impedances only, will be called *basic ground-return (zero-sequence) equivalent circuit of one span.*

## 5. Determination of basic network equations by means of topological methods

The basic zero-sequence equivalent circuit of one span is, according to the network theory, a *six-terminal network* with terminals $O_f O_v O_g$, and $P_f P_v P_g$. Now, the relations have to be found by means of which the currents and voltages of the six-terminal network become computable.

The six-terminal network to be solved is shown in Fig. 5. The impedances, as well as the currents and voltages, together with their respective positive directions are also indicated. Each node is marked for identification. The network diagram of the six-terminal circuit is represented in Fig. 6.

Considering the 4 potential sources, this network is built up of 12 branches ($B=12$). The number of nodes is 8 ($N=8$). As regards topology, the

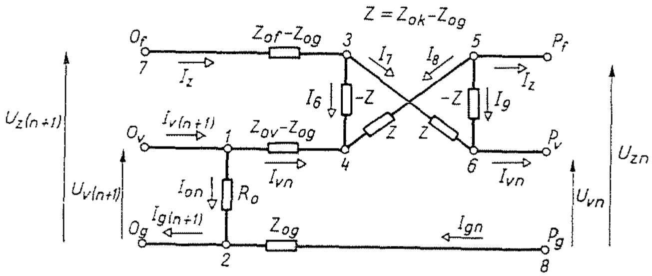



*Fig. 5*

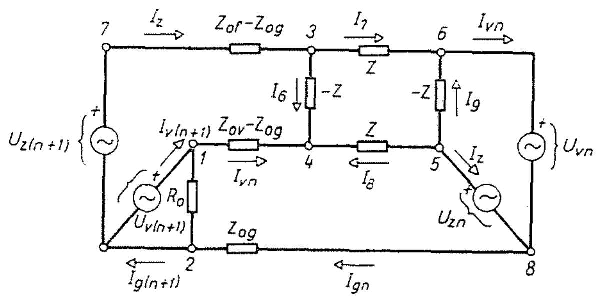
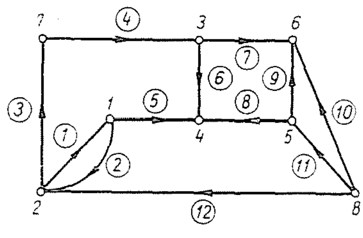



*Fig. 6*

circuit is a *complex* network, consisting of several *simplex* elements. The configuration of nodes and branches can be described by a *linear graph* (Fig. 7). When constructing the graph of the network, the potential sources in branches



*Fiq. 7*

1, 3, 10 and 11 have been short-circuited. The number of independent loops ($L$) can be calculated in the following way:

$$L = B - N + 1 = 12 - 8 + 1 = 5\,.$$

The equations referring to the independent loops can be set up by means of the $M$— or *tie-set matrix*. For that purpose one *tree* of the graph of the network must be found. This tree is a *subgraph* of the network's graph with all the nodes included, without producing a closed loop. The tree chosen from the many possible trees is represented in Fig. 8. The number of branches of the tree is $B-L = 12-5 = 7$, the number of links, i. e. that of the branches not belonging to the tree is $L = 5$.

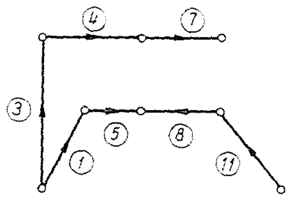



*Fig. 8*

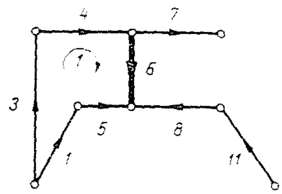



*Fig. 9*

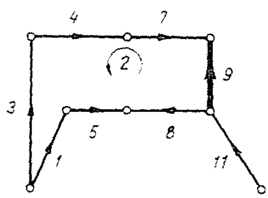



*Fig. 10*

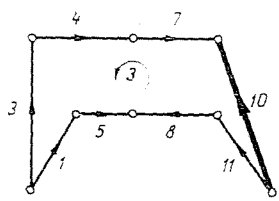



*Fig. 11*

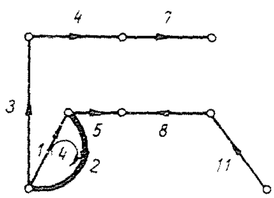



*Fig. 12*

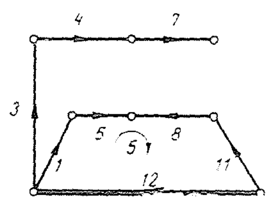



*Fig. 13*

To build up the tie-set matrix the 5 links of the given tree have to be coupled to the tree. Thus, five tie-sets are obtained, and the directions of loop currents flowing in these tie-sets are taken as equal to the respective directions of currents flowing in the links. According to the directions of branch-currents, the matrix elements can be either $+1$, or $-1$, or is 0, if the branch considered does not belong to the loop investigated.

