# AN ANALYSIS OF ELECTRIC POWER SYSTEMS ON THE DC NETWORK ANALYSER

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(Received March 30, 1962)

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## Introduction

The speedy development all over the world of electric energy production and consumption caused a similar development of the distribution networks. The cooperative power systems, constituting the basis of electric energy-distribution in most of the countries are, in recent days, already everywhere complicated, manifoldly looped networks, having energy fed into one part of their nodes and consumed in the other part of them. A widespread method in the whole world for studying the steady-state performances of these networks is based on the use of network analysers. They form, as it is well known, a special group of analog computers. All the elements of the examined network are represented in proper scale on these analysers and the different operation conditions of the network can be studied on them with the aid of direct measurement.

There are two kinds of network analysers: the *AC* or impedance, and the *DC* or resistance network analysers. On the former we can represent both the real and the imaginary part of the passive elements (complex impedances) of the network under consideration, and we can adjust not only the size, but also the phase angle of the voltages switched on the feed-in nodes. By the latter kind either the real or the imaginary parts of the network impedances can be represented (occasionally their absolute values or projections on one or the other direction), and the analyser is fed by direct current. The disadvantage of the *DC* analysers against the *AC* ones is that the results of measurements made on these contain certain mistakes in advance, unless the $R/X$ relations of the single branches in the represented network is the same. Its advantage is, however, that it is much cheaper and the measurements on it are simpler and faster. In this article we wish to discuss those problems, occurring in the practice of dispatcher centres, which can easily be solved and with the required accuracy on the *DC* network analysers.

## Problem- solving on the DC network analysers

1. The measurement of effective power-flow distribution. — The measurement of effective power-flow distribution on the *DC* network analyser is based on the fact that if we feed in, *i. e.* consume currents in the nodes of it proportional with the proper effective powers of the real network, then the currents flowing in the single branches of the analyser will nearly be proportional with the effective powers flowing in the studied network. That will be demonstrated in Appendix I.

It is characteristic for the degree of approach that according to literature data [3] the error committed is less than 5 percent. It should be mentioned that the effective power-flow distribution for two concrete states of loading of the Hungarian cooperative power system measured on the *DC* analyser of the National Electrical Dispatch Centre (OVT) was compared with those measured on the *AC* analyser of the Research Institute of Electrical Energetics (VILLENKI). The difference was under 1.5 MW in all the branches and was generally about 5%.

Measuring the power-flow distribution, care is to be taken of the fact that the power losses are neglected because effective power-flows are represented by proportional currents. They have to be taken into consideration by evaluating the presumable power losses of the single lines on the basis of former data, and adjusting them as plus loads in the limiting two nodes.

2. The measurement of reactive power-flow distribution. — In the preceding point it was already seen that in the measurement of effective power-flow distribution mistakes were caused by omitting the line resistivities and the reactive power-flows. However because of the fact that in high voltage networks on the one hand the inductivity of the single lines is far more than the resistivity, on the other hand the reactive power-flows of the network are generally altogether at about one third of the effective power-flows, the mistakes committed are negligible. The measurement of reactive power-flows could be made in the same way theoretically as that of the effective power-flows. This statement could be demonstrated on the basis of equation [17] in Appendix I, but now $R$ and $P$ is omitted. So it can be written:

$$U_S = U_R + \frac{QX}{U_R} = U_S + \Delta U_1 \tag{1}$$

In a closed loop of the network for the longitudinal voltage drops:

$$\Sigma \Delta\, U_1 = 0 \cong \frac{1}{U_R} \Sigma QX \tag{2}$$

Comparing equation (2) with equation (22a) of App. I. it can be seen that our above statement is true. Sorry to say we have already seen that the effective power-flows of a network are in general essentially greater than the reactive power-flows, therefore, their omission would cause serious mistakes. So we have to find other methods for the measurement of reactive power distribution. In the literature [2] and [5] there are two methods discussed in this relation, the advantage of which is that with their aid we can determine both the effective and reactive power distribution with an accuracy similar to that of the $AC$ network analyser. Their disadvantage is, however, that they consist of successive approximation, and in applying them we need a more qualified personal, more work and a far more expensive mechanism than by the simple effective power-flow measurement described above.

