THE EFFECT AND THE COMPENSATION OF THE DISTURBING VARIABLE IN SAMPLED-DATA CONTROL SYSTEMS WITH FINITE SETTLING TIME

By

Gy. FODOR

Department for Theoretical Electricity, Polytechnical University, Budapest
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Presented by Prof. Dr. K. SIMONYI

1. Formulation of the problem

Let us examine the effect of the disturbing variable in a linear control system, in which the actuating signal is of the sampled-data type. We are assuming that the period of sampling T is constant and that the sampling time τ_0 is so short in comparison with the former one that the sampled signals can be assumed to be Dirac impulses.

We are limiting our considerations to systems satisfying the following two conditions. 1. The controlled variable reaches the prescribed steady state condition within a finite settling time under the effect of a typical reference input (step signal, ramp function, acceleration signal). 2. The average value of the square of the difference between the controlled variable corresponding to a stationary stochastic reference input of determined statistical characteristics on the one hand, and the prescribed value of this on the other hand is minimum (with respect to the sampling instants). The pulse compensators of these systems can be directly designed [3].

The disturbing variable is assumed to be acting at the output of the controlled system. The effect of a disturbing variable acting at another point can be theoretically reduced to this case, in practice, however, this reduction meets with difficulties.

The effect of the disturbing variable is examined from two aspects. On the one hand we assume that the disturbing variable is changing as a step function and the change of the controlled variable in the sampling instants is examined. The requirement is that the effect of the disturbing variable should disappear as rapidly as possible. On the other hand we determine the statistical error corresponding to the stationary stochastic disturbing variable of given statistical characteristics, i.e. the average value of the square of the controlled variable with respect to the sampling instants. The performance of the system with respect to the disturbing variable is the better, the lower is the settling time and the statistical error of the output signal produced by the disturbing variable.

2. The examination of the basic system

Let us first consider a system which was designed only with respect to the reference input [3]. The block diagram is shown in Fig. 1. Here X denotes the reference input, Y the controlled variable, U the disturbing variable, D the pulse compensator, G_T the hold circuit, G_S the controlled system. If only the values occurring at the sampling instants are examined, then we can calculate with discrete transforms having the variable $Z = e^{-sT}$, where s denotes the variable of the Laplace transformation.

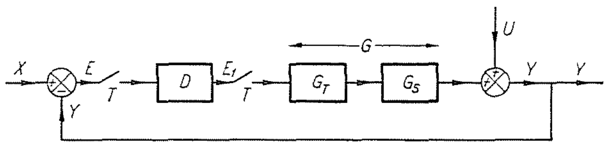

Fig. 1

The transfer function corresponding to the reference input is

$$W(Z) = \left[\frac{Y(Z)}{X(Z)}\right]_{U=0} = \frac{Y_X(Z)}{X(Z)} = \frac{D(Z)\,G(Z)}{1 + D(Z)\,G(Z)}\,. \tag{1}$$

The transfer function corresponding to the disturbing variable is

$$W_U(Z) = \left[\frac{Y(Z)}{U(Z)}\right]_{X=0} = \frac{Y_U(Z)}{U(Z)} = \frac{1}{1 + D(Z)\,G(Z)} = 1 - W(Z)\,. \tag{2}$$

Accordingly in the case of a follow-up system, the transfer function corresponding to the disturbing variable differs only in the sign from the error transfer function.

If the system has been designed to follow the m-order reference input

$$x_m(t) = 1(t)\,\frac{t^{m-1}}{(m-1)!} \tag{3}$$

without steady-state error, then the transfer function satisfies the requirement [5,3]

$$1 - W(Z) = (1 - Z)^m\,C_0(Z)\,, \tag{4}$$

where $C_0(Z)$ is a polynomial which can be determined on the basis of the other conditions. It follows from this, that

1. not only the effect of a step-like ($m = 1$), but even of an m-order disturbing variable disappears after a finite settling time in the sampling instants and

2. the settling time corresponding to the disturbing variable is identical with the settling time $T_s = rT$ corresponding to the reference input, where r is the degree of the transfer function.

