GROUP DELAY CORRECTOR IN THE MICROWAVE FREQUENCY RANGE

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1. Introduction

The condition of distorsionless transmission in the frequency domain can be written as $A(\omega) = \text{const.}$ and $\tau(\omega) = \text{const.}$ It is the task of the practical design to approach the required distorsionless transmission within the given frequency band and at the same time to keep the tolerances ensuing from the system specifications. The optimal solution of this task is a circuit design which simultaneously takes into account the requirements of the amplitude characteristic and those of the group delay characteristic. The computation problems arising in such a design work can only be solved by means of electronic computers and in many cases lead to circuit elements which are practically difficult to realize.

To approximate the condition of the distorsionless transmission group delay correctors are widely used. Connecting in chain these correctors with the original quadripole, the transmission characteristic of the resulting quadripole approaches to the condition of distorsionless transmission better. Thus, the original problem may be divided in two parts, namely to the design of a network with the wanted amplitude characteristic and to that of a group delay corrector. The group delay corrector is a so-called all-pass circuit, that is, its attenuation is a constant irrespective of the frequency, having the value of zero, and its group delay characteristic varies with the frequency in the prescribed way.

The correction of the group delay characteristic is of foremost importance in frequency-division microwave radiorelay systems with frequency modulation. The variations of the group delay time lead to waveform distorsions in the transmission of television signals and to intermodulation noises in multichannel telephone transmission. The group delay variations in microwave radiorelay systems are usually corrected at intermediate frequency (in the neighbourhood of 70 Mc/s). The calculation of the necessary corrector in bridge circuit offers no difficulty and the realization may be accomplished in bridged T-network.

In all-TWT-systems or in microwave waveguide links (pipe lines) the group delay correction can, of course, not be done at intermediate frequency

(70 Mc/s). It already occurs in the microwave IF system with 1920 channels that the group delay variations of the radio-frequency circuit elements should be corrected in the microwave frequency range.

The correction of the group delay variations in the microwave range meets with difficulties. This is partly due to the fact that the measurement of the group delay time in this range is very complicated [1, 2]. In our case the results of measurements were determined by means of the method reported in [3]. This method in substance means a point-to-point measurement of phase based on the comparison with the phase of a waveguide, and on numerical differentiation. Another cause of the difficulties is that no information is available in the literature concerning the design of group delay correctors operating in the microwave frequency range. According to our knowledge, this subject has been touched only in the studies of FEDIDA [4] and LARSEN [5].

In this study the principle of operation of the microwave group delay corrector and the method of its calculation are given. Further, results of measurements of the circuit realized at the Research Institute for Telecommunication (Budapest) will be reported.

2. Principle of operation

We come to the group delay corrector by a hybrid operating in the microwave range by means of the following considerations [6].

The equivalent bridge circuit of the all-pass network is shown in Fig. 1. The bridge impedances Z_I and Z_{II} are dual reactances, that is $Z_I Z_{II} = R_d^2 = Z_0^2 = 1$. The calculation of group delay correctors is based upon this equivalent scheme.

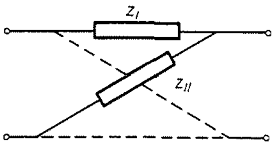

Fig. 1. Equivalent bridge circuit of the all-pass network

The circuit shown in Fig. 1 cannot be realized directly in the microwave range, therefore, a suitable equivalent circuit must be found. For this purpose the equivalent circuit of the bridge comprising a hybrid coil may be conveniently used. The equivalency is shown in Fig. 2 and its demonstration in terms of the Bartlett—Brune theorem can be found e.g. in the book of CAUER [7].

The equivalent circuit is also valid in the microwave range as the hybrid applied here can easily be realized by means of a 3 dB microwave directional coupler. The symbols for the microwave hybrid which will be used later on, are shown in Fig. 3. In case of matched termination of the terminal pair 4, the input of the terminal pair 1 will be divided in the hybrid between the terminal pairs 2 and 3, and no signal will arrive to the terminal pair 4. The arrangement of the hybrid coil and that of the microwave directional coupler can be found in reference [8].