The insertion of the 5 links can be followed by considering Figs. 9 to 13. Using the notations of the figures, matrix **M** takes the following form:

| loop currents | branch currents 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | −1 | 0 | +1 | +1 | −1 | +1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | +1 | 0 | −1 | −1 | +1 | 0 | −1 | −1 | +1 | 0 | 0 | 0 |
| 3 | +1 | 0 | −1 | −1 | +1 | 0 | −1 | −1 | 0 | +1 | −1 | 0 |
| 4 | +1 | +1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | +1 | 0 | 0 | 0 | +1 | 0 | 0 | −1 | 0 | 0 | −1 | +1 |

Denoting the column vector of the branch currents by $\mathbf{I}$, the transpose of $\mathbf{M}$ by $\mathbf{M}_t$, and the column vector of loop currents by $\mathbf{I}_h$, we can write

$$\mathbf{I} = \mathbf{M}_t \cdot \mathbf{I}_h \tag{7}$$

The impedances of network branches can be written into the form of a *diagonal matrix:*

$$\mathbf{Z} = \begin{bmatrix} Z_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{12} \end{bmatrix}.$$

The *matrix of sources*, which contains the voltages and currents takes the following form:

$$\mathbf{E}_f = \begin{bmatrix} E_{f1} \\ 0 \\ E_{f3} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E_{f10} \\ E_{f11} \\ 0 \end{bmatrix} \qquad \mathbf{I}_f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0\,.$$

The quantities appearing in matrices $\mathbf{M}$, $\mathbf{Z}$ and $\mathbf{E}_f$ are as given below, using the notations of Fig. 6:

$$
\begin{array}{llllll}
I_1 = I_{v(n+1)} & & Z_1 = 0 & & E_{f1} = U_{v(n+1)} & \\
I_2 = I_{0n} & & Z_2 = R_0 & & & \\
I_3 = I_z & & Z_3 = 0 & & E_{f3} = U_{z(n+1)} & \\
I_4 = I_z & & Z_4 = Z_{0f} - Z_{0g} & & & \\
I_5 = I_{vn} & & Z_5 = Z_{0v} - Z_{0g} & & & \\
I_6 = I_6 & & Z_6 = -Z & & & \\
I_7 = I_7 & & Z_7 = Z & & & \\
I_8 = I_8 & & Z_8 = Z & & & \qquad (8) \\
I_9 = I_9 & & Z_9 = -Z & & & \\
I_{10} = -I_{vn} & & Z_{10} = 0 & & E_{f10} = U_{vn} & \\
I_{11} = -I_z & & Z_{11} = 0 & & E_{f11} = U_{zn} & \\
I_{12} = I_{gn} & & Z_{12} = Z_{0g} & & & \\
 & \text{and} & Z = Z_{0k} - Z_{0g} & & &
\end{array}
$$

It is known that

$$\mathbf{E}_f + \mathbf{U} = \mathbf{Z}\,(\mathbf{I} + \mathbf{I}_f)$$

from which

$$\mathbf{M}.\mathbf{Z}.\mathbf{I} = \mathbf{M}\,(\mathbf{E}_f - \mathbf{Z}.\mathbf{I}_f) \tag{9}$$

Expressing $\mathbf{I}_v$ from Eq. 7 with $\mathbf{I}_f = 0$, we obtain

$$\mathbf{M}.\mathbf{Z}.\mathbf{M}_t.\mathbf{I}_h = \mathbf{M}.\mathbf{E}_f \tag{10}$$

The loop equations can be derived by expressing matrix equation 10. The steps are as follows:

$$
\mathbf{M} \cdot \mathbf{Z} = \begin{bmatrix}
0 & 0 & 0 & Z_4 & -Z_5 & Z_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -Z_4 & Z_5 & 0 & -Z_7 & -Z_8 & Z_9 & 0 & 0 & 0 \\
0 & 0 & 0 & -Z_4 & Z_5 & 0 & -Z_7 & -Z_8 & 0 & 0 & 0 & 0 \\
0 & Z_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Z_5 & 0 & 0 & -Z_8 & 0 & 0 & 0 & Z_{12}
\end{bmatrix}
$$

$$
\mathbf{M} \cdot \mathbf{Z} \cdot \mathbf{M}_t =
$$

$$
= \begin{bmatrix}
Z_4 + Z_5 + Z_6 & -Z_4 - Z_5 & -Z_4 - Z_5 & 0 & -Z_5 \\
-Z_4 - Z_5 & Z_4 + Z_5 + Z_7 + Z_8 + Z_9 & Z_4 + Z_5 + Z_7 + Z_8 & 0 & Z_5 + Z_8 \\
-Z_4 - Z_5 & Z_4 + Z_5 + Z_7 + Z_8 & Z_4 + Z_5 + Z_7 + Z_8 & 0 & Z_5 + Z_8 \\
0 & 0 & 0 & Z_2 & 0 \\
-Z_5 & Z_5 + Z_8 & Z_5 + Z_8 & 0 & Z_5 + Z_8 + Z_{12}
\end{bmatrix}.
$$

The diagonal elements of matrix $\mathbf{M} \cdot \mathbf{Z} \cdot \mathbf{M}_t$ are represented by the loop impedances of the tie-sets. The other elements correspond to the mutual impedances of the loops, their signs depending on the relative direction of the loops.