The first characteristic is apparently favourable, since the requirements for the reference input are generally more rigid, hence in most cases $m > 1$. However, it is known that the error signal of a system tuned for a reference input of higher order is of a strongly overshooting character. Accordingly the same applies for the controlled variable excited by the disturbing variable, which is disadvantageous. The second characteristic is similarly disadvantageous, since the possibly rapid elimination of the effect of the disturbing variable is desirable.

The statistical error corresponding to a stochastic disturbing variable is given by the formula

$$\zeta_u^2 = \lim \frac{1}{2N+1} \sum_{k=-N}^{N} y_u^2[k] = \sum_{|Z_i|<1} \operatorname*{Res}_{Z=Z_i} W_u(Z) W_u(Z^{-1}) \overline{R}_{uu}(Z), \tag{5}$$

where $\overline{R}_{uu}(Z)$ denotes the two-sided discrete transform of the autocorrelation function of the disturbing variable. In our case

$$W_u(Z) = 1 - W(Z) = 1 - \sum_{i=1}^{r} w_i Z^i. \tag{6}$$

In a manner quite similar to the calculation carried out in [3], it can be proved that

$$\zeta_u^2 = \sum_{i=1}^{r} \sum_{j=1}^{r-1} w_i w_{i+j} M_{uj} + \frac{1}{2} N_u \sum_{i=1}^{r} w_i^2 - \sum_{j=1}^{r} w_j M_j + \frac{1}{2} N_u. \tag{7}$$

The meaning of the coefficients is

$$M_{uj} = \frac{1}{j!} \overline{R}_{uu}^{(j)}(0) + \sum_h \frac{\mu_{uh}^j + \mu_{uh}^{-j}}{\mu_{uh}} \operatorname{Res} \overline{R}_{uu}(\mu_{uh}), \tag{8}$$

$$N_u = 2\left[\overline{R}_{uu}(0) + \sum_h \frac{1}{\mu_{uh}} \operatorname{Res} \overline{R}_{uu}(\mu_{uh})\right], \tag{9}$$

where μ_{uh} denotes the poles of the function $\overline{R}_{uu}(Z)$ inside the unit circle.

On designing the system we have assumed that [1, 3]

$$W(Z) = A(Z) B(Z) = \sum_{k=0}^{n} a_k Z^k \cdot \sum_{i=1}^{q} b_i Z^i, \tag{10}$$

where $B(Z)$ is a known polynomial, $A(Z)$ in turn will be determined in the course of designing. We can introduce the coefficients

$$A_{up} = \sum_{i=1}^{q} b_i \sum_{j=1}^{r} (b_{i+j+p} + b_{i+j-p}) M_{uj} + N_u \sum_{i=1}^{q} b_i b_{i+p}, \tag{11}$$

$$B_{uk} = \sum_{j=1}^{r} b_{j-k} M_{uj}. \tag{12}$$

By incorporating these, the formula for the statistical error is [3]

$$\zeta_u^2 = \frac{1}{2} A_{u0} \sum_{k=0}^{n} a_k^2 + \sum_{k=0}^{n} \sum_{p=1}^{n-k} A_{up} a_k a_{k+p} - \sum_{k=0}^{n} B_{uk} a_k + \frac{1}{2} N_u. \tag{13}$$

This is absolutely identical in form with the formula for the statistical error corresponding to the reference input.

3. Compensation for the disturbing variable

The performance of the controlled variable, as the effect of the reference input and of the disturbing variable, respectively, can be influenced separately, if two pulse compensators are applied as is shown in the block diagram of Fig. 2 [4]. The formulae for the transfer functions in this case, are

$$W(Z) = \frac{Y_X(Z)}{X(Z)} = \frac{D_1(Z)\,G(Z)}{1 + D_1(Z)\,D_2(Z)\,G(Z)}, \tag{14}$$

