CURRENT SOURCE WITH VERY GREAT RESISTANCE

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1. Introduction

The realisation of current stabilizers, integrators, time-recorders, linear sawtooth generators demands a current source with the greatest possible output resistance. High output impedance can be achieved by using either negative or positive feedback [1], [2], [3]. Although within certain limits the negative feedback makes it possible to realize arbitrarily high output resistances, it cannot, however, be used in making an extreme one. On the other hand, simple circuitry provides extremely high output resistance under certain conditions by using positive feedback. Difficulty arises, however, from the fact that the exact setting of the feedback depends on the parameters of the active elements used. The variational effects of the parameters on the resulting output resistance will be examined both theoretically and experimentally.

2. Realization of infinite output resistance with linear elements

Fig. 1 represents a vacuum-tube current source. A current, independent of the voltage, flows through the load impedance of V_1. This is realized by providing a positive feedback from plate to cathode of V_1 by means of the cathode follower V_2. A negative feedback stabilizes V_1 by way of a high value resistor in its cathode circuit.

The equivalent circuit for the analysis is given on Fig. 2. In place of the load impedance Z a voltage source of U voltage may be inserted into the plate circuit of V_1. The problem now is to determine the conditions for the case of infinite output resistance, that is, when $I_1 = 0$.

The equation for Loop I is:

$$U = I_1 R_{01} + I_1 R_k + I_1 R_1 + I_2 R_1 - \mu_1 U_1 \tag{1}$$

and

$$U_1 = -I_1 R_k - I_1 R_1 - I_2 R_1 \tag{2}$$

Similarly for Loop II:

$$\mu_2 U_2 = I_2 (R_2 + R_{02} + R_1) + I_1 R_1 \tag{3}$$

and

$$U_2 = -I_2 (R_2 + R_1) - I_1 R_1 + U \tag{4}$$

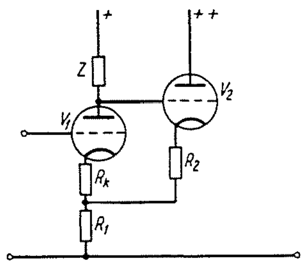

Fig. 1. Basic circuit to realize infinite inner resistance

It follows from Eq. (1)—Eq. (4), that:

$$\frac{U}{I_1} = \frac{[R_{01}+(1+\mu_1)(R_k+R_1)][R_{02}+(1+\mu_2)(R_1+R_2)] - (1+\mu_1)(1+\mu_2)R_1^2}{R_{02}+(1+\mu_2)(R_1+R_2) - R_1 (1+\mu_1) \mu_2} . \tag{5}$$

$I_1 = 0$, if

$$R_{02} + (1 + \mu_2)(R_1 + R_2) = R_1 (1 + \mu_1) \mu_2 \tag{6}$$

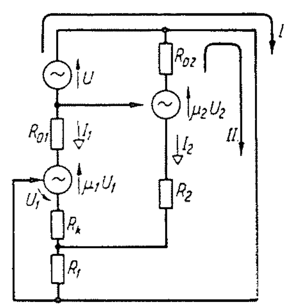

Fig. 2. Equivalent circuit for the circuit of Fig. 1

and so

$$R_1 = \frac{R_{02} + (1 + \mu_2) R_2}{\mu_1 \mu_2 - 1} . \tag{7}$$

By assuming that

$$R_{02} \ll (1 + \mu_2) R_2$$

$$\mu_1 > 1$$

$$\mu_2 \gg 1$$

a good approximation is:

$$R_1 \cong \frac{R_2}{\mu_1} . \tag{8}$$

Consequently the circuit balance depends on one parameter only: the amplification factor of V_1. If V_1 is a triode, then its amplification factor can be approximately held constant and have equalizing results within wide limits.

Having simplified the above equation for U/I, the output impedance can be determined for the general case.