The right side of matrix equation 10 is:

$$\mathbf{M} \cdot \mathbf{E}_f \begin{bmatrix} -E_{f1} + E_{f3} \\ E_{f1} - E_{f3} \\ E_{f1} - E_{f3} + E_{f10} - E_{f11} \\ E_{f1} \\ E_{f1} - E_{f11} \end{bmatrix}.$$

The complete form of the matrix equation can be found in Appendix 2, with the substitution of quantities summarized under 8. The five equations obtained after development of the matrix and the eight available node equations are also given in Appendix 2. These thirteen equations, together with equation $I_z = 1.0 + j0.0$ as fourteenth, are sufficient for the determination of the following fourteen unknown quantities: $U_{z(n+1)}$, $U_{zn}$, $U_{v(n+1)}$, $U_{vn}$, $I_z$, $I_{v(n+1)}$, $I_{vn}$, $I_{g(n+1)}$, $I_{gn}$, $I_{0n}$, $I_6$, $I_7$, $I_8$, $I_9$. By choosing for $I_z$ the value given, all currents are obtained as per-unit *complex values* referred to the pure real reference quantity of $I_z$.

Otherwise, based on Figs. 9 to 13, for the network shown in Fig. 6, the following *five independent loop equations* may be written:

Loop 1:

$$U_{z(n+1)} - I_z(Z_{0f} - Z_{0g}) - I_6(-Z) + I_{vn}(Z_{0v} - Z_{0g}) - U_{v(n+1)} = 0$$

Loop 2:

$$U_{v(n+1)} - I_{vn}(Z_{0v} - Z_{0g}) + I_8 Z - I_9(-Z) + I_7 Z + I_z(Z_{0f} - Z_{0g}) - U_{z(n+1)} = 0$$

Loop 3:

$$U_{v(n+1)} - I_{vn}(Z_{0v} - Z_{0g}) + I_8 Z - U_{zn} + U_{vn} + I_7 Z + I_z(Z_{0f} - Z_{0g}) - U_{z(n+1)} = 0$$

Loop 4:

$$U_{v(n+1)} - I_{0n} R_0 = 0$$

Loop 5:

$$U_{v(n+1)} - I_{vn}(Z_{0v} - Z_{0g}) + I_8 Z - U_{zn} - I_{gn} Z_{0g} = 0$$

In the course of solving the 14 simultaneous equations (Appendix 2) in 14 unknowns a stage is arrived at, in which the equations only contain $U_{z(n+1)}$, $U_{zn}$, $U_{v(n+1)}$, $U_{vn}$, $I_z$, $I_{v(n+1)}$ and $I_{vn}$. This stage consists of the following equations:

$$U_{z(n+1)} - I_z Z_{0f} - I_{vn} Z_{0k} - U_{zn} = 0 \tag{11}$$

$$U_{v(n+1)} - I_z Z_{0k} - I_{vn} Z_{0v} - U_{vn} = 0 \tag{12}$$

$$I_{v(n+1)} - I_z \frac{Z_{0k}}{R_0} - I_{vn}\left(1 + \frac{Z_{0v}}{R_0}\right) - \frac{U_{vn}}{R_0} = 0 \tag{13}$$

and

$$U_{zn} + I_z Z_{0f} + I_{v(n+1)} Z_{0k} - \frac{U_{v(n+1)}}{R_0} Z_{0k} - U_{z(n+1)} = 0 \tag{14}$$

$$U_{vn} + I_z Z_{0k} + I_{v(n+1)} Z_{0v} - \frac{U_{v(n+1)}}{R_0} Z_{0v} - U_{v(n+1)} = 0 \tag{15}$$

$$I_{vn} - I_{v(n+1)} + \frac{U_{v(n+1)}}{R_0} = 0 \tag{16}$$

## 6. Matrices containing the generalized constants of the six-terminal network

Relations 11 to 13 can be rewritten into a form to permit the left-side quantities $U_{z(n+1)}$, $I_z$, $U_{v(n+1)}$ and $I_{v(n+1)}$ of the six-terminal network to be expressed by the right-side quantities $U_z$, $I_z$, $U_v$ and $I_v$ of the six-terminal network:

$$\begin{bmatrix} U_{z(n+1)} \\ I_z \\ U_{v(n+1)} \\ I_{v(n+1)} \end{bmatrix} = \begin{bmatrix} 1 & Z_{0f} & 0 & Z_{0k} \\ 0 & 1 & 0 & 0 \\ 0 & Z_{0k} & 1 & Z_{0v} \\ 0 & \dfrac{Z_{0k}}{R_0} & \dfrac{1}{R_0} & 1 + \dfrac{Z_{0v}}{R_0} \end{bmatrix} \begin{bmatrix} U_{zn} \\ I_z \\ U_{vn} \\ I_{vn} \end{bmatrix}$$

or, denoting the column vector of the left-side quantities by **B**, that of the right-side quantities by **J**, and the matrix of the generalized constants of the calculation from the right to the left by **A**:

$$\mathbf{B} = \mathbf{A} \cdot \mathbf{J} \tag{17}$$

Relations 14 to 16 may, in an analogous way, be rewritten into a form to permit the right-side quantities of the six-terminal network to be expressed by the left-side quantities:

$$\begin{bmatrix} U_{zn} \\ I_z \\ U_{vn} \\ I_{vn} \end{bmatrix} \begin{bmatrix} 1 & -Z_{0f} & \dfrac{Z_{0k}}{R_0} & -Z_{0k} \\ 0 & 1 & 0 & 0 \\ 0 & -Z_{0k} & 1 + \dfrac{Z_{0v}}{R_0} & -Z_{0v} \\ 0 & 0 & -\dfrac{1}{R_0} & 1 \end{bmatrix} \begin{bmatrix} U_{z(n+1)} \\ I_z \\ U_{v(n+1} \\ I_{v(n+1)} \end{bmatrix}$$

or, introducing, in addition to the notations used so far, symbol $\mathbf{C}$ for denoting the matrix of constants of the calculation from left to right:

$$\mathbf{J} = \mathbf{C} \cdot \mathbf{B} \tag{18}$$

$\mathbf{A}$ and $\mathbf{C}$ are *inverse matrices*, i.e.

$$\mathbf{A} = \mathbf{C}^{-1} \quad \text{and} \quad \mathbf{C} = \mathbf{A}^{-1} \tag{19}$$

A checking of relations under 19 has shown that $\mathbf{A}$ and $\mathbf{C}$ are truly inverse matrices of each other.

## 7. Elaboration of the calculation method for the case of fault supplied from one side

By means of matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{J}$ dealt with in Chapter 6, the voltage and current values for the right and left or intermediate terminals of $n$, six-terminal networks in cascade can be calculated. Now, let the method be applied to the case of an overhead-line fault supplied *from one side only.*

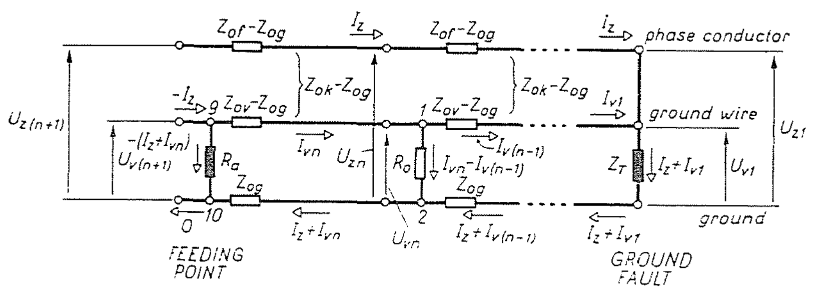



*Fig. 14*

The calculation method is described on the basis of Fig. 14, which is the equivalent circuit of an overhead-line section consisting of $n$ spans and confined by the feeding point and the fault. The phase conductor, ground wire and ground-return path, the tower groundings, as well as the mutual impedance between phase-conductor and ground-wire are shown in the figure. The values deviating from the elements of the basic equivalent circuit of one span are indicated as full quadrangles, such as the spreading resistance ($R_a$) of the grounding grid installed at the supplying substation, as well as the *resultant* ($Z_T$) of the footing resistance of the faulty tower and the resultant impedance of the ladder network extending beyond the fault. This ladder network consists of the ground-wire sections and tower-footing resistances. Fig. 14 is otherwise an equivalent circuit built-up of ground-return quantities.

The boundary conditions are as follows (Fig. 14): on the side of the fault, i.e. on the right side of the first span, the phase conductor and ground wire are connected, due to the phase-to-ground fault, thus $U_{z1} = U_{v1}$, further, the ground wire and ground-return path are connected through impedance $Z_T$.