$$W_U(Z) = \frac{Y_U(Z)}{X(Z)} = \frac{1}{1 + D_1(Z)\,D_2(Z)\,G(Z)}. \tag{15}$$

From these the transfer functions of the pulse compensators are

$$D_1(Z) = \frac{W(Z)}{W_U(Z)\,G(Z)}, \tag{16}$$

$$D_2(Z) = \frac{1 - W_U(Z)}{W(Z)} \tag{17}$$

Let

$$G(Z) \equiv \frac{G_1(Z)\,G_2(Z)}{G_3(Z)\,G_4(Z)}, \tag{18}$$

where all the functions $G_i(Z)$ are polynomials and the zeros of $G_1(Z)$ and $G_3(Z)$ are outside the unit circle, while the zeros of $G_2(Z)$ and $G_4(Z)$ are inside the unit circle. The transfer function can be written in the form

$$W(Z) = D_1(Z) \frac{G_1(Z)\,G_2(Z)}{G_3(Z)\,G_4(Z)} W_U(Z) \tag{19}$$

If the controlled system is not stable in itself, i.e. $G_4(Z) \neq 1$, then the poles resulting in the instability of the system can be eliminated by choosing

$$W_U(Z) = C_0(Z)\, G_4(Z)\,, \tag{20}$$

where $C_0(Z)$ is an arbitrary polynomial for the time being. The factor $G_4(Z)$ should, therefore, not be contained in $[1—W(Z)]$ in this case [4]. Thus the

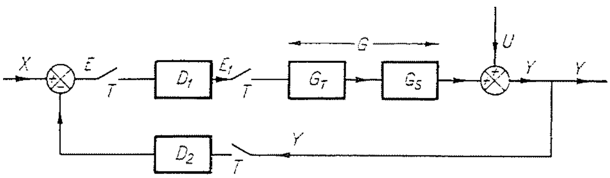

Fig. 2

determination of $W(Z)$ in the general case is the same as for a system containing a single pulse compensator, if the controlled system is then stable in itsefl [3].

The steady-state effect of a step formed, disturbing variable can be eliminated by choosing

$$W_U(Z) = (1 - Z)\, C_1(Z) = (1 - Z)\, C_2(Z)\, G_4(Z)\,,$$

where $C_1(Z)$ and $C_2(Z)$ are arbitrary polynomials for the time being. On the other hand, — since $W(Z)$ must contain the factor $G_2(Z)$, otherwise in view of (16) D_1 would not be stable, — therefore according to (17) $[1 - W_U(Z)]$ should also contain this so as to ensure the stability of D_2, consequently

$$W_U(Z) = 1 - C(Z)\, G_2(Z)\,. \tag{22a}$$

If after a step form disturbing variable a ripple-free settling is required, then it can be proved by the modified discrete transformation, — the calculations are not detailed here, — that $[1 - W_U(Z)]$ should contain the factor $G_1(Z)\, G_2(Z)$, hence then

$$W_U(Z) = 1 - C(Z)\, G_1(Z)\, G_2(Z) \tag{22b}$$

is necessary. In the two cases, let

$$G_2(Z) = \sum_{j=1}^{q_u} g_j\, Z^j \tag{23a}$$

resp.

$$G_1(Z)\, G_2(Z) = \sum_{j=1}^{q_u} g_j\, Z^j\,. \tag{23b}$$

Our task is to choose the degree of the polynomial

$$C(Z) = \sum_{k=0}^{n_u} c_k Z^k, \tag{24}$$

which is for the time being arbitrary, and to determine the coefficients. Thus, the expression for the transfer function corresponding to the disturbing variable is

$$W_U(Z) = 1 + \sum_{i=1}^{r_u} w_{ui} Z^i, \tag{25}$$

$$w_{u_i} = - \sum_{k=0}^{n_u} c_k g_{i-k}. \tag{26}$$

If the controlled system is stable in itself, then condition (21) is equivalent to condition

$$W_U(Z = 1) = 1 - C(Z = 1)\, G_2(Z = 1) = 0, \tag{27}$$

i.e. to the condition equation

$$\sum_{k=0}^{n_u} c_k = \gamma_0, \quad \frac{1}{\gamma_0} = \sum_{j=1}^{q_u} g_j. \tag{28}$$

If the only aspect is to eliminate the effect of the disturbing variable as rapidly as possible, then by choosing $c_0 = \gamma_0$, according to (22) $W_U(Z)$ is determined, and thus designing is finished. That which has so far been discussed in Chapter 3 is the adoption of results described in [4], with slight modifications.