Generally

$$R_{01} \ll (1+\mu_1) R_k$$

$$R_{02} \ll (1+\mu_2) R_2$$

$$\mu_2 \gg 1$$

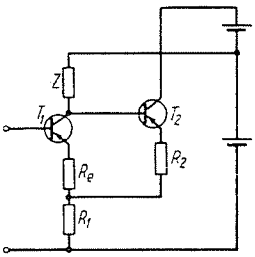

Fig. 3. Transistorized equivalent of the circuit of Fig. 1

and from Eq. (5):

$$\frac{U}{I_1} \cong \frac{(R_k+R_1)(R_1+R_2)-R_1^2}{\frac{R_1+R_2}{1+\mu_1}-R_1} = \frac{R_1(R_k+R_2)+R_k R_2}{R_2-\mu_1 R_1}(1+\mu_1). \tag{9}$$

3. Transistorized current source

Fig. 3 shows the transistorized analogue circuit of Fig. 1. The transistor T_1 secures the constant output current, while T_2 is in the feedback path. A significant difference between this and the tube circuit lies in the fact that T_2 needs a driving current, flowing through the load impedance. Therefore, the same negative resistance must be set between the collector of T_1 and the earth, as the one between the base of T_2 and the earth.

The general approximate equation for U/I (i.e. Eq. (9)), may be used for these calculations. First, however, the relationship between tube and transistor parameters must be determined.

The relation between input current I_i and input voltage U_i of a transistor (see Fig. 4a) is given by

$$I_i = \frac{U_i - h_{12} U_0}{h_{11}}$$

Expressing I_0 and U_0:

$$I_0 = h_{21} I_i = \frac{h_{21}}{h_{11}} (U_i - h_{12} U_0) \tag{11}$$

and

$$U_0 = -\frac{I_0}{h_{22}} = -\frac{h_{21}}{h_{11} h_{22}} (U_i - h_{12} U_0) \tag{12}$$

From these

$$U_0 = -\frac{h_{21} U_i}{h_{11} h_{22} - h_{21} h_{12}} = -\frac{h_{21}}{h_{11} h_{22}} \frac{U_i}{1 - \frac{h_{21} h_{12}}{h_{11} h_{22}}} \tag{13}$$

For small currents the second term in the denominator for alloyed low-frequency transistors is appr. 0.5, so:

$$\mu = -\frac{U_0}{U_i} \cong 2 \frac{h_{21}}{h_{11} h_{22}} = \frac{1}{h_{12}} . \tag{14}$$

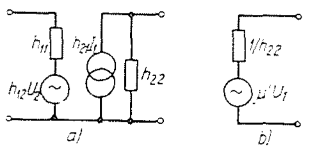

Fig. 4. a) Equivalent circuit of the transistor using the h parameters; *b)* Equivalent circuit of the collector circuit modified to conform with electron tube analogy

In general, however,

$$\mu = k \frac{h_{21}}{h_{11} h_{22}} \quad (k = 1{,}5 \ldots 3). \tag{15}$$

Fig. 4*b* shows the output of the transistor substituted by a voltage source and a series resistance, in order to make use of the tube analogy.

Consequently if

$$k \frac{h_{21}^{\mathrm{I}}}{h_{11}^{\mathrm{I}} h_{22}^{\mathrm{I}}} \text{ is written instead of } \mu_1 \text{, and}$$

$$R_e \text{ instead of } R_k \text{ ,}$$

then the balance condition of Eq. (9) can be reformulated as

$$\frac{U}{I_1} = \frac{R_1 (R_e + R_2) + R_e R_2}{R_2 - \mu^{\mathrm{I}} R_1} (1 + \mu^{\mathrm{I}}) = - R_{i2} \tag{16}$$

where

$$\mu^{\mathrm{I}} = k \frac{h_{21}^{\mathrm{I}}}{h_{11}^{\mathrm{I}} h_{22}^{\mathrm{I}}} . \tag{15}$$

The input resistance of the second stage, R_{i2} is given with good approximation by

$$R_{i2} = \frac{h_{21}^{\mathrm{II}} R_2}{1 + h_{22}^{\mathrm{II}} R_2} . \tag{17}$$

Expressing R_1:

$$R_1 = R_2 \frac{R_{i2} + R_e (1 + \mu^{\mathrm{I}})}{\mu^{\mathrm{I}} R_{i2} - (R_e + R_2)(1 + \mu^{\mathrm{I}})} \approx$$

$$\approx R_2 \frac{\dfrac{R_{i2}}{\mu^{\mathrm{I}}} + R_e}{R_{i2} - R_e - R_2} = R_2 \frac{\dfrac{h_{21}^{\mathrm{II}} R_2}{1 + h_{22}^{\mathrm{II}} R_2} \dfrac{h_{11}^{\mathrm{I}} h_{22}^{\mathrm{I}}}{k h_{21}^{\mathrm{I}}} + R_e}{\dfrac{h_{21}^{\mathrm{II}}}{1 + h_{22}^{\mathrm{II}} R_2} - R_e - R_2} =$$