The author has worked out a method for the measurement of reactive power flow distribution which could easily and rapidly be applied, while its accuracy is at about the same as that of the effective power-flow measurement discussed. Its essence is the following: On behalf of obtaining the influence of effective power-flows on the reactive power distribution, in equation (22) of App. I. let us multiply the $PX$ of all the lines limiting the closed loop by $R/X$; if the $R/X$ relation of the single lines is the same we can write:

$$\frac{1}{U_R} \Sigma PR = \frac{1}{U_R} \Sigma PX \cdot \frac{R}{X} = \frac{R}{X} \cdot \frac{1}{U_R} \cdot \Sigma PX = 0 \qquad (3)$$

So in equation (17) for the second part of the longitudinal voltage drop:

$$\Sigma \Delta U_2 = \frac{1}{U_R} \Sigma PR \cong 0 \qquad (4)$$

will be valid, too.

And as in closed loops we can write for the longitudinal voltage drops:

$$(\Sigma \Delta U_1 + \Sigma \Delta U_2) \cong 0 \qquad (5)$$

Therefore, after comparing (5) and (4) it is to be seen, that (2) remains correct in consequence of which the reactive power distribution does not change. (Equation (5) is an approximation, because the direction of the node voltage vectors is not the same, but there is only a slight declination between them, so the approximation is good.) If, however, the $R/X$ relations are different, than in closed loop (3) is not true, consequently (2) can be valid only, if there will be an accessory reactive power-flow ($Q'$) in the loop, for the voltage drop of which we can write:

$$\Sigma \Delta U_1' = \frac{1}{U_R} \cdot Q'R = - \Sigma \Delta U_2 = - \frac{1}{U_R} \cdot \Sigma PR \qquad (6)$$

Now in comparing (4), (5) and (6) we can see, that (2) is again valid.

On this ground the real reactive power flow will be the sum of the $Q$ and $Q^{*}$ flows (because of the above mentioned, naturally, with certain approximation), which can be measured independently. Thus, we can measure the reactive power-flow distribution in two parts: *a)* At first we have to measure the reactive power distribution of the network unloaded from the point of view of effective power.

*b)* At second we have to determine the influence of the effective power-flow distribution measured before on the reactive power distribution with regard to equation (6).

ad *a)* In adjusting the reactive power distribution attention should be paid to the fact that the network will produce *i. e.* consume a lot of reactive

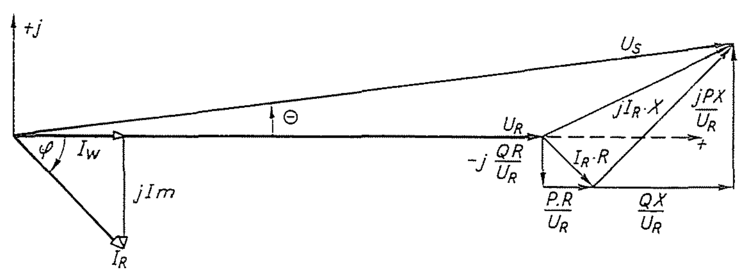


*Fig. 1.* The vector-diagramme of a transmission line loaded by the current $I_R$

energy partly by the capacity, and partly by the inductivity of the single lines. The capacitive reactive powers can be calculated with the nominal voltage. On the other hand, the reactive power consumed by the series inductivities can be calculated by dividing the estimated power losses of the single lines with their $R/X$ relations. After, we have to divide these inductive and capacitive reactive powers between the end-points the same way, as in the case of effective power-losses.

ad *b)* We can carry out the measurement as follows: (Fig. 1).

Let us denote the $\frac{P_i X_i}{U_R}$ voltage values measurable between the end-points of the single lines during effective power distribution measurement by $V_{k_i}$. In multiplying these with the $R/X$ relations of the proper lines on the ground of (3) we obtain:

$$V_{k_i} \cdot \left(\frac{R}{X}\right)_i = V'_{k_i} \tag{7}$$

where $V'_{k_i}$ will represent the $\frac{1}{U_2} P_i R_i$ values of the network.