Let us now examine the expression for the statistical error. We can see upon comparing formulae (22) and (6) that result (7) can be adopted by substituting $w_i = - w_{ui}$, i.e.

$$\zeta_u^2 = \sum_{i=1}^{r_u} \sum_{j=1}^{r_u - i} w_{u_i} w_{u_{i+j}} M_{uj} + \frac{1}{2} N_u \sum_{i=1}^{r_u} w_{ui}^2 + \sum_{j=1}^{r_u} w_{uj} M_{uj} + \frac{1}{2} N_u, \tag{29}$$

where the expressions for M_{uj} and N_u are given by (8) and (9), respectively. We may also adopt form (13), but now $a_k = c_k$ and $b_i = - g_i$, i.e.

$$\zeta_u^2 = \frac{1}{2} A_{u_0} \sum_{k=0}^{n_u} c_k^2 + \sum_{k=0}^{n_u} \sum_{p=1}^{n_u - k} A_{up} c_k c_{k+p} - \sum_{k=0}^{n_u} B_{uk} c_k + \frac{1}{2} N_u, \tag{30}$$

where the meaning of the coefficients, in the manner of (11) and (12) is

$$A_{up} = \sum_{i=1}^{q_u} g_i \sum_{j=1}^{r_u} (g_{i+j+p} + g_{i+j-p}) M_{uj} + N_u \sum_{i=1}^{q_u} g_i g_{i+p}, \tag{31}$$

$$B_{uk} = \sum_{j=1}^{n_u} g_{j-k} M_{uk}. \tag{32}$$

Coefficients c_k should be chosen in such a way, that ζ^2 should be minimum and at the same time the auxiliary conditions (21) and (28) should be satisfied. For the determination of the coefficients by the method described in [3] we obtain the following linear system of equations.

$$\sum_{k=0}^{n_u} A_{usk} c_k + \lambda = B_{us}, \quad s = 0, 1, \ldots, n_u, \tag{33}$$

$$\sum_{k=0}^{n_u} c_k = \gamma_0, \tag{34}$$

where $A_{usk} = A_{up}$, $p = |s - k|$. This means $(n_U + 2)$ linear equations for the determination of the $(n_U + 1)$ unknown coefficients c_k and of the Lagrange parameter λ which is of no interest. By solving the system of equations, $C(Z)$ and so also $W_U(Z)$ is known. We should check whether the performance of the controlled variable corresponding to the disturbing variable is satisfactory in the case of the chosen value n_U. If necessary, the calculation is repeated by another degree n_U. Finally the transfer function of the pulse compensators is determined by applying formulae (16) and (17).

4. An illustrative example

a) *Basic data*

Let the transfer function of the controlled system be

$$G_S(s) = \frac{K_0}{s(s+1)}. \tag{35}$$

Select the value $T = 1$ for the period of sampling, then

$$\begin{aligned} G(s) &= G_T(s)\, G_S(s) = K_1 \frac{1 - e^{-s}}{s} \frac{K_0}{s(s+1)} = \\ &= K_0 K_1 [1 - e^{-s}] \left[\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1} \right], \end{aligned} \tag{36}$$

$$G(Z) = \tau_0 K_0 K_1 [1 - Z] \left[\frac{Z}{(1-Z)^2} - \frac{1}{1-Z} + \frac{1}{1 - e^{-1} Z} \right]. \tag{37}$$