$$= \frac{R_2}{k h_{21}^{\mathrm{I}}} \frac{h_{21}^{\mathrm{II}} h_{11}^{\mathrm{I}} h_{22}^{\mathrm{I}} R_2 + (1 + h_{22}^{\mathrm{II}} R_2) R_e k h_{21}^{\mathrm{I}}}{h_{21}^{\mathrm{II}} R_2 - (R_e + R_2)(1 + h_{22}^{\mathrm{II}} R_2)} . \tag{18}$$

Thus it is quite clear that in spite of the approximations the balancing condition still depends, in a rather complex way, on the parameters of the transistors used. In practical cases the second term of the nominator, and the first term of the denominator dominates only, and so as a first, very coarse, approximation it can be written

$$R_1 \approx \frac{(1 + h_{22}^{\mathrm{II}} R_2) R_e}{h_{21}^{\mathrm{II}}} . \tag{18a}$$

4. The error caused by the variation of the parameters due to operating point settings

It is common knowledge that of the triode parameters the amplification factor depends the least on operating point setting. In the above application even this small dependence must be taken into account, since the resulting inner resistance depends greatly on μ_1. Expressing I_1 from Eq. (9):

$$I_1 = \frac{R_2 - \mu_1 R_1}{R_1 (R_k + R_2) + R_k R_2} . \tag{19}$$

The resistances can be chosen for fixed values; μ_1, however, depends on the interelectrode voltages of V_1, consequently I_1 will be zero only in certain cases.

The dependence of μ_1 on operating point setting is [4]:

$$\mu_1 = \mu_0 + a \frac{U_G}{U_A} \tag{20}$$

where U_G is the grid voltage, U_A the plate voltage (both in respect to the cathode), and $a > 0$ is a constant. According to this result the more negative the grid the less μ_1 becomes, which is the consequence of the island-effect.

By expressing the plate current of V_1 as

$$I_{a1} = I_{a1}(U_{\mathrm{eff}}) \tag{21}$$

then

$$U_{1\,\mathrm{eff}} = U_G + \frac{U_A}{\mu_1} \tag{22}$$

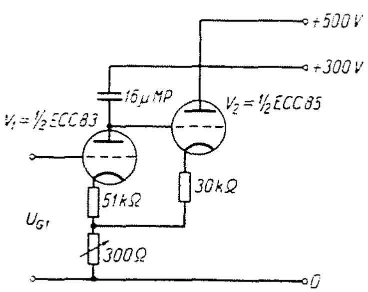

Fig. 5. Linear integrating circuit

and so

$$U_G = U_{1\,\mathrm{eff}} - \frac{U_A}{\mu_1} \approx U_{1\,\mathrm{eff}} - \frac{U_A}{\mu_0}. \tag{23}$$

Therefore,

$$\begin{aligned} \mu_1 = \mu_0 + a \frac{U_{1\,\mathrm{eff}} - \dfrac{U_A}{\mu_0}}{U_A} &= \mu_0 + a \frac{U_{1\,\mathrm{eff}}}{U_A} - \frac{a}{\mu_0} = \\ &= \mu_0' + a \frac{U_{1\,\mathrm{eff}}}{U_A} \end{aligned} \tag{24}$$

and by using Eq. (19):

$$I_1 = \frac{U}{1+\mu_1} \, \frac{R_1}{R_1(R_k + R_2) + R_k R_2} \left[\frac{R_2}{R_1} - \mu_0' - a \frac{U_{1\,\mathrm{eff}}}{U_A} \right]. \tag{25}$$

In case the circuit is balanced at one certain plate voltage and current, i.e. if infinite output impedance (zero conductance) has been set, then either an increase in plate current, or a decrease in plate voltage will become a negative output impedance.