Thereafter, we have to determine the independent loops of the network in question, and in all of them to sum up the $V_{k_i}$ values calculated before, according to a circulation-direction fixed in advance. We examine whether the voltage obtained after summing up in the single loops is greater, than 5% of the maximum voltage measurable between the end points of the lines limiting the loop in question. If in a loop this is to be so it means, that there the influence of the effective power distribution on the reactive power distribution is greater than 5%; therefore we have to determine $Q'$ here. This can be done in the way, that on the network analyser deprived of its feedings-in and consumers ("passive network") we switch into the limiting lines of the loop in question the proper $V'_{k_i}$ values in the form of voltage sources, and after it we measure the reactive power-flows produced.

The accuracy of the described method was tried in measuring the effective and reactive power-flow distribution of an arbitrarily loaded network, simultaneously on the *AC* network analyser of the VILLENKI and on the *DC* one of the Dispatcher Centre; in the latter case the $Q'$ flows described were also measured according to the author's method. The reactive power-flows obtained on the *DC* network analyser differed from those obtained on the *AC* one by 15—30%, if $Q'$ is not counted. After correction with $Q'$ the difference diminished to 3—6%.

3) The examination of voltage relations in the network.

The determination of voltages in the single nodes of the network is based on the measurements described in points 1. and 2.; from the results obtained there we can calculate the complex voltage-drops of the single transmission lines.

The calculation is based on the fact, that according to equation (17) in App. I. the voltage drop of an arbitrary line in *AC* networks has longitudinal and cross components. As we adjust only the reactances on the *DC* analyser, so we get by direct measurement the second part of the longitudinal and the first part of the cross voltage drops, that is to say:

*a)* The voltages, which we obtained in measuring the effective power distribution (Fig. 1.)

$$V_{k_i} = \left(\frac{PX}{U_R}\right)_i \tag{8}$$

*b)* The voltages, which we obtained in measuring the reactive power distribution:

$$V_{h_i} = \left(\frac{QX}{U_R}\right)_i \tag{9}$$

Where:

$$V_{h_i} = V^{\mathrm{I}}_{h_i} + V^{\mathrm{II}}_{h_i} \tag{10}$$

($V^{I}_{h_i}$ is the voltage between the end-points of the single lines at 2a, while $V^{II}_{h_i}$ the same at 2b.) The two other parts of the voltage drop can be calculated by multiplying with the $R/X$ relation of the transmission line in question. On this basis the total voltage drop of a transmission line can be obtained from the expression below:

$$V_{K_i} = \frac{R}{X} \cdot V_{K_i} + V^{I}_{h_i} + V^{II}_{h_i} + j\left[V_{K_i} - \frac{R}{X}\left[V^{I}_{h_i} + V^{II}_{h_i}\right]\right] \tag{11}$$

4) The determination of power-loss increment of a network.

The most economic state of the operation of a meshed network having several feed-in *i. e.* consuming nodes can always be determined, as is well known, on the basis of heat-consumption increment and the network-power-loss increment of the single power plants. The condition of the least heat consumption of an energy system with $k$ feed-in points for a fixed state of loading is expressed by the equations below (containing certain permissible neglections [6]):

$$\lambda_p = \frac{\frac{dh_i}{dp_i}}{1 - n_{pi}} \tag{12}$$

and

$$\frac{\lambda_q}{\lambda_p} = n_{qi} \tag{13}$$

In these equations the $\lambda_p$ and $\lambda_q$ are the Lagrange factors of the conditional edge-value calculations, $\frac{dh_i}{dp_i}$ the heat consumption increment of the $i^{th}$ power-plant, $n_{pi}$ the effective, and $n_{qi}$ the reactive power-loss increment of the network. The definition of $n_{pi}$ and $n_{qi}$ is:

$$n_{pi} = \frac{\partial V_p}{\partial P_i} \quad \text{and} \quad n_{qi} = \frac{\partial V_p}{\partial Q_i} \tag{14}$$