After reduction, by using the notation $K = e^{-1} \tau_0 K_0 K_1$,

$$G(Z) = K \frac{Z(1 + 0.718\, Z)}{(1 - Z)(1 - 0.368\, Z)}. \tag{38}$$

Let us assume that the autocorrelation function of the stochastic reference input and of the disturbing variable, respectively, is

$$r_{xx}(\tau) = c_x^2\, e^{-2|\tau|}, \quad \overline{R}_{xx}(Z) = \frac{c_x^2}{1 - e^{-2}Z} + \frac{c_x^2\, e^{-2}}{Z - e^{-2}}, \tag{39}$$

$$r_{uu}(\tau) = c_u^2\, e^{-|\tau|}, \quad \overline{R}_{uu}(Z) = \frac{c_u^2}{1 - e^{-1}Z} + \frac{c_u^2\, e^{-1}}{Z - e^{-1}}. \tag{40}$$

From these, the parameters necessary for the calculation,partly on the basis of [3] are

$$M_j = 2c_x^2\, e^{-2j}, \quad N = 2c_x^2, \quad P_i = 2c_x^2\, e^{-2i}, \quad Q = c_x^2\,; \tag{41}$$

$$M_{uj} = 2c_u^2\, e^{-j}, \quad N_u = 2c_u^2\,; \tag{42}$$

$$A_p = 2c_x^2\, e^{-2p}, \quad B_p = 2c_x^2\, e^{-2(p+1)}; \tag{43}$$

$$A_{up} = 2c_u^2\, e^{-p}, \quad B_{up} = 2c_u^2\, e^{-(p+1)}. \tag{44}$$

After this the transfer functions can be calculated.

b) *Designing for the reference input*

This step is not closely connected to the questions here raised, therefore the calculation was performed according to [3] only shortly.

We want to determine the transfer function $W(Z)$ relying upon the following conditions. 1. The controlled variable should even follow an output signal of the form of a ramp function without steady-state error, i.e. $m = 2$. 2. Settling is required only for the sampling instants; the system is not ripple-free, hence $B(Z) = G_2(Z) = Z$.

Let us choose the value $n = 3$, whereby the settling time is $T_s = 4$. By selecting one parameter later, the system of equations for the coefficients a_k is [3]

$$\begin{aligned} a_0 + e^{-2}a_1 + e^{-4}a_2 &= e^{-2} - e^{-6}a_3\,, \\ a_0 + a_1 + a_2 &= 1 - a_3\,, \\ a_1 + 2a_2 &= 1 - 3a_3\,. \end{aligned} \tag{45}$$

Upon solving the system of equations, by evaluating the error signal and the statistical error corresponding to the unit step, we find that the overshoot is minimum if $a_3 = -1.083$, while the statistical error is minimum if $a_3 = -1.594$. Let us select the value $a_3 = -1.200$, then we obtain the results

$$W(Z) = -0.151Z + 2.101Z^2 + 0.249Z^3 - 1.200Z^4\,, \tag{46}$$

$$\Psi_1(Z) = -1.000 - 1.151Z + 0.951Z^2 + 1.200Z^3 \tag{47}$$

$$\Psi_2(Z) = -Z - 2.151Z^2 - 1.200Z^3\,, \tag{48}$$

$$\zeta^2 = 5.346\, c_x^2. \tag{49}$$

Both the maximum overshoot and the statistical error are excessively high, hence in practice it would be more to the purpose to further diminish these characteristics at the expense of increasing the settling time. As an illustration of the process, however, we should be content with the above results.

c) *Systems not compensated for the disturbing variable*

If the system is not compensated in respect of the disturbing variable, then we obtain an arrangement having a block diagram as shown in Fig. 1. The unit step, the output signal excited by the disturbing variable is in accordance with (2) and (47),

$$Y_U(Z) = 1.000 + 1.151Z - 0.951Z^2 - 1.200Z^3 ; \quad (50)$$

$$T_{su} = 4.$$

This is illustrated in Fig. 3 by the curve marked 1. The continuous output signal was determined by the modified discrete transformation. The above solution only supplied the values arising at the sampling instants. This signal shape cannot at all be regarded as satisfactory. The satistical error, in view of (13), is

$$\zeta_u^2 = 5.917\, c_u^2 , \quad (51)$$

which is similarly a high value. The transfer function of the only pulse compensator of the system is given by