Hence, column vector $\mathbf{J}_1$ of the right-side quantities will be

$$\mathbf{J}_1 = \begin{bmatrix} (I_z + I_{v1}) Z_T \\ I_z \\ (I_z + I_{v1}) Z_T \\ I_{v1} \end{bmatrix} \tag{20}$$

On the side of feeding, i.e. on the left side of the $n$th span, the ground wire and ground-return path are connected through $R_a$. In matrices $\mathbf{A}_n$ and $\mathbf{C}_n$ referring to the $n$th span $R_0$ *is replaced by* $R_a$, whereas column vector $\mathbf{B}_n$ will be

$$\mathbf{B}_n = \begin{bmatrix} U_{z(n+1)} \\ I_z \\ -(I_z + I_{vn}) R_a \\ -I_z \end{bmatrix}$$

The equations (referring to the last, feeding-end span) gained by developing matrix equation $\mathbf{B}_n = \mathbf{A}_n \cdot \mathbf{J}_n$ are the following:

$$U_{z(n+1)} = U_{zn} + I_z Z_{0f} + I_{vn} Z_{0k} \tag{21}$$

$$0 = U_{vn} + I_z (Z_{0k} + R_a) + I_{vn} (Z_{0v} + R_a) \tag{22}$$

$$0 = \left(\frac{U_{vn}}{R_a} + 1\right) + I_z \left(\frac{Z_{0k}}{R_a} + 1\right) + I_{vn} \left(\frac{Z_{0v}}{R_a} + 1\right) \tag{23}$$

Relations 22, and 23 are identical, thus when writing the matrix equation applying to the last span, after development, the number of equations obtained is by one less than required. On the other hand, relation 22 may also be written in another form, making use of $U_{vn} = I_{0(n-1)} R_0$ and $I_{0(n-1)} = I_{vn} - I_{v(n-1)}$:

$$I_z (Z_{0k} + R_a) + I_{vn} (Z_{0v} + R_0 + R_a) - I_{v(n-1)} R_0 = 0 \tag{24}$$

Eq. 24, however, is the same as the loop equation of the $n$th span, referring to the loop consisting of nodes 1—2—9—10, since it can be written that

$$0 = -I_{vn}(Z_{0v} - Z_{0g}) - I_z(Z_{0k} - Z_{0g}) - [I_{vn} - I_{v(n-1)}] R_0 - (I_z + I_{vn}) (Z_{0g} + R_a)$$

and, after throwing the equation, the terms containing $Z_{0g}$ cancel each other:

$$I_z (Z_{0k} + R_a) + I_{vn} (Z_{0v} + R_0 + R_a) - I_{v(n-1)} R_0 = 0 \tag{24}$$

Accordingly, the calculation is simplified in such a way that matrix equation 17 has to be written for $n-1$ spans only, the $n$th span lending itself to be considered by loop equation 24 (anyhow, $\mathbf{A}_n$ is not identical to $\mathbf{A}$, since $R_0$ *is replaced by* $R_a$).

Based on the above statements, for the case of feeding from one side, the calculation method is the following: column vector $\mathbf{J}_1$ of the right-side quantities is already known (relation 20). The quantities associated with the left-side terminals of the $(n-1)$th span are: $U_{zn}$, $I_z$, $U_{vn} = [I_{vn} - I_{v(n-1)}]R_0$ and $I_{vn}$. Thus, the column vector of these left-side quantities will be:

$$\mathbf{B}_{n-1} = \begin{bmatrix} U_{zn} \\ I_z \\ [I_{vn} - I_{v(n-1)}] R_0 \\ I_{vn} \end{bmatrix} \tag{25}$$

Now, the following matrix equation can be written:

$$\mathbf{B}_{n-1} = \mathbf{A}^{(n-1)} \cdot \mathbf{J}_1 \tag{26}$$

Since, in the knowledge of $\mathbf{A}$, matrix $\mathbf{A}^{(n-1)}$ can be computed, the following unknown quantities are contained in *Eq.* 26 : $I_z$, $I_{v1}$, $I_{v(n-1)}$, $I_{vn}$ and $U_{zn}$. The fifth equation still required is represented by relation 26, which also contains $R_a$ of the feeding point.

With the aid of Eqs. 24 and 26 the unknowns can be computed. This means that, in the knowledge of $\mathbf{B}_{n-1}$, the determination of the quantities of the $(n-1)$th span on the side of the fault, i.e. the *right-side* quantities of said span, which constitute column vector $\mathbf{J}_{n-1}$, also becomes possible:

$$\mathbf{J}_{n-1} = \mathbf{C} \cdot \mathbf{B}_{n-1} \tag{27}$$

and, considering that when computing the $(n-2)$th span,

$$\mathbf{B}_{n-2} = \mathbf{J}_{n-1} \tag{28}$$

in the following, by proceeding towards the fault, all ground-wire currents are readily obtainable. The currents flowing through the ground-return path and towers can be calculated by means of Kirchhoff's node equations:

$$I_z + I_{vn} - I_{gn} = 0 \tag{29}$$

$$I_{vn} - I_{r(n-1)} - I_{0(n-1)} = 0 \tag{30}$$

## 8. Valuation of the calculation method

The calculation method lends itself to be applied to a digital computer. With respect to programming, some difficulties lie in throwing the equations required for solving matrix equations 24 and 26, on the one hand, and matrix equation 27, on the other. Therefore, to facilitate the programming of the problem of zero-sequence current distribution, another calculation method has also been developed, which will be published at a later date.