The circuit given on Fig. 5 was designed to represent a linear integrating circuit [3]. The variable resistance R_1 facilitates the setting of the infinite output resistance. Fig. 6 shows the relationship between plate current variation and plate voltage. It can be seen that the slope of the curves i.e. I_1/U actually becomes negative (Fig. 7), if the plate current is increased or the plate voltage

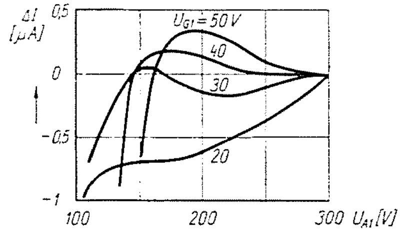

Fig. 6. Variation of plate current of V_1 plotted against U_{a_1}, referring to Fig. 5

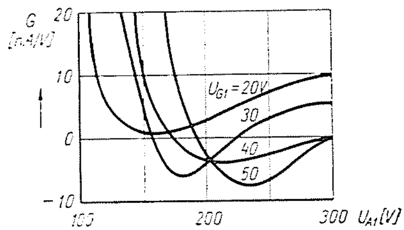

Fig. 7. Output conductance of the circuit of Fig. 5 plotted against U_{a_1}

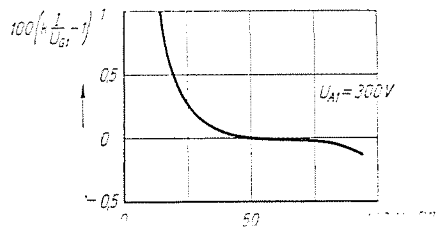

Fig. 8. The deviation of plate current-input voltage curve from the linear one

decreased. At low plate potential, however, the gradient of the curves turns to be positive since the plate current then demands appr. zero grid voltage, this causing grid current, which in turn brings non-linearity into the system.

The advantage of this system is the linear relationship between input voltage and output current. By assuming linear tube characteristics, the plate current — without the cathode feedback — is

$$I_A = g_m \frac{U_{G1} + \dfrac{U_A}{\mu_1}}{1 + g_m R_k} . \tag{26}$$

In case of $U_{G1} = 0$ the tube will not be totally cut off, a current of

$$I_A = \frac{g_m}{\mu_1} \; \frac{U_A}{1 + g_m R_k} \tag{27}$$

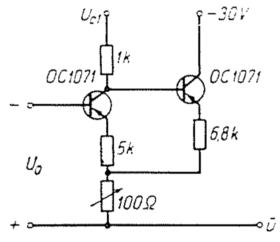

Fig. 9. Experimental transistorized circuit analogous to that of Fig. 5

will still flow. The feedback will, however, cause a voltage drop U_1 across R_1:

$$U_1 = \frac{U_A + U_K}{\mu_1} \approx \frac{U_A}{\mu_1} \tag{28}$$

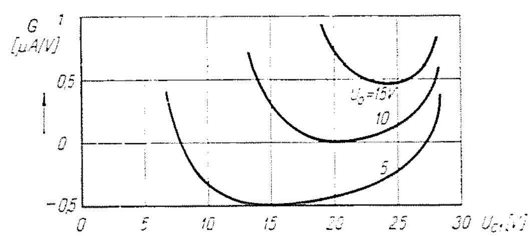

Fig. 10. The output conductance plotted against U_{c_1} of the circuit shown on Fig. 9

which is subtracted from U_{inp}, so

$$I_A \approx g_m \frac{U_{G1}}{1 + g_m R_k} \tag{29}$$

that is I_A is proportional to U_{G1} without any additive term. The deviation of the plate-current input-voltage curve from the linear one is illustrated on Fig. 8.

If a twin triode (i. e. ECC83) is used for U_1 (Fig. 5), and during operation the tube is often in a cut off condition, then it must be checked, whether the cathode of the other system emits electrons to the plate of V_1. From experiments made with twin triodes without internal shield (ECC82, ECC83) this current amounted to 0.1—0.5 μA, which is too high for certain applications.

In such cases it is more appropriate to use one system only and to leave the other one without heater supply.

The circuit shown on Fig. 9 is the transistorized equivalent of the tube circuit of Fig. 5. The value of the differential conductance G was measured in the circuit given on Fig. 2 and is plotted against U_{c1} on Fig. 10. Experience proved the good validity of Eq. (18-a) for determining the value of R_1.

5. Applications

The electron tube version of the circuit can be used first of all in extreme linear integrators. These include the different sawtooth generators, analogue circuits, analogue memory units, etc.