where: $V_p$ is the total effective power-loss of the network, while $P_i$ and $Q_i$ the effective *i. e.* reactive power flowing into the network from the $i^{th}$ power-plant. Equation (12) having greater importance in attaining the least heat-consumption, one used to take only this into consideration. The heat-consumption increment of the single power stations are generally at hand for the dispatcher centres controlling the operation of the system, but the effective power-loss increment factors are not known, and it is very difficult to obtain them by calculation. However, we can determine in a relatively easy manner the power-loss increment with the aid of *DC* analyser measurement. (It is to be remarked, that the above definition of power-loss increment does not fix, which kind of increase in consumption balances the increase of the feed-in of the $i^{th}$ power station. The condition could be assumed, for instance, that all the consumer nodes would together consume the increase in the fed-in effective power on the way, that the single consumer nodes would participate relatively unaltered. This definition would make the measurement very difficult, so we accept the definition according to which the power-loss increment is a value related to a referent node (being in the center of the consumer area) that is suitably selected. This conception approximates reality quite well.)

The basis of measurement is that we can calculate the power-loss increment of an arbitrary node $i$ from the following equation (see in App. II):

$$n_{p_i} = \frac{2}{U^2} \cdot \sum_k a_{i_k} R_k P_k \qquad (15)$$

where $R_k$ is the resistance of the $k^{th}$ line, $P_k$ the effective power flowing on it, $U$ the mean line voltage of the network, and $a_{i_k}$ the power-flow division factor related to the $i^{th}$ node and the reference point; this latter shows, how much MW power is flowing on the $k^{th}$ line, if we feed 1 MW into the $i^{th}$ node and consume 1 MW in the reference point. The $P_k$-s and $a_{i_k}$-s can be measured on the *DC* network analyser, so we easily calculate the power-loss increment on the basis of equation (15).

It is to be remarked, that instead of the power-loss increment the so-called power-loss increment factors are used, in general, because they can be immediately inserted in equation (12). Its expression is: $b_i = \dfrac{1}{1 - n_{p_i}}$.

Beside the network power-loss increment it is necessary, sometimes, to determine the total power losses of the network. So, for instance, by economic comparison it might be necessary to determine the growth of the network's total power losses in consequence of switching off an arbitrary $i$—$k$ transmission line carrying $P_{i-k}$ effective power (for instance because of its maintenance). The problem can be solved in an elementary way by measuring the power distribution for both states on the network analyser, computing the losses for all the transmission lines and adding them. This method is, however, very cumbersome and we would have to make the calculations for all cases separately. But there is another possibility for solving the problem as on the basis of App. III. the growth of the network's total power losses:

$$\Delta V_p = AP_{i-k} + BP_{i-k}^2 \qquad (16)$$

where $A$ and $B$ can be calculated according to (38). The quantities necessary for calculating $A$ and $B$ ($n_{p_{ik}}$, $a_{(ik)j}$, $a_{ik}$) can be determined by network analyser measurements, therefore, if we determine these in advance for all the transmission lines of the network, then the growth of power-losses because of switching off an arbitrary line can easily be calculated from (16).

It is finally to be mentioned that we can apply the method described above to calculate the $n_{q_i}$ reactive power increment too.

5. Closing the article it should be remarked that the *DC* network analyser is also suitable for short-circuit examinations. With regard to the fact, however, that this sort of application of the *DC* network analyser is quite well known, we do not intend to deal with it here.

## Appendix I

In *AC* networks the following equation can be written between the voltages ($U_S$ and $U_R$) of the end-points ($S$ and $R$) of an arbitrary line — neglecting the line capacities.

$$U_S = U_R + \frac{PR + QX}{U_R} + j\,\frac{PX - QR}{UR} \tag{17}$$

where $R$ is the resistivity and $X$ the inductive reactance of the line in question, while $P$ the effective, and $Q$ the reactive power flowing on it from $S$ into $R$ related to $U_R$ ($U_R$ is assumed here to lie in the real axis of the complex number-plane). Neglecting $R$ and $Q$ equation (17) alters into:

$$U_S \cong U_R + j\,\frac{PX}{U_R} \tag{18}$$

Then the angle of loading between $U_R$ and $U_S$:

$$\delta \cong \operatorname{arc\,tg} \frac{PX}{U_R^2} \tag{19}$$

If this angle is small we can write approximately:

$$\delta_i^{[\text{radian}]} \cong \frac{PX}{U_R^2} \tag{20}$$

After summarizing the $\delta_i$-s of the lines lying along a closed loop of the network with the proper sign it can be written:

$$\Sigma\,\delta_i = 0\,. \tag{21}$$

Substituting equation (20) into equation (22) we obtain:

$$\frac{1}{U_R^2} \sum_i P_i X_i \cong 0 \tag{22}$$

It can be written for an arbitrary closed loop of the *DC* analyser:

$$\sum_i R_i I_i = 0 \tag{22a}$$

Where $R_i$ is the resistivity of the $i^{\text{th}}$ branch, and $I_i$ the current flowing therein. Comparing (22a) and (22) we can understand that the current flowing in an arbitrary branch of the analyser is proportional to the effective power flowing in the proper line of the examined network (obviously only with certain approximations because of neglecting $R$ and $Q$).

## Appendix II

The total power loss of a three-phase network, as it is well known:

$$V_p = 3 \sum_k I_k^2 R_k \tag{23}$$

We can rewrite this substituting the apparent power ($S_k$):

$$V_p = 3 \sum_k \left(\frac{S_k}{U\sqrt{3}}\right)^2 \cdot R_k = \sum_k \frac{P_k^2 + Q_k^2}{U^2} \cdot R_k \tag{24}$$

Where $P_k$ is the effective and $Q_k$ the reactive power flowing on the $i^{\text{th}}$ transmission line. Let us write the power-loss growth according to (14):

$$n_{pi} = \frac{\partial}{\partial P_i} \left[\frac{1}{U^2} \cdot [P_k^2 + Q_k^2] \cdot R_k\right] \tag{25}$$

The effective power-flows of the lines are functions of feed-in powers ($P_i$). As we are seeking the power-loss increment of the $i^{\text{th}}$ feed-in point, let us write the partial derivative according to $P_i$ of this function:

$$\frac{\partial P_k}{\partial P_i} = a_{ik} \tag{26}$$

where $P_i$ is the effective power fed into the $i^{\text{th}}$ node, the increase of which, — as already mentioned — is balanced by the increase in the consumption of the chosen reference node. This equation is evident on the ground of the above definition of $a_{ik}$.

Now comparing (25) and (26), furthermore, with the permissible neglection of taking the reactive powers of the lines independent from $P_i$ we can write:

$$n_{pi} = \frac{2}{U^2} \sum_k a_{ik} R_k P_k \tag{27}$$

## Appendix III

The effect of switching off a transmission line from the network (carrying a current $i_{i-k}$ from node $i$ into node $k$) is equivalent, for the network with

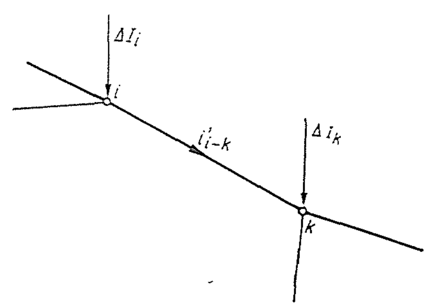


*Fig. 2.* The representation of switching out the line $i$—$k$

feeding, into the node $i$ a current $\Delta I_i$ and into the node $k$ a current $\Delta I_k$, for which we can write:

$$\Delta I_i = -\Delta I_k = i'_{i-k} \tag{28}$$

where $i'_{i-k}$ is the new current in the branch $i$—$k$ brought about by feeding in $\Delta I_i$ and $\Delta I_k$. If, namely, equation (28) is valid, we can see on Fig. 2. that the new state of the network is equivalent with that after the switching off of the line $i$—$k$. But the increase in the current of the branch $i$—$k$ is to be expressed in the following way:

$$i'_{i-k} - i_{i-k} = a_{i-k} \cdot \Delta I_i \tag{29}$$

Comparing equation (28) and (29) we can write:

$$i'_{i-k} = \Delta I_i = \frac{i_{i-k}}{1 - a_{i-k}} . \tag{30}$$

Where $a_{i-k}$ is the current division factor of the branch $i—k$ — being numerically equal to the current which would flow in the branch $i—k$, if we fed unity current into node $i$ and consumed unity current in node $k$, the network being unloaded for the rest.