A problem given in Appendix 3 has been solved to an accuracy of nine decimals using an *Elliott* 803 B digital computer, also applying the latter method, which is more suitable for digital computer processing. The results obtained by the two computation methods are compiled in Table I.

**Table I**

| | $I_{v1}$ | $I_{v2}$ |
|---|---|---|
| Matrix calculus . . . . . . . . . . . . . . . . . . . . . . | $-0.4765+j0.0098$ | $-0.4313+j0.0173$ |
| Calculation worked out for computer processing . . . . . . . . . . . . . . . . . . . . . . . . . . . | $-0.4765+j0.0099$ | $-0.4313+j0.0175$ |

The results gained by means of two methods and from equations based on two entirely different theories show complete identity. This proves the correctness of both the calculation methods and calculations. Another proof of the correctness of the method is the fact that $Z_{0g}$ used as auxiliary quantity has dropped out during the course of the calculation.

A further checking has been provided by programming the basic data of the field measurements described in [23] and computing the current distribution by means of the digital computer. A very good coincidence of measured and computed results has been found.

As a summary, it may be stated that the tasks outlined in Chapter 2 have successfully been solved by developing a calculation method based on equations lending themselves to be set up in an extremely simple form.

*

In the references, in addition to those [5], [23] already mentioned, and without claiming to be complete, a few books and papers dealing with the calculation of zero-sequence current distribution are given.

## Appendix 1. List of symbols

| | |
|---|---|
| *Voltages* | Dimension: volt, r.m.s., considered as complex magnitudes. |
| $U_0$ | zero-sequence voltage rise in the phase conductor with respect to ground (at the site of measurement), |
| $U_{vn}$ | a.c. voltage rise with respect to ground measured in the ground-wire sections, |
| $U_{zn}$ | a.c. voltage rise with respect to ground measured in the phase-conductor sections. |
| *Currents* | Dimension: ampere, r. m. s., considered as complex magnitudes. By "current" the a.c. component of subtransient short-circuit current is to be understood. |
| $I_z = 3I_0$ | treble value of zero-sequence current flowing in the phase conductor and through the fault, respectively (fault current), |
| $I_g$, $I_{gn}$ | current flowing through the ground-return path (in the $n$-th span, as counted from the fault), |
| $I_0$, $I_{0n}$ | tower current (current flowing in the $n$-th tower, as counted from the fault), |
| $I_v$, $I_{vn}$ | ground-wire current (in the $n$-th span, as counted from the fault), |
| $I_6$, $I_7$, $I_8$, $I_9$ | auxiliary quantities. |
| *Impedances* | considered as complex magnitudes. Dimension: ohm. Ground-return and zero-sequence impedances have been computed by means of the simplified Carson-Clem formulae. These impedance values are reduced to one span. |
| $Z = Z_{0k} - Z_{0g}$ | auxiliary quantity used with the basic equivalent circuit, |
| $Z_T$ | resultant of $R_0$ and $Z_{0e}$ connected parallelly, |
| $Z_{0e}$ | resultant impedance of the ladder network, |
| $Z_{0f}$ | zero-sequence self-impedance of phase-conductors, |
| $Z_{0g}$ | impedance of ground-return path in the zero-sequence circuits, |
| $Z_{0k}$ | zero-sequence mutual impedance between a phase conductor and ground wires, |
| $Z_{0v}$ | zero-sequence self-impedance of ground wires, |
| $R_a$ | resistance of substation grounding grid, |
| $R_0$ | tower footing resistance. |

*Other symbols:*

| | |
|---|---|
| $x$, $y$ | coefficients of a simultaneous equation, |
| $B$ | number of branches of a network, |
| $D$ | determinant of simultaneous equations, |
| $L$ | number of independent loops, |
| $N$ | number of nodes in a network. |

*Matrices:*

| | |
|---|---|
| $\mathbf{A}$ | matrix of generalized constants of a six-terminal network (calculation from right to left), |
| $\mathbf{B}$ | column vector of left-side boundary conditions, |
| $\mathbf{C}$ | matrix of generalized constants of a six-terminal network (calculation from left to right), |
| $\mathbf{E}_j$ | column vector of voltage sources, |
| $\mathbf{I}$ | column vector of branch currents, |
| $\mathbf{I}_j$ | column vector of current sources, |
| $\mathbf{I}_h$ | column vector of loop currents, |
| $\mathbf{J}$ | column vector of right-side boundary conditions, |
| $\mathbf{M}$, $\mathbf{M}_t$ | loop matrix and its transpose, |
| $\mathbf{U}$ | column vector of branch voltages, |
| $\mathbf{Z}$ | diagonal matrix of branch impedances. |