$$D(Z) = -\frac{0.151}{K}\,\frac{(1 - 0.368Z)(1 - 13.952\,Z - 1.656\,Z^2 + 7.968\,Z^3)}{(1 + 0.718\,Z)\,(1 - Z)\,(1 + 2.151\,Z + 1.200\,Z^2)} . \quad (52)$$

d) *Minimum settling time*

If the main aspect is that the effect of the step-form disturbing variable should disappear at the sampling instants as rapidly as possible, then in view of (28), by choosing $c_0 = \gamma_0 = 1$, we obtain

$$W_U(Z) = 1 - Z. \quad (53)$$

The change of the controlled variable now is

$$Y_U(Z) = 1, \ T_{su} = 1 , \quad (54)$$

i.e. the disturbing variable has already no effect in the case $k = 2$ (Figs 3 and 4, curve 2a). At the same time the statistical error is

$$\zeta_u^2 = 1.264\, c_u^2 . \quad (55)$$

These values are far more favourable than those obtained in the previous case. The transfer function of the pulse compensators is, by force of formulae (16) and (17),

$$D_1(Z) = -\frac{0.151}{K}\,\frac{1-0.368\,Z}{1+0.718\,Z}\,(1-13.952\,Z-1.656\,Z^2+7.968\,Z^3)\,, \tag{56}$$

$$D_2(Z) = -\frac{1}{0.151}\,\frac{1}{1-13.952\,Z-1.656\,Z^2+7.968\,Z^3}\,. \tag{57}$$

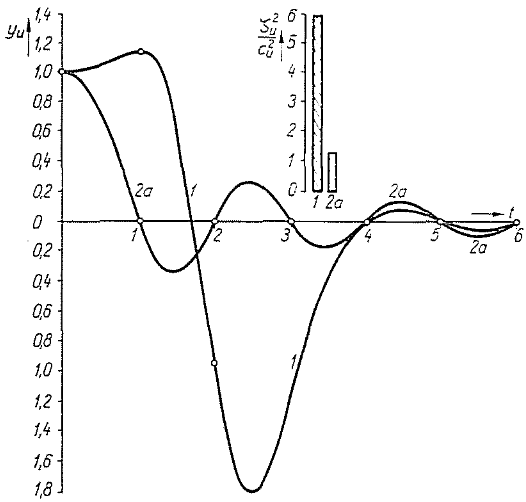

Fig. 3

Both transfer functions are simpler than $D(Z)$ determined above, whereby realization is facilitated in most cases.

e) *Reduction of the statistical error*

The statistical error corresponding to the disturbing variable can be reduced at the expense of increasing settling time. Let e.g. $n_U = 1$, then the equations serving to calculate the parameters c_0 and c_1 are according to (33) and (34),

$$\begin{aligned} c_0 + 0.368c_1 + \lambda &= 0.368\,, \\ 0.368\,c_0 + c_1 + \lambda &= 0.135\,, \\ c_0 + c_1 &= 1\,. \end{aligned} \tag{58}$$

The solution of this system of equations is $c_0 = 0.684$; $c_1 = 0.316$ and thus

$$W_U(Z) = 1 - 0.684\,Z - 0.316\,Z^2 = (1-Z)\,(1+0.316\,Z)\,, \tag{59}$$

$$Y_U(Z) = 1 + 0.316\,Z\,,\quad T_{su} = 2\,, \tag{60}$$

(Fig. 4, the curve marked by 3a.)

$$\zeta_u^2 = 1.138 c_u^2 \,. \tag{61}$$

The transfer function of the pulse compensators is

$$D_1(Z) = -\frac{0.151}{K}\,\frac{(1-0.368\,Z)\,(1-13.952\,Z-1.656\,Z^2+7.968\,Z^3)}{(1+0.718\,Z)\,(1+0.316\,Z)} \tag{62}$$

$$D_2(Z) = -\,4.541\,\frac{1+0.462\,Z}{1-13.952\,Z-1.654\,Z^2+7.968\,Z^3}\,. \tag{63}$$