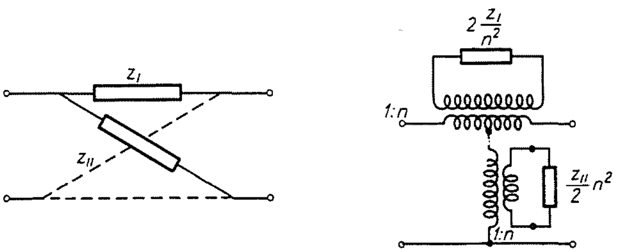

Fig. 2. Equivalent scheme of the bridge circuit, comprising an ideal transformer

In terms of the foregoing, to the diagrammatic layout of the realization with hybrid a 3 *dB* directional coupler and two terminating reactances are necessary.

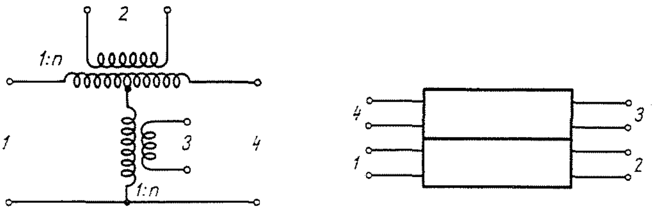

Fig. 3. Analogous networks of the hybrid coil and the microwave directional coupler

In the practical realization a so-called quadratic hybrid was used. The scattering matrix of this hybrid is

$$\mathrm{S} = \frac{e^{-j2\beta l}}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix} \tag{1}$$

It may be seen that the transformer ratio is $n = \sqrt{2}$, further that between the outputs 2 and 3 a phase shift of $\pi/2$ exists. Thus, the terminations are not duals of each other any longer but identical reactances. The final circuit layout is shown in Fig. 4.

This realization is also practically advantageous as hybrids occur in microwave radiorelay systems anyway. A further advantage of the realization with hybrid is that it ensures a theoretically reflectionless matching. Therefore, the reflection attenuation is zero and the dissipative attenuation can be held at a low value.

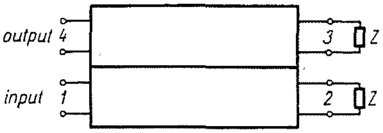

Fig. 4. Diagrammatic layout of the group delay corrector with hybrid, operating in the microwave-frequency range

3. The calculation of the group delay corrector

In the calculation of the group delay corrector operating in the microwave frequency range we proceed from the scattering matrix of the circuit shown in Fig. 4.

Writing out in detail the equation $\bar{b} = \mathbf{S}\,\bar{a}$ by means of the scattering matrix 1, we obtain

$$b_1 = \frac{e^{-j2\beta l}}{\sqrt{2}}(a_2 + ja_3) \tag{2}$$

$$b_2 = \frac{e^{-j2\beta l}}{\sqrt{2}}(a_1 + ja_4) \tag{3}$$

$$b_3 = \frac{e^{-j2\beta l}}{\sqrt{2}}(ja_1 + a_4) \tag{4}$$

$$b_4 = \frac{e^{-j2\beta l}}{\sqrt{2}}(ja_2 + a_3) \tag{5}$$

The arms 2 and 3 of the hybrid are terminated by an admittance with the reflection coefficient $r = \dfrac{1-jB}{1+jB}$. It follows from this that

$$a_2 = rb_2 \tag{6}$$

$$a_3 = rb_3\,. \tag{7}$$

Let us write Eq. (6) and (7) into Eq. (3) and (4):