## Appendix 2. Complete form of matrix equation given in relation 10 and the simultaneous equations in 14 unknowns used for the solution

Complete form of the matrix equation:

$$\begin{bmatrix} (Z_{0f}-Z_{0g})-Z+(Z_{0v}-Z_{0g}) & -(Z_{0f}-Z_{0g})-(Z_{0v}-Z_{0g}) & -(Z_{0f}-Z_{0g})-(Z_{0v}-Z_{0g}) & 0 & -(Z_{0v}-Z_{0g}) \\ -(Z_{0f}-Z_{0g})-(Z_{0v}-Z_{0g}) & (Z_{0f}-Z_{0g})+(Z_{0v}-Z_{0g})+Z-Z+Z & (Z_{0f}-Z_{0g})+(Z_{0v}-Z_{0g})+Z+Z & 0 & (Z_{0v}-Z_{0g})+Z \\ -(Z_{0f}-Z_{0g})-(Z_{0v}-Z_{0g}) & (Z_{0f}-Z_{0g})+(Z_{0v}-Z_{0g})+Z+Z & (Z_{0f}-Z_{0g})+(Z_{0v}-Z_{0g})+Z+Z & 0 & (Z_{0v}-Z_{0g})+Z \\ 0 & 0 & 0 & R_0 & 0 \\ -(Z_{0v}-Z_{0g}) & (Z_{0v}-Z_{0g})+Z & (Z_{0v}-Z_{0g})+Z & 0 & (Z_{0v}-Z_{0g})+Z+Z_{0g} \end{bmatrix} \begin{bmatrix} I_6 \\ I_9 \\ I_{vn} \\ I_{0n} \\ I_{gn} \end{bmatrix} = \begin{bmatrix} -U_{v(n+1)}+U_{z(n+1)} \\ U_{v(n+1)}-U_{z(n+1)} \\ U_{v(n+1)}-U_{z(n+1)}+U_{vn}-U_{zn} \\ U_{v(n+1)} \\ U_{v(n+1)}-U_{zn} \end{bmatrix}$$

Simultaneous equations in 14 unknowns:

Loop 1:
$$U_{z(n+1)} - I_6(Z_{0f} + Z_{0v} - 2Z_{0g} - Z) - I_9(-Z_{0f} - Z_{0v} + 2Z_{0g}) - I_{vn}(-Z_{0f} - Z_{0v} + 2Z_{0g}) - I_{gn}(-Z_{0v} + Z_{0g}) - U_{v(n+1)} = 0$$

Loop 2:
$$-U_{z(n+1)} - I_6(-Z_{0f} - Z_{0v} + 2Z_{0g}) - I_9(Z_{0f} + Z_{0v} - 2Z_{0g} + Z) - I_{vn}(Z_{0f} + Z_{0v} - 2Z_{0g} + 2Z) - I_{gn}(Z_{0v} - Z_{0g} + Z) + U_{v(n+1)} = 0$$

Loop 3:
$$-U_{z(n+1)} - U_{zn} - I_6(-Z_{0f} - Z_{0v} + 2Z_{0g}) - I_9(Z_{0f} + Z_{0v} - 2Z_{0g} + 2Z) - I_{vn}(Z_{0f} + Z_{0v} - 2Z_{0g} + 2Z) - I_{gn}(Z_{0v} - Z_{0g} + Z) + U_{v(n+1)} + U_{vn} = 0$$

Loop 4:
$$-I_{0n} R_0 + U_{v(n+1)} = 0$$

Loop 5:
$$-U_{zn} - I_6(-Z_{0v} + Z_{0g}) - I_9(Z_{0v} - Z_{0g} + Z) - I_{vn}(Z_{0v} - Z_{0g} + Z) - I_{gn}(Z_{0v} + Z) + U_{v(n+1)} = 0$$

$$
\begin{array}{llllll}
\text{Node 1:} & I_{v(n+1)} & -I_{vn} & -I_{0n} & = & 0 \\
\text{Node 2:} & -I_{g(n+1)} & +I_{gn} & +I_{0n} & = & 0 \\
\text{Node 3:} & I_z & -I_6 & -I_7 & = & 0 \\
\text{Node 4:} & I_{vn} & +I_6 & +I_8 & = & 0 \\
\text{Node 5:} & -I_z & -I_8 & -I_9 & = & 0 \\
\text{Node 6:} & -I_{vn} & +I_7 & +I_9 & = & 0 \\
\text{Node 7:} & I_{g(n+1)} & -I_{v(n+1)} & -I_z & = & 0 \\
\text{Node 8:} & I_z & +I_{vn} & -I_{gn} & = & 0 \\
\text{and} & I_z & =1.0+j0.0 & & &
\end{array}
$$