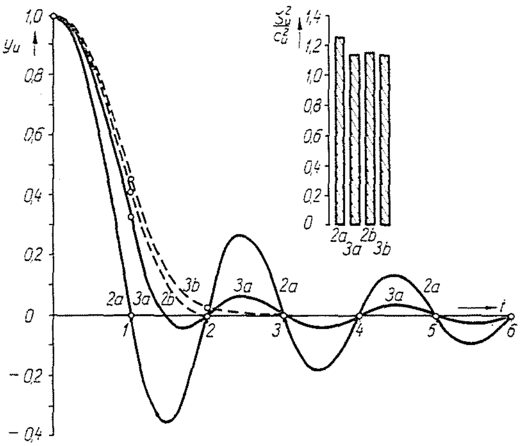

Fig. 4

These are of a somewhat more complicated structure than in the previous case, but are simpler than the transfer function corresponding to a single pulse compensator.

At the expense of increasing settling time, the statistical error can further be reduced.

For the sake of comparison we have drawn in Fig. 4 also the change of the controlled variable in a system which is ripple-free in respect to the disturbing variable. Curve 2b represents the case of minimum settling time, while curve 3b the one parameter reduction of the statistical error. In Figs. 3 and 4 the statistical errors corresponding to the various cases were drawn.

5. Conclusions

In the following a system which was not designed separately for the disturbing variable (Chapter 2, Fig. 1) will be named N-system, while a system compensated for the disturbing variable (Chapter 3, Fig. 2) K-system. In accordance with theoretical considerations and with the example the following conclusions can be drawn.

1. Both in the N- and the K-system the settling time is finite with respect to the disturbing variable.

2. The settling time T_{sU} corresponding to the disturbing variable is identical with the settling time corresponding to the reference input in the N-system, while in the K-system it can be selected independently. Accordingly the K-system is more advantageous, since a possibly low value is desirable in the case of T_{sU}, while for T_s often a higher value is allowed so as to satisfy other requirements.

3. In the N-system the limit ordinal number of the disturbing variable is identical with that of the reference input, while in the K-system m_U can be selected independently. Generally, the choice of $m_U = 1$ is satisfactory. If $m > 1$, the advantage of the N-system obtained in this way is only apparent, since considerable oscillations produced by the disturbing variable of step form mean an increased disadvantage.

4. The N-system is ripple-free from the aspect of the disturbing variable only in the case when it is ripple-free from the aspect of the reference input as well. In the K-system the absence of ripples can be separately ensured for the disturbing variable.

5. In the N-system the statistical error caused by the stochastic disturbing variable can be influenced only to a small extent. In the K-system the statistical error is considerably lower than even in the simplest case.

6. The N-system only contains one pulse compensator, while the K-system two. The transfer function of these two compensators is generally of a simpler build-up, accordingly their realization is generally more easy. Nevertheless, this is the only drawback of the K-system.

7. From the point of view of calculation technique, the designing of the K-system does not necessitate considerably more work.

Upon considering the above aspects, the following final conclusion can be drawn. If the disturbing variable arising at the output of the control system is considerable, then it is worth reducing its effect by the application of two pulse compensators. It should be decided in every case whether compensation should be made first of all for the step signal (short settling time, absence of ripples), or for the stochastic signal (low statistical error).

Summary

In sampled-data control systems with finite settling time the settling time is finite also in the case of a step-like disturbing variable arising at the output. If two pulse compensators are applied in accordance with HUNG, then the effect of the disturbing variable can be influenced independently of the reference input. A method has been given for determining the transfer function of pulse compensators. The system designed in this way has a finite settling time, both with respect to the reference input and the disturbing variable, further in the case

of known statistical characteristics (autocorrelation function) of these signals the statistical erorr is minimum. The drawback of this system is that two pulse compensators must be applied. However, it is advantageous that the transient and the stationary behaviour is much more favourable with respect to the disturbing variable.

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DR. GY. FODOR, Budapest XI., Műegyetem rkp. 3. Hungary