$$a_2 = r\frac{e^{-j2\beta l}}{\sqrt{2}}(a_1 + ja_4) \tag{8}$$

$$a_3 = r\frac{e^{-j2\beta l}}{\sqrt{2}}(ja_1 + a_4) \tag{9}$$

Setting the expressions (8) and (9) in Eq. (2), we obtain

$$b_1 = jre^{-j4\beta l}\, a_4 \,. \tag{10}$$

Comparing these with the equation

$$b_1 = S_{11}a_1 + S_{12}a_4 \tag{11}$$

of the corrector quadripole shown in Fig. 4, the relations

$$\begin{aligned} S_{11} &= 0 \\ S_{12} &= j\, r\, e^{-4j\beta l} \end{aligned} \tag{11/a}$$

will be obtained. Setting the expression

$$r = \frac{1 - jB}{1 + jB} = e^{-j2 \operatorname{arctg} B} \tag{12}$$

of the reflection coefficient in S_{12} we obtain

$$S_{12} = je^{-j(4\beta l + 2 \operatorname{arctg} B)} \tag{13}$$

whence

$$\psi = -\frac{\pi}{2} + 4\,\beta\, l + 2 \operatorname{arctg}\ \mathrm{B}\,. \tag{14}$$

The group delay time is

$$\tau = \frac{d\psi}{d\omega} = 4l\frac{d\beta}{d\omega} + \frac{2}{1 + B^2}\,\frac{dB}{d\omega}\,. \tag{15}$$

Let us neglect the expression $4\, l\, \dfrac{d\beta}{d\omega}$ giving the group delay time of the hybrid and choose for the termination a parallel resonant circuit whose admittance in the neighbourhood of the resonance frequency is

$$jB \approx j2\sqrt{\frac{C}{L}}\,\frac{\varDelta\omega}{\omega_0} = j4\, Q_m\,\frac{\varDelta\omega}{\omega_0} \tag{16}$$

Here

$$Q_m = \frac{1}{2}\sqrt{\frac{C}{L}} \tag{17}$$

is the quality factor [9]. In these terms from the expression 15, the formula

$$\tau = \frac{8\,Q_m}{\left[1 + 16\,Q_m \left(\frac{\Delta\omega}{\omega_0}\right)^2\right] \omega_0} \tag{18}$$

will be obtained. The resonance frequency of the resonant circuit is equal to the middle frequency of the corrector operating range. In the middle of the frequency band $\Delta\omega$ equals zero and thus for the middle band group delay (signed by τ_0) we obtain the formula

$$\tau_0 = \frac{8\,Q_m}{\omega_0}. \tag{19}$$

By use of this, the group delay characteristic as function of $\Delta\omega$ will be

$$\tau = \frac{\tau_0}{1 + \frac{1}{4}\,\tau_0^2\,\Delta\omega^2}. \tag{20}$$

Let be notated the group delay at the edge of the band at frequency $\Delta\omega_+$ by τ_+. The group delay difference has the value

$$\Delta\tau = \tau_0 - \tau_+ = \frac{1}{4}\;\frac{\tau_0^3\,\Delta\omega_+^2}{1 + \frac{1}{4}\,\tau_0^2\,\Delta\omega_+^2} \tag{21}$$

Arranging 21, for the value of τ_0 we obtain an equation of the third degree

$$\Delta\tau + \frac{1}{4}\Delta\tau\,\Delta\omega_+^2\,\tau_0^2 - \frac{1}{4}\Delta\omega_+^2\,\tau_0^3 = 0\,. \tag{22}$$

For the practical design as initial value is given the group delay difference $\Delta\tau$ which is to be achieved in the range of $\omega_0 \pm \Delta\omega_+$. We obtain τ_0 from the solution of Eq. (22). In the knowledge of τ_0 and ω_0 the Q_m value of the parallel resonant circuit can be determined from Eq. (19).

The problem remains to realize in microwaves a parallel resonant circuit with the loaded quality factor Q_m. This problem has already been solved in connection with microwave filters in such manner that two rows of inductive stabs (posts) were placed. The rows of stabs formed a cavity resonator from the waveguide section between them. It may be adjusted to the specified design value of the resonance frequency of the cavity resonator by the separation of the stab rows, and the loaded quality factor Q_m of the cavity by the number of stabs in the row and the stab diameter. For the fine tuning of the cavity a capacitive plug in the median plane of the section serves between the rows. The realization is described in greater details, for instance, in reference [9].