## Appendix 3. Checking of the calculation method on a numerical problem

To check the calculation method a numerical problem has been worked out for the case of a fault supplied from one side and for a distance of two spans between feeding point and fault. The basic data have been computed for a 120-kV, single-circuit, three-phase, overhead transmission line with 110/20 sq.mm ACSR phase conductors and 2 × 50 sq.mm steel ground wires mounted on H-frame steel towers. Soil resistivity has been taken as 200 ohm.m, average span 250 meters. The basic data applying to a ground-return circuit (referring, instead of three, to one of the phase-conductors, in this case to the middle one):

$$
\begin{aligned}
Z_{0f} &= 0.0775 + j0.1928 \text{ ohm/span} \\
Z_{0v} &= 0.6725 + j0.3035 \text{ ohm/span} \\
Z_{0k} &= 0.0124 + j0.0800 \text{ ohm/span} \\
Z_T &= 1.0289 + j0.1633 \text{ ohm} \\
\text{and } R_0 &= 5 \text{ ohm} \\
R_a &= 0.1 \text{ ohm}
\end{aligned}
$$

The calculations were performed on an office-type desk computer to an accuracy of 4 decimals.

The matrix equation corresponding to Eq. 26:

$$
\begin{bmatrix} U_{z2} \\ I_z \\ (I_{v2}-I_{v1})R_0 \\ I_{v2} \end{bmatrix} = \begin{bmatrix} 1 & Z_{0f} & 0 & Z_{0k} \\ 0 & 1 & 0 & 0 \\ 0 & Z_{0k} & 1 & Z_{0v} \\ 0 & \dfrac{Z_{0k}}{R_0} & \dfrac{1}{R_0} & 1+\dfrac{Z_{0v}}{R_0} \end{bmatrix} \begin{bmatrix} (I_z+I_{v1})Z_T \\ I_z \\ (I_z+I_{v1})Z_T \\ I_{v1} \end{bmatrix} .
$$

The equation corresponding to Eq. 24:

$$I_z(Z_{0k} + R_a) + I_{v2}(Z_{0v} + R_0 + R_a) - I_{v1} R_0 = 0\,.$$

The unknowns are: $I_z$, $I_{v1}$, $I_{v2}$, and $U_{z2}$, and the available equations after development and rearrangement:

$$\begin{aligned}
I_z(Z_T + Z_{0j}) &= U_{z2} + I_{v1}(-Z_T - Z_{0k})\\
I_z(Z_T + Z_{0k}) &= I_{v1}(-Z_T - Z_{0v} - R_0) + I_{v2} R_0\\
I_z(-R_a - Z_{0k}) &= I_{v1}(-R_0) + I_{v2}(Z_{0v} + R_0 + R_a)\\
I_z &= 1.0 + j0.0.
\end{aligned}$$

Together with the last equation the above relations constitute a set of simple, linear, inhomogeneous simultaneous equations in 3 unknowns, from which $I_{v1}$ and $I_{v2}$ can be directly calculated by applying Cramer's rule. That is just the reason for choosing a two-span distance between feeding point and fault, since in that case there is no need to make use of relations 27 and 28.

Thus, the above set of equations may be written in the following form:

$$\begin{aligned}
x_{11} U_{z2} + x_{12} I_{v1} \qquad\qquad &= y_1\\
x_{22} I_{v1} + x_{23} I_{v2} &= y_2\\
x_{32} I_{v1} + x_{32} I_{v2} &= y_3
\end{aligned}$$

and the determinant of the simultaneous equations will be:

$$D = x_{11}(x_{22}\, x_{33} - x_{32}\, x_{23}) = -13.5421 - j4.7285$$

while the currents flowing in the ground wire are:

$$I_{v1} = \frac{1}{D}(y_2\, x_{33} - y_3\, x_{23}) = \frac{1}{D}(6.4991 + j2.1204)$$

$$I_{v2} = \frac{1}{D}(x_{22}\, y_3 - x_{32}\, y_2) = \frac{1}{D}(5.9224 + j1.8051)\,.$$

## Summary

The zero-sequence current distribution among ground wires and the ground return path of overhead transmission lines is subject to change in the surroundings of the fault and feeding point. The paper deals with the concepts used in connection with the end-effect phenomena of transmission lines, the necessity of solving the zero-sequence current distribution, the problems encountered in the calculations, the factors influencing the end-effect current distribution and the assumptions adopted for the computations. A new system of calculation using topological methods and matrices is described, and the procedure to be followed in the case of ground-faults supplied from one side is given in detail. A numerical example is included in the paper. The method is suitable for digital computer applications.

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