The circuit shown on Fig. 5 was used, for instance, for statistical data processing. A series of measured data, $x_1, x_2, \dots x_n$ — having been transformed into a corresponding voltage series $U_1, U_2, \dots U_n$ — were fed one after the other to the grid of V_1 each one having the same duration. After the "n"-th term the voltage appearing across the plate circuit capacitor is

$$U_c = k \sum_{=1}^{n} U_i = Ak \frac{1}{n} \sum_{i=1}^{n} x_i \tag{30}$$

which is proportional to the average value of the data. Here k is the constant of the integrator (including also the switch-on time of V_1), n is the number of the data to be measured, and A is the constant of the apparatus (depending on n). If another similar stage is fed by voltages U_i' this time proportional to x_1^2, then having made n measurements the voltage appearing across the capacitor of this latter will be

$$U_c' = k \sum_{i=1}^{n} U_i' = Bk \frac{1}{n} \sum_{i=1}^{n} x_i^2 \tag{31}$$

By subtracting U_C'' from U_C', the former being proportional to the square of the average value,

$$U_c'' = CU_c^2 = C \left(Ak \frac{1}{n} \sum_{i=1}^{n} x_i \right)^2 \tag{32}$$

then the voltage difference is

$$\Delta U = U_c' - U_c'' = Bk \frac{1}{n} \sum_{i=1}^{n} x_i^2 - C \left(Ak \frac{1}{n} \sum_{i=1}^{n} x_i \right)^2 . \tag{33}$$

Since the variance of a series measurement can, in general, be written as

$$s^2 = \frac{1}{n} \sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n} \sum_{i=1}^{n} x_i \right)^2 \tag{34}$$

then ΔU will be proportional to s^2, if

$$B \cdot k = C \cdot A^2 \cdot k^2 . \tag{35}$$

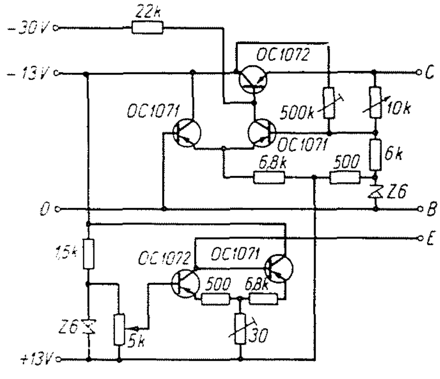

Fig. 11. Power supply used in high frequency transistor parameter measuring apparatus

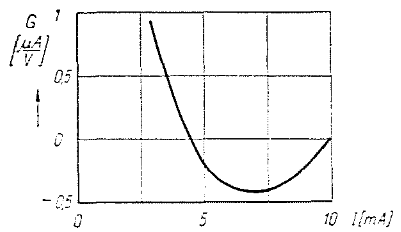

Fig. 12. The output conductance plotted against the load current of the circuit shown on Fig. 11

The circuit shown on Fig. 5 can be used as an analogue data storage unit, its accuracy was better than 1 % even when the leakage current of the storage capacitor was taken into consideration.

A transistorized circuit for the same purpose (Fig. 3) cannot be made with the types of transistors used at present, because the residual current of transistor T_1 and the driving current of transistor T_2 constantly flows through the impedance of the collector circuit. Consequently, if the collector circuit contains a capacitor, its charging would go on even after T_1 had been cut off. The previously mentioned two currents can be limited to an acceptable low value only by using silicon planar transistors.

In other applications where the current need never be equal to zero but its independence from the load resistance is an important factor, transistorized circuits can be advantageously used. Constant current is often needed for chemical and physical experiments, and for electronoptical magnetic lenses [5]. The efficiency of the current stabilizers is a great deal higher with transistors than with tubes, taking the usual current values into consideration. Fig. 11 shows a transistor power supply unit, used in transistor parameter (h, y, f_a, etc) measuring devices. Between terms C and B stabilized voltage is provided in the usual way, while between terms E and B a constant current flows, quite independent of the transistor to be measured. In this way the emitter current is kept constant and the stability of the parameter measurement is secured. The differential resistance is plotted against I in Fig. 12, corresponding to the circuit given on Fig. 11.

Summary

A current source with extremely high inner resistance is needed in current stabilizers, integrators, timing circuits, sawtooth generators. By using positive feedback the realization of infinite inner resistance is possible even with simple circuits. The dependence of the inner resistance on the parameters of active elements — electron tubes and transistors — is examined and the error caused by the dependence of these parameters on the operating point is expressed. Of the numerous applications two are discussed in detail: an electron tube circuit for automatic statistical quality control, and a transistorized circuit used in transistor parameter measuring equipments.

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