Therefore, if there is an effective power $P_{i-k}$ flowing on the line to be switched off, then the effect of switching off can be represented by a $\Delta P_i$ power fed into point $i$ and consumed in point $k$, where similarly to (30) the $\Delta P_i$ is:

$$\Delta P_i = \frac{P_{i-k}}{1 - a_{i-k}} \tag{31}$$

Let us write the total power losses of the network after switching off the line in question:

$$V'_p = \frac{1}{U^2} (\sum_j S_j'^2 \cdot R_j - \Delta P_j^2 \cdot R_{i-k}) \tag{32}$$

Where $S'_j$ is the power flowing on the line $j$ after switching off the line $i—k$. That member, which has to be substracted is necessary for the following:

The representation mentioned before of the switching off of the line $i—k$ is valid only for the network behind, but it is not valid for the line itself, because by feeding in *i.e.* consuming a power $\Delta P_i$ the power-flow of this line increases to $\Delta P_i$ and the $\frac{1}{U^2} \cdot \Delta P_i^2 \cdot R$ brought about by this is coming into the summing-up. In reality, however, there can be no power loss of the line switched off, so we have to substract in expression (32) the mentioned member from the result of the summing-up. $S'_j$ can be written as follows:

$$S'_j = P_{j0} + a_{(ik)j} \cdot \Delta P_i + jQ_{j0}$$

Where $P_{j0}$ is the effective, and $Q_{j0}$ the reactive power flowing in the $j^{th}$ branch of the network before; $a_{(i-k)j}$ the current which would flow in the branch $j$, if we fed-in unity current into node $i$ and consumed the same in node $k$. With the aid of (33) the equation (32) can be rewritten:

$$V'_p = \frac{1}{U^2} [\sum_j R_j (P_{j0}^2 + 2a_{(ik)j} \cdot P_{j0} \Delta P_i + a_{(ik)j} \Delta P_i^2 - R_{j0}^2) - \Delta P_i^2 R_{ik}] \tag{34}$$

If we compare in this expression the second member with the equation (27) we can recognise that it is the power-loss increment of the point $i$ related to point $k$, before switching off the line $i—k$, multiplied by $P_i$.

Let us write the total losses before switching off the line $i—k$:

$$V_p = \frac{1}{U^2} \sum_j R_j (P_{j0}^2 + Q_{j0}^2) \tag{35}$$

Comparing (35) and (36) we can write the increase of the network's total power-losses in consequence of switching off the line $i$—$k$:

$$\Delta V_p = V_p' - V_p = n_{pik} \cdot \Delta P_i + \left[\frac{1}{U^2} \cdot \sum_j R_j a^2_{(ik)j} - \frac{1}{U^2} \cdot R_{ik}\right] \Delta P_i^2 \quad (36)$$

As $P_i$ can be calculated from the power-flow of the line to be switched off according to equation (31), so the loss-increase can be expressed as follows:

$$\Delta P = A P_{i-k} + B P^2_{i-k} \quad (37)$$

Where the constants $A$ and $B$ can be calculated on the base of (36):

$$A = \frac{n_{pik}}{1 - a_{ik}} \quad \text{and} \quad B = \frac{1}{n^2 (1 - a_{ik})^2} \left[\sum_j R_j a^2_{(ik)j} - R_{ik}\right] \quad (38)$$


## Summary

Summarizing the above discussion we can state that the following problems, occurring in the practice of dispatcher centres can be solved on the DC network analyser:

1. The determination of effective power-flow distribution in the network.
2. The determination of reactive power-flow distribution in the network.
3. The examination of voltage relations.
4. The determination of the network power losses and the power-loss increment factors.
5. The measurement of the networks' short-circuit currents.

The accuracy of the detailed measurements is somewhat less than that of the AC analyser, but at the same time the handling of the DC analyser is simpler, the measurements can be made quicker than on the AC one, which is a very important point of view in dispatcher practice.

On this basis the DC analyser is an important stand-by in the work of dispatcher centres commanding the operation of electrical energy systems.


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