4. Example and results of measurement

Let the task be to design a group delay corrector for the frequency range of 5900 ± 12 Mc/s. The corrector shall give a group delay difference of $\Delta\tau = 34\ ns$. With the values

$$\Delta\tau = 34\ ns = 3.4 \cdot 10^{-8}$$

$$\Delta\omega_{\div} = 2\,\pi \cdot 12 \cdot 10^{6} = 7.55 \cdot 10^{7}$$

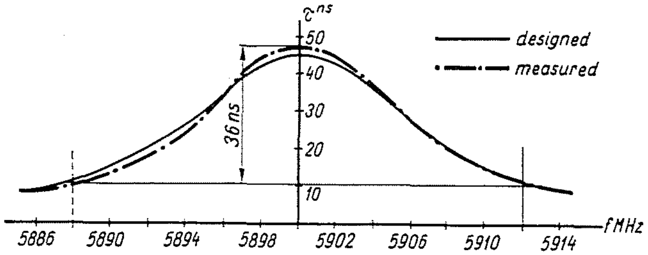

Fig. 5. Calculated and measured group delay characteristics of the microwave group delay corrector

Eq. (22), will have the form

$$3.4 \cdot 10^{-8} + \frac{1}{4}\, 3.4 \cdot 10^{-8} \cdot 5.7 \cdot 10^{15} \tau_0^2 - \frac{1}{4}\, 5.7 \cdot 10^{15} \tau_0^3 = 0\,.$$

After reduction

$$1 + 1.41 \cdot 10^{15}\, \tau_0^2 - 0.415 \cdot 10^{23}\, \tau_0^3 = 0\,.$$

The approximate solution of the equation is

$$\tau_0 = 4.5 \cdot 10^{-8} = 45\ ns\,.$$

From Eq. (19), the Q_m value of the parallel resonant circuit is

$$Q_m = \frac{\tau_0\, \omega_0}{8} = 208.$$

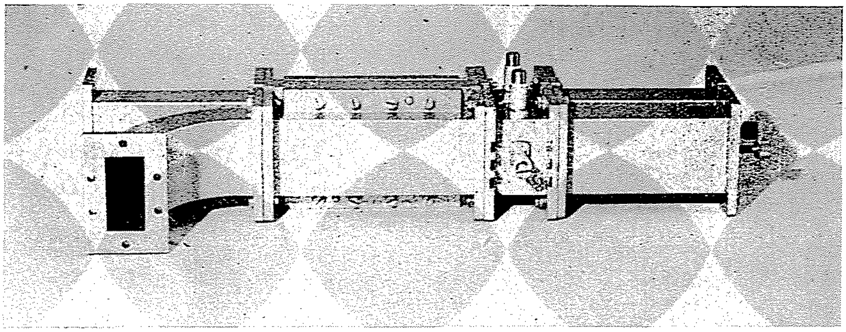

Fig. 6. Photograph of the realized corrector

The measured group delay characteristic of the realized circuit is shown in Fig. 5. The measured group delay difference is $\Delta\tau = 36$ *ns*. In the figure the result of the calculation is also indicated. In Fig. 6 the photograph of the corrector is shown.

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Summary

In microwaves, the group delay corrector may be conveniently realized by using a hybrid. Terminating the hybrid by [dual] reactances an all-pass network will be obtained. The solution of the equation given by relation (22) permits the design of a quadratic corrector producing a given group delay difference in the specified frequency band. The calculation is demonstrated on the design of a corrector giving a group delay difference of 34 *ns* in the range of 5900 ± 12 Mc/s. Results of measurements on the circuit realized in the Research Institute for Telecommunication are reported and the photograph of the corrector is shown.

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